EVOLUTION OF LARGE SCALE DISTURBANCES AND THEIR INTERACTION WITH MEAN FLOW IN A ROTATING BAROTROPIC ATMOSPHERE——PART I

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ABSTRACT

The problems on evolution of large-scale disturbances and their interaction with mean flow recently attract much effort of meteorologists due to their practical importance in weather and climate predictions. In this paper, some theoretical results obtained in current investigations of these problems will be reviewed. A barotropic atmosphere is taken in this paper, and the baroclinic atmosphere is left in our second paper.

The following aspects are reviewed: First, the general properties of two-dimensional barotropic motion both in the nonlinear and linearized equations and both in the quasi-geostrophic and non-geostrophic models. Second, the evolution and the structure of Rossby wave packet superimposed on a zonal or non-zonal basic flow.

In this part, only the above two problems are reviewed. The remanent problems, i.e., the normal modes and continuous spectra of both quasi-geostrophic and non-geostrophic models, weakly nonlinear theory and the fully nonlinear theory will be discussed in part II (another paper).

I. INTRODUCTION

In the atmosphere of a rotating planet there is always a jet-like zonal flow around the axis of the planet. There are also pronounced nonzonal disturbances superimposed on this zonal flow in the atmosphere of some planets such as our (earth's) atmosphere. There are some very intensive large-scale vortexes observed in other atmospheres such as Jovian atmosphere, although its wave-like disturbances are very weak. In all cases the generation and maintenance of the zonal flow, the evolution (propagation and changes in the intensity, structure and shape) of disturbances, waves and vortexes, and their interactions with the zonal flow are basic theoretical problems in the dynamics.

Undoubtedly, a permanent zonal flow in our atmosphere is generated by the energy imput of solar radiation and controlled by the earth's rotation and other factors. This zonal flow is disturbed by the non-homogeneous characters of the earth's surface such as orography and the thermal difference between the continent and ocean, and results in some quasi-steady nonzonal disturbances. This zonal flow, or plus forced quasi-steady nonzonal disturbances, is the basic flow, on which there are superimposed ceaseless transient disturbances. Atmospheric statistics shows that, in general, large-scale transient disturbances are amplifiedand maintained by the baroclinic conversion from zonal to eddy available potential energy and the internal heating, while they are damped by the barotropic conversion from eddy to zonal kinetic energy. The last mechanism, namely, fulfils the requirement of angular momentum transfer to maintain the zonal circulation (see, for example, Starr[1] (1948), Yeh and Zhu[2] (1958), and Lorenz[3] (1967). However, a daily individual situation can greatly differ from the statistical mean. A disturbance can be maintained for a very long time, or grow and decay rapidly, depending on its structure and the characteristics of zonal flow; and the energy cascade can be very different, there exists continuous feedback of eddy energy to the zonal in some cases, but the transfer of energy from eddy to zonal and the reversed alternatively go on in other cases. The statistical properties of general atmospheric circulation result from two factors, one is that the conditions for transformation of eddy energy into zonal occur more often, and the other is that a disturbance superimposed on a jet-like zonal flow is more easily changed into a decaying one. Therefore to study the conditions and mechanism of growing, decaying and maintenance of disturbances and their transport properties are the basic problems. They are important for weather prediction as well as for the understanding of the statistical nature of general atmospheric circulation.

Recently, long-range weather prediction (or short-term climate prediction) attracts more attention of meteorologists. Blocking and the other anomalistic phenomena in the atmospheric circulation are some intensive disturbances, having very long time scale or even being quasi-steady. They interact with basic zonal flow, normal or abnormal, and the transient nonzonal disturbances with relatively high frequency. These anomalistic flows can be considered as some disturbances with respect to the time mean zonal flow on one hand, and must be considered as a basic but nonzonal flow with respect to the transient disturbances on the other hand. Therefore, the topics for investigations have been extended. At present there are some developments in the investigations, both observational and theoretical, on such extended problems.

Besides, the discovery of QBO phenomenon in the equatorial stratosphere and its relation to the momentum transport of gravity waves indicates that the interaction of gravity waves and basic flow might be important in the change of general circulation. Much effort from dynamical meteorologists has been devoted to this problem in the past ten to twenty years, and promoted the study of interaction problems to a large extent. Currently, the importance of gravity waves in the meso-scale dynamics has also been pointed out. Maybe, the interaction of tidal waves with basic flow is also interesting.

We try to give a review of theoretical results obtained in some aspects of these extended evolutionary and interaction problems, in which Chinese meteorologists have been interested, making the emphasis on the Chinese works. In this paper we will deal with the barotropic atmosphere, leaving the baroclinic atmosphere to the second paper.

II. GENERAL PROPERTIES OF TWO-DIMENSIONAL BAROTROPIC MOTION

1. *Integral Properties of Barotropic Quasi-Geostropic Motion on a Sphere*

We first take the barotropic quasi-geostrophic model on a sphere as follows

$$\frac{d}{dt}q \equiv \left(\frac{\partial}{\partial t} + v_\lambda \frac{\partial}{a\sin\theta\partial\lambda} + v_\theta \frac{\partial}{a\partial\theta}\right)q = 0, \tag{1}$$

where q is the absolute potential vorticity

$$q = \left\{\frac{1}{a^2\sin\theta}\left(\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} + \frac{\partial^2}{\sin\theta\partial\lambda^2}\right) - K\frac{f_0^2}{\bar{\phi}}\right\}\psi + 2\omega\cos\theta, \tag{2}$$

$$v_\lambda=\frac{\partial\psi}{a\partial\theta},\quad v_\theta=-\frac{\partial\psi}{a\sin\theta\partial\lambda},\tag{3}$$

$f_0=2\omega\cos\theta_0$ is an average Coriolis parameter, $\tilde{\Phi}$ is a constant (the average potential of the free surface), and $K=0$ or 1. If $K=0$ is taken, Eq. (1) is degenerated into the non-divergent model. The other symbols are commonly used.

This model possesses conservations of total angular momentum M, total energy E and total squared absolute potential vorticity F:

$$M(t)=\iint_S\left(av_\lambda\sin\theta-K\frac{f_0^2a^2}{\tilde{\Phi}}\psi\cos\theta\right)\cdot a^2\sin\theta d\theta\lambda=M(0),\tag{4}$$

$$E(t)=\iint_S\frac{1}{2}\left[\left(\frac{\partial\psi}{a\partial\theta}\right)^2+\left(\frac{\partial\psi}{a\sin\theta\partial\lambda}\right)^2+K\frac{f_0^2a^2}{\tilde{\Phi}}\left(\frac{\psi}{a}\right)^2\right]$$
$$\times a^2\sin\theta d\theta d\lambda=E(0),\tag{5}$$

$$F(t)=\iint_S\frac{1}{2}q^2a^2\sin\theta d\theta d\lambda=F(0),\tag{6}$$

where S denotes the whole spherical surface. Combining Eqs. (4) and (6) yields the conservation of enstrophy (total squared relative potential vorticity) Q:

$$Q(t)=\iint_S\frac{1}{2}\left\{\frac{1}{a^2\sin\theta}\left(\frac{\partial}{\partial\theta}\sin\theta\frac{\partial\psi}{\partial\theta}\right)+\frac{\partial^2\psi}{a^2\sin^2\theta\partial\lambda^2}-K\frac{f_0^2}{\tilde{\Phi}}\psi\right\}^2$$
$$\times a^2\sin\theta d\theta d\lambda=Q(0).\tag{7}$$

By making expansion of ψ into a series of spherical functions

$$\psi(\theta,\lambda,t)=\sum_{n=1}^{\infty}\sum_{m=0}^{n}\psi_{mn}(t)e^{im\lambda}P_n^m(\cos\theta)\tag{8}$$

from Eqs. (5) and (7), we obtain the conservation of mean scale l^{**} of the motion or somewhat mean wave number n^{**},

$$n^{**}(n^{**}-1)+K\frac{f_0^2a^2}{\tilde{\Phi}}=\frac{a^2Q}{E}=\text{const}.\tag{9}$$

The conservation of mean scale l^{**} of non-divergent flow on a non-rotating sphere ($\omega=0$) is first discovered by Fjϕrtoft[4] (1953). However, in the case of rotating sphere, the atmospheric motion possesses some anisotropy. The Fjϕrtoft's types of energy transfer, i. e., L→M←S and L←M→S, take place when L, M and S only refer to the composed relative long, medium and short waves, respectively. As to the wave lengths decomposed along the latitudinal and meridional circles, the change in the spectral distribution of energy can differ from the Fjϕrtoft's types. For example, during the period of transition from low to high index of circulation the mean longitudinal wave number m^* decreases, and the mean meridional wave number $n^*=n^{**}-m^*$ increases; while during the inverse transition the m^* usually increases to some extent, and n^* decreases. The important thing is that, on a rotating sphere, m^* can decrease monotonically and become zero in many cases, but $n^*=n^{**}-m^*$ can never be zero, or some zonal flow always remains unless $n^*=0$ or $M=0$ initially, as can be seen in the following paragraph. The above mentioned anisotropy and possibility of monotonic decrease of m^* have been first emphasized by Rhines[5] (1975) and Zeng[6,7] (1979). A beta-plane has been used by the first author.

In order to investigate the interaction between the disturbances and zonal flow, it is convenient to decompose ψ and the correspondent properties into two parts. Denoting the zonal part of ψ by $\bar{\Psi}(\theta, t)$, We get

$$\psi(\theta,\lambda,t)=\bar{\Psi}(\theta,t)+\psi'(\theta,\lambda,t). \tag{10}$$

From Eq. (1) or (4)—(7) we have

$$\begin{aligned} &\bar{M}=M,\ M'=0,\\ &\bar{E}+E'=E,\\ &\bar{Q}+Q'=Q, \end{aligned} \tag{11}$$

where the bar refers to the zonal flow, and the prime the disturbances.

The change of $\bar{E}$ or E' is given by the following equation

$$\frac{\partial \bar{E}}{\partial t}=\frac{-\partial E'}{\partial t}=\iint_S v'_\lambda v'_\theta a\sin\theta\frac{\partial\bar{\lambda}}{a\partial\theta}ds, \tag{12}$$

where $\bar{\lambda}=\bar{v}_\lambda/a\sin\theta$. Similarly,

$$\frac{\partial\bar{Q}}{\partial t}=-\frac{\partial Q'}{\partial t}=\iint_S q'v'_\theta\frac{\partial\bar{q}}{a\partial\theta}ds. \tag{13}$$

We call a disturbance as developing if $\partial E'/\partial t>0$ and as decaying if $\partial E'/\partial t<0$. It is well known that the most favorable structure of disturbances for developing (decaying) is that, the trough or ridge lines are tilted eastward (westward) with the increasing of $|\theta-\theta_j|$, where θ_j denotes the axis of the jet of westerlies, i. e., $\partial\bar{\lambda}/\partial\theta=0$ at θ_j.

If $\partial\bar{\lambda}/\partial\theta=0$ everywhere, we have $\partial\bar{E}/\partial t=\partial E'/\partial t=0$. Moreover, it is not difficult to prove, that a flow with $\partial\bar{\lambda}/\partial\theta\equiv0$ has minimum zonal enstrophy $\bar{Q}$ and minimum zonal energy $\bar{E}=\bar{E}_0=4\pi aA^2/3$ for a fixed M, where

$$A=-\frac{3M}{4\pi a^2}\left(2+K\frac{f_0^2a^2}{\tilde{\Phi}}\right)^{-1}.$$

This means, that if a flow with $\partial\bar{\lambda}/\partial\theta\equiv0$ initially, its eddy energy will be bounded by its initial value, i. e., $E'(t)\leqslant E'(0)$. For an arbitrary flow the possible maximum portion of zonal energy which can be drawn by the disturbance is

$$\bar{E}(0)-\bar{E}_0\equiv\bar{E}_a. \tag{14}$$

Thus, $\bar{E}_a$ can be called as available zonal energy.

The $\bar{E}_a$ of a jet-like profile of $\bar{\Psi}$, which differs very much from $\partial\bar{\lambda}/\partial\theta\equiv0$, may be large. However, only a limited amount of $\bar{E}_a$ can be released. This is due to the stabilizing effect of the rotating sphere. In fact, multiplying Eq. (1) by $v_\theta\sin\theta$ and integrating it over the whole sphere, we obtain

$$\iint_S \hat{q}\ddot{\eta}ds=\iint_S\frac{2\omega}{a}v_\theta^2\sin^2\theta ds, \tag{15}$$

where $\eta=a\cos\theta$, $\ddot{\eta}=d^2\eta/dt^2$ and $\hat{q}=q-2\omega\cos\theta$ (the relative potential vorticity). Thus, the average "acceleration" $\ddot{\eta}$ over the region of cyclonic potential vorticity ($\hat{q}>0$) must be larger than that for anticyclonic potential vorticity ($\hat{q}<0$) if $\omega>0$. This is a favorable factor for separation of cyclonic vorticities from anticyclonic vorticities and slowing down the development of eddies. Eq. (15) is the extension of Kuo's formula[8] (1950) (Zeng[9], 1982).

On the contrary, the eddy energy E' can be completely or almost completely absorbed by the zonal flow in many cases. This is called by Zeng[6,7](1979) as rotational adaptation.

The conditions and mechanism for existence of such complete absorption of disturbances will be studied later.

2. *Integral Properties of Disturbances in a Linearized Model*

When the nonzonal flow is not very strong, the linearized equation

$$\left(\frac{\partial}{\partial t}+\bar{v}_\lambda\frac{\partial}{a\sin\theta\partial\lambda}\right)q'=-v'_\theta\frac{\partial\bar{q}}{a\partial\theta} \tag{16}$$

can be used. From Eq. (16) we obtain also (12)—(13). This means that the interaction aspect can be investigated by (16) together with

$$\frac{\partial\bar{q}}{\partial t}=-\frac{\partial\overline{v'_\theta q'}\sin\theta}{a\sin\theta\partial\theta} . \tag{17}$$

The structures of developing and decaying disturbances are the same as mentioned in the foregoing paragraph. Besides, from (16) we obtain the following two invariant integrals

$$\iint_S\left(\frac{\sin\theta}{\partial\bar{q}/a\partial\theta}\right)\frac{\partial q'^2}{\partial t}ds=0, \tag{18}$$

$$\iint_S\left\{\frac{\partial}{\partial t}\left[\left(\frac{\partial\psi'}{a\partial\theta}\right)^2+\left(\frac{\partial\psi'}{a\sin\theta\partial\lambda}\right)^2+K\frac{f_0^2a^2}{\tilde{\phi}}\left(\frac{\psi'}{a}\right)^2\right]\right.$$
$$\left.+\frac{(\bar{\lambda}-\bar{\lambda}_0)\,a\sin\theta}{\dfrac{\partial\bar{q}}{a\partial\theta}}\frac{\partial q'^2}{\partial t}\right\}ds=0, \tag{19}$$

where $\bar{\lambda}_0$ is an arbitrary constant (a reference angular velocity). It is not difficult to prove that both integrals on the left hand side of Eqs. (18) and (19) exist even if $\frac{\partial\bar{q}}{\partial\theta}=0$ at some latitudes.

From (19) we conclude, that the sufficient criterion for stability is the existence of a constant $\bar{\lambda}_0$ such that

$$\frac{\bar{\lambda}-\bar{\lambda}_0}{\dfrac{\partial\bar{q}}{\partial\theta}}\geqslant 0 \qquad \text{everywhere}, \tag{20}$$

and that the necessary criterion for instability is existence of an interval (θ_1,θ_2) depending on $\bar{\lambda}_0$ for any arbitrary constant $\bar{\lambda}_0$ such that

$$\frac{\bar{\lambda}-\bar{\lambda}_0}{\dfrac{\partial\bar{q}}{\partial\theta}}<0. \qquad (\theta_1<\theta<\theta_2) \tag{21}$$

If $\partial\bar{q}/\partial\theta$ does not change its sign in $0\leqslant\theta\leqslant\pi$ the flow is stable. Besides, if $\bar{v}_\lambda$ is also steady, then $E'+Q'_w$ is a constant, where

$$Q'_w\equiv\iint_S\left(\frac{\bar{\lambda}-\bar{\lambda}_0}{\dfrac{\partial\bar{q}}{\partial\theta}}\sin\theta\right)\frac{q'^2}{2}ds \tag{22}$$

and $(\bar{\lambda}-\bar{\lambda}_0)/(\partial\bar{q}/\partial\theta)\geqslant 0$. We can determine a weighted average scale l_w as follows

$$\left(\frac{l_w}{l_\beta}\right)=\frac{E'}{Q'_w}, \tag{23}$$

where l_β is a generalized stationary Rossby wavelength

$$l_\beta \equiv \left[\frac{1}{4\pi a^2}\iint_S a^2 \frac{\bar{\lambda}-\bar{\lambda}_0}{\frac{\partial \bar{q}}{\partial \theta}} \sin\theta ds\right]^{1/2}. \tag{24}$$

When the disturbances develop, E' increases, and Q'_w decreases simultaneously, hence l_w increases. The reversal is true for decaying disturbances. An increase of l_w can be achieved either by the increase of actual mean scale l of disturbances or by the moving of disturbances from the region of larger $(\bar{\lambda}-\bar{\lambda}_0)/(\partial\bar{q}/\partial\theta)$ to that of smaller $(\bar{\lambda}-\bar{\lambda}_0)/(\partial\bar{q}/\partial\theta)$. Both take place simultaneously, as can be seen later by using WKB method.

As (17) is used, i. e., the interaction is taken into account, $\bar{v}_\lambda$ is no longer steady. However, $\bar{v}_\lambda$ is a slowly varying function of t, so far as the jet is not too narrow and the disturbances are not too strong. Thus, our conclusion above the change of mean scale of disturbances is valid approximately. When $\bar{v}_\lambda$ is at the unstable state, $\partial\bar{q}/\partial\theta=0$ and $(\bar{\lambda}-\bar{\lambda}_0)/(\partial\bar{q}/\partial\theta)<0$ at some latitudes, the change of mean scale can not be investigated by integral properties alone.

Note that according to (18) and (19), in the case of complete absorption of eddy energy the eddy enstrophy does not approach to zero as $t\to\infty$ if initially not zero. This means that q' becomes an ideal function due to the infinitesimal mean wavelength as $t\to\infty$.

3. *General Properties of Barotropic Motions without Quasi-Geostrophic Approximation*

A barotropic model without quasi-geostrophic approximation, i. e. the shallow water equation on a rotating sphere, is usually taken as follows

$$\left[\frac{d\mathbf{v}}{dt}\right]_h = -\nabla\phi - [2\boldsymbol{\omega}\times\mathbf{v}]_h, \tag{25}$$

$$\frac{\partial\phi}{\partial t} = -\nabla\cdot(\phi\mathbf{v}), \tag{26}$$

where $\mathbf{v}=\boldsymbol{\theta}^0 v_\theta+\boldsymbol{\lambda}^0 v_\lambda$ is the relative velocity of the fluid, ϕ the geopotential of the free surface, $\boldsymbol{\omega}$ the angular velocity of the rotating sphere, the symbol $[\quad]_h$ denotes the projection of vector standing in the brackets onto the horizontal surface, and ∇ the gradient operator on the spherical surface. From (25) and (26) we get the conservation of potential vorticity, $dq/dt=0$, where

$$q=\frac{\tilde{\phi}}{\phi}\left[2\omega\cos\theta+\frac{1}{a\sin\theta}\left(\frac{\partial v_\lambda\sin\theta}{\partial\theta}-\frac{\partial v_\theta}{\partial\lambda}\right)\right], \tag{27}$$

and the conservation of total mass, total angular momentum, total energy and the total squared absolute potential vorticity as follows

$$M_{as}(t)\equiv\iint_S \phi ds=4\pi a^2\tilde{\phi}=M_{as}(0), \tag{28}$$

$$M(t)\equiv\iint_S \phi a\sin\theta(v_\lambda+\omega a\sin\theta)ds=M(0), \tag{29}$$

$$E(t)\equiv\iint_S \frac{1}{2}(\phi|\mathbf{v}|^2+\phi^2)ds=E(0), \tag{30}$$

$$F(t)\equiv\iint_S \frac{\phi}{2}q^2 ds=F(0). \tag{31}$$

In a manner similar to (9) we can introduce an invariant scale l^{**} as follows

$$l^{**}=(E/F)^{1/2}=\text{const}. \tag{32}$$

Therefore, similar conclusions about the energy transfer can be drawn. But l^{**}, as well as the conclusions drawn primarily, is relevant to the rotational part of the motion, because only the vorticity is included in F. For the motions such as inertio-gravity waves, in which the irrotational part is dominant or as large as the rotational part, l^{**} determined by (32) by no means indicates the actual scale of the motion because the scale determined by the divergence of velocity is also important and is not included in F. In such case the physical interpretation of (32) is not simple.

Linearizing (25) and (26) with respect to a zonal basic flow $(\bar v_\lambda, \bar v_\theta=0, \bar\Phi)$, we have the following three integrals

$$\frac{\partial}{\partial t}\iint_S \frac{\bar\Phi}{2}\left[(v_\theta')^2+(v_\lambda')^2+\frac{1}{\bar\Phi}\phi'^2\right]ds=-\iint_S \bar\Phi v_\theta' v_\lambda' a\sin\theta\frac{\partial\bar\lambda}{a\partial\theta}ds, \tag{33}$$

$$\iint_S\left[\frac{\bar\Phi^2 a\sin\theta}{\bar\Phi(\partial\bar q/a\partial\theta)}\frac{\partial}{\partial t}\left(\frac{q'^2}{2}\right)+a\sin\theta\frac{\partial\phi' v_\lambda'}{\partial t}\right]ds=0, \tag{34}$$

$$\frac{\partial}{\partial t}\iint_S \frac{\bar\Phi}{2}\left\{\left[1-\frac{(\bar v_{\lambda 0}-\bar v_\lambda)^2}{\bar\Phi}\right]v_\lambda'^2+v_\theta'^2+\frac{1}{\bar\Phi}[\phi'-(\bar v_{\lambda 0}-\bar v_\lambda)v_\lambda']^2\right\}ds$$
$$+\iint_S \frac{(\bar\lambda_0-\bar\lambda)\bar\Phi^2 a\sin\theta}{\bar\Phi(-\partial\bar q/a\partial\theta)}\frac{\partial}{\partial t}\left(\frac{q'^2}{2}\right)ds=0, \tag{35}$$

where $\bar v_{\lambda 0}\equiv\bar\lambda_0 a\sin\theta$, and $\bar q=\frac{\tilde\Phi}{\bar\Phi}\left[2\omega\cos\theta+\frac{1}{a\sin\theta}\frac{\partial\bar v_\lambda\sin\theta}{\partial\theta}\right]$ is the potential vorticity of the basic zonal flow. Note that (35) is valid only if the basic flow is a steady one. Eqs. (34) and (35) have been obtained by Zeng[5] (1979) on a beta-plane.

These integrals are the extensions of (12), (18) and (19) respectively. Again, Eq. (33) determines the dependence of development on the structure and relative location of the disturbances, and the criterion for stability or instability is drawn from (35). For the large scale motion we have always $\bar\Phi\gg\bar v_\lambda^2$, hence we get the same criterion for stability and instability as (20) and (21), and also (23) but with E' and Q_w' replaced by the first and second integrals (without the symbol $\partial/\partial t$) in (35) respectively. All the conclusions obtained in above subsection are valid qualitatively if the rotational part of the velocity field is dominant no matter whether the disturbance is quasi-geostrophic. However, l_w by no means indicates the scale of inertio-gravity waves. Moreover, besides barotropic instability there exists another type of instability, the inertial instability, which causes a rapid development of pure inertio-gravity waves.

When the influence of disturbance on the basic flow is taken into account the basic flow is no longer steady, and (35) does not take place. Especially, a nonzero meridional circulation, $\bar{v}_\theta \neq 0$, will be induced by the interaction. Therefore, we need to write perturbation equations linearized with respect to a basic zonal flow with $\bar{v}_\theta \neq 0$ and the equations describing the evolution of the basic flow in the weakly nonlinear interaction theory. These will be given in section V. (part II).

Finally, we point out that it is better to adopt a more accurate equation

$$\left[\frac{d\mathbf{v}}{dt}\right]_h = -\nabla\phi - [2\boldsymbol{\omega}\times\mathbf{v}]_h - [\boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r})]_h \tag{36}$$

instead of (25), where $\mathbf{r}$ is the vector radius, and the centrifugal force is included rather than the visible horizon is introduced. $dq/dt=0$ and Eqs. (27), (28), (29) and (30) are all valid, but (31) is replaced by

$$E(t)\equiv\iint_S \frac{1}{2}(\phi|\mathbf{v}+\boldsymbol{\omega}\times\mathbf{r}|^2+\phi^2)ds=E(0). \tag{37}$$

Besides, we have also the conservation of angular momentum vector

$$\mathbf{M}_v(t)\equiv\iint_S[\phi\mathbf{r}\times(\mathbf{v}+\boldsymbol{\omega}\times\mathbf{r})]ds=M_v(0), \tag{38}$$

consequently, the conservation of inertial axis of the fluid, i. e. the direction of $\mathbf{M}_v$. M in (29) is the projection of $\mathbf{M}_v$ onto the axis of the rotating sphere. The use of (38) will be interpreted in section VI.

III. EVOLUTION AND STRUCTURE OF BAROTROPIC WAVE PACKET

1. *General Formulas Describing the Evolution of Rossby Wave Packet*

The concepts of wave packet and group velocity were early applied by Rossby[10] (1945) and Yeh[11] (1949) to the study of atmospheric disturbances in a uniform basic flow. Later, taking wave-packet representation and more or less using the WKB method, Dickinson, Zeng[6] (his book was submitted to the press in 1977), Hoskins and many others have investigated various dynamic aspects of synoptic disturbances[6,12-27]. Especially, Dickinson[12], Grose and Hoskins[14], Hoskins and Karoly[22,24], and Hayashi[23] have paid attention to the propagation or the rays of Rossby wave; Chao and Yeh[13] and the others[15-17] have paid attention to the spiral structure of the Rossby waves; while the evolutional aspects of wave packets in non-uniform basic flow have been thoroughly studied by Zeng [9,24-26], Chen[17], Lu and Zeng[18,20], Lu[21], Young and Rhines[19]. Note that it is unnecessary for using the WKB method to take uniform and zonal flow as the basic flow.

According to Zeng[9] (1982), taking a Mercator projection of the sphere but a nonzonal basic flow $(\bar{v}_\lambda, \bar{v}_\theta)$, introducing characteristic velocity U^*, horizontal length L^*, and time $t^*=L^*/U^*$ for the perturbation, denoting the nondimensional time by t and the nondimensional coordinates as follows

$$x=\frac{a\lambda}{L^*},\quad y=\frac{a}{L^*}\ln\frac{1+\cos\theta}{\sin\theta}, \tag{39}$$

we transform the linearized nondimensional quasi-geostrophic potential vorticity equation into the following

$$\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}+\bar{v}\frac{\partial}{\partial y}-2\left(\frac{L^*}{a}\right)\bar{v}\cos\theta\right)\left[\frac{\partial^2\psi'}{\partial x^2}+\frac{\partial^2\psi'}{\partial y^2}-\left(\frac{L^*}{L_0}\right)^2\sin^2\theta\psi'\right]$$

$$+\bar{\beta}_y\frac{\partial\psi'}{\partial x}-\bar{\beta}_x\frac{\partial\psi'}{\partial y}=0, \tag{40}$$

where ψ' is the perturbation of nondimensional stream function, $rU^*\bar{u}\equiv\bar{v}_\lambda/\sin\theta$, $rU^*\bar{v}\equiv-\bar{v}_\theta/\sin\theta$, r is the ratio of scaling velocity for the basic flow to that for the perturbation, $L_0\equiv\tilde{\phi}/2\omega\cos\theta_0$ is the Rossby deformation radius, $\bar{\beta}_x\equiv R_0^{-1}\partial\bar{q}/\partial x$, $\bar{\beta}_y\equiv R_0^{-1}\partial\bar{q}/\partial y$, $Ro\equiv rU^*/L^*2\omega\cos\theta_0$ is the Rossby Number, and $\bar{q}$ is the nondimensional geostrophic potential vorticity of the basic flow whose nondimensional stream function is $\bar{\Phi}$

$$\bar{q}=\frac{\cos\theta}{\cos\theta_0}+\frac{Ro}{\sin\theta}\left[\frac{\partial^2\bar{\Phi}}{\partial x^2}+\frac{\partial^2\bar{\Phi}}{\partial y^2}-\left(\frac{L^*}{L_0}\right)\sin^2\theta\cdot\bar{\Phi}\right]. \tag{41}$$

For simplicity we have taken $K=1$ in (40) and (41).

Assuming that $\bar{u}$, $\bar{v}$, $\bar{\beta}_x$ and $\bar{\beta}_y$ are slowly varying functions of (x, y, t), and that the scales of perturbation are small as compared with those of the basic flow, i. e. ε, the ratio of these two scales, is a small parameter, we introduce a slow time T and enlarged coordinates (X, Y) as follows

$$X=\varepsilon x,\ Y=\varepsilon y,\ T=\varepsilon t, \tag{42}$$

and represent the perturbation in the form of a wave packet

$$\Psi'(x,y,t)=\mathrm{Re}[\Psi(X,Y,T)e^{i\alpha(X,Y,T)/\varepsilon}], \tag{43}$$

$$\Psi(X,Y,T)=\Psi_0(X,Y,T)+\varepsilon\Psi_1(X,Y,T)+\cdots, \tag{44}$$

then the partial derivatives of $\bar{u}$, $\bar{v}$, $\bar{\beta}_x$, $\bar{\beta}_y$, α and Ψ_j $(j=0, 1, 2,\ldots)$ with respect to the arguments (X, Y, T) are all of O (1). Substituting (42)—(44) into (40) and expand every term into a power series of ε, we get the dispersion relation

$$(\sigma-m\bar{u}-n\bar{v})\nu^2+\bar{\beta}_y m-\bar{\beta}_x n=0, \tag{45}$$

and an equation governing the evolution of amplitude

$$\left(\frac{\partial}{\partial T}+\bar{u}\frac{\partial}{\partial X}+\bar{v}\frac{\partial}{\partial Y}\right)\nu^2\Psi_0-(\sigma-m\bar{u}-n\bar{v})\left[2\left(m\frac{\partial}{\partial X}+n\frac{\partial}{\partial Y}\right)\Psi_0\right.$$

$$\left.+\Psi_0\left(\frac{\partial m}{\partial X}+\frac{\partial m}{\partial Y}\right)\right]-\bar{\beta}_y\frac{\partial\Psi_0}{\partial X}+\bar{\beta}_x\frac{\partial\Psi_0}{\partial Y}=0, \tag{46}$$

where σ, m, n are the instantaneous and local frequency, wave numbers along X and Y respectively, i. e.

$$\sigma\equiv-\frac{\partial\Theta}{\partial T},\qquad m=\frac{\partial\Theta}{\partial X},\qquad n=\frac{\partial\Theta}{\partial Y}, \tag{47}$$

$$\nu^2\equiv m^2+n^2+\rho^2 \quad \text{and}$$

$$\rho^2\equiv\left(\frac{L^*}{L_0}\right)^2\sin^2\theta,$$

where Θ is a phase function.

Besides, we have also the following kinematic relationships

$$D_g\sigma/DT=-\left(m\frac{\partial\bar{u}}{\partial T}+n\frac{\partial\bar{v}}{\partial T}\right)-\nu^{-2}\left(n\frac{\partial\bar{\beta}_x}{\partial T}-m\frac{\partial\bar{\beta}_y}{\partial T}\right), \tag{48}$$

$$D_g m/DT = -\left(m\frac{\partial \bar{u}}{\partial X}+n\frac{\partial \bar{v}}{\partial X}\right)-\nu^{-2}\left(n\frac{\partial \bar{\beta}_x}{\partial X}-m\frac{\partial \bar{\beta}_y}{\partial X}\right), \quad (49)$$

$$D_g n/DT = -\left(m\frac{\partial \bar{u}}{\partial Y}+n\frac{\partial \bar{v}}{\partial Y}\right)-\nu^{-2}\left(n\frac{\partial \bar{\beta}_x}{\partial Y}-m\frac{\partial \bar{\beta}_y}{\partial Y}\right)-\frac{\partial \sigma}{\partial \rho^2}\frac{\partial \rho^2}{\partial Y}, \quad (50)$$

where

$$D_g/DT \equiv \frac{\partial}{\partial T}+C_{gX}\frac{\partial}{\partial X}+C_{gY}\frac{\partial}{\partial Y}, \quad (51)$$

$C_{gX}\equiv\partial\sigma/\partial m$, $C_{gY}\equiv\partial\sigma/\partial n$ are the components of the group velocity, and $\partial\sigma/\partial\rho^2=(m\bar{\beta}_y-n\bar{\beta}_x)/\nu^4$. Eqs. (48)—(50) determine the change in σ, m and n. Therefore, we have obtained all the equations describing all the characteristics of evolutionary process of the wave packet.

Integrating (46) over the whole region occupied by the wave packet w, we get the energy equation

$$\frac{\partial}{\partial T}E' = \iint_w |\Psi_0|^2\left[m^2\frac{\partial \bar{u}}{\partial X}+mn\left(\frac{\partial \bar{u}}{\partial Y}+\frac{\partial \bar{v}}{\partial X}\right)\right.$$
$$\left.+n^2\frac{\partial \bar{v}}{\partial Y}\right]dXdY, \quad (52)$$

where

$$E'\equiv\iint_w \frac{1}{2}\nu^2|\Psi_0|^2 dXdY \quad (53)$$

is the total energy of disturbances represented in the form of wave packet. Eq. (52) is the wave packet representation of energy equation

$$\frac{\partial}{\partial t}\iint_S \frac{1}{2}\left[v_\lambda'^2+v_\theta'^2+\frac{f_0^2a^2}{\bar{\phi}}\left(\frac{\phi'}{a}\right)^2\right]ds=\iint_S\left[v_\lambda' v_\theta'\left(\sin\theta\frac{\partial\bar{\lambda}}{\partial\theta}+\frac{\partial\bar{\theta}}{\sin\theta\partial\lambda}\right)\right.$$
$$\left.+v_\lambda' v_\lambda'\sin\theta\frac{\partial\bar{\lambda}}{\sin\theta\partial\lambda}+v_\theta' v_\theta'\frac{\partial\bar{\theta}}{\partial\theta}-v_\lambda'^2\,\mathrm{ctg}\,\theta\cdot\bar{v}_\theta\right]ds, \quad (54)$$

but the last term in (54) is neglected due to the smallness of $\bar{v}_\theta$.

Eq. (52) is a useful equation indicating the dependency of the development on the structure and location of the disturbance. Especially, since the disturbance is a long wave (Rossby wave), the mean flow $\bar{\psi}$ must consist of a zonal flow and some ultra-long waves, if it is not a purely zonal one, hence (42) can be simplified as follows

$$\frac{\partial E'}{\partial T}\approx\iint_w |\Psi_0|^2\left[(m^2-n^2)\frac{\partial \bar{u}}{\partial X}+mn\frac{\partial \bar{u}}{\partial Y}\right]dXdY, \quad (55)$$

because $|\partial\bar{v}/\partial X|$ is usually smaller than $|\partial\bar{u}/\partial Y|$ in our atmosphere. More analyses will be given later.

Next, from (49) and (50) we obtain an equation describing the temporal variation of two-dimensional wave number $(m^2+n^2)^{1/2}$. When the jet profile is not too sharp, this equation takes the following approximate form

$$\frac{D_g}{DT}(m^2+n^2)\approx-2\left[m\left(m\frac{\partial \bar{u}}{\partial X}+n\frac{\partial \bar{u}}{\partial Y}\right)+n\left(m\frac{\partial \bar{v}}{\partial X}+n\frac{\partial \bar{v}}{\partial Y}\right)\right]. \quad (56)$$

Therefore we have

$$\frac{\partial E'}{\partial T} \approx -\iint_w \frac{|\Psi_0|^2}{2} \frac{D_g(m^2+n^2)}{DT} dXdY. \tag{57}$$

This means, that the scale (wavelength) of a disturbance is enlarged as it develops, while its scale becomes smaller as it decays. This has been discovered first in Zeng and Lu's works[18,20,21].

2. *Rossby Wave Packet in a Jet-like Zonal Flow*

Rossby wave packet in a jet-like zonal flow has been analyzed in detail by Zeng and Lu[20,21] (1981) and Zeng[9] (1982).

Note that in this case the disturbances not necessarily have a wave packet form along every latitudinal circle, i. e., m can be a const., and n, σ, Ψ_0 be functions of Y and T.

Let $v=0$, from (45)—(50) we obtain

$$\frac{\partial}{\partial T}\iint_w \frac{1}{2} \nu^2|\Psi_0|^2 dXdY = \iint_w |\Psi_0|^2 mn \frac{\partial \bar{u}}{\partial Y} dXdY, \tag{58}$$

$$\iint_w \frac{1}{\bar{\beta}_y} \frac{\partial}{\partial T} (\nu^2|_0\Psi|)^2 dXdY = 0, \tag{59}$$

$$\iint_w \left[\frac{\partial}{\partial T} (\nu^2|\Psi_0|^2) + \frac{\bar{u}_0 - \bar{u}}{\bar{\beta}_y} \frac{\partial}{\partial T} (\nu^2|\Psi_0|)^2 dXdY = 0, \tag{60}$$

where $\bar{u}_0$ is a const., these three integrals namely are the analogues of (50), (60) and (61) respectively. So that the criteria for stability and instability are the same as mentioned in section II-2 but relevant to the local region occupied by the wave packet.

If $\bar{u}$ is a steady zonal flow, (46) can be transformed into a more compact form

$$\frac{\partial}{\partial T}\left[\frac{(\nu^2|\Psi_0|)^2}{2\bar{\beta}_y}\right] + \frac{\partial}{\partial X}\left[C_{gX} \frac{(\nu^2|\Psi_0|)^2}{2\bar{\beta}_y}\right] + \frac{\partial}{\partial Y}\left[C_{gY} \frac{(\nu^2|\Psi_0|)^2}{2\bar{\beta}_y}\right] = 0. \tag{61}$$

According to (45), we have

$$-\frac{(\nu^2|\Psi_0|)^2}{2\bar{\beta}_y} = \frac{e'}{\frac{\sigma}{m} - \bar{u}}, \tag{62}$$

where $e' \equiv \nu^2|\Psi_0|^2/2$ is the energy density of the disturbance and $e'/((\sigma/m)-\bar{u})$ is called as wave action (Bretherton[28], 1969). So that (61) is the conservation of wave action. According to (48) and (49), σ and m do not vary with time along $\mathbf{C}_g$, the group velocity, if the basic flow is zonal and steady. Therefore, Eq. (61) together with (62) is very convenient for computing the evolution of e' along $\mathbf{C}_g$. However, the wave action might not exist at a line, where $\bar{\beta}_y = 0$, hence $\sigma - m\bar{u} = 0$, if there is such a line within the wave packet, but (46) (58) and (59) are still valid. Besides, in the nonzonal case, the conservation of wave action breaks down (Yamagata[29], 1976). Instead, the conservation of total enstrophy approximately takes place if $O(\bar{\beta}_x) = 0(\bar{\beta}_y) = O(\varepsilon)$ and $\partial\bar{u}/\partial X + \partial\bar{v}/\partial Y = 0$ (Zeng and Lu[18], 1981, and Young and Rhines[19], 1980). Even this conservation also breaks down if $\partial\bar{u}/\partial X + \partial\bar{v}/\partial Y \neq 0$ (Zeng and Lu[18], 1981).

In a zonal, but not necessarily steady, basic flow we have two equations, one describes the variation of two-dimensional wave number and the other the variation of the tilt of the trough-ridge lines, as follows

$$\frac{D_g}{DT}(m^2+n^2)=\frac{D_g n^2}{DT}=-2mn\left(\frac{\partial \bar{u}}{\partial Y}+\nu^{-4}\bar{\beta}_y\frac{\partial \rho^2}{\partial Y}-\nu^{-2}\frac{\partial \bar{\beta}_y}{\partial Y}\right), \tag{63}$$

$$\frac{D_g}{DT}\left(\frac{n}{m}\right)=-\left(\frac{\partial \bar{u}}{\partial Y}+\nu^{-4}\bar{\beta}_y\frac{\partial \rho^2}{\partial Y}-\nu^{-2}\frac{\partial \bar{\beta}_y}{\partial Y}\right). \tag{64}$$

Eq. (63) can be transformed into a more compact form

$$\bar{\beta}_y\frac{D_g}{DT}\frac{\nu^4}{\bar{\beta}_y}=-2mn\frac{\partial \bar{u}}{\partial Y}, \qquad \text{if } \bar{\beta}_y\neq 0. \tag{65}$$

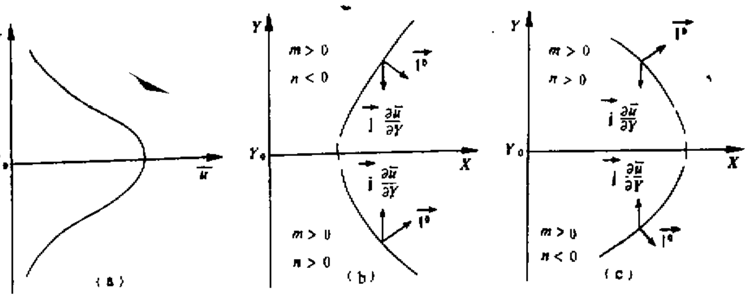

Fig. 1. (a) An ideal jet profile $\bar{u}(Y)$. (b) A typical trough or ridge line of barotropically growing disturbance. (c) The same as in (b), but for decaying disturbance. $\vec{l}^{\circ}$ is the direction of the ray.

Suppose that there are westerlies with a jet whose axis (the maximum of $\bar{u}$) is located at $Y=Y_0$ in the Northern Hemisphere. The typical structure of developing and decaying trough–ridge line is given in Fig. 1b and 1c. respectively and some typical flow patterns composed of basic flow and disturbances are given in Fig. 2.

In our atmosphere $L_0\cong 3\times 10^3$ km, $L^*=10^3$ km for long wave disturbance, and $L^*<10^3$ km for short one, so that O $(L_0/L^*)=O(1)$, $O(R_0)<O(1)$ and $O(\bar{\beta}_y)=O(1)$ for the long wave; and $O(L_0/L^*)>O(1)$ and $O(\bar{\beta}_y)<O(1)$ for the short wave. In all cases, ρ^2 is small, and $\partial\rho^2/\partial Y$ can be neglected in (63) and (64). When the jet is moderate or weak, $\bar{\beta}_y$ is positive, and the leading term of $\bar{\beta}_y$ is $R_0^{-1}\,\partial(\cos\theta/\cos\theta_0)/\partial y$ hence $\partial\bar{\beta}_y/\partial Y$ is negative. Therefore in the south of the jet, where $\partial\bar{u}/\partial Y>0$, two terms remained in the right hand side of (63) have the same sign, and are negative for developing disturbance ($mn>0$) and positive for decaying disturbance ($mn<0$). This indicates that when a disturbance develops (decays) in the south of the jet, its wavelength enlarges (shortens), and its axis, i. e., the trough or ridge line, becomes less (more) tilted, in accordance with (63) and (64). On the other hand, when a disturbance is located in the north of the jet, where $\partial\bar{u}/\partial Y<0$, the leading term in (45) and (46) is $\partial\bar{u}/\partial Y$, providing the jet is not too weak ($\partial\bar{v}/\partial Y$ is not too small), thus the conclusion about the change of scale and axis of disturbances is also valid.

As $\bar{\beta}_y>0$, $C_{gY}=2mn\bar{\beta}_y\nu^{-4}$ has the same sign as mn, therefore the maximum amplitude of developing (decaying) disturbance moves toward (out from) the jet. Taking $\bar{u}_0=\bar{u}(Y_0)$ in (60), $(\bar{u}_0-\bar{u})/\bar{\beta}_y\geqslant 0$ and has minimum at $Y=Y_0$ (the axis of the jet). Thus, the two ways mentioned in section II–2 to enlarge or shorten l_w take place simultaneously.

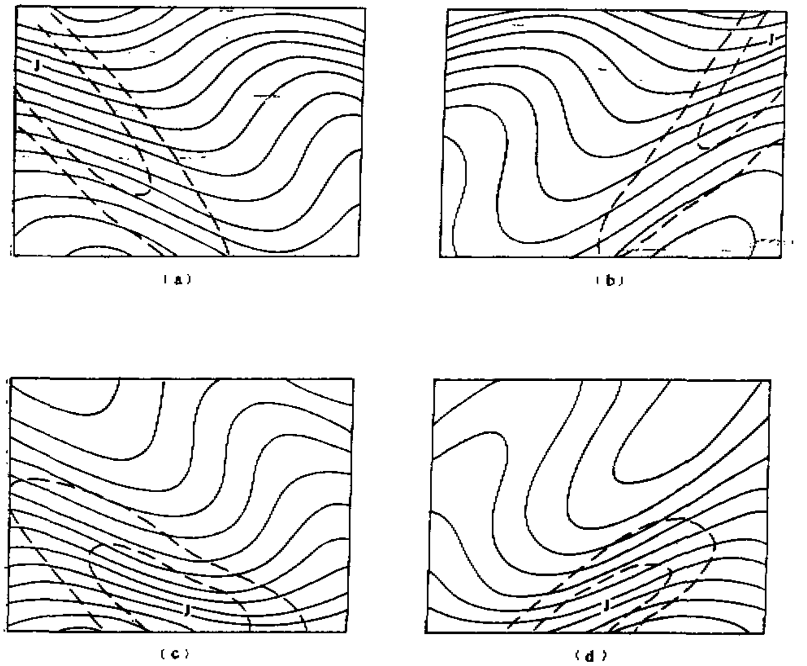

Fig. 2. Typical patterns of barotropic developing and decaying disturbances. (a) developing trough on the southside of the jet stream, (b) decaying trough on the southside of the jet stream, (c) decaying trough on the northside of the jet stream. Dashed lines are isotaches within the area with strong wind, and "J" denotes the location of maximum wind speed.

Suppose that initially we have a developing wave packet, in the first stage this wave packet develops, draws energy from the zonal basic flow; and its trough–ridge lines gradually become less tilted due to the shear of the zonal flow. Then, the trough–ridge lines are directed nearly along a meridian, the energy of the wave packet reaches its maximum. After that the trough–ridge lines become more and more tilted in the opposite direction, the wave packet decays, and its energy is continuously absorbed by the zonal flow. This is the whole life cycle of a Rossby wave packet, and can be schematically shown in Fig. 3.

Let a wave packet locate initially at $Y=Y^{(0)}$ and have local wave number $m^{(0)}$, $n^{(0)}$, hence $\sigma^{(0)}=m^{(0)}\bar{u}^{(0)}-m^{(0)}\bar{\beta}_y^{(0)}/(\nu^{(0)})^2$, where $\bar{u}^{(0)}\equiv\bar{u}(Y^{(0)})$, $\bar{\beta}_y^{(0)}\equiv\bar{\beta}_y(Y^{(0)})$ and $(\nu^{(0)})^2\equiv(m^{(0)})^2+(n^{(0)})^2+\rho^2(Y^{(0)})$. $Y=Y_c$ is called as a critical line with respect to the wave packet if $\bar{u}(Y_c)=\bar{u}^{(0)}-\bar{\beta}_y^{(0)}/(\nu^{(0)})^2$. When a wave packet approaches its critical line, it keeps its σ and m unchanged, but its $e'\to 0$, $|\Psi_0|\to 0$ and $\nu^2\to\infty$ due to $\sigma-m\bar{u}(Y_c)\to 0$ in accordance with the conservation of wave action, (61) and (62), and the local dispersion relation. This means that the wave packet will be finally and completely absorbed by the zonal basic flow. This is the case when the wave packet consists of long wave (hence, $\bar{\beta}_y/\nu^2$ is not large) and the westerlies have a single but strong enough jet ($\bar{u}$ is a monotonic function of Y on every side of the jet axis and has an increasement large enough to make $\bar{u}=\bar{u}^{(0)}-\bar{\beta}_y^{(0)}/(\nu^{(0)})^2$ satisfied at some Y).

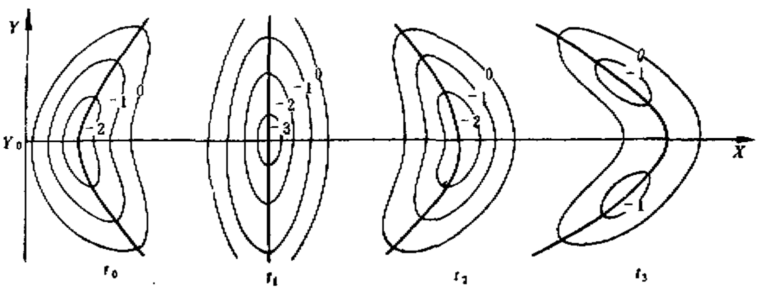

Fig. 3. Life cycle of a wave packet. Heavy lines are the trough lines, thin lines the isopleths of disturbances. $t_0<t_1<t_2<t_3$.

If the wave packet consists of ultra-long wave with very small $|m^{(0)}|$ and $|n^{(0)}|$, $\sigma^{(0)}$ is negative, hence a strong enough east wind is needed to completely absorb the wave packet. Besides, if there are double jets, and if there is no critical line in the area between these two jets, a decaying wave packet will be amplified again after propagating across the minimum of $\bar{u}$, then reflected at some latitude, and then alternatively undergoes a grow and decay. Therefore, some portion of energy of the wave packet will remain, and there exists a wave guide in this area (see, Fig. 4).

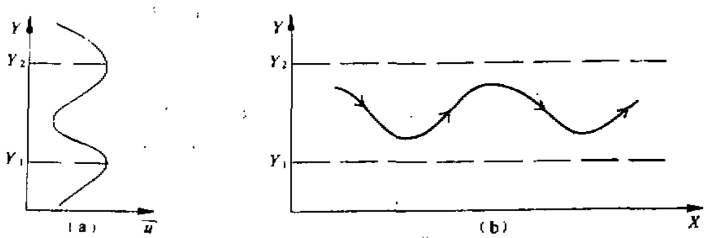

Fig. 4. A zonal flow with double jets, (a), and a wave guide (b). The heavy curve with arrows indicates the wave ray.

Finally, when the jet is too weak, ($|\partial\bar{u}/\partial Y|$ is too small), the conclusions about the disturbances in the north of westerlies jet can only be drawn under additional or detailed analysis, for example, for short wave the term $\nu^{-2}\partial\bar{\beta}_y/\partial Y$ is negligibly small due to smallness of O ($\bar{\beta}_y$) or largeness of n^2 or m^2, hence ν^2. When the jet is too strong and narrow, especially when $\bar{\beta}_y$ even changes its sign, there is no simple law governing the variation of scale of the distrubance, further investigation is needed.

Note that the wave packet representation of a disturbance is very simple and clear in the geometric picture, and the WKB method gives mathematically simple and excellent form of formulas, but, the first, the real disturbance can hardly be represented by a single wave packet with sufficient accuracy, hence a combination of several wave packets and the study of their interference are needed; and the second, WKB method introduces certain approximations to the original equation, hence the conclusions such obtained might be only approximately valid, so that a more exact method is desirable, this will be described in section IV (another paper).

3. *The Case of Nonzonal Basic Flow*

In our atmosphere there are always some forced ultra-long waves. Thus to study the evolution of disturbances superposed on a nonzonal flow and their interaction is more important in practice. This problem needs further investigation. Here, we are able only to show some examples.

Suppose that there are a zonal flow $\bar{\Phi}_z$ with jet axis located at Y_0 and an ultra-long wave whose stream function is

$$\bar{\Phi}_a = A\cos MX \cdot \cos N(Y - Y_a), \tag{66}$$

where M and N are the wave numbers of the ultra-long wave, $N > M > 0$ and $A > 0$, $Y_a < Y_0$ (on the southern side of the jet), and A or M is small enough. The composed stream function $\bar{\Phi} = \bar{\Phi}_z + \bar{\Phi}_a$ is given in Fig. 5. Denoting $Y_s = \min\ (Y_0,\ Y_a + \pi/2N)$, we divide XY plane into four regions: regions I, II, III and IV, corresponding to the northeast

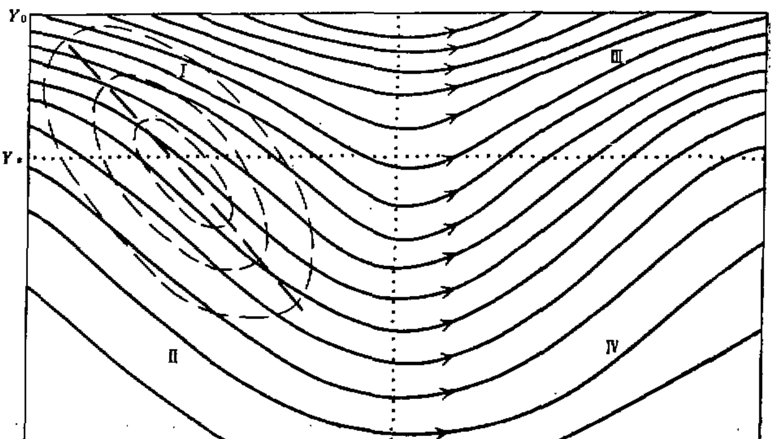

Fig. 5. A nonzonal basic flow (solid lines) and a growing disturbance (dashed thin lines). The axis of the disturbance (trough line) is indicated by the dashed heavy line.

side of the basic ridge ($0<X<\pi/2M, Y_a<Y<Y_s$), the southeast side of the basic ridge ($0<X<\pi/2M,\ Y_a>Y>Y_a-\pi/2N$), the northeast side of the basic trough ($\pi/2M<X<\pi/M,\ Y_a<Y<Y_s$), and the southeast side of the basic trough ($\pi/2M<X<\pi/M,\ Y_a>Y>Y_a-\pi/2N$) respectively. So that we have $\partial\bar{u}/\partial Y>0$ in all these four regions, while $\partial\bar{u}/\partial X<0$ in regions I and IV, and $\partial\bar{u}/\partial X>0$ in regions II and III.

Suppose that there is a leading wave packet ($mn>0$) located in regions I and II with $n^2>m^2$ (disturbance is shallow in the meridional direction, and its trough-ridge lines cross Y-axis with an angle $\delta>\pi/4$, see Fig. 5), according to (55), this is a growing disturbance due to the fact that leading term, $mn\ \partial\bar{u}/\partial Y$, is positive in these two regions, but the northern part (in region I) develops more rapidly than the southern part (in region II) because the two terms $(m^2-n^2)\partial\bar{u}/\partial X$ and $mn\partial\bar{u}/\partial Y$ are positive in region I, while $(m^2-n^2)\ \partial\bar{u}/\partial X$ gives a negative contribution, hence cancels some positive contribution of $mn\ \partial\bar{u}/\partial Y$.

Suppose that a trailing wave packet ($mn<0$) with $n^2>m^2$ is located in regions III and IV. Its northern part decays more rapidly than the southern part.

When m^2-n^2 is zero or very small so is the additional contribution.

Besides, Eqs. (49) and (50) are simplified as follows

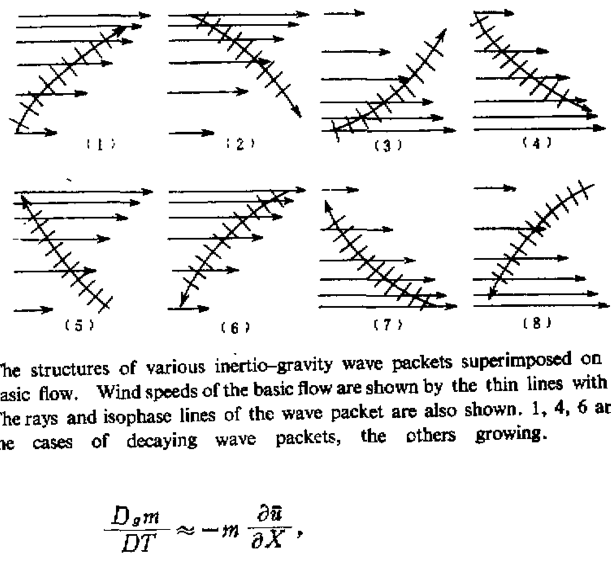

Fig. 6. The structures of various inertio-gravity wave packets superimposed on a shear basic flow. Wind speeds of the basic flow are shown by the thin lines with arrows. The rays and isophase lines of the wave packet are also shown. 1, 4, 6 and 7 are the cases of decaying wave packets, the others growing.

$$\frac{D_g m}{DT}\approx -m\frac{\partial\bar{u}}{\partial X}, \tag{67}$$

$$\frac{D_g n}{DT}\approx -m\frac{\partial\bar{u}}{\partial Y}-n\frac{\partial\bar{u}}{\partial X}+\frac{m}{\nu^2}\frac{\partial\beta_y}{\partial Y}\ . \tag{68}$$

According to these equations, for the leading wave packet both m and n decrease in regions II, while m increases but n decreases in region I. Similarly, for the trailing wave packet m decreases in region IV but increases in region III.

If Y_a is closed to Y_0, and if m^2-n^2 is small, a leading trough ($mn>0$) on the east side of the basic ridge develops and simultaneously enlarges its longitudinal and meridional scales, then it becomes a deep and wide trough replacing the old basic trough. This is one of the typical types of the so-called retrograde propagation of ultra-long wave. On the other hand, a trailing trough ($mn<0$) on the east side of the basic trough decays, and its longitudinal and meridional scales simultaneously decrease.

4. *Non-Geostrophic Wave Packets*

There are three types of wave packets, the Rossby or vortic wave, forward propagating inertio-gravity wave and backward propagating inertio-gravity wave, each of them is a solution to the primitive equations (25) and (26) linearized. Because of the linearity these wave packets do not interact each other.

The behaviour of Rossby wave packet obtained by solving the linearized primitive equations slightly differs from that by the quasi-geostrophic model (see, Zeng and Lu,[18] 1981) due to the fact, that the terms $\bar{\beta}_x$ and $\bar{\beta}_y$ are excluded in the dispersion relation (45), and that $\bar{\beta}_y$ is replaced by a const. in (59) (hence the total enstrophy is conserved as $\partial\bar{u}/\partial X+\partial\bar{v}/\partial Y=0$), if v'_λ, v'_θ and ϕ' are directly expanded into power series of ε.

Using a local standard coordinate system and neglecting the small metric terms in the linearized primitive equations, we get the same equations governing the evolution and the structure of inertio-gravity wave packet, consequently, the same conclusions as in our previous works (Zeng[6], 1979, Zeng and Lu[18], 1981, and Zeng[*], 1983). Namely, introducing characteristic scales $t^*=f_0^{-1}$, $L^*=L_0=\sqrt{\bar{\phi}}/f_0$, U^* and $\phi^*=f_0L^*U^*$ for the perturbation, and $\varepsilon^{-1}t^*$, $\varepsilon^{-1}L_0$, rU^* and $\varepsilon^{-1}rf_0L_0U^*$ for the basic flow, we have the perturbation equations as follows

$$\begin{cases} \dfrac{\partial\hat{u}'}{\partial t}+U\dfrac{\partial\hat{u}'}{\partial x}+V\dfrac{\partial\hat{u}'}{\partial y}-f\hat{v}'+\mu^2\dfrac{\partial\phi'}{\partial x}+\varepsilon\Big[\Big(\hat{u}'\dfrac{\partial U}{\partial X}+\hat{v}'\dfrac{\partial U}{\partial Y}\Big) \\ \quad -\hat{u}'\Big(\dfrac{\partial U}{\partial X}+\dfrac{\partial V}{\partial Y}\Big)\Big]=0, \\ \dfrac{\partial\hat{v}'}{\partial t}+U\dfrac{\partial\hat{v}'}{\partial x}+V\dfrac{\partial\hat{v}'}{\partial y}+f\hat{u}'+\mu^2\dfrac{\partial\phi'}{\partial y}+\varepsilon\Big[\Big(\hat{u}'\dfrac{\partial V}{\partial X}+\hat{v}'\dfrac{\partial V}{\partial Y}\Big) \\ \quad -\hat{v}'\Big(\dfrac{\partial U}{\partial X}+\dfrac{\partial V}{\partial Y}\Big)\Big]=0, \\ \dfrac{\partial\phi'}{\partial t}+U\dfrac{\partial\phi'}{\partial x}+V\dfrac{\partial\phi'}{\partial y}+\dfrac{\partial\hat{u}'}{\partial x}+\dfrac{\partial\hat{v}'}{\partial y}+\varepsilon\phi'\Big[\dfrac{\partial U}{\partial X}+\dfrac{\partial V}{\partial Y}\Big]=0. \end{cases} \tag{69}$$

where all the variables are nondimensional, including f; $X\equiv\varepsilon x$, $Y\equiv\varepsilon y$,

$$\begin{aligned} &\mu^2=1+\frac{rf_0L_0U^*}{\varepsilon\bar{\phi}}\tilde{\phi}, \\ &(\hat{u}',\hat{v}')\equiv\mu^2(u',v'), \\ &(U,V)\equiv\frac{rU^*}{f_0L^*}(\bar{u},\bar{v}), \end{aligned} \tag{70}$$

and $\tilde{\phi}$ is the departure of geopotential of the basic flow from its maen. Taking $t^*=f_0^{-1}$ and $\phi^*=f_0L^*U^*$ means that we are studying inertio-gravity wave rather than the pure gravity wave. Taking $L^*=L_0$ means that the perturbation is of meso-scale and basic flow of large scale if $L_0\approx10^5$m, and that the perturbation is of large scale and basic flow of planetary scale if $L_0\approx10^6$ m. $rU^*/f_0L^*=1$ as $r=10$, $U^*=1$ m sec^{-1}, $L^*=10^5$ m and $f_0=10^{-4}$ sec^{-1};

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and $rU^*/f_0L^*=0.4$ as $r=40$, $U^*=1$ m sec^{-1}, $L^*=10^6$ m and $f_0=10^{-4}$ sec^{-1}. In both cases (U, V) and μ^2 are all slowly varying functions, and we have also $\left(\frac{\partial}{\partial T}+U\frac{\partial}{\partial X}+V\frac{\partial}{\partial Y}\right)\mu^2+\mu^2\left(\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}\right)=0$, where $T\equiv\varepsilon t$ (the closed set of equations for the basic flow will be given in section V)

Let

$$\begin{cases} \hat{u}'=[\hat{u}_0(X,Y,T)+\varepsilon\hat{u}_1(X,Y,T)+\cdots]\exp[i\theta(x,y,t)], \\ \hat{v}'=[\hat{v}_0(X,Y,T)+\varepsilon\hat{v}_1(X,Y,T)+\cdots]\exp[i\theta(x,y,t)], \\ \phi'=[\phi_0(X,Y,T)+\varepsilon\phi_1(X,Y,T)+\cdots]\exp[i\theta(x,y,t)], \end{cases} \tag{71}$$

we have the dispersion relation, group velocity and equations governing the change of the frequency and the wave numbers for the inertio-gravity wave packets as follows:

$$\xi^2=f^2+\mu^2(m^2+n^2),\quad \xi\equiv-\sigma+mU+nV,$$

$$C_{gX}=\frac{\partial\sigma}{\partial m}=U-\mu^2m/\xi,\quad C_{gY}=\frac{\partial\sigma}{\partial n}=V-\mu^2n/\xi,$$

$$\begin{cases} \frac{D_g\sigma}{DT}=m\frac{\partial U}{\partial T}+n\frac{\partial U}{\partial T}-\frac{m^2+n^2}{2\xi}\frac{\partial\mu^2}{\partial T}, \\ \frac{D_gm}{DT}=-\left[m\frac{\partial U}{\partial X}+n\frac{\partial V}{\partial X}-\frac{m^2+n^2}{2\xi}\frac{\partial\mu^2}{\partial X}-\frac{1}{2\xi}\frac{\partial f^2}{\partial X}\right], \\ \frac{D_gn}{DT}=-\left[m\frac{\partial U}{\partial Y}+n\frac{\partial V}{\partial Y}-\frac{m^2+n^2}{2\xi}\frac{\partial\mu^2}{\partial Y}-\frac{1}{2\xi}\frac{\partial f^2}{\partial Y}\right], \end{cases} \tag{72}$$

$$\frac{D_g}{DT}\ln\mu^2(m^2+n^2)=-\left(\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}\right)-\frac{1}{m^2+n^2}\left[m^2\frac{\partial U}{\partial X}+n^2\frac{\partial V}{\partial Y}+mn\left(\frac{\partial U}{\partial Y}+\frac{\partial V}{\partial X}\right)\right]+\frac{1}{\mu^2(m^2+n^2)}\left[\frac{\mu^2m}{\xi}\frac{\partial f^2}{\partial X}+\frac{\mu^2m}{\xi}\frac{\partial f^2}{\partial Y}\right]. \tag{73}$$

The internal relation between the velocity and geopotential fields is given by

$$u_0=\frac{-m\xi+inf}{m^2+n^2}\phi_0,\quad v_0=\frac{-n\xi-imf}{m^2+n^2}\phi_0. \tag{74}$$

From the equation governing the evolution of ϕ_0 we obtain the energy integral

$$\frac{\partial}{\partial T}\iint_w e_0'dXdY=-\iint_w\frac{e_0'}{2\xi^2}\left\{(\xi^2+f^2)\left(\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}\right)+2\mu^2\left[m^2\frac{\partial U}{\partial X}+n^2\frac{\partial V}{\partial Y}+mn\left(\frac{\partial U}{\partial Y}+\frac{\partial V}{\partial X}\right)\right]\right\}dXdY, \tag{75}$$

and an equation

$$\frac{\partial A}{\partial T}+\nabla\cdot(A\mathbf{C}_g)=-A\left\{\frac{f^2}{\xi^2}\left(\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}\right)+\frac{1}{2\xi^2}\frac{Df^2}{DT}\right\}, \tag{76}$$

where e_0' and A are the energy density and the wave action respectively,

$$e_0' \equiv \frac{1}{2}[\mu^{-2}(|\hat{u}_0|^2+|\hat{v}_0|^2)+|\phi_0|^2] = \frac{\xi^2}{\mu^2(m^2+n^2)}|\phi_0|^2, \tag{77}$$

$$A \equiv e_0'/\sqrt{\xi^2}, \tag{78}$$

and $\frac{D}{DT} \equiv \frac{\partial}{\partial T} + U\frac{\partial}{\partial X} + V\frac{\partial}{\partial Y}$.

Eq. (75) is the analogue of (33). Note that there should be more terms on the right hand side in (33) if the basic flow is not a nondivergent zonal one (see section V).

Suppose that there are westerlies with $V \approx 0$ in the Northern Hemisphere, hence $\partial U/\partial X = \partial V/\partial X = 0$ and $\partial V/\partial Y \approx 0$. From (73) and (75) we have: (a) When the wave packets are generated near the jet axis, the downstream propagating waves ($\xi > 0$) in both sides of the jet are amplified, while the upstream propagating waves ($\xi < 0$) are decaying. (b) When the source emitting the wave packets is located far away from the jet axis the downstream propagating wave packets first propagate toward the jet with gradual decrease of energy; and then either are reflected or penetrate from one side into the other side of the jet axis, in both cases they become amplified; but the upstream propagating waves first propagate toward the jet with gradual increase of energy, and then penetrate the jet axis and propagate away with continuous decrease of energy. (c) The weighted wave number, $\mu(m^2+n^2)^{1/2}$ of a southward propagating and decaying wave packet decreases, while it increases with a northward propagating and growing one; the change of weighted wave number is unclear with northward propagating and decaying waves or southward propagating and growing waves due to the influence of $\partial f^2/\partial Y$. The structure of the various wave packets is sketched in Fig. 6. Note that the word "wave" used here means only the large-scale inertio-gravity wave.

From (77) we conclude that a barotropic zonal flow is not efficient to absorb the large-scale inertio-gravity waves completely. In fact, we have the conservation of wave action if $V=0$ and $\partial U/\partial X + \partial V/\partial Y = 0$. However, ξ^2 is never zero, hence, e_0' neither. In a barotropic atmosphere the interaction of ultra-long wave (basic flow) and large-scale inertio-gravity waves or interaction of large scale divergent basic flow and meso-scale inertio-gravity waves are needed to absorb the inertio-gravity waves more efficiently.

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