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• Research Highlight •

A New Semi-Lagrangian Finite Volume Advection Scheme Combines the Best of Both Worlds

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Advection, or transport by wind, is fundamental to numerical weather and climate modeling. It is especially important to solve the equations governing the distribution of heat, moisture, pollutants, and so on. Two classes of advection schemes have become dominant in recent years. Semi-Lagrangian (SL; Diamantakis, 2013) methods form the advection equation in its "Lagrangian" or flow-following form and use the winds (either prescribed or predicted) to trace trajectories backwards from a grid point to their original location on the previous time step. The value of the variable at that upwind point, usually interpolated from the surrounding grid points, is the advected value of the variable at the destination grid point. The "semi-" part of the name comes from the use of a fixed set of grid points at which the solution is represented, rather than the fully-Lagrangian solution which would continually follow the trajectories. Meanwhile, finite volume (FV; LeVeque, 2002) methods dispose of grid points and instead consider the atmosphere as a set of cells and then solves the cell-integrated flux-form advection equation. The update to the solution, which is the mean value on each cell, is computed from the net flux into a cell over a time step. An FV method typically computes these fluxes by assuming subgrid reconstruction within the cell and then integrating it over the area of the cell flowing outward during a time step.

Both methods have increasingly supplanted the older spectral and finite-difference dynamics that were common throughout much of the first half-century of numerical weather and climate modeling. SL methods are used by the European Centre for Medium-Range Weather Forecasting (ECMWF) Integrated Forecast System (IFS), in the United Kingdom Met Office (UKMO) Unified Model, and in the Environment and Climate Change Canada Global Environmental Multiscale Model (GEM), while the Finite-Volume Cubed-Sphere Dynamical Core (FV3; Putman and Lin, 2007) is used across the US National Oceanic and Atmospheric Administration (NOAA) and for global models in the National Aeronautics and Space Administration (NASA).

SL and FV methods present complementary advantages over the older methods. Semi-Lagrangian methods are unconditionally stable: they have no timestep restriction for their stability. This alone would make SL methods very attractive—most notably ECMWF adopted an SL method when it was found to be far faster than a Eulerian spectral method (Simmons, 1991)—but they are also highly accurate when paired with an accurate method for computing the upstream trajectories and in interpolating from the surrounding grid points. SL methods also easily generalize to two and three dimensions. FV methods are automatically mass-conservative since mass leaving one grid cell enters its neighbor. FV methods can also easily incorporate physical constraints on the solution, particularly shape-preserving constraints such as monotonicity—that in a nondivergent flow of passive tracers no new extrema are created—and positivity—that mass or mixing ratio never becomes negative—through either modifying the subgrid reconstructions or by limiting the fluxes to enforce these constraints.

Conversely, most SL methods are not mass-conservative, and shape preservation is only approximate unless piecewise-constant interpolation is used everywhere. Most FV methods have time step restrictions similar to finite-difference methods and are more challenging to extend beyond a single dimension. That the disadvantages of one map directly onto the advantages of the other has led some researchers to create finite-volume semi-Lagrangian methods, although these either can be very complex (Nair and Machenhauer, 2002; Zerroukat et al., 2002, Harris et al., 2011) or are restricted to being

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semi-Lagrangian in one direction only (Lin and Rood, 1996).

The scheme presented in Tang et al. (2021), a modification of the authors' Conservative Semi-Lagrangian with Rational Function (CSLR) called CSLR1-M, uses a few novel ideas to greatly simplify the implementation of a FV-SL scheme. CSLR1-M computes advection using a FV method to retain exact conservation, but the values used to compute the FV flux are computed from the upstream points as in a SL method. To implement this method, a "multi-moment" version of an FV scheme was used, in which both the volume-integrated averages (VIAs) defined on grid cells and the point values (PVs) defined on cell corners are prognostic variables. The PVs are first updated using a two-dimensional SL method and then used to compute the fluxes needed to update the VIAs, which correspond to the conventional FV solution. The multi-moment approach gives an approximation of the usual upstream grid cell used in many FV-SL schemes. Doing so avoids having to compute complicated integrals over arbitrary regions while retaining the full two-dimensionality of the semi-Lagrangian computation. This design also allows CSLR1-M to use a simpler and highly accurate method at the edges of the cubed-sphere grid used in this scheme, greatly reducing grid imprinting.

While CSLR1-M is a novel extension to existing advection schemes, it remains to be seen whether it would improve upon the multitude of existing advection schemes; a literature search could lead the careless scholar to conclude that everyone working in numerical methods has developed an advection scheme. Further, it isn't known whether this marks an improvement over existing SL methods, led by Temperton et al. (2001), or existing FV methods (Lin and Rood, 1996). Many schemes have claimed improved accuracy over these schemes, but the improvement is rarely sufficient to justify the added numerical expense or complexity, which in many cases is quite substantial. Tang et al. (2021) also only apply CSLR1-M to two-dimensional advective problems. While this is useful for transport modeling with prescribed flow fields, there is no application of the scheme to three-dimensional dynamical problems with evolving flow fields, which is necessary for the method to be useful for constructing a full weather or climate model. Such an implementation may expose additional challenges needing to be overcome before becoming a useful model.

CSLR1-M is a promising method that combines the best features of both SL and FV methods while remaining relatively simple and overcoming some of the problems of earlier SL-FV methods. The results of Tang et al. (2021) show that this may be a useful method for future transport and modeling applications.

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