ON THE LOW-FREQUENCY, PLANETARY-SCALE MOTION IN THE TROPICAL ATMOSPHERE AND OCEANS

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ABSTRACT

Scale analyses for long wave, zonal ultralong wave (with zonal scale of disturbance $L_1 \sim 10^4$ km and meridional scale $L_2 \sim 10^3$ km) and meridional ultralong wave ($L_1 \sim 10^3$ km, $L_2 \sim 10^4$ km) are carried out and a set of approximate equations suitable for the study of these waves in a dry tropical atmosphere is obtained. Under the condition of sheared basic current, frequency analyses for the equations are carried out. It is found that Rossby waves and gravity waves may be separated for $n \geq 1$ where n is the meridional wave number, whereas for $n=0$ and $L_1 \sim 1000$ km, the mixed Rossby-gravity wave will appear. Hence it is confirmed that the above results of scale analyses are correct. The consistency between frequency analysis and scale analysis is established.

The effect of shear of basic current on the equatorial waves is to change their frequencies and phase velocities and hence their group velocities. It increases the velocity of westward travelling Rossby waves and inertia-gravity and mixed waves, but decelerates the eastward inertia-gravity waves and the Kelvin wave. The recently observed low-frequency equatorial ocean wave may be interpreted as an eastward Kelvin wave in a basic current with shear.

I. INTRODUCTION

In recent years, much improvement has been achieved in the field of large-scale tropical motion. For the purposes of general global circulation modeling, monsoon research and numerical prediction of tropical atmospheric motion, it is necessary to elucidate the basic dynamic characteristics of the planetary-scale and low-frequency motion in the tropics. Charney (1963) first proposed that the large-scale motion in the tropical atmosphere is nondivergent and has weak communication in the vertical. But this suggestion disagrees with observed facts in the tropics, especially with those of planetary waves. Figure 1 shows that motions in upper and lower layers are considerably coupled. On the other hand, Matsuno (1966) pointed out that the principle characteristic of large-scale motion of the tropical atmosphere is that there are some kinds of low-frequency inertia-gravity waves, such as mixed Rossby-gravity waves and Kelvin waves, which disappeared in Charney's nondivergent model. They did so because in the nondivergent vorticity equation the gravity wave has been filtered with only the Rossby wave left. This case may be easily shown by using Charney's model of the form

$$\frac{\partial^2 v}{\partial t \partial x} + \bar{u} \frac{\partial^2 v}{\partial x^2} + \beta v = 0 ,$$

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where β is $\frac{\partial f}{\partial y}$, v the meridional component of velocity, and $\bar{u}$ the basic current assumed to be a constant. If we substitute the wave solution $v=v'e^{ik(x-ct)}$ into the above equation, it gives $c=\bar{u}-\beta/k^2$. Here c is wave velocity and k is wave number. Clearly it is the famous formula of Rossby wave velocity in the nondivergent case. All gravity waves including Kelvin waves and mixed waves have been filtered. Thus there is a contradiction between Charney's result and Matsuno's about large-scale motion in the tropics. In order to overcome this disagreement, Li and Yao (1979, 1980, 1981) have shown that the difference between the two theories mentioned above is owing to different meridional scales of motion used by the two authors. We noticed that the mixed wave can appear only in the case $y\to\pm\infty, v\to 0$ in Matsuno's paper. Here n is the number of zero points of the v solution in the meridional direction. This case corresponds to infinite meridional wavelength; in other words, the meridional scale of motion is at least one order larger than the zonal scale in Matsuno's model. Then if we take meridional scale L_y as O (10^7 m) instead of O (10^6 m), which was used by Charney, the divergent term is the same order as the βv term, and it is still retained in the vorticity equation of tropical motion. In fact, Murakami (1971) has made some computations. Using the boundary condition $v\to 0, y_w < 3000$ km instead of Matsuno's condition $v\to 0, y_w\to\infty$, he obtained the result that the frequencies of Rossby waves and gravity waves could be separated from each other and no mixed waves appear. This result confirms that very long meridional scale is the most important condition for the appearance of mixed waves.

In the present paper Section I gives some scale analysis and frequency analysis for tropical motions. We want to check the results by those two different methods against each other, emphasizing the characteristics of planetary-scale motion in the tropics. In Section II, the author uses the scale analysis to elucidate the characteristics of planetary-scale motion in the tropics. Section III deals with the effect of meridional shear of basic current on the frequencies and velocities of tropical waves.

II. SCALE ANALYSIS FOR THE TROPICAL ATMOSPHERIC MOTION

The momentum, hydrostatic, and continuity equations together with the first law of thermodynamics may be written in pressure coordinates and the β plane as

$$\frac{\partial u}{\partial t}-(\zeta+f)v+\omega\frac{\partial u}{\partial p}=-\frac{\partial P}{\partial x}, \tag{1}$$

$$\frac{\partial v}{\partial t}+(\zeta-f)u+\omega\frac{\partial v}{\partial p}=-\frac{\partial P}{\partial y}, \tag{2}$$

$$\frac{\partial^2\phi}{\partial t\partial p}+u\frac{\partial^2\phi}{\partial x\partial p}+v\frac{\partial^2\phi}{\partial y\partial p}+s\omega=0 \tag{3}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial\omega}{\partial p}=0, \tag{4}$$

where

$$P=\frac{u^2+v^2}{2}+\phi, \quad \omega=\frac{dp}{dt},$$

$$s=-\frac{RT}{p\theta}\frac{\partial\theta}{\partial p},f=\beta y,\beta=\frac{\partial f}{\partial y}.$$

Assume that

$$\phi=\phi_0(p,y)+\phi'(x,y,p,t), \tag{5}$$

where $\phi_0(p,y)$ is the geopotential height of the standard atmosphere and $\phi'(x,y,p,t)$ is fluctuation of geopotential height. We define u_0, v_0, L_1, and L_2 as the typical zonal and meridional velocities and space scales and ω_0, t, s_0, and P_0 as the scale of vertical motion, time, the stability factor, and vertical space scale in the p coordinate, and we select $T\sim L_1/u_0$. In the tropics the relation between wind and geopotential height is usually not geostrophic. Taking Eqs. (1) and (2) into account, we find

$$\begin{aligned}\delta\phi_x&\sim L_1(\zeta_0+f_0)v_0+\frac{u_0^2+v_0^2}{2},\\ \delta\phi_y&\sim L_2(\zeta_0+f_0)u_0+\frac{u_0^2+v_0^2}{2}.\end{aligned} \tag{6}$$

Here $\delta\phi_x$ and $\delta\phi_y$ indicate the magnitudes of turbulence ϕ' in the x and y coordinates, respectively, and ζ_0 is the typical scale of vorticity $\left(\zeta_0\sim\frac{V_0}{L_1}-\frac{u_0}{L_2}\right)$. Relation (6) suits all areas including middle latitudes and equatorial areas (where $f_0=0$). In the tropics ($f_0\sim10^{-5}$ s^{-1}), usually the first term on the right-hand side of (6) is larger than the last one, except in the case of Kelvin waves. Thus we have the following approximate relations:

$$\begin{aligned}\delta\phi_x&\sim L_1(\zeta_0+f_0)v_0,\\ \delta\phi_y&\sim L_2(\zeta_0+f_0)u_0.\end{aligned} \tag{7}$$

From (7) we can see that the order of variation of pressure in the tropics is one order smaller than that in middle latitudes because f_0 is one order smaller than it is therein. But if ζ_0 is larger than $f_0\sim10^{-5}$, the pressure may still be large, such as, in tropical storms. The variation magnitudes especially relate to scales and direction of tropical motion. The variation in planetary-scale is one order larger than in synoptic-scale. This result agrees with observed facts.

From thermodynamic equation (3) we have

$$\begin{aligned}\omega_0&\sim Ri^{-1}P_0(\zeta_0+f_0),\\ D_0&\sim Ri^{-1}(\zeta_0+f_0).\end{aligned} \tag{8}$$

Here R_i is the Richardson number in scale magnitudes. It has the form

$$Ri=\frac{s_0P_0^2}{u_0v_0}.$$

Charney (1963) first indicated the relative smallness of w_0 or P_0 in dry tropical atmosphere; we can appreciate that it is true for long wave scale motion. But it is changed for planetary-scale. This difference may be confirmed by the scaling of vorticity equation. From Eqs. (1) and (2), the form of the nondimensional vorticity equation is

$$\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+\frac{L_1v_0}{L_2u_0}\frac{\partial}{\partial y}\right)\zeta+\frac{2\omega\cos\phi}{a}\frac{L_1v_0}{u_0\zeta_0}\beta'v+\frac{2\omega\cos\phi}{a}\frac{L_1L_2}{u_0\zeta_0}Ri^{-1}\beta'yD$$

$$=\frac{\zeta_0+f_0}{u_0}L_1Ri^{-1}\left(\zeta D+\omega\frac{\partial\zeta}{\partial p}+\frac{v_0}{L_1\zeta_0}\frac{\partial\omega}{\partial x}\frac{\partial v}{\partial p}-\frac{u_0}{L_2\zeta_0}\frac{\partial\omega}{\partial p}\frac{\partial u}{\partial p}\right). \quad (9)$$

Here ϕ is latitude, a is the radius of the earth, and β' is a nondimensional value of β the typical value is $\beta\sim\frac{2\Omega\cos\phi}{a}\sim10^{-11}\mathrm{m}^{-1}\mathrm{s}^{-1}$. In the baroclinic atmosphere, the velocity of internal gravity waves is $c_0=(p_0^2s_0)^{1/2}\sim50$—$60$ m/s. Next we shall go into detail about (9).

(1) *Zonal planetary waves*

$$L_1\sim10^7\mathrm{m},\ L_2\sim10^6\mathrm{m},\ u_0\sim10^1\ \mathrm{m/s},\ v_0\sim10^0\ \mathrm{m/s}$$

and then

$$\zeta_0\sim\frac{v_0}{L_1}-\frac{u_0}{L_2}\sim\frac{u_0}{L_2}\sim10^{-5}\mathrm{s}^{-1},Ri\sim3\times10^2\ .$$

Then (9) gives

$$\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)\zeta+\beta v=0\ . \quad (10)$$

The divergence term is two orders smaller than the β term. This result is exactly the same as the result obtained by Charney for longwave-scale motion.

(2) *Meridional planetary waves*

These waves have the following typical scale:

$$L_1\sim10^6\mathrm{m},\ L_2\sim10^7\mathrm{m},\ u_0\sim10\ \mathrm{m/s},\ v_0\sim10\ \mathrm{m/s},$$
$$\zeta_0\sim v_0/L_1\sim10^{-5}\mathrm{s}^{-1},Ri\sim3\times10^1.$$

From (9) we have

$$\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}\right)\zeta+\beta v+\beta yD=0. \quad (11)$$

In this case the divergent term is the same order as the β term. But the advection of vorticity in the y direction is one order smaller than that in the x direction.

(3) *Very long waves (planetary waves in both the x and y directions)*

These waves have the following typical scale:

$$L_1\sim L_2\sim10^7\mathrm{m},\ u_0\sim v_0\sim10\ \mathrm{m/s},$$
$$\zeta_0\sim10^{-6},Ri\sim3\times10^1.$$

Then from (9) the divergent term is one order larger than the advective term but is the same order as the β term,

$$\beta yD=\beta v. \quad (12)$$

(4) *Long waves*

This case is synoptic-scale motion. It has the following typical scale;

$$L_1 \sim L_2 \sim 10^6\text{m},\ u_0 \sim v_0 \sim 10\ \text{m/s},$$
$$\zeta_0 \sim 10^{-5}\text{s}^{-1},\ Ri \sim 3\times 10^1.$$

Then (9) gives

$$\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)\zeta+\beta v=0. \tag{13}$$

This result is exactly the same as (10) and Charney's (1963) result.

(5) *Kelvin waves*

We can see that a kind of very important wave is the Kelvin wave which is not included in (10)—(13). Because this wave has the characteristic with $v=0$, in other words, v has the scale 10^{-1} m/s, the approximate relation, (7), is not correct any more in this case as in the one mentioned earlier. We should select another approximate relation. From (6) we have

$$\delta\phi_x \sim \delta\phi_y \sim u_0^2. \tag{14}$$

Substituting (14) into (3) gives

$$\omega_0 \sim Ri^{-1}P_0 u_0/L_1. \tag{15}$$

Here

$$Ri=\frac{P_0 s_0^2}{u_0^2}.$$

Then

$$D_0 \sim Ri^{-1}u_0/L_1. \tag{16}$$

Kelvin waves usually have the following typical scale:

$$L_1 \sim 10^7\text{m},\ L_2 \sim 10^6\ \text{m},\ u_0 \sim 10\ \text{m/s},\ v_0 \sim 10^{-1}\ \text{m/s}.$$

Substituting these scales and (14)—(16) into (1)—(4) gives:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}=-\frac{\partial\phi}{\partial x},$$

$$\beta y u=-\frac{\partial\phi}{\partial y},$$

$$\frac{\partial\phi}{\partial t\partial p}+u\frac{\partial^2\phi}{\partial x\partial p}+s\omega=0, \tag{17}$$

$$\frac{\partial u}{\partial x}+\frac{\partial\omega}{\partial p}=0.$$

As is well-known, (17) is a set of typical equations which describe Kelvin waves in the tropics. We can see that a Kelvin wave has a very long zonal wavelength, but it is confined to the tropics. Thus it is a special zonal planetary wave.

From (10)—(13) we can see that mixed Rossby-gravity waves can appear only in the meridional planetary wave. We can conclude that the very much larger meridional scale is the main characteristic of mixed Rossby-gravity waves; in the next section we confirm this conclusion by frequency analysis.

For planetary waves we have obtained Eq. (12) which is the same as for middle latitudes and obeys the steady vorticity equation. It is not surprising, because the geostrophic relation is also correct for planetary-scale motion in the tropics.

III. FREQUENCY ANALYSIS FOR THE TROPICAL ATMOSPHERIC MOTION

1. *Frequency Analysis of Wave Motion in the Tropics*

We consider $u=U(y)+u'(x,y,p,t), \phi=\bar{\phi}(y)+\phi'(x,y,p,t)$. Here $U(y)$ is the basic current. The linearization of Eqs. (1)—(4) results in

$$\begin{aligned} &\frac{\partial u}{\partial t}+U\frac{\partial u}{\partial x}-\beta y^* v=-\frac{\partial \phi}{\partial x}, \\ &\frac{\partial v}{\partial t}+U\frac{\partial v}{\partial x}+\beta y u=-\frac{\partial \phi}{\partial y}, \\ &\frac{\partial^2 \phi}{\partial t \partial p}+U\frac{\partial^2 \phi}{\partial x \partial p}+v\frac{2\bar{\phi}}{\partial y \partial p}+sw=0, \\ &\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial p}=0. \end{aligned} \tag{18}$$

Following More and Philander (1977), we seek a separable solution of the form

$$\left.\begin{aligned} u&=u'(x,y,t) \\ v&=v'(x,y,t) \\ \phi&=\phi'(x,y,t) \end{aligned}\right\} G(p). \tag{19}$$

If (19) is substituted into the first three equations of (18), then, using the continuity equation, we obtain

$$\begin{aligned} &\frac{\partial u'}{\partial t}+U\frac{\partial u'}{\partial x}-\beta y^* v'=-\frac{\partial \phi'}{\partial x}, \\ &\frac{\partial v'}{\partial t}+U\frac{\partial u'}{\partial x}+\beta y u'=-\frac{\partial \phi}{\partial y}, \end{aligned} \tag{20}$$

and

$$\frac{\partial \phi'}{\partial t}+U\frac{\partial \phi'}{\partial x}+\beta y U v'+c_0^2\left(\frac{\partial u'}{\partial x}+\frac{\partial v'}{\partial y}\right)=0.$$

We assume

$$\beta y U=-\frac{\partial \bar{\phi}}{\partial y}. \tag{21}$$

Here $c_0=\left[\frac{R^2 T}{g}(\gamma_d-\gamma)\right]^{\frac{1}{2}}$ is the speed of internal gravity waves; γ is vertical lapse rate; γ_d, the adiabetic lapse rate.

Now we define the typical space and time scale as

$$T=\frac{1}{\beta L}=(c_o\beta)^{-1/2}, L=(c_o/\beta)^{1/2}. \tag{22}$$

Again let u, $\bar{U}$, and $c_o u$ be the scale of the velocity of basic current, and turbulence (ϕ') respectively. Now we consider the wave solution

$$u, v, \phi \sim e^{i(kx+wt)}, \tag{23}$$

where u, v, and ϕ are nondimensional and k and ω are the nondimensional wave number and frequency, respectively. Then we have

$$k=k'L,\ \omega'=\beta L\omega,\ \text{and}\ R=U/c_0. \tag{24}$$

Furthermore, if U_y is the shear of basic current $U(y)$, we may write

$$U(y)=U_y(y-y_c)=U_0+U_y y. \tag{25}$$

Substituting Eqs. (22)—(25) into the linearized primary equations (20) of large-scale motion of the tropical atmosphere and neglecting the high-order term of R, we obtain

$$\frac{d^2v}{dy^2}+[a_0+b_0y+c_0y^2]\frac{dv}{dy}+[a_1+b_1y-y^2]v=0 \tag{26}$$

where

$$b_0=-RU_0,c_0=-RU_y,a_1=\frac{k}{\sigma_0}-RU_0-\Delta_0,$$

$$b_1=\left(\frac{2k^{1/2}}{\Delta_0}-\frac{k^2}{\sigma_0^2}+2\sigma_0k-1\right)RU_y, \tag{27}$$

$$\sigma_0=\omega+RkU_0,\ \Delta_0=k^2-\sigma_0^2,\ a_0=\frac{2\sigma_0k}{\Delta_0}RU_y.$$

Eq. (26) is the same as that obtained by Bennet and Young (1971). If we set $R=0$ (no shear of basic current U), Eq. (26) will become the Schrodinger equation investigated by Matsuno (1966) and Kuo (1977). If we set

$$W=v\exp\left(\frac{c_0}{6}y^3+\frac{2+b_0}{4}y^2+\frac{a_0+c_0-b_0y}{2}\right),$$

$$Z=\frac{1}{2}\left[\sqrt{2}y-\frac{\sqrt{2}}{2}(b_1^2-c_0^2)\right]^2 \tag{28}$$

then Eq. (26) may be written as

$$Z\frac{d^2W}{dZ^2}-\left(\frac{1}{2}-Z\right)\frac{dW}{dZ}+\frac{\alpha}{2}W=0, \tag{29}$$

in which

$$\alpha=\frac{1}{2}\left[\left(\frac{k}{\sigma_0}-RU_0-\Delta_0\right)+\frac{1}{2}RU_0-1\right]. \tag{30}$$

Eq. (29) is a special case of a confluent hypergeometric equation. Its general solution may be written as

$$W=k_1M\left(-\frac{\alpha}{2},\frac{1}{2},Z\right)+k_2M\left(-\frac{\alpha}{2}+\frac{1}{2},\frac{3}{2},Z\right), \tag{31}$$

where α is the eigenvalue of the equation and M makes use of the boundary condition

$$y=\pm y_W,\ v=0. \tag{32}$$

We may determine α according to whether Eq. (26) is being considered a symmetric or asymmetric solution (the eigenvalue of the asymmetric case is α). In the symmetric case,

$$M\left\{-\frac{\alpha}{2},\frac{1}{2},\frac{1}{2}\left[\sqrt{2}y-\frac{\sqrt{2}}{2}(b_1-c_0)\right]^2\right\}=0, \tag{33}$$

while in the asymmetric case

$$M\left\{-\frac{\alpha^*}{2}+\frac{1}{2},\frac{3}{2},\frac{1}{2}\left[\sqrt{2}\,y-\frac{\sqrt{2}}{2}(b-c_0)\right]^2\right\}=0. \qquad (34)$$

If we take $\beta=2.29\times10^{-11}\mathrm{m}^{-1}\mathrm{s}^{-1}$, $R=0.1$, and $\bar{U}=10$ m s^{-1}, the first two eigenvalues of α computed by Eqs. (33) and (34) are shown in Table 1.

Table 1. The Relationship between α and y_w

y_W (km)	Symmetric Case		Asymmetric Case	
	α_1	α_2	α_1^*	α_2^*
1×10^3	4.37	23.29	14.10	46.67
2×10^3	1.09	7.61	4.28	14.28
3×10^3	0.49	4.60	4.46	7.85
4×10^3	0.27	3.51	1.82	57.3
5×10^3	0.17	2.99	1.52	4.75
6×10^3	0.12	2.70	1.36	4.22
7×10^3	0.09	2.52	1.27	3.89
8×10^3	0.06	2.40	1.20	3.63
9×10^3	0.05	2.32	1.16	3.54
10×10^3	0.04	2.26	1.12	3.43

From Table 1, we may see that when y_W is very large, $\alpha_1\to0,\alpha_2\to2,\alpha_1^*\to1$, and $\alpha_2^*\to3$. These eigenvalues are completely equivalent to those obtained from the special boundary condition $y_W\to\pm\infty, v\to0$. In order to explore the effects of the shear of basic current and meridional scale, we may select the finite value of y_W. Now we analyze the effects of y_W on the frequency. From Eq. (30) we have

$$\frac{k}{\sigma_0}-k^2+\sigma_0^2-\frac{1}{2}RU_0=2\alpha+1. \qquad (35)$$

The approximate solution of Eq. (35) for $\alpha\geqslant1$ is

$$\omega_1\simeq\frac{k}{k^2+2\alpha+1}+\left[1-\frac{1}{2(k^2+2\alpha+1)^2}\right]RkU_yy_c, \qquad (36)$$

$$\omega_{2,3}\simeq\pm\left(k^2+2\alpha+1+\frac{1}{2}RU_yy_c\right)^{\frac{1}{2}}+RkU_yy_c, \qquad (37)$$

where ω_1 is the frequency of Rossby waves and $\omega_{2,3}$ are the frequencies of inertia-gravity waves. Eqs. (36) and (37) express that these two kinds of waves can separate completely for the case $\alpha\geqslant1$. For the case $\alpha=0$, the two solutions of Eq. (35) are

$$\omega_1\simeq-\left(\frac{k^2}{4}+1+\frac{1}{2}RU_yy_c\right)^{\frac{1}{2}}-\frac{k}{2}+RkU_yy_c, \qquad (38)$$

$$\omega_2\simeq\left(\frac{k^2}{4}+1+\frac{1}{2}RU_yy_c\right)^{\frac{1}{2}}-\frac{k}{2}+RkU_yy_c, \qquad (39)$$

where ω_1 and ω_2 are, respectively, the frequency of east propagating inertia-gravity waves and west propagating mixed Rossby-gravity waves.

Therefore we conclude that only for the case $\alpha=0$ can these two kinds of waves be mixed. They are completely separable from each other, for the case $\alpha\geqslant 1$. From Table 1 we may see that either case may be separable for $y_w<3000$ km. The mixed wave can not be present for $y_w>7000$ km and all $\alpha\geqslant 1$ either. This fact shows that when $y_w>7000$ km the mixed wave can be present only with the first eigenvalue of symmetry. Because α_1 is the first eigenvalue of symmetry when $y=y_w$, $v=0$, y_w is equivalent to half a wavelength; therefore it is a long-wave disturbance for the case, $y_w<3000$ km. As for $y_w>7000$ km, it is an ultralong wave. Therefore we can conclude that, when the scale of disturbance *in the meridional direction is a long wave scale in the tropics,* both Rossby waves and gravity waves may be clearly separated. But the mixed Rossby-gravity waves *will appear only in the case of ultralong waves in the meridional direction.*

At the same time, from Eqs. (38) and (39), ω_2 is the frequency of west propagating inertia-gravity waves for the case $0\leqslant k\leqslant\frac{1}{\sqrt{2}}$; and ω_2 is the frequency of Rossby waves for the case $k\geqslant\frac{1}{\sqrt{2}}$. These two kinds of waves may thus be completely mixed for the case $k=\frac{1}{\sqrt{2}}$. The zonal wavelength corresponding to $k=\frac{1}{\sqrt{2}}$ is 10^3 km, i. e., the zonal characteristic scale is about 10^3 km. This is a long-wave scale.

Thus we conclude that the mixed Rossby-gravity waves may be present only if the meridional characteristic scale is about 10^4 km and the zonal scale is about 10^3 km, i. e., only when the meridional ultralong wave is in the unique weather situation, and the scale analysis made in the previous section is in accordance not only with Matsuno's (1966) linear analysis but also with the analysis considering the effects of shear of basic current in the tropics.

In the case of zonal ultralong waves, $k\ll 1$ and $\alpha\gg 1$, from (35) we have ($U=0$)

$$\omega\sim\frac{k}{2\alpha+1}.$$

This is a Rossby wave's frequency, and gravity waves have been filtered. This result is also in agreement with scale analysis.

2. *The Effect of the Latitudinal Shear of Basic Current on Wave Velocity*

The expressions of wave velocity corresponding to Eqs. (36) and (37) are

$$c_1=U_0-\left\{\frac{\beta}{k^2+\frac{\beta}{c}(2\alpha+1)}\right\}-\left\{\frac{\beta^2}{c^2\left[k^2+\frac{\beta}{c}(2\alpha+1)^2\right]}\right\}U_y y_c, \tag{40}$$

$$c_{2,3}=U_0\mp c_0\left(1+\frac{\beta}{k^2c}(2\alpha+1)+\frac{1}{2}\frac{\beta}{k^2c}U_y y_c\right)^{\frac{1}{2}}. \tag{41}$$

Clearly c_1 is the velocity of east propagating Rossby waves and $c_{2,3}$ are the velocities of west propagating inertia-gravity waves. Eqs. (40) and (41) show that the shear of basic current increases the velocity of west propagating Rossby waves and inertia-gravity waves, but decreases the velocity of east propagating inertia-gravity waves.

The velocities corresponding to Eqs. (38) and (39) are

$$c_1=U_0+c_0\left[\left(\frac{1}{4}+\frac{\beta}{k^2c}+\frac{1}{2}\frac{\beta}{k^2c}U_y y_c\right)^{\frac{1}{2}}+\frac{1}{2}\right], \tag{42}$$

$$c_2=U_0-c_0\left[\left(\frac{1}{4}+\frac{\beta}{k^2c}+\frac{1}{2}\frac{\beta}{k^2c}U_y y_c\right)^{\frac{1}{2}}-\frac{1}{2}\right], \tag{43}$$

where c_1 is the velocity of inertia-gravity waves, c_2 is that of mixed waves. Eqs. (42) and (43) show that due to the effect of basic current, the velocity of west propagating mixed waves increases.

It may be shown that in the case $U=0$, the equations mentioned above are the same as those obtained by Matsuno (1966) and Kuo (1977). So the effect of shear of basic current leads to great changes of dynamical characteristics in the tropics.

Finally, if we take frequency analysis on (17) just the same as we have done on Eq. (18), the frequency of Kelvin waves in tropical motion under the effect of shear of basic current is

$$\omega=-k[1+RU_y(y-y_c)]. \tag{44}$$

Thus the relation between wavenumber and frequency varies with latitude. But if $k=0$, we also have $\omega=0$. This is a special case of Kelvin waves; when $R=0$, then $\omega=-k$. From Eq. (44) the shear of basic current will decrease the velocity of eastward propagating Kelvin waves.

The velocity corresponding to Eq. (44) is

$$c=U_0+c_0[1+RU_y(y-y_c)]. \tag{45}$$

3. *Standing Equator Waves*

Both in ocean and in atmosphere, there is a kind of quasi-standing low-frequency equator wave. Gent (1979) pointed out that it may be a short eastward propagating Rossby wave. Chang (1977), Luther (1980), and Mofjeld (1981) suggested that it is a viscous Kelvin wave. But Cane and Moore (1981) suggested that it is the basic mode of the sum of a Kelvin wave and its eastern boundary reflection. We think the shear of basic current may play the role of standing low-frequency waves.

At the equator in both atmosphere and ocean the basic currents are easterlies (Fig. 1 and 2) with magnitudes O (10 m/s) and O (10^0 m/s) in the atmosphere and ocean, respectively. From Eq. (25) we can see $U_0<0$, and $U_y y_c>0$. In the dimensional case,

$$U_0\approx -U_y y_c\sim\begin{cases}-10 \text{ m/s in the atmosphere,}\\ -10^0 \text{ m/s in the ocean,}\end{cases}$$

so that in the nondimensional case $U_0\sim -U_y y_c\sim 1$. We now pay attention only to eastward propagating waves, since this standing low-frequency wave propagates very slowly. From Eqs. (40)—(45), only two kinds of waves are eastward propagating. They are propagating inertia-gravity waves and Kelvin waves. Owing to the term $U_y y_c>0$ in (41) and (42), the decreases of velocity of eastward propagating inertia-gravity waves due to the effect of shear of basic current are not very remarkable. The velocity of Kelvin waves at the equator is

$$c=U_0+c_0\left[1-\frac{\beta}{k^2c}U_y y_c\right]. \tag{46}$$

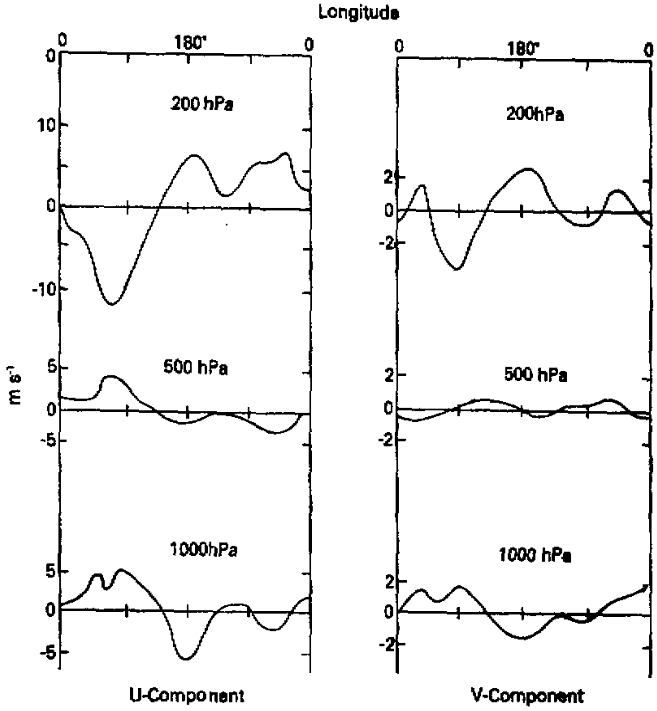

Fig. 1. The distribution of tropical atmospheric winds in the troposphere by longitude.

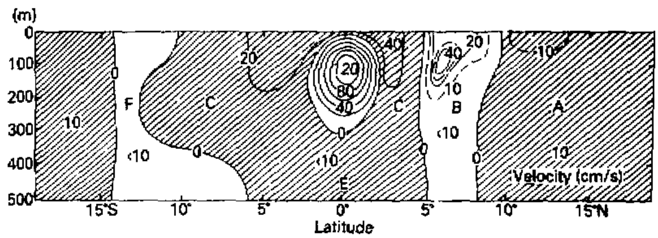

Fig. 2. The zonal velocity distribution in the ocean. Shaded areas are the easterly flows; unshaded areas, westerly. Units in cm s⁻¹.

For $U_y y_e > 0$, $U_0 < 0$, the c is very small both for the atmosphere and for the ocean. The magnitude of c is shown in Table 2.

Table 2. The Velocity in Atmosphere and Ocean

	U_s(m/s)	c_0(m/s)	$U_y y_e$(m/s)	c(m/s)
ocean	−1.0	2.0	1	
atmosphere	−10	15	10	1.5

In the atmosphere $\begin{cases} c_o = 15\text{m/s}(\text{Madden and Julia, 1978}) \\ U_0 = 10\text{m/s}. \end{cases}$

In the ocean $\begin{cases} c_0 = 2.0\text{m/s}(\text{McWilliam and Gent, 1978}) \\ U_0 = 1\text{m/s}. \ (\text{Philander, 1978}) \end{cases}$

For the frictionless case, both in the ocean and in the atmosphere, the Kelvin waves travel very slowly, especially in the ocean, where they are almost trapped. Most recently Gent (1980), Cane et al. (1981) and Lau (1981) discussed these waves, but they did not consider this effect. It is in agreement with observations.

4. *The Effects of Latitudinal Shear of Basic Current on the Frequency*

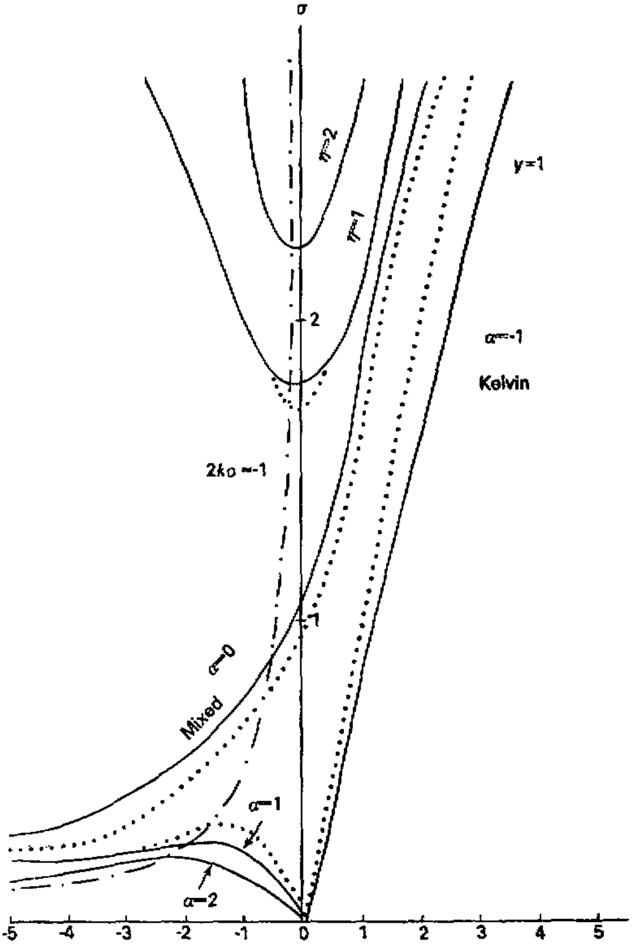

Fig. 3. The dispersion relation for equatorial waves (from pedlosky, 1980).
The dotted lines are the mixed and Kelvin wave dispersion curves as affected by the basic current shear.

Figure 3 shows the dispersion relation for equatorial waves under the effects of latitudinal shear of basic current. The σ is the relative frequency, $\sigma=w+kKU$. We can see that for low values of α the effect is considerable. The increase of mixed wave frequency is very remarkable, and it increases the minimum frequency of gravity waves and decreases the maximum frequency of Rossby waves slightly. This effect can be realized from the dispersion relation. Differentiating (35) with respect to k, we find

$$\frac{\partial\sigma}{\partial k}=\frac{2k\sigma-1}{2\sigma^2-k/\sigma}.$$

Then when

$$2\sigma k=1, \tag{47}$$

we obtain

$$\sigma_{min}=\frac{(2\alpha+1-RU_0/2)^{1/2}}{2}+\frac{(4\alpha^2+4\alpha-RU_0/2)^{1/4}}{2}, \tag{48}$$

which is the minimum frequency for the αth gravity wave mode under the effects of shear of the basic current in the tropics, and

$$\sigma_{max}=\frac{(2\alpha+1-RU_0/2)^{1/2}}{2}-\left(\frac{4\alpha^2+4\alpha-RU_0/2}{2}\right)^{1/4}, \tag{49}$$

which is the maximum frequency of the αth Rossby mode. Thus the shear of basic current decreases the maximum frequency of the Rossby mode and increases the minimum frequency of gravity waves, especially for low α values.

There is no effect of the shear of the basic current on Kelvin waves at the equator ($y=0$), but there is a considerable effect on Kelvin waves in the case of $y=1$.

In the case of the zonal ultralong wave, $\alpha>1$ and $k^2\leqslant 1$. From (35) we have

$$\sigma=-\frac{k}{2\alpha+1+k^2+\frac{1}{2}U_0k}. \tag{50}$$

This is the Rossby wave's frequency in the basic current shear. When $k^2\ll 1$, then we have

$$\omega\approx\frac{k}{2\alpha+1+\frac{1}{2}U_0k}. \tag{51}$$

This longwave approximation has been shown (Lighthill 1969, Lau 1982) to be equivalent to assuming nondivergence and geostrophic balance in the zonal direction. When $\alpha=-1$ and $U=0$, then

$$\omega=-k. \tag{52}$$

This is the Kelvin wave mode (Pedlosky 1980, Lau 1982). Hence when the geostrophic balance is assumed in the zonal direction, although the mixed waves and other high-frequency gravity waves are lost, the most important low-frequency waves (low-frequency Rossby waves, Kelvin waves) can still be retained. This approximation is a good model for studying tropical planetary motion (Lau 1982, Lighthill 1969).

For ultralong waves, where $k^2\ll 1$ and $\alpha=0$, from (50) we have

$$\omega\sim\frac{k}{1+\frac{1}{2}U_0k}. \tag{53}$$

When $U_0=0$, then we have

$$\omega=k. \tag{54}$$

It is opposite in sign to (52). Like those in middle latitudes, ultralong waves in the tropics are also undispersed.

IV. CONCLUSION

The planetary-scale and low-frequency waves have been studied by both scale analysis and frequency analysis in the tropics. It has been shown that zonal planetary waves which have the same characteristics as normal long waves are nondivergent, and this result coincides with the result firstly obtained by Charney in scale analysis. Kelvin waves are also zonal planetary waves, but $v=0$ everywhere. However, meridional planetary waves are divergent, with strong vertical couples. Mixed waves belong to this kind of wave. Ultralong waves (in both zonal and meridional planetary scales) are geostrophic but have divergent motion and strong vertical couples. These two kinds of waves are very important for monsoon air-sea interactions and cross-equator motion. We will further analyse effects of external forces on this low-frequency planetary-scale motion, and spherical geometry needs to be included in the analysis of these planetary-scale motions.

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