EQUILIBRIUM STATES OF PLANETARY WAVES FORCED BY TOPOGRAPHY AND PERTURBATION HEATING AND BLOCKING SITUATION

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ABSTRACT

By using a two-level, highly truncated spectral model, the equilibrium states of ultra-long waves induced by topographic and thermal forcing are obtained, and the instabilities of the states are studied. It is found that there exist some stable equilibrium states possessing typical characteristics of blocking situation. Some inferences about the dynamic mechanism of blocking phenomenon are deduced since the solutions are analytically obtained.

1. MODEL

Blocking is an important atmospheric phenomenon and possesses some conspicuous features. In a previous paper[1], it is proposed that the forcing of longitudinally asymmetric heating or topography and the nonlinearity of advection are the two fundamental factors in the dynamic mechanism of blocking. Using a highly truncated spectral method similar to that used by Charney and Devore[2], we obtained a kind of low-index, stable equilibrium state induced by thermal forcing, which possesses some main features of blocking, including the typical pattern and the position of the high. Now the model used in Ref. [1] is extended to incorporate topographic forcing.

The model in this paper is similar to Ref. [1] except that the vertical velocity at the lower boundary is due to the joint effects of Ekman pumping and topographic forcing. Suppose

$$\omega_4 = -\frac{cP_4}{H}\{\delta\nabla^2(\psi-\varepsilon)+J(\psi-\varepsilon,h)\},$$

where h is the height of orography, H the scalar height of homogeneous atmosphere and c a proportional constant. Through the same procedure as in Ref. [1] the non-dimensional equations of two-level model can be obtained by using L, f_0^{-1} and $c^{-1}H$ as the unit of length, time and height respectively:

$$\begin{aligned}\frac{\partial}{\partial t}\nabla^2\psi = &-R_0\frac{\partial}{\partial x}\nabla^2\psi - R_T\frac{\partial}{\partial x}\nabla^2\varepsilon-\beta\frac{\partial\psi}{\partial x}-\gamma\nabla^2(\psi-\varepsilon)\\ &-J(\psi-\varepsilon,h),\\ \frac{\partial}{\partial t}(\nabla^2-\lambda^*)\varepsilon = &J(\psi,\lambda^*\varepsilon)+R_0\lambda^*\frac{\partial\varepsilon}{\partial x}-R_T\lambda^*\frac{\partial\psi}{\partial x}\end{aligned} \quad (1)$$

$$-R_0\frac{\partial}{\partial x}\nabla^2\varepsilon-R_T\frac{\partial}{\partial x}\nabla^2\psi-\beta^*\frac{\partial\varepsilon}{\partial x}$$

$$+\gamma\nabla^2(\psi-\varepsilon)+J(\psi-\varepsilon,h)-Q^*, \tag{2}$$

where

$$R_0=\bar{u}/f_0L,\quad R_T=\bar{u}_T/f_0L,\quad \beta^*=L\beta/f_0,$$

$$\gamma=\sqrt{\frac{\nu}{2f_0}}\Big/H,\quad \lambda^*=2f_0^2L^2/\sigma p_2^2,\quad Q^*=RQ/c_p\sigma p_2^2f_0.$$

Here $\bar{u}$ and $\bar{u}_T$ are the velocities of basic zonal wind and thermal wind respectively, Q is diabatic heating rate. Choosing the orthogonal functions:

$$F_A=\sqrt{2}\cos y,\quad F_K=2\cos nx\sin y,\quad F_L=2\sin nx\sin y,$$

the distributions of diabatic heating and topography can be specified in the following idealized forms

$$Q^*=Q_A^*F_A+Q_K^*F_K,\quad h=h_KF_K. \tag{3}$$

When $n=2$ Eq. (3) illustrates the idealized distribution of two oceans and two continents in the Northern Hemisphere. Then the highly truncated spectral equations can be deduced as follows:

$$-\dot{\psi}_A=\gamma(\psi_A-\varepsilon_A)-n\alpha h_K(\psi_L-\varepsilon_L), \tag{4}$$

$$-(1+n^2)\dot{\psi}_K=-a_1\psi_L+c_1\varepsilon_L+b_1(\psi_K-\varepsilon_K), \tag{5}$$

$$-(1+n^2)\dot{\psi}_L=a_1\psi_K-c_1\varepsilon_K+b_1(\psi_L-\varepsilon_L)+n\alpha h_K(\psi_A-\varepsilon_A), \tag{6}$$

$$-(1+\lambda^*)\dot{\varepsilon}_A=n\alpha\lambda^*(\psi_L\varepsilon_K-\psi_K\varepsilon_L)-\gamma(\psi_A-\varepsilon_A)$$
$$+n\alpha h_K(\psi_L-\varepsilon_L)-Q_A^*, \tag{7}$$

$$-(1+n^2+\lambda^*)\dot{\varepsilon}_K=n\alpha\lambda^*(\psi_A\varepsilon_L-\psi_L\varepsilon_A)-a_1\varepsilon_L+c_1\psi_L$$
$$-b_1(\psi_K-\varepsilon_K)+nR_0\lambda^*\varepsilon_L-nR_T\lambda^*\psi_L-Q_K^*, \tag{8}$$

$$-(1+n^2+\lambda^*)\dot{\varepsilon}_L=n\alpha\lambda^*(\psi_K\varepsilon_A-\psi_A\varepsilon_K)+a_1\varepsilon_K-c_1\psi_K$$
$$-b_1(\psi_L-\varepsilon_L)-nR_0\lambda^*\varepsilon_K+nR_T\lambda^*\psi_K$$
$$-n\alpha h_K(\psi_A-\varepsilon_A), \tag{9}$$

where

$$\alpha=8\sqrt{2}/3\pi,\quad a_1=n[\beta^*-(1+n^2)R_0],$$

$$b_1=\gamma(1+n^2),\quad c_1=n(1+n^2)R_T.$$

The steady solutions of Eqs. (4)—(9) can be obtained through complicated algebraic operations:

$$\bar{\varepsilon}_K=-\mu_1\left\{\frac{2(c_1-a_1)}{n\alpha\lambda^*}+\frac{R_0-R_T}{\alpha}\right\}\left(\mu_1^2+\frac{b_1}{l}\mu_2^2\right)^{-1}\pm\left(\mu_1^2+\frac{b_1}{l}\mu_2^2\right)^{-1}$$
$$\times\sqrt{\left(\mu_1^2+\frac{b_1}{l}\mu_2^2\right)\frac{\mu_2^2Q_A^*}{B_2n\alpha\lambda^*}-\frac{b_1}{l}\mu_2^2\left\{\frac{2(c_1-a_1)}{n\alpha\lambda^*}+\frac{R_0-R_T}{\alpha}\right\}^2}, \tag{10}$$

$$\bar{\varepsilon}_L=\pm\sqrt{Q_A^*(n\alpha\lambda^*B_2)^{-1}-b_1l^{-1}\bar{\varepsilon}_K^2}, \tag{11}$$

$$\bar{\varepsilon}_A=[(A_2-1)\bar{\varepsilon}_K-B_2\bar{\varepsilon}_L]^{-1}\{D_1\bar{\varepsilon}_K^2+D_2\bar{\varepsilon}_K\bar{\varepsilon}_L+(c_1-a_1)(n\alpha\lambda^*)^{-1}$$
$$\times[(1+A_2)\bar{\varepsilon}_K-B_2\bar{\varepsilon}_L]+R_0\alpha^{-1}\bar{\varepsilon}_K-R_T\alpha^{-1}(A_2\bar{\varepsilon}_K-B_2\bar{\varepsilon}_L)\}, \tag{12}$$

$$\bar{\psi}_K=A_2\bar{\varepsilon}_K-B_2\bar{\varepsilon}_L, \tag{13}$$

$$\bar{\psi}_L=A_1\bar{\varepsilon}_K+A_2\bar{\varepsilon}_L, \tag{14}$$

$$\bar{\psi}_A=\bar{\varepsilon}_A+D_1\bar{\varepsilon}_K+D_2\bar{\varepsilon}_L, \tag{15}$$

where

$$l = b_1 + (nah_K)^2/\gamma,\quad A_1 = b_1(c_1 - a_1)(a_1^2 + b_1 l)^{-1},$$
$$A_2 = (a_1 c_1 + b_1 l)(a_1^2 + b_1 l)^{-1},\quad B_2 = lA_1/b_1,$$
$$D_1 = nah_K A_1/\gamma,\quad D_2 = a_1 D_1/b_1,$$
$$\mu_1 = D_1 + (A_2 - 1)Q_K^* Q_A^{*-1},\quad \mu_2 = D_2 - B_2 Q_K^* Q_A^{*-1}.$$

From Eqs. (10)—(15) it can be seen that there are four kinds of equilibrium states under the joint effect of thermal and topographic forcing. They will be refered as state 1—4, according to signs (+, +) (+, −), (−, +), (−, −) in Eqs. (10) and (11) respectively. The instability of these equilibrium states to disturbances of ultra-long waves and shorter waves is studied with a numerical method presented in Ref. [1].

II. EQUILIBRIUM STATES INDUCED BY TOPOGRAPHY

The equilibrium states induced by topography can be obtained from Eqs. (10)−(15) with $Q_K^* = 0$. There are four kinds of equilibrium states in this case, whereas there are only two in the case of considering merely thermal forcing[1]. It is interesting to find that the four kinds of equilibrium states are also obtained by Charney and Straus[4], whose method is numerical other than analytical.

To reveal the characteristics of the equilibrium states,many examples of the states and their instability are calculated by taking various values of $\bar{u}, \bar{u}_T$ and Q_A^*. The other parameters are as follows: $n=2$, $\beta^*=0.35$, $L=a/2$ (a is the earth's radius), $\lambda^*=36$, and $h_K=0.015$. Shown in Fig. 1 is the distribution of orography in which the maximum difference of height between the ocean and continent is about 750 m.

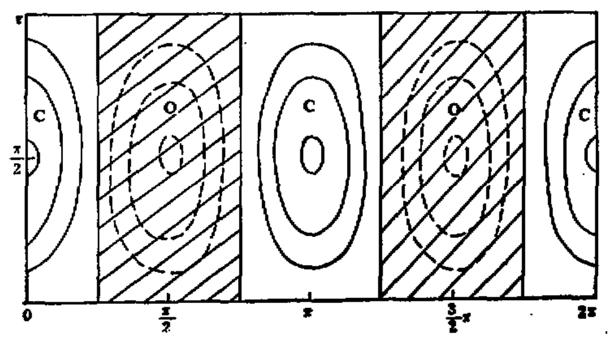

Fig. 1. Distribution of orography. The letters C and O indicate the words continent and ocean respectively.

From these calculated examples it is found that the characteristics of the four kinds of equilibrium states are very different. States 1 and 2 are usually unstable. State 3 may be stable but it is a sort of high-index flow with the ridge located in west of the ocean. State 4 can be stable when the values of $\bar{u}, \bar{u}_T$ and Q_A^* are reasonable. It is noticeable that the flow pattern of this kind of equilibrium state possesses some typical features of blocking situation as shown in Fig. 2. The pattern is Ω-shaped. The high and warm centers on 500 hPa level are almost coincident and situated in east of the ocean. In fact, it is in the east of Atlantic and Pacific ocean that the blocking highs appear most frequently. Therefore state 4 can be named as the blocking type of equilibrium state induced by topography.

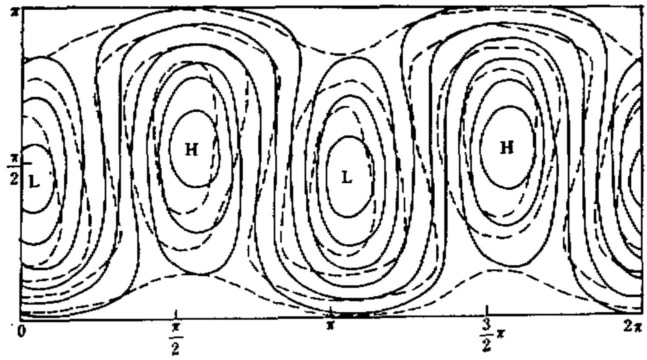

Fig. 2. Equilibrium state 4 induced by topography.
$R_0=0.045$, $R_T=0.04$, $Q^*_A=0.00371$, max. $\sigma_{real}=0.00074$.

Two inferences about the mechanism of blocking induced by topographic forcing are deduced from the solutions as follows:

(1) From the requirement of existing real roots in Eq. (10) the necessary condition for blocking is

$$Q^*_A \geqslant \left\{ \frac{\gamma^2 b_1}{(nah_K)^2} + \gamma \right\} \frac{na\lambda^*}{c_1-a_1} \left\{ \frac{2(c_1-a_1)}{na\lambda^*} + \frac{R_0-R_T}{a} \right\}^2 \equiv Q^*_{AC}, \tag{16}$$

where Q^*_{AC} is a critical value of zonal heating difference Q^*_A. Only when $Q^*_A \geqslant Q^*_{AC}$, can the equilibrium states induced by topography exist. From Eq. (16) we see that the larger the height differece h_K, the smaller the critical value Q^*_{Ac}. Consequently, the requirement for the existence of blocking will be satisfied more easily. The value of Q^*_{Ac} also depends on the parameters $\bar{u}$ and $\bar{u}_T$. Calculations indicate that only when $\bar{u}_1$ is close to the Rossby critical velocity $\bar{u}_c$ and $\bar{u}_3$ is small, can Q^*_{Ac} range over a reasonable value, otherwise it will be too large for actual circumstances. Furthermore, it is found that Q^*_{Ac} is more sensitive to the value of zonal wind than Q^*_{Kc}, which is a critical value of perturbation heating in the case considering merely thermal forcing[1]. This may account for that in the case considering merely topographic forcing blocking can form and maintain only when the zonal flow is in quite favourable conditions.

(2) From Eqs. (4)—(7) we obtain the relationship

$$\bar{\varepsilon}_K^2 + [1+(1+n^2)(nah_K)^2]\bar{\varepsilon}_L^2 = \frac{Q^*_A(a_1^2+b_1^2)}{na\lambda^* b_1(c_1-a_1)} + \frac{(1+n^2)(nah_K)^2 Q^*_A}{na\lambda^* b_1(c_1-a_1)}. \tag{17}$$

Then we see that the amplitude of blocking wave forced by topography is proportional to $\sqrt{Q_A^*}$. This is consistent with the fact that the intensity of blocking in winter is usually much stronger than that in summer since the meridional heating difference is greater in winter. From Eq. (17) another conclusion can be drawn, i. e. when $\bar{u}_1$ is close to $\bar{u}_c$, corresponding to $c_1 - a_1 \approx 0$, the amplitude of the stationary wave forced by topography will be so large that blocking forms. This indicates that the flow and the forcing approach the resonant state in the case of blocking. This conclusion is similar to that in Ref [1]. Therefore it can be concluded that blocking situation is a sort of stable, nonlinear equilibrium state near resonant character.

III. EQUILIBRIUM STATES INDUCED BY BOTH THERMAL AND TOPOGRAPHIC FORCING

In Ref. [1] and the previous section of this work we have studied the equilibrium states of blocking type induced by either topography or stationary heat sources. Either of them can be, partially but not perfectly, used to explain the position of actual blocking highs in various seasons, e. g. blocking highs over the East Atlantic in summertime can be hardly explained in terms of the equilibrium state induced by thermal forcing alone. Actually, the flow is affected by the two kinds of forcings simultaneously. We shall further study the characteristics of the equilibrium states excited by both forcings, utilizing Eqs. (10)—(15). For simplicity we shall investigate the two idealized cases, i. e. the summertime and wintertime respectively corresponding to the same and reverse signs for both Q_K and h_K. As for their absolute values, Q_A is taken to be 0.005 in winter case and 0.001 in summer case, but h_K to be 0.015 all the time.

1. *Winter Case*

In winter the ocean is a heat source and the continent a heat sink. From many calculated examples it is found that the main characteristics of the four kinds of equilibrium states resemble roughly those of states forced by topography alone. In this case the states 1 and 2 are unstable, state 3 can be stable but it actually is a sort of high-index flow field with the trough located in west of the land. Only state 4 is usually stable and possesses typical blocking features as shown is Fig. 3.

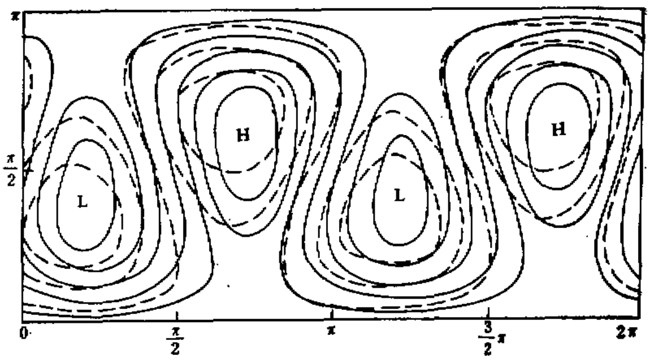

Fig. 3 (a) State 4 in winter case. $R_0 = 0.05$, $R_T = 0.035$, $Q_A^* = 0.005$, $Q_K^* = -0.00693$ and $\max.\sigma_{real} = -0.002347$.

It can be seen from Fig.3 that the high of this state is situated in the east of the ocean, near the west coast of the continent. When the heat source is stronger, the high will be stronger and its position will further west (see Fig. 3 (a)) compared to Fig. 3 (b). This characteristic is coincident with the statistics for blocking situations[7], in which it is pointed that the position of blocking highs in high winter is usually further west than other time in winter, and the intensity of sea-land heat difference is also stronger in high winter.

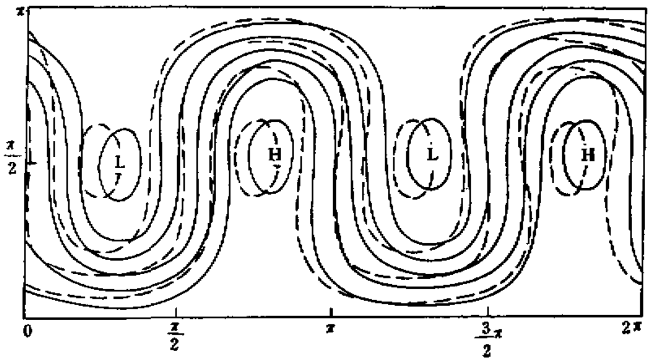

Fig. 3 (b). State 4 in winter case when Q_K^* is smaller. As in Fig. 3 (a) except for $Q_K^* = -0.00439$ and max. $\sigma_{real} = -0.000002$.

It may be concluded that in wintertime the effects of two kinds of forcings on the position and intensity of blocking highs are nearly in phase, because in the case of considering merely diabatic heating the high of equilibrium state is located at about 1/4 wavelength east of the heat source, whereas in the case of considering merely topography the high is situated in the east of ocean which is the heat source in winter. Therefore the two kinds of forcings consistently cause the blocking highs to form and maintain in the east of oceans in this season.

2. *Summer Case*

In summertime the ocean becomes a cooling source and the land a heat source, so that the characteristics of the equilibrium states are quite different from those in wintertime. In this case states 3 and 4 become unstable, states 1 and 2 can be stable and sometimes possess blocking features.

State 2 is the main sort of blocking type state in summer case. As shown in Fig. 4 (a), its high is situated in the east of ocean, near coast. From the statistics by Rex[8] and by Sumner[9], the areas where blocking highs appear most frequently in summer are 10°—20° W and 130—140° W, just the east of two oceans. The blocking highs in these areas may be interpreted by equilibrium state 2.

State 1 is usually a sort of wave-like flow field, but it can be of blocking type when Q_K^* is larger, as shown in Fig. 4 (b) with the high near the west coast of the ocean. Actually, in summertime blocking highs also appear over the sea of Okhotsk, northeast of Asia. This position of the highs is consistent with that of state 1.

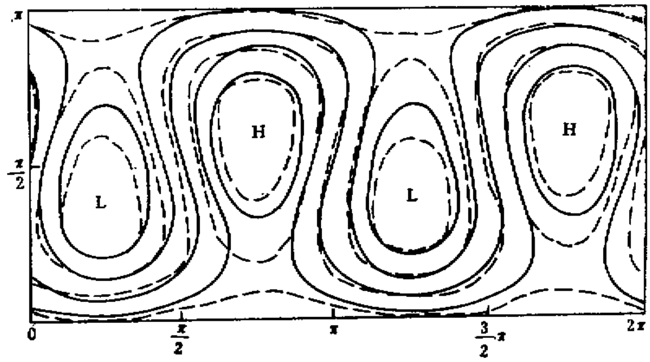

Fig. 4 (a). State 2 in summer case. $R_0=0.04$, $R_T=0.035$, $Q_A{}^*=0.001$, $Q_K{}^*=0.00771$ and max. $\sigma_{real}=-0.00027$.

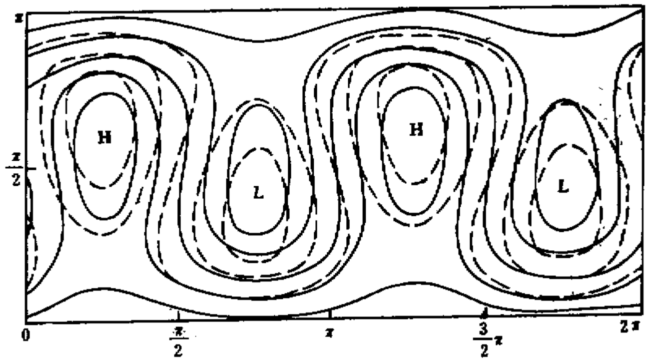

Fig. 4 (b). State 1 in summer case. As in Fig. 4 (a) except for max. $\sigma_{real}=-0.00057$.

Therefore, there may be two kinds of equilibrium states of blocking type in summer case, but only one in winter case. The joint effects of thermal and topographic forcings are more complex than that in winter. This may be due to the fact that in summer case the effects of two kinds of forcings on the position and intensity of blocking highs are not consistent, and that the ocean becomes cooling source in this season.

3. *Discussion*

From Eq.(10) a necessary condition for the existence of blocking induced by two kinds of forcings can be deduced:

$$Q_K^{*2} \geqslant Q_{KC}^{*2} - \frac{b_1^2\left(D_1^2 + \frac{A_1}{B_1} D_2^2\right) Q_A^{*2}}{A_1^2(a_1^2 + b_1 l)} \equiv Q_{KCT}^{*2}, \tag{18}$$

where Q_{KC}^* and Q_{KCT}^* are defined as the critical values of Q_K^* in the case of considering merely thermal forcing[1] and two kinds of forcings respectively. It can be seen from Eq.(18) that when $h_K=0$, then $Q_{KCT}^*=Q_{KC}^*$, whereas if $h_K \neq 0$, $Q_{KCT}^* < Q_{KC}^*$. Therefore the critical value of thermal forcing in the case of incorporating topography is less than that with considering merely stationary heat sources. Thus, it can be concluded that the existence of topographic forcing is a favourable condition for the formation and maintenance of blocking because the necessary condition is more easily satisfied. However, whether it is satisfied is mainly determined by the intensity of heat sources.

The large-scale topographic forcing is an important factor in the dynamics of blocking. It makes blocking highs trend to form and maintain in the east of oceans, and makes the necessary condition for the existence of blocking type of equilibrium states be satisfied more easily. In this sense the topographic forcing can be considered as a background condition in the mechanism of blocking. On the other hand, compared with the diabatic heating, the topographic forcing is less changeable. The seasonal and annual veriabilities of blocking activities in their position and intensity and maintaining interval can not be explained with the topographic forcing only. It may be more reliable that the variations in distribution and intensity of diabatic heating are mainly caused by sea-land contrast, which is changeable with seasons. As stated above, the equilibrium states of blocking type can exist only when the intensity of longitudinally asymmetric heating reaches a critical value. And the differences of intensity and position of blocking highs in winter and summer are due to the variation of thermal forcing. The thermal forcing may play a more active and variable role in the dynamics of blocking activities. This conclusion about the controlling effects of thermal forcing on blocking activity is somewhat consistent with that proposed by Namias[11] and more recently by White and Clark. Namias pointed out that the sea surface temperature distribution of the North Atlantic was characterized by large and persistent anomalies during all seasons from late 1958 through 1959 to 1960. At the same time the blocking activity was extremely frequent and persistent over North Europe. Thus he assumed that the physical causes of blocks lie in feedback machanisms between the atmosphere and the surface.

IV. SUMMARY

So far the nonlinear equilibrium states of ultra-long waves induced by thermal and topographic forcings are studied by using the highly truncated spectral method. It is found that there may be some characteristic blocking types of equilibrium states in the cases of incorporating either or both of two kinds of forcings. For latter case we have investigated the characteristics of the states in idealized winter and summer conditions. It is revealed that the characteristics of the blocking types of the states in these cases are roughly consistent with that of actual blocking situations in these seasons not only in the main features of the flow field and the stability, but also in the positions of the highs. It is heuristic that the results of the highly simplified model can reflect the main features of blocking. This may be due to that the model incorporates the two main factors in dynamics of blocking, i. e. the forcing of topography and stationary heat sources and the nonlinearity of flow.

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