THE HEATING FIELD IN AN ASYMMETRIC HURRICANE — PART I: SCALE ANALYSIS

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Received October 20, 1984

ABSTRACT

A closed system of equations describing an asymmetric disturbance in cylindrical geometry is expanded about a small parameter. The small parameter describes the ratio of the magnitude of divergence in the boundary layer to that above that layer. A low order system describes a gradient wind balance in the radial direction and is quasi-symmetric with respect to the pressure and temperature fields. This system can be solved as an inverse problem for a mature steady state hurricane. The procedure entails asking the questions what structure and heating distributions are required to maintain a given asymmetric distribution of the tangential velocity (i. e. the angular momentum) in steady state. The method of characteristics enables us to solve for the vertical motion. That in turn determines the radial motion from the mass continuity equation. Application of the hydrostatics to the cylindrical thermal wind equation determines the pressure and the thermal fields and finally the required heating fields are deduced from the first law. This entire system of inverse dynamics is linear although no nonlinear terms are dropped from the original direct set of equations. The real data applications of this procedure will be described in part II (to be published in the next issue).

I. INTRODUCTION

In recent years a considerable number of studies have been devoted to the investigations of the full three dimensional typhoon (hurricane) problem, see the review by Anthes (1982)[1]. These studies have provided a better understanding of the life cycle of typhoons (hurricane). Hawkins and Rubsam (1968)[2] provided the most detailed synoptic analysis of an asymmetric hurricane. This study provided detailed vertical cross sections of the motion, thermal and the mass fields across a mature hurricane, Hilda of 1964. The research aircraft penetrations of hurricanes have provided unique data sets for the analysis of the structure of the inner rain area. Modelling studies of asymmetric hurricanes have mostly used idealized inital state where an incipient disturbance is usually in a gradient wind balance, see Anthes (1971 a, b)[3,4], Tuleya and Kurihara (1975)[5], Jones (1977 a, b)[6,7], Madala and Piaesek (1975)[8] and several others. Real data numerical weather prediction experiments have been attempted by Mathur (1974)[9], Fiorino (1978)[10], Madala and Hodur (1977)[11] and several others. These studies have shown considerable success in the simulation or prediction of various features such as the structure of the mass, motion and thermal fields. These include the asymmetries of the outflow layer, spiral rain bands, eye wall convection and general descent within the eye wall. Energetically the role of cumulus convection in generating eddy available potential energy and its release to eddy kinetic energy have been addressed by Tuleya and Kurihara (1975)[5]. More recently Kurihara and Tuleya (1981)[12] and Tuleya and Kurihara (1981)[13] examined the role of convection, horizontal and vertical shear in the development of a tropical storm.These studies show the importance of weak easterly shear (in the vertical) for the development of tropical storms.

The present study is an extension of an asymmetric hurricane model, Krishnamurti (1961, 1962)[14,15]. Here the maintenance of a mature asymmetric steady hurricane is addressed. It is shown that given the momentum distribution(i. e. the tangential winds) and a formulation of the frictional forces, it is possible to construct a consistent dynamical and thermodynamical structure of the hurricane. In particular, it is possible to construct the diabatic heating fields along the rain bands and the eye wall by an inverse approach starting from the tangential velocity. This problem is approached in two parts. Part I of the paper presents a mathematical treatment and a scale analysis of the inverse problem. In part II of this paper we shall show a step by step analysis of the results for a hurricane.

Fig. 1 shows the typical streamline and isogon field that can be obtained from the analysis of the flight data in a hurricane. Aircraft flight data is not synoptic but most observations are made in a period of about 3 hours, hence a synoptic analysis of the data can be made with some degree of confidence. The Doppler navigation system is capable of giving wind speeds with errors within 10% and direction within 10°. These are based on estimates of probable errors in the air speed, ground speed, heading and the drift angle of the aircraft. The aircraft flight was made near the 250 hPa surface, the analyzed streamlines depict the high level outflow in hurrican Helene of 1958. The outflow is rather marked and one can obtain a good measure of the tangential and radial winds at this level from the isotach field shown in Fig. 2. In general, however, the flows above the boundary layer are more tangential than radial, hence the tangential motions are better known from flight data than the radial motions. NOAA research flight facility aircrafts are equipped to measure among the various parameters wind speeds, D-values, air temperatures and moisture in a hurricane.

The flight data is generally of good quality. The isotach of the total wind speed can generally be analyzed without much difficulty and one frequently finds the smooth crescent shaped geometry in the analyzed maps. All that can be said about the temperature and D-values is that they are more symmetric about the storm center than the velocity field. The moisture field is generally very poorly defined.

In order to draw an internally consistent three dimensional picture of a mature tropical storm an alternate manner of description of the flows is proposed in this paper. We assume that the tangential velocity distribution is prescribed in a storm. For momentum and mass balance a first order partial differential equation for the vertical motion must be solved which in turn describes the radial motion. The temperature, D-value and heating distribution required for steady state maintenance of the storm are obtained by solving the radial, vertical and the thermodynamic energy equations. An obvious advantage of this scheme lies in that all of the differential equations are rendered linear by the prescribed momentum distribution. Furthermore, since the calculations are anchored to an observed distribution of momentum the other fields will be expected to be realistic.

A three dimensional hurricane model can be of value in our understanding of the interaction of a hurricane with the environment. Furthermore besides having a forecasting value such studies will be important for understanding the role of the eye wall and the spiral rain bands in the momentum, heat and energy balance of the storm.

Our steady state three dimensional model is somewhat limited in its scope but we feel that this approach will answer some of the same questions that one may hope to answer by

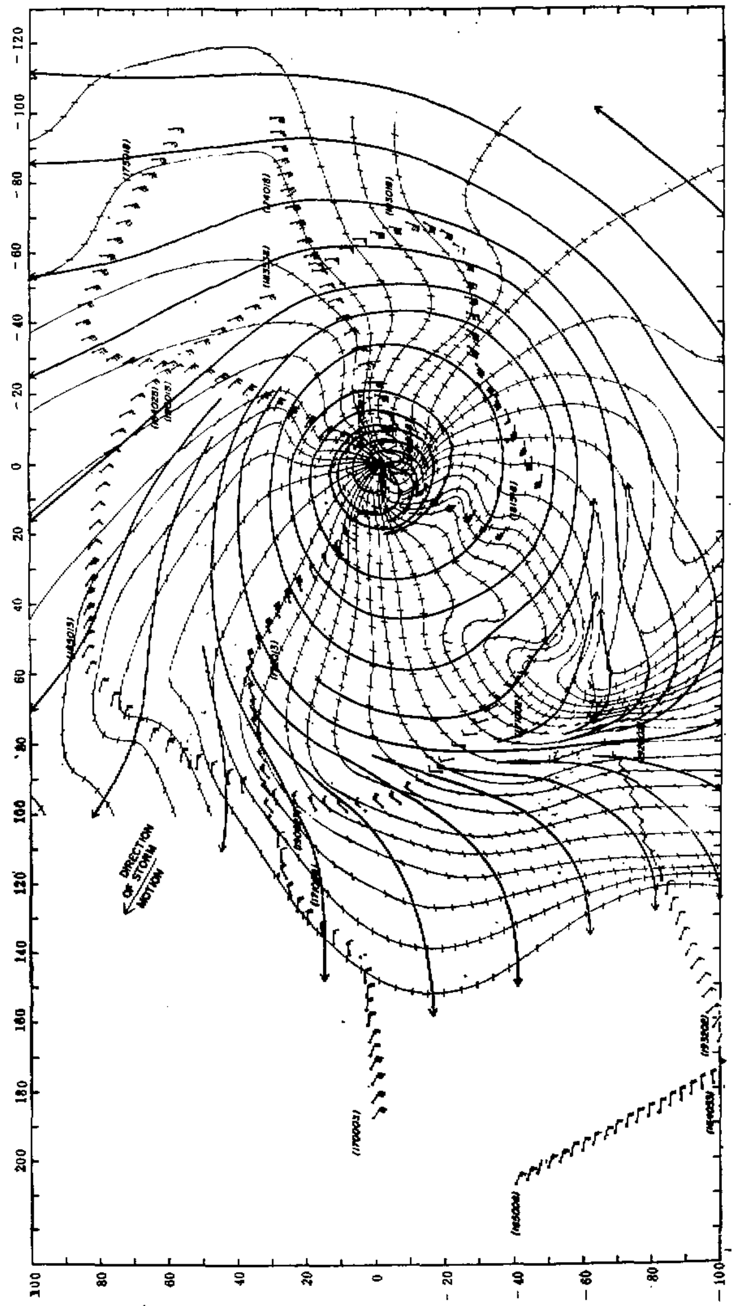

Fig. 1. The observed distribution of isogons (thin solid lines) and streamlines (heavy solid lines) in Hurricane Helene, September 26, 1958 near the 250 hPa surface. The flight time is marked in parenthesis in hours, minutes, and seconds in Greenwich time. Plotted winds are about 30 seconds apart. The *horizontal scale* is in *nautical miles*; *0–0* marks a fix on the center of the storm.

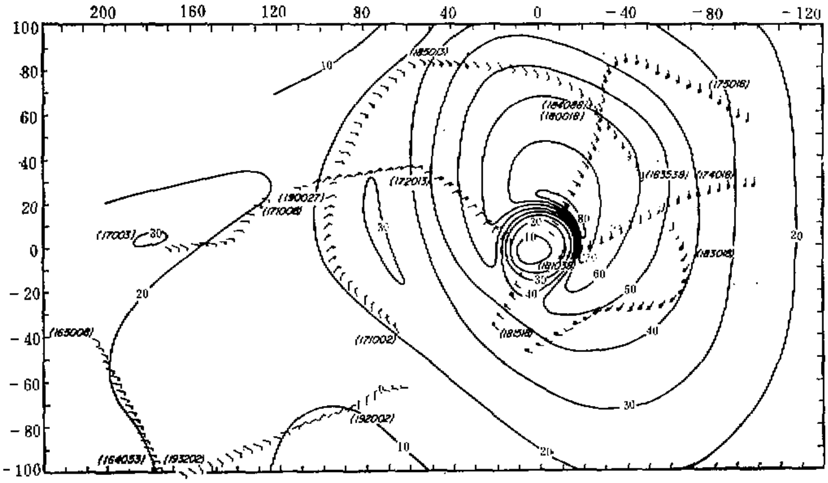

Fig. 2. Isotach analysis of the total winds at the 250 hPa surface in Hurricane Helene, September 26, 1958 (units: knots).
The horizontal scale is in nautical miles; 0–0 marks a fix on the center of the storm.

solving the more general initial value problem.

We shall first show that if fluid parcels conserve their mass and absolute angular momentum for steady state motions, then for a prescribed tangential motion the radial and vertical motions are exactly determined from solutions of a first order partial differential equation.

(A list of symbols is included in Table 1, placed at the end of the paper). *Conservation* of mass is expressed by the relation

$$\frac{1}{r}\frac{\partial U}{\partial \theta}+\frac{1}{r}\frac{\partial Vr}{\partial r}+\frac{\partial \omega}{\partial p}=0, \tag{1}$$

and conservation of absolute angular momentum M, is expressed by the relation

$$\frac{U}{r}\frac{\partial M}{\partial \theta}+V\frac{\partial M}{\partial r}+\omega\frac{\partial M}{\partial p}=0, \tag{2}$$

where $M=Ur+f_0r^2/2$. Eq. (2) may be rewritten in the form

$$V=-\frac{\frac{U}{r}\frac{\partial M}{\partial \theta}}{\frac{\partial M}{\partial r}}-\frac{\omega\frac{\partial M}{\partial p}}{\frac{\partial M}{\partial r}}$$

$$=-\frac{\frac{U}{r}\frac{\partial U}{\partial \theta}}{\zeta_a}-\frac{\omega\frac{\partial U}{\partial p}}{\zeta_a},$$

or $V=b+a\omega$, (3)

Substituting in (1) we obtain

$$\frac{\partial\omega}{\partial p}+\frac{\omega}{r}\frac{\partial ar}{\partial r}+a\frac{\partial\omega}{\partial r}+\frac{1}{r}\frac{\partial br}{\partial r}+\frac{1}{r}\frac{\partial U}{\partial\theta}=0. \tag{4}$$

This first order partial differential equation in ω can in principle be solved by the method of characteristics. If one makes the further assumption that flows are quasi-barotropic, namely $\partial U/\partial p$ is small, then convergence, $\partial\omega/\partial p$, is given by the relation

$$\frac{\partial\omega}{\partial p}=\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{U\frac{\partial U}{\partial\theta}}{\zeta_a}\right)-\frac{1}{r}\frac{\partial U}{\partial\theta}. \tag{5}$$

The right hand side is zero for a symmetric tangential velocity distribution, hence convergence is related to the asymmetries of the tangential velocity by Eq. (5). The barotropy assumption is not essential for the present study.

If we consider the following form for U,

$$U=U_0(r)+\sum_{n=1}^{M}(A_n(r)\sin n\theta+B_n(r)\cos n\theta), \tag{6}$$

convergence $\partial\omega/\partial p$ may be expressed by a relation of the form

$$\frac{\partial\omega}{\partial p}=\sum_{n=1}^{2M}(C_n(r)\sin n\theta+D_n(r)\cos n\theta). \tag{7}$$

It follows then that if the tangential velocity field contains a single harmonic, the convergence field will contain more than one harmonic.

A hypothetical vortex of the type considered here, with several harmonics in its tangential velocity distribution will contain several bands of convergence and divergence. This banded form is a consequence of conservation of mass and absolute angular momentum.

An examination of the distribution of absolute angular momentum in a hurricane leads one to conclude that the isoline of the angular momentum could not possibly describe either the trajectories or the streamlines. Frictional torques are rather significant for motions on many scales including the cumulus scale. Indeed a parametarization of the cumulus scale motion would describe the gross effects of the frictional torques. The foregoing analysis is presented to illustrate the relation between the momentum distribution and a banded convergence distribution. In the following section we shall formally show that the inclusion of frictional torques alters the picture considerably near the center of the storm but to a lesser degree away from the center.

II. SCALE ANALYSIS OF THE STEADY STATE EQUATIONS

The scale theory is based on an expansion of the dependent variables of the governing equations in powers of a parameter $\varepsilon=R/\lambda_0$, where R and λ_0 are measures of the inverse of the gradients of a typical variable in the radial and tangential directions in a hurricane, i. e..

$$\frac{\partial}{\partial r}\equiv\frac{1}{R}\frac{\partial}{\partial r'}, \tag{8}$$

$$\frac{\partial}{\partial\lambda}\equiv\frac{1}{\lambda_0}\frac{\partial}{\partial\lambda'}, \tag{9}$$

R and λ_0 may be considered as two fundamental length scales larger than the cumulus scale.

In a hurricane variations in the radial direction are somewhat greater than in the tangential direction both in the boundary layer and above. Hence, it is safe to state that

$$\varepsilon = \frac{R}{\lambda_0} < 1. \tag{10}$$

This is equivalent to the statement that a hurricane is quasi-symmetric.

We shall divide the flows into two regions, I: the boundary layer and, II: above the boundary layer. In the regions I and II we shall further assume:

Region I

$$U = V_0 U', \tag{11}$$

$$V = V_0 V', \tag{12}$$

Region II

$$U = V_0 U', \tag{13}$$

$$V = V_0 \frac{R}{\lambda_0} V', \tag{14}$$

where the primes refer to non-dimensional quantities of order unity. Here V_0 may be considered a typical measure of the tangential flow in the storm. In the boundary layer the radial and tangential velocity are of the same order, thus permitting inflow angles of the order of $\pi/4$. Above the boundary layer we have assumed that the radial motions are smaller than the tangential motions.

(1) *Convergence in non-dimensional form*

The equation

$$\frac{\partial U}{\partial \lambda} + \frac{1}{r} \frac{\partial V r}{\partial r} + \frac{\partial \omega}{\partial p} = 0 \tag{15}$$

for Region I may be written in the form:

$$\frac{V_0}{\lambda_0} \frac{\partial U'}{\partial \lambda'} + \frac{V_0 R}{R^2} \frac{1}{r'} \frac{\partial V' r'}{\partial r'} + \frac{\partial \omega}{\partial p} = 0,$$

$$\frac{V_0}{R} \left(\frac{R}{\lambda_0} \frac{\partial U'}{\partial \lambda'} + \frac{1}{r'} \frac{\partial V' r'}{\partial r'} \right) + \frac{\partial \omega}{\partial p} = 0. \tag{16}$$

The primed quantities are of order unity, hence, the principal term in the convergence in the boundary layer is

$$\left. \frac{\partial \omega}{\partial p} \right|_{\mathrm{I}} = - \frac{V_0}{R} \frac{1}{r'} \frac{\partial V' r'}{\partial r'}, \tag{17}$$

and the magnitude of convergence in the boundary layer $\sim V_0/R$.

For region II Eq. (8) may be written as

$$\frac{V_0}{\lambda_0} \frac{\partial U'}{\partial \lambda'} + \frac{V_0 R}{R \lambda_0} \frac{R}{R} \frac{1}{r'} \frac{\partial V' r'}{\partial r'} + \frac{\partial \omega}{\partial p} = 0,$$

$$\frac{V_0}{\lambda_0} \left(\frac{\partial U'}{\partial \lambda'} + \frac{1}{r'} \frac{\partial V' r'}{\partial r'} \right) + \frac{\partial \omega}{\partial p} = 0. \tag{18}$$

Above the boundary layer convergence

$$\frac{\partial\omega}{\partial p}\Big|_{11} = -\frac{V_0}{\lambda_0}\left(\frac{\partial U'}{\partial\lambda'} + \frac{1}{r'}\frac{\partial V'r'}{\partial r'}\right),$$

and its magnitude $\approx V_0/\lambda_0$.

Hence, the expansion parameter $\varepsilon = R/\lambda_0$ may be re-expressed as the ratio:

$$\frac{\text{Convergence above the boundary layer}}{\text{Convergence in the boundary layer}}.$$

(2) *Vorticity in non-dimensional form*

The expression $\zeta_a = \frac{\partial U}{\partial r} + \frac{U}{r} - \frac{\partial V}{\partial\lambda} + f_0$ may be non-dimensionalized in the boundary layer as follows

$$\zeta_a = \frac{V_0}{R}\left(\frac{\partial U'}{\partial r'} + \frac{U'}{r'}\right) - \frac{V_0}{\lambda_0}\frac{\partial V'}{\partial\lambda'} + f$$

$$= \frac{V_0}{R}\left(\frac{\partial U'}{\partial r'} + \frac{U'}{r'}\right) - \frac{R}{\lambda_0}\frac{\partial V'}{\partial\lambda'} + f.$$

Let $$\zeta'_a = \zeta_a/(V_0/R) = \frac{\partial U'}{\partial r'} + \frac{U'}{r'} + \frac{Rf}{V_0} - \frac{R}{\lambda_0}\frac{\partial V'}{\partial\lambda'}, \tag{20}$$

hence, to orders of $(R/\lambda_0)^0$ in the boundary layer

$$\zeta'_a = \frac{\partial U'}{\partial r'} + \frac{U'}{r'} + \frac{fR}{V_0}$$

$$= \frac{\partial U'}{\partial r'} + \frac{U'}{r'} + \frac{1}{R_0}. \tag{21}$$

Above the boundary layer we obtain

$$\zeta_a = \frac{V_0}{R}\frac{\partial U'}{\partial r'} + \frac{U'}{r'} - \frac{V_0 R}{\lambda_0^2}\frac{\partial V'}{\partial\lambda'} + f$$

and

$$\zeta'_a = \frac{\partial U'}{\partial r'} + \frac{U'}{r'} + \frac{fR}{V_0} - \frac{R^2}{\lambda_0^2}\frac{\partial V'}{\partial\lambda'},$$

again to powers of $\left(\frac{R}{\lambda_0}\right)^0$, $\zeta'_a = \frac{\partial U'}{\partial r'} + \frac{U'}{r'} + \frac{1}{R_0}$.

The expressions for vorticity to the lowest order are similar in the boundary layer and above. However, it is of interest to note that the tangential variations of the radial motions are of the order R/λ_0 in the boundary layer and $(R/\lambda_0)^2$ above the boundary layer.

(3) *Scale analysis of the equations of motion and the thermodynamic energy equation above the boundary layer*

The basic equations are, the tangential equations of motion

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial\lambda} + V\left(\frac{\partial U}{\partial r} + \frac{U}{r} + f_0\right) + \omega\frac{\partial U}{\partial p} = -g\frac{\partial Z}{\partial\lambda} + F_\theta, \tag{24}$$

the radial equation of motion

$$\frac{\partial V}{\partial t}+U\frac{\partial V}{\partial \lambda}+V\frac{\partial V}{\partial r}-\frac{U^2}{r}-f_0U+\frac{\omega\partial V}{\partial p}=-g\frac{\partial Z}{\partial r}+F_r, \tag{25}$$

the hydrostatic relation

$$\frac{R_GT}{p}=-\frac{g\partial Z}{\partial p}, \tag{26}$$

and the thermodynamic energy equation

$$\frac{\partial T}{\partial t}+U\frac{\partial T}{\partial \lambda}+V\frac{\partial T}{\partial r}+\omega\frac{\partial T}{\partial p}=\omega\frac{R_GT}{C_pp}+\frac{H}{C_p}. \tag{27}$$

We shall perform a scaling of the Eq. (24) through (27) as follows:

$$U=V_0U',$$

$$V=V_0\frac{R}{\lambda_0}V',$$

$$\omega=\Omega\omega',$$

$$\frac{\partial}{\partial r}=\frac{1}{R}\frac{\partial}{\partial r'},$$

$$\frac{\partial}{\partial \lambda}=\frac{1}{\lambda_0}\frac{\partial}{\partial \lambda'},$$

$$\frac{\partial}{\partial p}=\frac{1}{P}\frac{\partial}{\partial p'},$$

$$T=T_0T',T_0=\frac{gH^*}{R_G},$$

where V_0 is a characteristic tangential velocity. Ω is a characteristic vertical velocity. Time is scaled by the inverse of the magnitude of convergence above the boundary layer. P is a characteristic pressure above the boundary layer, defined by the equation,

$$\frac{\partial\omega}{\partial p}=\frac{\Omega}{P}\frac{\partial\omega'}{\partial p'} \tag{28}$$

where the dimensional ratio $\frac{\Omega}{P}$ is of the order $\frac{V_0}{\lambda_0}$, as defined earlier. T_0 is a characteristic temperature and H^* a corresponding scale height of the atmosphere. The primed quantities are non-dimensional.

The horizontal pressure gradient force $-g\nabla Z$ is scaled with the tacit assumption that

$$\frac{g\partial Z}{\partial r}=\frac{V_0^2}{R}\frac{\partial Z'}{\partial r'}. \tag{29}$$

This is equivalent to stating that the pressure gradient force is of the same order as the centrifugal force for scaling purposes. This is based on the observational studies of Gray (1961)[16].

Let ΔZ_r denote the variation of geopotential height along the radial direction, then

$$\Delta Z_r\approx\frac{V_0^2}{g}.$$

Furthermore $\Delta Z_r/\Delta Z_\lambda$ is of the order λ_0/R hence ΔZ_λ is of the order $\frac{R}{\lambda_0}\cdot\frac{V_0^2}{g}$

or

$$\frac{g\partial Z}{\partial\lambda}=\frac{V_0^2}{\lambda_0^2}R\frac{\partial Z'}{\partial\lambda'}. \tag{30}$$

The friction terms F_θ and F_r in the tangential and radial equations are written in the following form:

$$F_\theta=\frac{\partial}{\partial\lambda}\left(\nu_\lambda\frac{\partial U}{\partial\lambda}\right)+\frac{\partial}{\partial r}\left(\nu\left(\frac{\partial U}{\partial r}+\frac{U}{r}\right)\right)+\frac{\partial}{\partial p}\left(K\frac{\partial U}{\partial p}\right), \tag{31}$$

$$F_r=\frac{\partial}{\partial\lambda}\left(\nu_\lambda\frac{\partial V}{\partial\lambda}\right)+\frac{\partial}{\partial r}\left(\nu_r\left(\frac{\partial V}{\partial r}+\frac{V}{r}\right)\right)+\frac{\partial}{\partial p}\left(K\frac{\partial V}{\partial p}\right), \tag{32}$$

where ν_λ, ν_r and K are eddy momentum exchange coefficients above the boundary layer. From dimensional considerations we shall scale these coefficients as follows:

$$\nu_r=R^2\frac{V_0}{\lambda_0}\nu_r',$$

$$\nu_\lambda=\lambda_0^2\frac{V_0}{\lambda_0}\nu_\lambda',$$

$$K=P_2^2\frac{V_0}{\lambda_0}K',$$

where V_0/λ_0 is a characteristic unit for the inverse of time scales, and R, λ_0, P_2 are the length scales along the coordinates, r, θ, and p respectively. This implies that R^2V_0/λ_0, $\lambda_0^2V_0/\lambda_0$ and $P_2^2V_0/\lambda_0$ are measures of the eddy momentum exchange coefficients. The reason for this scaling lies in the following intuitive argument. If subgrid scale motions like the cumulus scale are important for transfer of momentum in a storm of fundamental scales R, λ_0, and P_2 then ν_r, ν_λ and K must be related to these scales in a simple manner.

Eq. (24) through (27) can be non-dimensionalized by the scaling parameters discussed above, and we obtain the following equations

$$\begin{aligned}&\frac{V_0^2}{\lambda_0}\left[U'\frac{\partial U'}{\partial\lambda'}+V'\left(\frac{\partial U'}{\partial r'}+\frac{U'}{r'}+\frac{1}{R_0}\right)+\omega'\frac{\partial U'}{\partial p'}\right]=\frac{V_0^2}{\lambda_0}\left(\frac{R}{\lambda_0}\frac{\partial Z'}{\partial\lambda'}\right)\\&\quad+\frac{V_0^2}{\lambda_0}\left\{\frac{R^2}{\lambda_0^2}\frac{\partial}{\partial\lambda'}\left(\nu_\lambda'\frac{\partial U'}{\partial\lambda'}\right)+\frac{\partial}{\partial r'}\left[\nu_r'\left(\frac{\partial U'}{\partial r'}+\frac{U'}{r'}\right)\right]+\frac{\partial}{\partial p'}\left(K'\frac{\partial U'}{\partial p'}\right)\right\},\end{aligned} \tag{33}$$

$$\begin{aligned}&\frac{V_0^2}{\lambda_0}\left[\frac{R}{\lambda_0}U'\frac{\partial V'}{\partial\lambda'}+V'\frac{\partial V'}{\partial r'}+\omega'\frac{\partial V'}{\partial p'}-\frac{\lambda_0}{R}\left(\frac{U'^2}{r'}+\frac{U'}{R_0}\right)\right]=-\frac{V_0^2}{R}\frac{\partial Z'}{\partial r'}\\&\quad+\frac{V_0^2}{\lambda_0}\left(\frac{R}{\lambda_0}\frac{R^2}{\lambda_0^2}\frac{\partial}{\partial\lambda'}\left(\nu_\lambda'\frac{\partial V'}{\partial\lambda'}\right)+\frac{\partial}{\partial r'}\left[\nu_r'\left(\frac{\partial V'}{\partial r'}+\frac{V'}{r'}\right)\right]+\frac{\partial}{\partial p'}\left(K'\frac{\partial V'}{\partial p'}\right)\right\},\end{aligned} \tag{34}$$

$$T'=-p'\frac{\partial Z'}{\partial p'}, \tag{35}$$

$$V_0\frac{gH^*}{R_G\lambda_0}U'\frac{\partial T'}{\partial\lambda'}+V'\frac{\partial T'}{\partial r'}-\omega'\frac{\partial T'}{\partial p'}-\frac{R_G}{C_p}\omega'\frac{T'}{p'}=\frac{H}{C_p}. \tag{36}$$

We shall next expand the non-dimensional dependent variables U', V', ω', Z', and T' in powers of R/λ_0

$$U'=U_0'+U_1'(R/\lambda_0)+U_2'(R/\lambda_0)^2+\cdots=\sum_{n=0}^{\infty}U_n'(R/\lambda_0)^n, \tag{37}$$

$$V'=V_0'+V_1'(R/\lambda_0)+V_2'(R/\lambda_0)^2+\cdots=\sum_{n=0}^{\infty}V_n'(R/\lambda_0)^n, \tag{38}$$

$$\omega'=\omega_0'+\omega_1'(R/\lambda_0)+\omega_2'(R/\lambda_0)^2+\cdots=\sum_{n=0}^{\infty}\omega_n'(R/\lambda_0)^n, \tag{39}$$

$$Z'=Z_0'+Z_1'(R/\lambda_0)+Z_2'(R/\lambda_0)^2+\cdots=\sum_{n=0}^{\infty}Z_n'(R/\lambda_0)^n, \tag{40}$$

$$T'=T_0'+T_1'(R/\lambda_0)+T_2'(R/\lambda_0)^2+\cdots=\sum_{n=0}^{\infty}T_n'(R/\lambda_0)^n. \tag{41}$$

A substitution of the series expansion of U', V', ω', Z' and T' in Eq. (33) through (36) may be made to obtain ordered sets of equations to powers of $(R/\lambda_0)^n$.

The lowest order set of equations is obtained by retaining terms of order $(R/\lambda_0)^0$. This is given by the following:

Equation of motion:

$$U'\frac{\partial U'}{\partial \lambda}+V'\frac{\partial U'}{\partial r'}+\frac{U'}{r'}+\frac{1}{R_0}+\omega'\frac{\partial U'}{\partial p'}=\frac{\partial}{\partial r'}\nu'\frac{\partial U'}{\partial r'}+\frac{U'}{r'}$$
$$+\frac{\partial}{\partial p'}K'\frac{\partial U'}{\partial p'}, \tag{42}$$

$$\frac{U'^2}{r'}+\frac{U'}{R_0}=\frac{\partial Z'}{\partial r'}, \tag{43}$$

$$T'=-p'\frac{\partial Z'}{\partial p'}. \tag{44}$$

Continuity equation:

$$\frac{\partial U'}{\partial \lambda'}+\frac{1}{r'}\frac{\partial V'r'}{\partial r'}+\frac{\partial \omega'}{\partial p'}=0. \tag{45}$$

Thermodynamic energy equation:

$$U'\frac{\partial T'}{\partial \lambda'}+V'\frac{\partial T'}{\partial r'}+\omega'\frac{\partial T'}{\partial p'}=\frac{\omega' T'}{p'}\frac{R_G}{C_p}+\frac{H}{C_p}\frac{R_G}{gH^*}\frac{\lambda_0}{V_0}. \tag{46}$$

The following properties of this lowest order system of equations above the boundary layer are of interest. The vortex is in gradient wind balance. Pressure torques do not influence the momentum distribution implying that the asymmetries in the D-value field and the temperature field are not very large. Angular momentum is not conserved and hence a net heating H is required to maintain this storm in steady state. The foregoing analysis would describe a storm where the boundary layer convergence is much larger than that just above, i. e., $\varepsilon \ll 1$. As ε approaches 1 departures from gradient wind balance become significant in Eq. (34).

III. CONCLUDING REMARKS

A simple framework for estimating vertical motions and the diabatic heating distributions in a hurricane is presented in this paper. The approach is relatively straightforward in that the inverse problem is linear. Given the distribution of angular momentum, a sequence of linear equations in turn describes the radial, vertical, thermal and heating distributions although the equations are in fact complete. Our aim is to seek a relationship between the angular momentum distribution and the asymmetric heating distribution such as the eye wall and rain bands. It should also be possible to examine the relative intensity of heating in the eye wall and in the rain bands of a hurricane.

Although the scaled system permits azimuthal asymmetries in the motion and heating fields, the pressure and the thermal fields to the lowest order are quasi-symmetric. The validity of that feature in the inner rain area has been pointed out in the observational studies of Hawkins and Rubsam(1968)[2] and several others. The relative flexibility of this inverse approach is due to the absence of pressure torques in the azimuthal direction. That makes the stepwise construction of the hurricane possible. It should be noted that although the inverse system is apparantly linear at every stage the fully scaled so-called nonlinear equations are in fact used everywhere.

The entire evolution of the asymmetries in the heating critically depends on the asymmetries of the angular momentum distribution. Thus the success of this method requires a good data base for the definition of the asymmetric tangential velocity. Since that is the larger of the velocity components, the present methods of observations, via research aircraft, are capable of providing sufficient accuracy of this field. The formulation of the friction terms in the momentum equations determines the sinks of momentum for this problem. In the second part of this paper we shall show that the essential asymmetries of the vertical motion and the heating are not critically dependent on the formulation of friction. It is the asymmetries of the observed momentum distribution that seems vital for the structure determined by the proposed method.

The research reported here was supported by the National Science Foundation (NSF) grant No. ATM 83040809. The computations reported here were carried out at the NCAR CRAY. NCAR is sponsored by the National Science Foundation. Mr. Sheng Jian would like to convey his appreciation to Nanjing University for permitting him to study at Florida State University.

Table 1 List of Symbols

r	radial coordinate, positive outward
θ	tangential coordinate, positive in the cyclonic sense
p	pressure, the vertical coordinate
U	tangential component of velocity
V	radial component of velocity
ω	vertical component of velocity
M	absolute angular momentum per unit mass
f_0, f	Coriolis parameter
ζ_a	absolute vorticity
a	$-\frac{1}{\zeta_a}\partial U/\partial P$
b	$-\frac{U}{\zeta_a}\frac{\partial U}{r\partial\theta}$

R	characteristic length scale in the radial direction
λ_s	characteristic tangential velocity
U'	non-dimensional tangential velocity
V'	non-dimensional tangential velocity
Z	geopotential height
F_θ	frictional force per unit mass in the tangential direction
F_r	frictional force per unit mass in the radial direction
R_G	gas constant
H	non-adiabatic heating per unit mass
C_p	specific heat of air at constant pressure
H^*	scale height of the atmosphere
Ω	characteristic vertical velocity
g	acceleration of gravity
ν_r	radial eddy-momentum exchange coefficient
ν_λ	tangential eddy-momentum exchange coefficient
K	vertical eddy-momentum exchange coefficient
P	characteristic pressure
r_m	radius of the zone of maximum tangential velocity
I	symbol for the boundary layer
II	symbol for region above the boundary layer

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