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SOME CATASTROPHE PROPERTIES OF TWO-LAYER SHEAR FLOW

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ABSTRACT

In this paper a simple current system which consists of two stratified incompressible layers is examined. For the basic equations of the motion of fluid a lower order spectrum model is established by means of Galerkin method. Adopting the difference of wind velocity between the upper and lower layers, $\Delta\bar{u}=\frac{\rho_1\bar{u}_1-\rho_2\bar{u}_2}{\rho_2}$ as a control parameter, the bifurcation and stability of the solution of the dynamical system are discussed. It is found that the flow states in the lower layer will have a catastrophe, when $|\Delta\bar{u}|>2\frac{\delta^2}{f^2}C_g^3$, where C_g is the phase velocity of the internal inertio-gravitational wave in a geostrophic current. These results may give a reasonable explanation for the mechanism of the catastrophe phenomena, including the "pressure-jump" in the atmosphere.

I. INTRODUCTION

For a stable stratified fluid, if the velocity of basic wind exceeds the phase velocity of the gravitational wave, a pressure-jump will happen. This concept is often used to explain the generation of squall in the atmosphere. Chao et al. have given a model[1] for the motion of fluid consisting of two stratified incompressible layers. They analysed some dynamical properties of the gravitational wave in geostrophic currents, when the fluid is at rest in the upper layer and the basic geostrophic wind exists in the lower layer. In this paper, the model is modified. We assume that the basic geostrophic wind also exists in the upper layer. By using a lower order spectrum truncation, the effects of the basic wind velocity difference between the upper and lower layers on bifurcations and the stability of the solutions of the spectrum equations are discussed.

II. BASIC EQUATIONS AND SPECTRUM MODEL

The motion of fluid shown in Fig. 1 is considered, where ρ is the density and $\bar{u}$ is the basic wind, both of them being constant, and H is the interface height of the fluid before it is disturbed. Labels 1 and 2 stand for the fluid in the upper and lower layers respectively, u, v and h are the horizontal velocity components in the lower layer and the disturbance height respectively.

When the thickness of fluid in the upper layer is large enough, we can use Tapper's assumption[2] that the motion of fluid in the lower layer has no influence on the motion in the upper layer. Therefore, the equations of motion for the fluid in the lower layer are

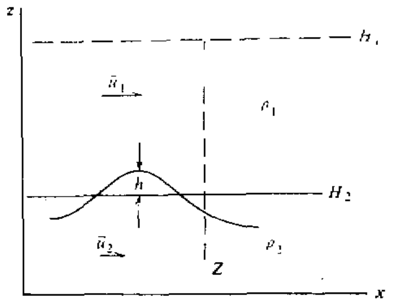

Fig. 1. The model of motion of fluid.

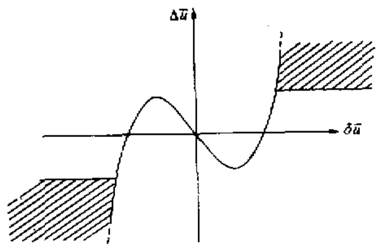

Fig. 2. The existential domain (exclusive of hatched areas) of the nontrivial solution.

given by

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho_2}\frac{\partial p}{\partial x}+fv, \tag{1}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho_2}\frac{\partial p}{\partial y}-fu, \tag{2}$$

$$\frac{\partial h}{\partial t}+u\frac{\partial(H_2+h)}{\partial x}+v\frac{\partial(H_2+h)}{\partial y}+(H_2+h)\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=0, \tag{3}$$

$$p=\rho_1 g(H_1-H_2-h)+\rho_2 g(H_2+h-z). \tag{4}$$

In addition, it is assumed that the basic flow in the upper and lower layers satisfies the geostrophic equilibrium relations, i. e. for the fluid in the upper layer

$$\begin{cases} g\dfrac{\partial H_1}{\partial x}=0, \\ g\dfrac{\partial H_1}{\partial y}=-f\bar{u}_1; \end{cases} \tag{5}$$

and for the fluid in the lower layer

$$\begin{cases} \dfrac{g}{\rho_2}\left[\rho_1\left(\dfrac{\partial H_1}{\partial x}-\dfrac{\partial H_2}{\partial x}\right)+\rho_2\dfrac{\partial H_2}{\partial x}\right]=0, \\ \dfrac{g}{\rho_2}\left[\rho_1\left(\dfrac{\partial H_1}{\partial y}-\dfrac{\partial H_2}{\partial y}\right)+\rho_2\dfrac{\partial H_2}{\partial y}\right]=-f\bar{u}_2. \end{cases} \tag{6}$$

Substituting Eq. (5) into (6), we obtain

$$\begin{cases} \dfrac{\partial H_2}{\partial x}=0, \\ \dfrac{\partial H_2}{\partial y}=\dfrac{f}{g^*}\dfrac{\rho_1\bar{u}_1-\rho_2\bar{u}_2}{\rho_2}=\dfrac{f}{g^*}\Delta\bar{u}, \end{cases} \tag{7}$$

where

$$g^*=\frac{\rho_2-\rho_1}{\rho_2}g, \qquad \Delta\bar{u}=\frac{\rho_1\bar{u}_1-\rho_2\bar{u}_2}{\rho_2}.$$

Substituting Eqs. (4) and (7) into (1), (2) and (3), we have

$$\begin{cases} \dfrac{\partial u}{\partial t}+u\dfrac{\partial u}{\partial x}+v\dfrac{\partial u}{\partial y}=-g^*\dfrac{\partial h}{\partial x}+fv, \\ \dfrac{\partial v}{\partial t}+u\dfrac{\partial v}{\partial x}+v\dfrac{\partial v}{\partial y}=-g^*\dfrac{\partial h}{\partial y}+f\bar{u}_2-fu, \\ \dfrac{\partial h}{\partial t}+u\dfrac{\partial h}{\partial x}+v\dfrac{\partial h}{\partial y}+(H_2+h)\left(\dfrac{\partial u}{\partial x}+\dfrac{\partial v}{\partial y}\right)+\dfrac{f}{g^*}\Delta\bar{u}\, v=0. \end{cases}$$

Let $u=\bar{u}_2+u', v=v'$ and substitute them into the above equations. Then we have

$$\begin{cases} \dfrac{\partial u'}{\partial t}+u'\dfrac{\partial u'}{\partial x}+\bar{u}_2\dfrac{\partial u'}{\partial x}+v'\dfrac{\partial u'}{\partial y}=-g^*\dfrac{\partial h}{\partial x}+fv', \\ \dfrac{\partial v'}{\partial t}+u'\dfrac{\partial v'}{\partial x}+\bar{u}_2\dfrac{\partial v'}{\partial x}+v'\dfrac{\partial v'}{\partial y}=-g^*\dfrac{\partial h}{\partial y}-fu', \\ \dfrac{\partial h}{\partial t}+u'\dfrac{\partial h}{\partial x}+\bar{u}_2\dfrac{\partial h}{\partial x}+v'\dfrac{\partial h}{\partial y}=-(H_2+h)\left(\dfrac{\partial u'}{\partial x}+\dfrac{\partial v'}{\partial y}\right)-\dfrac{f}{g^*}\Delta\bar{u}\, v'. \end{cases}$$

Assuming that u', v' and h are independent of y and neglecting label "2" in the equations, we obtain

$$\frac{\partial u'}{\partial t}+u'\frac{\partial u'}{\partial x}+\bar{u}\frac{\partial u'}{\partial x}=-g^*\frac{\partial h}{\partial x}+fv', \tag{8}$$

$$\frac{\partial v'}{\partial t}+u'\frac{\partial v'}{\partial x}+\bar{u}\frac{\partial u'}{\partial x}=-fu', \tag{9}$$

$$\frac{\partial h}{\partial t}+u'\frac{\partial h}{\partial x}+\bar{u}\frac{\partial h}{\partial x}=-(H+h)\frac{\partial u'}{\partial x}-Rv', \tag{10}$$

where $R=\dfrac{f}{g^*}\Delta\bar{u}$. These are the basic equations which we will discuss.

It has been assumed that u', v', and h possess periodic boundary condition, therefore the net flux along x direction is equal to zero in the interval of the wavelength L.

As a preliminary discussion, let us consider the following lower order spectrum truncations:

$$\begin{cases} u'=A_0(t)+A_1(t)\sin\delta x+A_2(t)\cos\delta x, \\ v'=B_0(t)+B_1(t)\sin\delta x+B_2(t)\cos\delta x, \\ h=C_0(t)+C_1(t)\sin\delta x+C_2(t)\cos\delta x, \end{cases} \tag{11}$$

where $\delta=2\pi/L$. Substituting (11) into (8)—(10), and then multiplying them by 1, $\sin\delta x$ and $\cos\delta x$ respectively and intergrating for x from zero to $2\pi/\delta$, we obtain an ordinary differential equation system satisfied by the amplitudes, i. e.

$$\begin{cases} \dot{A}_1=\delta(A_0+\bar{u})A_2+fB_1+\delta g^*C_2, \\ \dot{A}_2=-\delta(A_0+\bar{u})A_1+fB_2-\delta g^*C_1, \\ \dot{B}_1=\delta(A_0+\bar{u})B_2-fA_1, \\ \dot{B}_2=-\delta(A_0+\bar{u})B_1-fA_2, \\ \dot{C}_1=\delta(A_0+\bar{u})C_2-RB_1+\delta(C_0+H)A_2, \\ \dot{C}_2=-\delta(A_0+\bar{u})C_1-RB_2-\delta(C_0+H)A_1, \end{cases} \tag{12}$$

$$\begin{cases} \dot{A}_0 = fB_0, \\ \dot{B}_0 = \frac{1}{2}\delta(A_1B_2 - A_2B_1) - fA_0, \\ \dot{C}_0 = -RB_0. \end{cases}$$

Let $C_0=0$, we have $B_0=0$ and $\dot{A}_0=0$. Thus A_0 is independent of t and represents a "direct current" term. In this case the system of equations expresses a flow in seven-dimension phase space. Using the matrix operator they can be written as

$$\dot{\mathbf{A}} = \delta A_0 \mathbf{A}^* - M(\delta\bar{u})\mathbf{A}, \tag{13}$$

$$A_0 = \frac{1}{2f}\delta(A_1B_2 - A_2B_1), \tag{14}$$

where

$$\mathbf{A} = [A_1, A_2, B_1, B_2, C_1, C_2]^T,$$
$$\mathbf{A}^* = [A_2, -A_1, B_2, -B_1, C_2, -C_1]^T,$$
$$\mathbf{A}\mathbf{A}^* = 0,$$
$$M(\delta\bar{u}) = \begin{pmatrix} 0 & -\delta\bar{u} & -f & 0 & 0 & -\delta g^* \\ \delta\bar{u} & 0 & 0 & -f & \delta g^* & 0 \\ f & 0 & 0 & -\delta\bar{u} & 0 & 0 \\ 0 & f & \delta\bar{u} & 0 & 0 & 0 \\ 0 & -\delta H & R & 0 & 0 & -\delta\bar{u} \\ \delta H & 0 & 0 & R & \delta\bar{u} & 0 \end{pmatrix}.$$

Eq. (12) is the spectrum models that we will discuss. For the convenience of analysis, the modules are defined as

$$\|u'\|^2 = A_0^2 + \frac{1}{2}(A_1^2 + A_2^2),$$

$$\|v'\|^2 = B_0^2 + \frac{1}{2}(B_1^2 + B_2^2),$$

$$\|h\|^2 = C_0^2 + \frac{1}{2}(C_1^2 + C_2^2),$$

which characterize the disturbance kinetic energy of the system.

III. EQUILIBRIUM POINTS OF THE SYSTEM

When $B_0 = C_0 = 0$, letting the right-hand side of (13) equal zero, we may obtain the equations satisfied by the equilibrium point of the system. i. e.

$$\begin{cases} \delta A_0 \mathbf{A}^* - M\mathbf{A} = 0, \\ A_0 = \frac{1}{2f}(A_1B_2 - A_2B_1). \end{cases} \tag{15}$$

Obviously, $A_0 = 0$ and $\mathbf{A} = 0$ are a solution of Eq. (15). In addition, it also has nontrivial solutions. When $A_1 = B_2 = C_1 = 0$, Eq. (15) becomes

$$\begin{cases} \delta(A_0 + \bar{u})A_2 + fB_1 + \delta g^* C_2 = 0, \\ \delta(A_0 + \bar{u})B_1 + fA_2 = 0, \\ \delta(A_0 + \bar{u})C_2 - RB_1 + \delta H A_2 = 0, \\ A_0 = -\frac{1}{2f}\delta A_2 B_1. \end{cases} \tag{16}$$

Let $y=\delta(A_0+\bar{u})$, the condition that Eq. (16) has a nontrivial solution is

$$\begin{vmatrix} y & f & \delta g^* \\ f & y & 0 \\ \delta H & -R & y \end{vmatrix} = y^3-(\delta^2 g^* H+f^2)y-\delta f g^* R=0.$$

Solving it, we obtain

$$y=\begin{cases} X_1+X_2, \\ -\frac{1}{2}(X_1+X_2)+i\frac{\sqrt{3}}{2}(X_1-X_2), \\ -\frac{1}{2}(X_1+X_2)-i\frac{\sqrt{3}}{2}(X_1-X_2), \end{cases} \tag{17}$$

where

$$\begin{cases} X_{1,2}=\left[\frac{\delta f^2}{2}\left(\Delta\bar{u}\pm\sqrt{\Delta\bar{u}^2-\Delta\bar{u}^2_c}\right)\right]^{1/3}, \\ \Delta\bar{u}_c=\frac{2}{\delta f^2}\left(\frac{\delta^2 g^* H+f^2}{3}\right)^{3/2}. \end{cases} \tag{18}$$

Substituting y into (16), it is easy to obtain the following two equilibrium points

$$\begin{cases} \delta A_0=y-\delta\bar{u}, \\ \delta A_2=\pm\sqrt{2y(y-\delta\bar{u})}, \\ \delta B_1=\mp\frac{f}{y}\sqrt{2y(y-\delta\bar{u})}, \\ \delta C_2=\pm\frac{f^2-y^2}{\delta g^* y}\sqrt{2y(y-\delta\bar{u})}. \end{cases} \tag{19}$$

Similarly, we can obtain another two equilibrium points

$$\begin{aligned} & A_2=B_1=C_2=0, \\ & \delta A_0=y-\delta\bar{u}, \\ & \delta A_1=\pm\sqrt{2y(y-\delta\bar{u})}, \\ & \delta B_2=\mp\frac{f}{y}\sqrt{2y(y-\delta\bar{u})}, \\ & \delta C_1=\pm\frac{f^2-y^2}{\delta g^* y}\sqrt{2y(y-\delta\bar{u})}, \end{aligned} \tag{20}$$

It is obvious that the nontrivial solutions intersect with the trivial one, when $y=\delta\bar{u}$ or

$$\Delta\bar{u}=\frac{1}{\delta f^2}[(\delta\bar{u})^3-(\delta^2 g^* H+f^2)\delta\bar{u}].$$

The condition, under which the nontrivial solutions (19) and (20) exist, should be

$$\begin{cases} y-\delta\bar{u}>0 \quad \text{or} \quad y<0, \quad \text{when } \delta\bar{u}>0; \\ y-\delta\bar{u}<0 \quad \text{or} \quad y>0, \quad \text{when } \delta\bar{u}<0; \end{cases}$$

For the parameter $\Delta\bar{u}$, the existential domain of the nontrivial solution is shown in Fig. 2.

According to the definition of the modules, the four nontrivial solutions (19) and (20) have the same module value. The modules plotted against $\Delta\bar{u}$ are given in Fig. 3. Two catastrophe points at $\Delta\bar{u}=\pm\Delta\bar{u}_c$ can be seen. If the state of motion varies along branch I and $\Delta\bar{u}$ is increasing, then at $\Delta\bar{u}=\Delta\bar{u}_c$ the state of motion will suddenly jump from the state shown in point P into the state shown in point M on branch II. Thereafter the state of motion will vary with $\Delta\bar{u}$ along branch II. On the contrary, if $\Delta\bar{u}$ is decreasing, then at $\Delta\bar{u}=-\Delta\bar{u}_c$ the state will jump back to branch I (from point N to point Q). The occurrence of these sudden changes provides a reasonable explanation for the genesis mechanism of the pressure-jump and sudden shift of the wind.

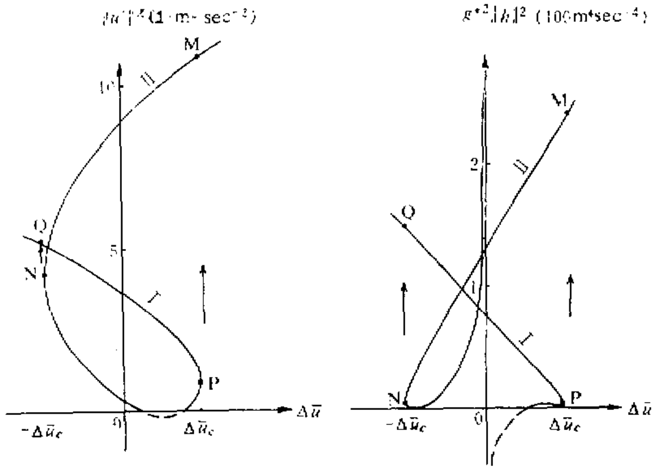

Fig. 3. The curves of module vs. $\Delta\bar{u}$, where $f=10^{-4}$ sec^{-1}, $g^*H=10$ m^2 sec^{-2}, $\delta^2=4\times10^{-9}$ m^{-2} and $\bar{u}=-\frac{1}{2}C_g$.

IV. BIFURCATION AND STABILITY OF THE TRIVIAL SOLUTION

In this paper the discussion about bifurcation and stability is confined to the trivial solution. Therefore, we consider the eigen-equation of (12) at the origin of the phase space. It is

$$\begin{vmatrix} \lambda & -\delta\bar{u} & -f & 0 & 0 & -\delta g^* & & & \\ \delta\bar{u} & \lambda & 0 & -f & \delta g^* & 0 & & & \\ f & 0 & \lambda & -\delta\bar{u} & 0 & 0 & & 0 & \\ 0 & f & \delta\bar{u} & \lambda & 0 & 0 & & & \\ 0 & -\delta H & R & 0 & \lambda & -\delta\bar{u} & & & \\ \delta H & 0 & 0 & R & \delta\bar{u} & \lambda & & & \\ & & & & & & \lambda & -f & 0 \\ & & & 0 & & & f & \lambda & 0 \\ & & & & & & 0 & R & \lambda \end{vmatrix}=0.$$

The left-hand side of this expression is a nine-degree polynomial. The above expression may be reduced to

$$\lambda(f^2+\lambda^2)\{\lambda^2[\lambda^2+(\delta^2 g^* H+f^2)-3\delta^2\bar{u}^2]^2+[3\delta\bar{u}\lambda^2+\delta\bar{u}(\delta^2 g^* H+f^2)-(\delta\bar{u})^3 + \delta f g^* R]^2\}=0.$$

Solving it, we obtain

$$\begin{cases} \lambda_1=0, \\ \lambda_{2,3}=\pm if, \\ \lambda_{4,5}=\pm i(X_1+X_2-\delta\bar{u}), \\ \lambda_{6,7}=-\frac{\sqrt{3}}{2}(X_1-X_2)\pm i\frac{1}{2}(X_1+X_2+2\delta\bar{u}), \\ \lambda_{8,9}=\frac{\sqrt{3}}{2}(X_1-X_2)\pm i\frac{1}{2}(X_1+X_2+2\delta\bar{u}). \end{cases}$$

According to the definitions (18) of X_1 and X_2, the real parts of the last two roots are positive, i. e. Re $(\lambda_{8,9})>0$, when $\Delta\bar{u}^2>\Delta\bar{u}_C^2$. Under this condition the motion will become unstable. When $\Delta\bar{u}^2<\Delta\bar{u}_C^2$, λ will be pure imaginary, and the motion will become boundary stable. So that the criterion for the unstability of the trivial solution is given by

$$\Delta\bar{u}>\Delta\bar{u}_C \qquad \text{or} \qquad \Delta\bar{u}<-\Delta\bar{u}_C.$$

If $C_g=(f^2\Delta\bar{u}_C/2\delta^2)^{1/3}=[(g^*H+f^2/\delta^2)/3]^{1/2}$, the condition will become $|\Delta\bar{u}|> 2\delta^2 C_g^3/f^2$. When $\bar{u}_1=0$, it becomes $|\bar{u}_2|>2\delta^2 C_g^3/f^2$ which agrees with Chao's criterion. Here C_g is the phase velocity of the internal inertio-gravitational wave in the atmosphere. From $\lambda_{8,9}$, we may get a Hopf bifurcation. The bifurcation points and the angle frequencies are

$$\begin{cases} \Delta\bar{u}=\Delta\bar{u}_C, & \omega=\pm\delta(C_g+\bar{u}); \\ \Delta\bar{u}=-\Delta\bar{u}_C, & \omega=\pm\delta(-C_g+\bar{u}). \end{cases} \tag{21}$$

For the angle frequency ω, when the right-hand sides are positive, the corresponding solutions are a forward propagation wave, whereas when they are negative, the solutions are a backward wave.

An example that shows the relation of the bifurcation point $\Delta\bar{u}_C$ with δ and g^*H is shown in Fig. 4. It can be seen that $\Delta\bar{u}_C$ reaches its minimum near $\delta=2\times10^{-5}$ m^{-1}. This indicates that the internal inertio-gravitational waves are more likely excited near meso-wavelength.

Formula (21) gives two bifurcation points. Now we only discuss the periodic solution with angle frequency $\omega=\delta(C_g+\bar{u})$, which is bifurcated at $\Delta\bar{u}=\Delta\bar{u}_C$.

Assuming that Eq. (12) has a solution in the following form

$$\begin{cases} A_0=A_0^{(1)}, & A_1=A^{(1)}\sin(\omega t+\alpha), & A_2=A^{(1)}\cos(\omega t+\alpha); \\ B_0=0, & B_1=B^{(1)}\sin(\omega t+\beta), & B_2=B^{(1)}\cos(\omega t+\beta); \\ C_0=0, & C_1=C^{(1)}\sin(\omega t+\gamma), & C_2=C^{(1)}\cos(\omega t+\gamma). \end{cases} \tag{22}$$

Substituting them into (12) and letting $y=\delta(A_0^{(1)}-C_g)$ we obtain

$$\begin{cases} M(y)\mathbf{A}=0, \\ A_0^{(1)}=\frac{1}{2f}\delta(A_1B_2-A_2B_1). \end{cases} \tag{23}$$

Obviously, if $\mathbf{A}$ and $A_0^{(1)}$ are not all equal to zero, then it is necessary that

$$\det|M(y)|=0.$$

This condition is equivalent to

$$y^3-(\delta^2 g^* H+f^2)y-\delta f g^* R=0.$$

Its roots are given by (17). Given y, we directly get

$$\delta A_0^{(1)}=y+\delta C_g. \tag{24}$$

The curve of $\delta A_0^{(1)}$ vs. $\Delta\bar{u}$ is shown in Fig. 5.

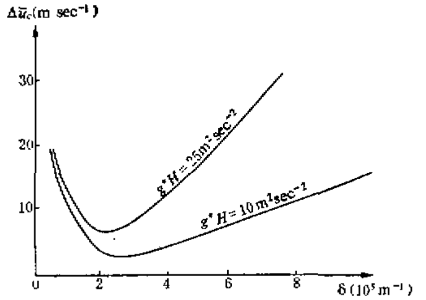

Fig 4. The relation of $\Delta\bar{u}$ with δ and g^*H where $f=10^{-4}$ sec^{-1}.

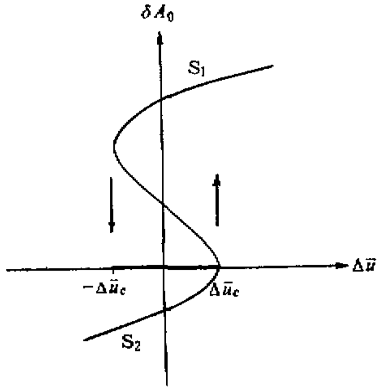

Fig. 5. The curve of $A_0^{(1)}$ vs. $\Delta\bar{u}$, where $f=10^{-4}$ sec^{-1}, $\delta^2=4\times10^{-9}$ m^{-2} and $g^*H=10$ m^2 sec^{-2}

Using (23) and (24), we obtain

$$\begin{cases} \delta^2 A^{(1)2}=\delta^2(A_1^2+A_2^2)=2y(y+\delta C_g), \\ \delta^2 B^{(1)2}=\delta^2(B_1^2+B_2^2)=2\dfrac{f^2}{y}(y+\delta C_g), \\ \delta^2 C^{(1)2}=\delta^2(C_1^2+C_2^2)=2\dfrac{(f^2-y^2)^2}{\delta^2 g^{*2} y}(y+\delta C_g), \end{cases} \tag{25}$$

and

$$\begin{cases} \sin(\alpha-\beta)=1, \\ \cos(\alpha-\gamma)=1. \end{cases} \tag{26}$$

Eqs. (24)—(26) are the variables to be calculated in Eq. (22). In this case only one phase needs to be determined. An example of the bifurcation is shown in Fig. 6.

Figs. 5 and 6 show that (24)—(26) are a subcritical Hopf bifurcation with hysteresis. In the interval $[-\Delta\bar{u}_c, \Delta\bar{u}_c]$ there are two stable states, which are represented by the heavy curves S_1 and S_2. In other words, when the stratified atmosphere described by the model is at rest and the basic wind velocity difference between the upper and lower layers is within the above interval, the rest state is stable for a small disturbance, and may be unstable for a disturbance with finite amplitude. In the latter case the rest state may jump into other states shown by the curve S_1 or S_2. When $\Delta\bar{u}>\Delta\bar{u}_c$, the rest state is unstable. Any trivial disturbances will develop in catastrophe manner, i. e. if the system is disturbed, the state of motion will change from the rest into branch S_1, and thereafter the state of motion will vary along curve S_1. When $\Delta\bar{u}=-\Delta\bar{u}_c$, the state will jump into branch S_2 from S_1.

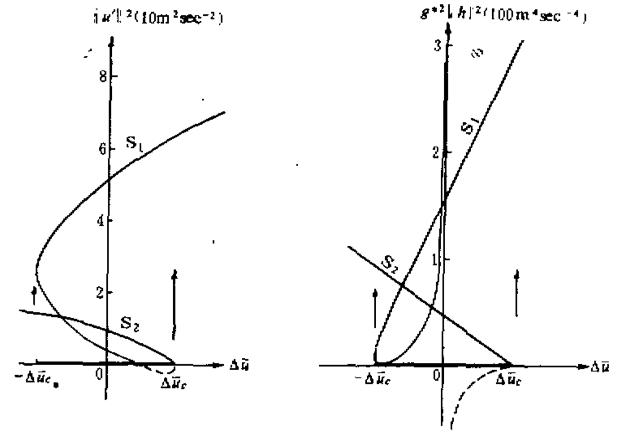

Fig. 6. An example of the bifurcation, where $f=10^{-4}$ sec^{-1}, $\delta^2=4\times10^{-9}$m^{-2} and $g^*H=10$ m^2 sec^{-2}. (Note: the first arrow from the left should be corrected to be upside down like ↓)

It is easy to calculate the mean kinetic energy for a period or a wavelength, i. e.

$$E_1=\frac{\omega}{2\pi}\int_0^{2\pi/\omega}u'^2dt=\frac{\delta}{2\pi}\int_0^{2\pi/\delta}u'^2dx=A_0^{(1)2}+\frac{1}{2}A^{(1)2}=\|u'\|^2,$$

$$E_2=\frac{\omega}{2\pi}\int_0^{2\pi/\omega}v'^2dt=\frac{\delta}{2\pi}\int_0^{2\pi/\delta}v'^2dx=\frac{1}{2}B^{(1)2}=\|v'\|^2,$$

$$E_3=\frac{\omega}{2\pi}\int_0^{2\pi/\omega}\left(\frac{\partial h}{\partial t}\right)^2dt=\frac{\delta}{2\pi}\int_0^{2\pi/\delta}\left(\frac{\partial h}{\partial t}\right)^2dx=\frac{\omega^2}{2}C^{(1)2}=\omega^2\|h\|^2.$$

It is shown that, when the scale parameter δ, the stratified parameter g^*H and the geographical parameter f have been determined, the kinetic energy of fluid motion depends only on the shear of the basic wind, $\Delta\bar{u}$, i. e. the disturbance energy can be obtained from the ambient wind field. An example of this relation is also plotted in Fig. 6.

In additon, it is not difficult to see that when the basic wind velocity in the lower layer is equal to the phase velocity of the internal inertio-gravitational wave but with opposit direction, i. e. $\bar{u}_2=-C_g$, the periodic solutions will degenerate into stationary solutions, and their modules are the same as that of the nontrivial solution described in the last section, i. e. the expression of the kinetic energy of disturbance of the system is all the same.

V. CONCLUSIONS

In summary, we have the following results.

(1) The equilibrium points of system (12) have four nontrivial solutions in addition to the trivial solution. If the difference of the basic wind velocity between the upper and lower layers satisfies the condition $|\Delta\bar{u}|>2\delta^2C_g^2/f^2$, then the states of motion corresponding the nontrivial solution will have a jumping-form catastrophe.

(2) If the above condition occurs, the trivial solution will also loss stability, and will

generate a subcritical Hopf bifurcation with hysteresis. It is shown that under the same condition the rest state will change in catastrophe form as well, and internal inertio-gravitational wave will be excited in the interface of the two layers.

(3) The properties of the subcritical bifurcation also indicate that the rest atmosphere could be unstable for a disturbance with finite amplitude. A jumping-form catastrophe may happen when $|\Delta \bar{u}| < 2\delta^2 C_0^3 / f^2$.

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