

## AN EXACT CALCULATION OF INFRARED COOLING RATE DUE TO WATER VAPOR

Xu Li (许黎) and Shi Guangyu (石广玉)

Institute of Atmospheric Physics, Academia Sinica, Beijing

Received October 15, 1984

### ABSTRACT

The longwave ( $0-2380\text{ cm}^{-1}$ ) cooling rate due to water vapor in the troposphere and the stratosphere has been calculated by a new infrared transmission model in this paper. An exact scheme is used for treating the integration over wavenumber and the inhomogeneous path in the atmosphere. It is shown that the atmospheric window region ( $730-1200\text{ cm}^{-1}$ ) (mainly water vapor continuum) plays an important role in the total cooling near the surface, about 72% of the total cooling lying in this region at the height of 1 km; the CG approximation used for an inhomogeneous path is fairly applicable for calculating the cooling rate due to water vapor, with a maximum error of 0.16 K/day throughout the troposphere and the stratosphere; on the other hand, the error due to the diffusivity factor of 1.66 appears to be slightly larger near the surface. In this study, the influences on the calculation of above infrared cooling rate, of the temperature-dependence of the absorption coefficients of water vapor, the upper level cutoff and the integration step for altitude, and the substitution of the quasi-grey approximation for the exact integration over wavenumber, are also examined.

### 1. INTRODUCTION

Absorption spectra of water vapor spread all over the entire infrared spectral range. Absorption, emission and transmission of thermal infrared radiation by water vapor have a very strong effect on the exchange of radiation energy between atmospheric layers, on the longwave radiation energy arriving at the surface and on the loss of radiation energy from earth-atmosphere system to space. Because of the influence of these radiative processes on the thermodynamic structure of the atmosphere, it is important both for modeling the general circulation and for climate modeling to accurately compute the terms involved, specifically the atmospheric cooling rate.

Using the Goody random model, Rodgers & Walshaw (1966)<sup>[1]</sup>, denoted RW, computed the infrared cooling rate in planetary atmospheres, in which water vapor was included. The accuracy of their method was better than any previous method (for example, radiation charts). The CG approximation was used for the inhomogeneous atmosphere and a diffusivity factor of 1.66 for computing diffuse radiation. In addition, they allowed for the  $\text{H}_2\text{O}$  8-13  $\mu\text{m}$  continuum in a crude manner by taking a constant absorption coefficient of 0.1  $\text{cm}^2/\text{g}$  and neglected the strong self-broadening and temperature effects of the continuum. Since then, similar computations have been carried out by a number of authors, e. g. Ellingson & Gille (1978)<sup>[2]</sup>, Roewe & Liou (1978)<sup>[3]</sup>; Liou & Ou (1981)<sup>[4]</sup>, in which the used method is still limited to the band model except that the number of the divided spectral

intervals and the chosen spectral data are different from one another. Compared to line-by-line calculation, however, Chou & Arking (1980)<sup>[5]</sup> have found that the commonly used method based on the Goody band model (RW (1966)<sup>[1]</sup>) introduces errors up to 11%. It should be pointed out that the altitude involved in their study is limited to below 30 km, i. e. to a region in which the Lorentz line shape is appropriate to the pressure broadening.

Shi (1981)<sup>[6]</sup> has developed a scheme to accurately calculate and represent the infrared transmission function of the atmospheric gaseous constituents. Basic idea of the method is that the processes of radiation are independent of each other in wavenumber space in the radiative transfer problems under consideration. Thus we can rearrange the absorption coefficients according to their numerical values in the wavenumber space and transform from the integration over wavenumber into that over absorption coefficient. The approach to the inhomogeneous atmosphere is similar to the correlated  $k$ -distribution method. As using this approach for calculating the atmospheric infrared cooling rate the accuracy is favourably comparable with that of the exact line-by-line integration<sup>[7]</sup>. Because the Voigt profile for a combined Lorentz and Doppler broadening is appropriately involved in this transmission model, it can be used to upper stratosphere. The purpose of the present investigation is to use the new transmission scheme for computing the infrared cooling rate due to water vapor and to examine the errors introduced by several approximations used in band models in more detail.

## II. CALCULATION OF COOLING RATE DUE TO WATER VAPOR

### 1. Basic Equations

With the usual assumptions used for calculating cooling rate<sup>[2]</sup>, the equations for the spectral radiance  $I_\nu(z; \mu)$  for wavenumber  $\nu$  at altitude  $z$  and direction  $\mu$  may be written as

$$I_\nu(z; \mu) = B_\nu(z) - \int_0^z T_\nu(z, z'; \mu) \frac{dB_\nu(z')}{dz'} dz' + [B_\nu(G) - B_\nu(0)] T_\nu(z, 0; \mu) \quad (0 \leq \mu < 1), \quad (1a)$$

$$I_\nu(z; \mu) = B_\nu(z) + \int_z^Z T_\nu(z, z'; \mu) \frac{dB_\nu(z')}{dz'} dz' - B_\nu(Z) T_\nu(z, Z; \mu) \quad (-1 \leq \mu \leq 0), \quad (1b)$$

where  $B_\nu$  is the Planck function,  $B_\nu(G)$  the effective emission of the surface,  $\mu$  the cosine of the local zenith angle,  $Z$  the effective top of the atmosphere under consideration, and  $T_\nu(z, z'; \mu)$  the intensity transmission defined as

$$T_\nu(z, z'; \mu) = \exp[-\tau_\nu(z, z')/\mu], \quad (2)$$

where  $\tau_\nu(z, z')$  is the optical thickness computed from

$$\tau_\nu(z, z') = \int_z^{z'} k_\nu(z'') \rho(z'') dz''. \quad (3)$$

Here  $k_\nu(z')$  and  $\rho(z')$  are the absorption coefficients of water vapor and water vapor density, respectively. If we define the flux transmission  $T_\nu(z, z')$  as

$$T_\nu(z, z') = 2 \int_0^1 T_\nu(z, z'; \mu) \mu d\mu, \quad (4)$$

from the exponential integral

$$E_n(\tau) = \int_1^\infty \exp(-\tau t) t^{-n} dt, \quad (5)$$

we have

$$T_v(z, z') = 2E_3[\tau_v(z, z')]. \quad (6)$$

In our "exact" calculation (see the next)  $E_3(\tau)$  is evaluated by the series expansion of  $E_1(\tau)$  and the recurrence formula for  $E_n(\tau)$  (if  $\tau \leq 1.1$ ) and by the Gaussian quadrature (if  $\tau > 1.1$ ).

Thus, the equations for the upward and the downward fluxes,  $F_v^\uparrow(z)$  and  $F_v^\downarrow(z)$ , may be written as

$$F_v^\uparrow(z) = \pi B_v(z) - 2\pi \int_0^z E_3[\tau_v(z, z')] \frac{dB_v(z')}{dz'} dz' + 2\pi [B_v(G) - B_v(0)] E_3[\tau_v(z, 0)] \quad (7a)$$

and

$$F_v^\downarrow(z) = \pi B_v(z) + 2\pi \int_z^Z E_3[\tau_v(z, z')] \frac{dB_v(z')}{dz'} dz' - 2\pi B_v(Z) E_3[\tau_v(z, Z)], \quad (7b)$$

and the net flux,  $F_v(z)$ , at altitude  $z$  is given by

$$F_v(z) = F_v^\uparrow(z) - F_v^\downarrow(z). \quad (8)$$

Finally, we can write the cooling rate  $C_{rv}(z)$  as

$$C_{rv}(z) = -\frac{1}{C_p \rho_a(z)} \cdot \frac{\partial F_v(z)}{\partial z}, \quad (9)$$

where  $C_p$  is the specific heat at constant pressure and  $\rho_a(z)$  the air density at altitude  $z$ .

## 2. Wavenumber Integration

To obtain the cooling rate over a spectral region, we have to integrate the monochromatic flux expressed by Eqs. (7) or (8) over wavenumber  $\nu$ . Based on the special variation of the Planck function with wavenumber, the infrared region ranged from 0 to 2380  $\text{cm}^{-1}$  is divided into 21 spectral intervals. Of them, 13 intervals are included in the rotational band of water vapor and 8 the 6.3  $\mu\text{m}$  vibration-rotation band. In addition, we divide the water vapor continuum in the 8–14  $\mu\text{m}$  atmospheric window into 4 intervals which overlap with 2 intervals of the rotational band and 2 of the 6.3  $\mu\text{m}$  band, as shown in Table 1 in detail. From Ref. [6], the mean transmission function,  $\bar{T}_i(P, T, W)$ , for  $i$ th spectral interval with a width of  $\Delta\nu_i$  can be fit by a sum of  $N$  exponential terms, i. e.

Table 1 The Divided Spectral Intervals (unit:  $\text{cm}^{-1}$ )

Rotational Band	0–40, 40–80, 80–120, 120–160, 160–220, 220–280, 280–350, 350–430, 430–530, 530–610, 610–730, 730–810, 810–940
Continuum	730–810, 810–940, 940–1110, 1110–1200
6.3 $\mu\text{m}$ Band	940–1110, 1110–1200, 1200–1350, 1350–1430, 1430–1590, 1590–1810, 1810–2110, 2110–2380

$$\begin{aligned} \bar{T}_i(P, T, W) &= \frac{1}{\Delta\nu_i} \int_{\Delta\nu_i} \exp[-k_\nu(P, T)W] d\nu \\ &= \sum_{n=1}^N P_n \exp[-k_n(P, T)W], \end{aligned} \quad (10)$$

where  $W$  is the amount of water vapor,  $P$  and  $T$  the pressure and the temperature, respectively, and the  $P_n$  a set of weights to be associated with the  $k_n$  which may be thought of as a set of equivalent absorption coefficients. Thus, we may reduce the transfer of radiation over the interval  $\Delta\nu_i$  to  $N$  pseudo-monochromatic calculations for each value of  $k_n$  and the net flux,  $F_i(z)$  integrated over the interval may be written as<sup>[6,7]</sup>

$$F_i(z) = \sum_{n=1}^N P_n F_n(z) = \sum_{n=1}^N P_n [F_n^{\uparrow}(z) - F_n^{\downarrow}(z)], \quad (11)$$

with  $N=9$  in this study.

### 3. Inhomogeneous Atmosphere and Curtis-Godson (CG) Approximation

To accurately deal with the inhomogeneous path of the atmosphere, firstly, the separability between temperature-dependence and pressure-dependence of the absorption coefficients were shown in Ref. [6] by the author; then an exact line-by-line technique was used for calculating the absorption coefficients at five pressures of 1000, 333.3, 50, 5, and 0.1 hPa and at three temperatures of 200, 296, and 325 K. After that, above absorption coefficients at the five pressures have been simultaneously rearranged, using the absorption coefficients at 50 hPa as a reference, and a polynomial regression analysis has been used for fitting the equivalent absorption coefficients at the five pressures. Finally, the formula for computing the equivalent absorption coefficient  $k_n(P, T)$  at any pressure and temperature in the atmosphere can be written as

$$k_n(P, T) = k_n(P_r, T_r) \exp(A_n x + B_n x^2 + C_n x^3 + D_n x^4 + E_n x^5 + F_n x^6) \left(\frac{T}{T_r}\right)^{AA + BB \cdot T} \quad (12)$$

with

$$x = \ln(P/P_r), \quad (13)$$

where  $P_r$  and  $T_r$  are the pressure and temperature used for a reference in the polynomial regression fitting and do not mean anything physically. They are taken as 0.1 hPa and 296 K respectively in this study. If some other pressure or temperature is taken as reference, the coefficients representing the pressure and the temperature-dependence of  $k_n(P, T)$ ,  $A_n, \dots, F_n, AA$  and  $BB$ , should be taken as other numerical values, which are cited from Ref. [6] for the present work.

Obviously, by using Eqs. (12), (10) and (11) we can exactly calculate the vertical inhomogeneity of the atmosphere, without other approximations. Commonly, however, a one-parameter scaling approximation or CG approximation, the most useful two-parameter method, is used for approaching the problem in the band model techniques. According to RW (1966)<sup>[1]</sup>, the CG approximation can be stated as follows: The transmission of an inhomogeneous atmospheric path is approximately equal to that of a constant pressure (effective) path with the absorber amount  $\int dm$  and half-width  $\bar{\alpha} = \int \alpha dm / \int dm$ . Because the half-width of a spectral line is well-known to be directly proportional to the pressure in the region where the Lorentz line shape is valid, the effective pressure  $\bar{P}$  of

the constant pressure path must be

$$\bar{P} = \int P dm / \int dm = \int P \rho dz / \int \rho dz. \quad (14)$$

Substituting the pressure level determined by Eq. (14) and absorber amount found from  $\int dm$  into Eqs. (13), (12), (10) and (11) is used for approaching the pressure effect of the CG approximation in our comparison calculations. Houghton (1963)<sup>[6]</sup> have used a similar approach to a non-constant pressure path in the calculation of the absorption of solar infrared radiation by the lower stratosphere. Although we have used the Lorentz profile as a basis for our derivation of Eq. (14), Goody (1964)<sup>[9]</sup> has shown that restrictions on the use of the CG approximation are few and that it may be used for other profiles.

#### 4. Integration over Angle

Because the exponential integral,  $2E_1(\tau)$ , behaves somewhat similarly to  $\exp(-r\tau)$ , where  $r$  is a numerical factor, many workers have therefore attempted to write

$$T_r(z, z') = \exp[-r\tau_v(z, z')], \quad (15)$$

substituting for the integration over zenith angle as shown in Eq. (4).  $r$  is known as the diffusivity factor and its optimum value is 1.66.

#### 5. Continuum Absorption by Water Vapor in the 8–14 $\mu\text{m}$ Window

Down to date it is not clear that the continuum absorption in the 8–14  $\mu\text{m}$  window may be due to the extreme wings of lines and/or due to dimers, such as  $(\text{H}_2\text{O})_2$ , and/or due to pressure-induced bands, but several theoretical studies<sup>[3,4]</sup> have indicated that the continuum absorption by water vapor in the 8–14  $\mu\text{m}$  window has important influences on calculations of longwave fluxes and cooling rates, especially at the surface.

Roberts et al. (1976)<sup>[10]</sup> expressed the extinction coefficient  $\sigma_c$  due to water vapor as

$$\sigma_c = C^0(\nu, T) \cdot W \cdot [P + r(P - p)], \quad (16)$$

where  $W$  is the amount of water vapor,  $P$  the total atmospheric pressure in atmospheres,  $p$  the partial pressure of water vapor,  $r$  is a relative measure of the ambient to self-broadened water vapor continuum term, with a value between 0–0.005, and  $C^0(\nu, T)$  is the self-broadening absorption coefficient by  $(\text{H}_2\text{O})_2$ . From the best fitting to the experimental data of Burch et al. (1974)<sup>[11]</sup>, Roberts et al. (1976)<sup>[10]</sup> found that at  $T=296$  K,

$$C^0(\nu, 296) = a + b \cdot \exp(-\beta\nu), \quad (17)$$

where  $a$ ,  $b$  and  $\beta$  are the three fit parameters with  $a=1.25 \times 10^{-22} \text{ mol}^{-1} \text{ cm}^2 \text{ atm}^{-1}$ ,  $b=1.67 \times 10^{-19} \text{ mol}^{-1} \text{ cm}^2 \text{ atm}^{-1}$ ,  $\beta=7.87 \times 10^{-3} \text{ cm}$ , for the 8–30  $\mu\text{m}$  region. The temperature dependence of  $C^0(\nu, T)$  may be written as

$$C^0(\nu, T) = C^0(\nu, 296) \exp[T_0(1/T - 1/296)], \quad (18)$$

based upon dimer contributions, here  $T_0$  is a temperature parameter with a value of 1800 K<sup>[10]</sup>.

### III. RESULTS AND DISCUSSIONS

#### 1. "Exact" Result

On the basis of the basic equations outlined in section II, the infrared cooling rate due to water vapor has been exactly computed for a clear tropical atmosphere (McClatchey et al. (1972))<sup>[12]</sup> from the surface to the altitude of 60 km, as shown in Fig. 1. The word "exact" means that the integration with respect to wavenumber is treated by our infrared transmission model obtained from the exponential sum fitting of transmission function; that such

as CG approximation or scaling approximation is abandoned in approaching the inhomogeneities of atmosphere; and that the diffuse radiation is computed by an exact exponential integration in the stead of the diffusivity factor approximation of 1.66. It has been shown that the accuracy of this method is comparable with that of an exact line-by-line integration technique<sup>[7]</sup>. The temperature and water vapor profiles used in our calculations are also shown in Fig. 1 for the tropical atmosphere.

As can be seen from the figure, longwave cooling due to water vapor destabilizes the lower regions of the atmosphere, thereby causing supercritical lapse rates. Because longwave cooling by gases other than  $H_2O$  is insignificant in whole troposphere, the water vapor opacity is the most important cause for the existence of the troposphere. The infrared cooling rate by water vapor is up to 3–4 K/day in regions of 1–2 km from the surface. The

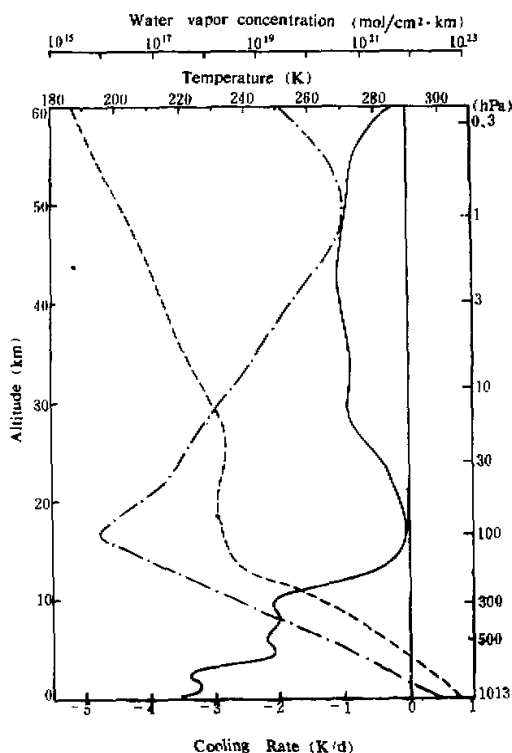


Fig. 1. Longwave cooling rate due to water vapor for the tropical atmosphere. The profiles of temperature (—) and water vapor density (---) are also shown here. The right ordinate is for pressure.

continuum absorption by  $(H_2O)_2$  in the atmospheric window is mainly responsible for it (see the next).

We also see from Fig. 1 that although the longwave cooling rates due to water vapor are less than that due to  $CO_2$  and  $O_3$  at some levels of middle-upper stratosphere, they still are of the order of 1 K/day. Therefore if we expect to exactly investigate the radiative

heat exchange in the middle atmosphere, we have to take account of the contribution from water vapor.

## 2. Errors Introduced from the CG, Diffusivity Factor and Quasi-Grey Approximations

As mentioned before, generally, the scaling or CG approximation, the diffusivity factor of 1.66, and band model which is a quasi-grey approximation essentially, are used for treating the inhomogeneous path of the atmosphere, the diffuse radiations and wavenumber integrations, respectively, in the calculations of longwave cooling rates. Those shown in Fig. 2 are the errors from above approximations in computing the infrared cooling rate due to water vapor. Note that here so-called quasi-grey approximation refers to that instead of using Eq. (11) for computing fluxes and cooling rates, in each spectral interval, we first calculate the mean transmission function over the interval at every level by means of Eq. (10); after that, cooling rates are computed in a regular manner. This approach makes it possible to separate the effect of quasi-grey approximation from that of the CG and diffusivity factor approximations.

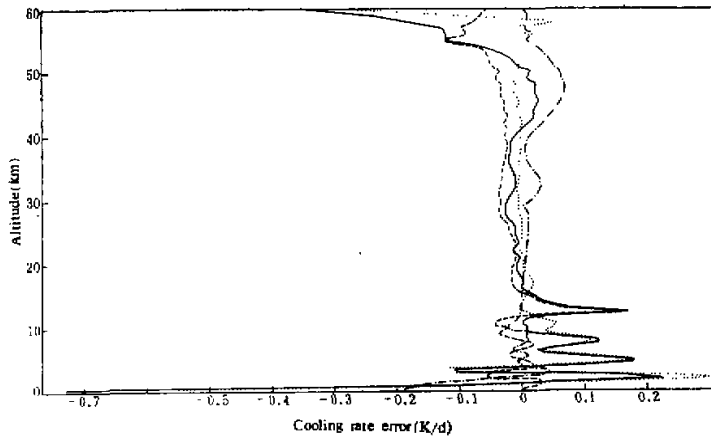


Fig. 2. Cooling rate errors from various approximations.  
 ---- CG; - · - 1.66 ..... Gray; — CG+1.66+Gray.

We can see that the CG approximation used for an inhomogeneous path is fairly applicable for calculating the longwave cooling rate due to water vapor, with a maximum error less than 0.16 K/day. This is in agreement with the results of other authors. The errors introduced by the diffusivity factor of 1.66 are slightly larger in the lower layers of the atmosphere. It should be noted that the error by the quasi-grey approximation appears to be significant in the upper and lower layers of the atmosphere. In fact, the use of a mean transmission (or average absorption coefficient) through a spectral interval in the stead of the exponential transmission in Eq. (10) for an exponential sum fitting of the transmission function (ESFT) underestimates the absorption by line center, while it overestimates that by wings of lines. Because of the small density of absorber in the upper layers of the atmosphere, the wings of line are almost transparent and the transmission depends essentially upon the absorption by the center of line. On the other hand, as a result of the larger density of absorber in the lower layers of the atmosphere, the center of line becomes

quite "black" for a long path and the wings dominate the spectrum. Consequently, the use of the mean transmission (i. e. quasi-grey approximation) may be responsible for the greater errors of cooling rates in the upper and lower layers of the atmosphere.

The cooling rate error by simultaneously using the CG, diffusivity factor and quasi-grey approximations (e. g. in usual band models) is also shown in Fig. 2 by the solid line. Because the above three errors may be partially constructive or destructive, the resultant error is not their simple sum. It should be noted that the error is up to 20% (0.7 K/day) at the level of 1 km from the surface. Obviously, the more exact transmission model is necessary to investigating the physical processes of radiations involved in the layers nearby the surface.

### 3. Influences of Cut-off Altitude and Vertical Step

As can be seen from Eq. (7b), the cut-off level,  $Z$ , used in the numerical calculations affects downward fluxes and then cooling rates. Theoretically, it is needless to say, we ought to take  $Z$  as infinite. In fact, however, neither possible nor necessary is this, it is meaning to examine the influence of cut-off level  $Z$  on calculations of infrared cooling rates. The problem is examined by our exact transmission model, as shown in Fig. 3 (a). The curves labeled by I, II, III, IV show the cooling rates with  $Z=15, 28, 40$  and  $60$  km, respectively. It can be seen from the figure that the cooling rates over a region of 5–10 km below  $Z$  are affected, more or less, by the value of  $Z$ . The contribution from the layers above  $Z$  was already included in these comparison calculations. An unreasonable result may be led out without taking this contribution into account (the curve V).

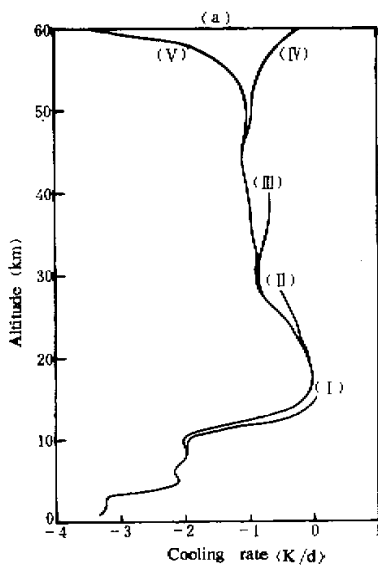


Fig. 3(a). Effect of cut-off altitude  $Z$  on cooling rate (see the text).

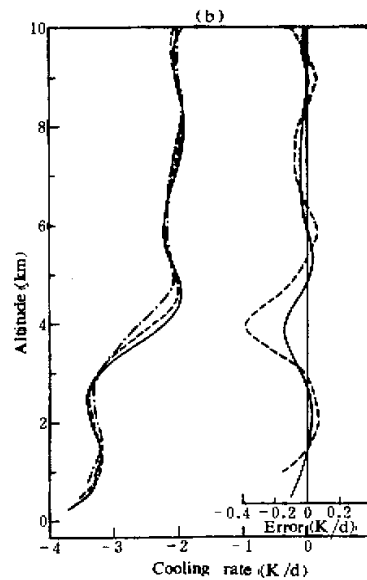


Fig. 3(b). Effect of integration step in altitude  $dz$  on cooling rate (see the text).  
 —  $dz=0.25$  km; - - -  $dz=0.5$  km; - · -  $dz=1$  km.



That shown in Fig. 3 (b) is the influence of altitude integration step,  $dz$ , on cooling rates. Because the amount of water vapor in the atmosphere decreases rapidly with altitude, we have to take a small value of  $dz$  in the lower layers of the atmosphere and a larger value of  $dz$  in the upper layers, in order to achieve a satisfactory compromise between accuracy and computing time. Fig. 3 (b) shows the cooling rate errors for  $dz=0.5$  and 1 km, by taking that of  $dz=0.25$  km as a reference. As in the figure, if  $dz=0.5$  km, the maximum error is 0.14 K/day at a height of 4 km. However, the errors decrease with increasing altitude, hence the results above 10 km are omitted in the figure.

#### 4. Influences of Temperature Dependence of Absorption Coefficient on Cooling Rate

It is difficult and complex to exactly include the temperature dependence of absorption coefficients into calculations of cooling rates. This is not only owing to the fact that both line half-width and intensity depend upon temperature, but that the temperature dependence of line intensities is not the same for all lines. Therefore, some approximations are used as an approach to the problem. The transmission model developed in Ref. [6] makes it possible to exactly investigate influences of temperature dependence of absorption coefficients on cooling rate. The cooling rates due to water vapor with (solid line) and without (dashed line) taking account of temperature dependences of absorption coefficients are shown in the left half of Fig. 4 and the right half of the figure is the error of cooling rate introduced by neglecting the temperature dependences, where (a) is for the line absorption +

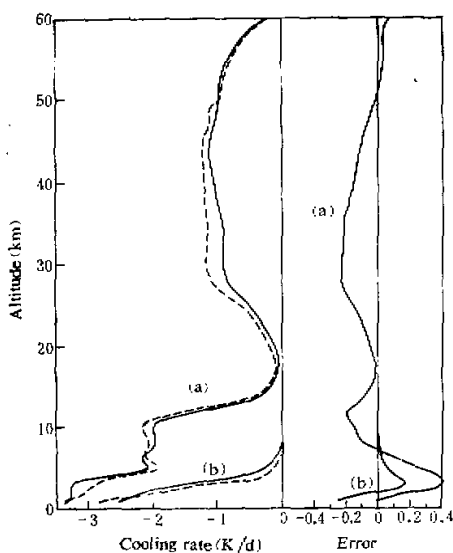


Fig. 4. Effect of temperature dependence of water vapor absorption coefficient on longwave cooling rate. (a) line absorption-continuum (b) continuum only (see the text).

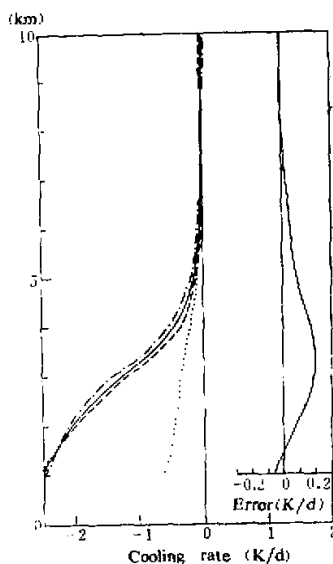


Fig. 5. Effect of the value of  $r$  in Eq. (16) on cooling rate due to continuum. --- for  $r=0.001$ ; — for  $r=0.002$ ; - · - for  $r=0.005$  and ..... for line absorption only. Cooling rate due to line absorption in 8–14  $\mu\text{m}$  window is also shown in the figure (see the text).

continuum and (b) is for continuum only. It can be seen from the curves (a) in the figure that the neglect of temperature dependence of absorption coefficients underestimates the cooling rates below a height of about 7 km, with the maximum error of 0.4 K/day at the altitude of 3 km. On the other hand, the above neglect would overestimate the cooling rates in the region from 7 to 50 km, with the maximum error of 0.2 K/day. As for the case of continuum absorption only, the cooling rate error is about  $\pm 0.2$  K/day. These results show that it is necessary to include simultaneously the temperature dependences of line absorption and continuum in calculations, if we want to obtain an exact cooling rate due to water vapor.

Table 2 Longwave Cooling Rates (K/day) due to Water Vapor in 730–1200  $\text{cm}^{-1}$  Window

Height (km)	Continuum			Line Absorption	Continuum+Line Absorption	
	$r = 0.001$	$r = 0.002$	$r = 0.005$		"Exact" Algorithm	Simple Sum
1	-2.5253	-2.5122	-2.4649	-0.6159	-2.4196	-3.1281
2	-2.0028	-2.0310	-2.0993	-0.4637	-2.0876	-2.4947
3	-1.1960	-1.2520	-1.4040	-0.3753	-1.4637	-1.6273
4	-0.3790	-0.4234	-0.5484	-0.2290	-0.6166	-0.6494
5	-0.1190	-0.1414	-0.2070	-0.1208	-0.2585	-0.2622
6	-0.0469	-0.0593	-0.0968	-0.0760	-0.1373	-0.1353
7	-0.0128	-0.0179	-0.0340	-0.0450	-0.0656	-0.0629
8	-0.0015	-0.0025	-0.0066	-0.0256	-0.0300	-0.0281
9	0.0012	0.0018	0.0028	-0.0140	-0.0132	-0.0121
10	0.0014	0.0023	0.0046	-0.0060	-0.0042	-0.0036

##### 5. Longwave Cooling Rate due to Water Vapor in the 8–14 $\mu\text{m}$ Window

In the 8–14  $\mu\text{m}$  window exist both line absorption and continuum absorption by water vapor. Experiments indicate the line absorption is much less than the continuum. In addition, the variation of continuum absorption coefficient with wavenumber is much slower than that of line absorption coefficients. Thus the overlapping of line absorptions and continuum can be treated by the transmission multiplication law, which is named as "exact algorithm". Cooling rates for several cases are shown in Table 2 and Fig. 5. We can see from the table and figure that (1) the longwave cooling rate due to line absorption is one-fourth of that due to the continuum in the near-surface layers below a height of 3 km. It is of interest to note that the cooling rates with neglecting the line absorptions and taking  $r$  in Eq. (16) as 0.005 is same as that from the exact method with taking both line absorption and continuum into account, with errors less than 0.07 K/day throughout the troposphere; (2) the longwave cooling due to water vapor over 730–1200  $\text{cm}^{-1}$  window region has a considerable contribution to total cooling, being 72%, 64%, and 45% at the heights of 1, 2, and 3 km, respectively; and (3)  $r$  in Eq. (16), as stated above, which is a relative measure of the ambient to self-broadened water vapor continuum term, is taken as 0.005 in LOWTRAN 3 by Selby and McClatchey (1975)<sup>[13]</sup>. However, measurements by Long's group, see Roberts et al. (1976)<sup>[10]</sup>, suggest that this value is a substantial overestimate and that a more

reasonable estimate is between 0 and 0.002 and recently LOWTRAN 5 routine sets  $r=0.002$  (Kneizys et al. (1980)<sup>(14)</sup>). To examine the influence of the value of  $r$  on cooling rate due to continuum, numerical experiments are carried out by taking  $r=0.001$ , 0.002, and 0.005, respectively, in this study, as shown in Table 2 and Fig. 5. The maximum difference of cooling rate between  $r=0.001$  and  $r=0.005$  is about 0.2 K/day at a height of 3 km. For lack of available data on  $r$  and the fact that the result with  $r=0.002$  is moderate,  $r$  is taken as 0.002 in our calculations.

## VI. CONCLUSIONS

The longwave cooling rate due to water vapor has been calculated by use of an accurate infrared transmission model, the accuracy of which is comparable to that of line-by-line method, but it is fairly economical of computing time-consuming. Based on the model, we have examined the errors due to various approximations used usually in the calculation of longwave cooling rate. Our main conclusions are as follows. (1) The CG approximation and the diffusivity factor of 1.66 introduce quite a small error into cooling rates due to water vapor. (2) The quasi-gray approximation which is similar to band models may lead to considerable errors at the lower layers of the atmosphere, especially at the near-surface layers. For example, the errors from the approximation is 0.55 K/day at the height of 0.5 km, accounting for about 16% of the total cooling rate. (3) The longwave radiation from the 8–14  $\mu\text{m}$  window (mainly water vapor continuum) has an important contribution to the total longwave cooling at lower layers of the atmosphere. It is necessary to accurately treat the continuum in order to investigate the radiative energy budget in lower layers of the atmosphere. (4) The temperature dependences of absorption coefficients including both line absorption and continuum have a significant influence on longwave cooling due to water vapor. If the temperature dependence of absorption is ignored, that may lead the errors in cooling rate rising to 12% at the altitude of 3 km and up to 20% at the better part of the stratosphere. So the profiles of longwave fluxes and then the cooling rate are affected remarkably.

## REFERENCES

- [ 1 ] Rodgers, C. D. & Walshaw, C. D., *Quart. J. Roy. Met. Soc.*; **92** (1966), 67–92.
- [ 2 ] Ellingson, R. G. & Gille, J. C., *J. Atmos. Sci.*, **35** (1978), 523–545.
- [ 3 ] Roewe, D. & Liou, K. N., *J. Appl. Met.*, **17** (1978), 92–106.
- [ 4 ] Liou, K. N. & Ou, S. C., *J. Atmos. Sci.*, **38** (1981), 2707–2716.
- [ 5 ] Chou, M. D. & Arking, A., *ibid.*, **37** (1980), 855–867.
- [ 6 ] Shi, G. Y., *An Accurate Calculation and Representation of the Infrared Transmission Function of the Atmospheric Constituents*, D. thesis, Dept. of Science, Tohoku University of Japan, 1981, pp. 191.
- [ 7 ] —, *Scientia Sinica, Series B.* **27** (1984), 947–957.
- [ 8 ] Houghton, J. T., *Quart. J. Roy. Met. Soc.*, **89** (1963), 319–331.
- [ 9 ] Goody, R. M., *Atmospheric Radiation. I. Theoretical Basis*, Oxford University Press, 1964, 436pp.
- [ 10 ] Roberts, R. E. et al., *Appl. Opt.*, **15** (1976), 2085–2090.
- [ 11 ] Burch, D. E. et al., AFCRL-TR-74-0377.
- [ 12 ] McClatchey, R. A. et al., *Optical Properties of the Atmosphere*, 3rd ed., AFCRL-72-0497, 1972.
- [ 13 ] Selby, J. & McClatchey, R., *Atmospheric Transmittance from 0.25 to 28.5 micron: Computer Code LOWTRAN 3*, Report No. AFCRL-TR-75-0255, Air Force Geophysics Lab., Hanscom AFB, MA, 1975.
- [ 14 ] Kneizys, F., et al., *Atmospheric Transmittance Radiance: Computer Code LOWTRAN 5*, Report No. AFGL-TR-80-0067, Air Force Geophysics Lab, Hanscom AFB, MA, 1980.