Predictability of the Atmosphere

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ABSTRACT

This paper makes a review on the predictability of the atmosphere. The essential problems of predictability theory, i.e., how a deterministic system changes to an undeterministic system (chaos) and how is the opposite (order within chaos), are discussed. Some applications of predictability theory are given.

I. INTRODUCTION

If the influence of human activity on weather and other similar effects are not considered, the atmospheric system can be regarded as a deterministic system. Its future state is completely determined by physical laws (differential equations) governing the system evolution, environmental conditions (boundary conditions) and initial situation (initial conditions). Numerical weather prediction is based on this viewpoint. It seems that if once there were a fully accurate model and an observational system, one could do sufficiently accurate weather forecast over an arbitrarily long period. In the circumstances, however, small errors, which are unavoidable, at initial moment may still produce very large deviations at a certain moment with increasing time.

During early 1940s, in a series of reports, Kolmogoroff demonstrated that the small errors of initial atmosphere state may lead to different atmosphere state in a long time. However, it was Thompson(1957) who first raised the problem of predictability. Lorenz(1969a) showed that there are three approaches to the study of atmospheric predictability. Up to date, most predictability studies have been based upon numerical models. Solutions originating from a "correct" initial state are determined, and the rate at which these solutions diverge is observed. It has become customary to summarize the results in terms of a doubling time for small-amplitude errors. In fact, whenever there is a model, this can be done. Some famous examples of numerical experiments have been done by Lorenz (1965, 1969a, 1982), Charney et al. (1966) and Smagorinsky (1969). The main achievement is the finding that weather forecast has a limit of predictability. The doubling time of small initial errors is 2–5 days. Estimates of the limit to the predictability of the total flow range from 8 to 16 days. Predictability decreases with decreasing horizontal scale. It is lower in the tropics than in the middle and the high latitudes, and higher in winter than in summer for the Northern Hemisphere. Predictability also relies on synoptic systems, i.e., some atmospheric states are more predictable than others. Ageostrophic, random initial perturbations develop more slowly than geostrophic, systematic perturbations do. This method suffers from the fact that the growth of errors is model-dependent, therefore different models yield different estimates of error growth for predictability. Another method is to calculate the growth of errors in homogeneous turbulence models (Leith, 1972). This method suffers from the limitation that the real atmosphere does not always behave as the idealized models, especially in the presence of forcing at the lower boundary and diabatic heating. The method suggested by Lorenz (1969c) is to examine the rate of divergence of pairs of close analogs in the real atmosphere. This method, which is the most attractive one from the conceptual point of view because it makes use of real atmospheric behaviors, suffers from the absence of close analogs.

After exceeding the limit of predictability, individual synoptic–scale perturbation goes from determinant to undeterminant. In this case, it is more helpful to study the forecasting of spatial and temporal mean values. Monin (1972) considered that it is not a constructive problem to determine the limit of predictability in itself (it itself must not be the goal). To solve the problem of long–term predictability constructively, it is required to point out the characteristic of meteorological fields predicted in this period. However, Marchuk (1974) believed that it is more suitable to do every ten–day averaged forecasts for near a monthly weather forecast, i.e., the means of the first, the second and the last ten days of a month. If we are interested in the weather forecasts of the next season, then we do the time–mean forecasts of the first, second, and third month of the season. The characteristic scale of forecast area (i.e., the range of the average of area) increases with the increase of forecast time limitation. For instance, the scale is 1000 × 1000 km for ten–day average, and 3000 × 3000 km for monthly mean. In the recent years, most researches were made for the predictability and the limit of predictability of spatial and temporal mean fields. A series of work of Shukla (1981, 1982, 1983, 1986) proposed the concepts of dynamic and forcing predictability for monthly mean fields and showed that the predictability of atmospheric dynamics exceeds a month, but the anomaly of underlying surface plays greater roles in two months or seasonal long–term forecasts. In this respect, the seasonal variation also becomes an important element. Based on experiments by using GFDL model, Miyakoda (1980) found that ten–day mean is better than daily forecast, and twenty–day mean is better than ten–day mean forecast. Mansfield(1986) and Miyakoda et al.(1987) studied prediction of monthly mean circulation by using numerical model, and showed that the predictive accuracy is seemingly dependent on flow pattern. Madden (1982) first proposed the concept of climate noise, and discussed the potential predictability of long–term forecast. These works are equivalent to the works which treats from an undeterministic system to a deterministic system. The essential problem of predictability theory is how a deterministic system changes to an undeterministic system (chaos) and how is the opposite (order within chaos). The following section will discuss this essential problem. Finally, some applications of predictability theory are given.

II. MATHEMATICAL THEORY

1. *An Intuitive Example*

The future evolution of a deterministic system is uniquely determined by initial values. However, under certain conditions, small initial differences can lead to large ones at a certain time. Since initial values may not be known accurately, its evolution then can not be predicted. Chou (1986) gave an intuitive example, which vividly demonstrates the production of this situation, using the following famous maximum simplified model proposed by Lorenz (1960) and expressed in the form of the dynamic equations, thus their most essential properties are maintained:

$$\frac{dX}{dt} = -\frac{1}{\sqrt{2}}\left(\frac{1}{k^2} - 1/(k^2 + l^2)\right) klYZ \tag{1}$$

$$\frac{dY}{dt}=\frac{1}{\sqrt{2}}\left(\frac{1}{l^2}-1/(k^2+l^2)\right)klXZ \tag{2}$$

$$\frac{dZ}{dt}=-\frac{1}{\sqrt{2}}\left(\frac{1}{l^2}-1/k^2\right)klXY. \tag{3}$$

It maintains the characteristics of conservation of mean kinetic energy E and of mean vorticity squared V of primary barotropic vorticity equation. When $\alpha=k/l>1$, these conservative properties can be written as

$$X^2+Y^2+Z^2=X_0^2+Y_0^2+Z_0^2, \tag{4}$$

$$\frac{\alpha^2-1}{2}X^2-\frac{1}{2(1+\alpha^2)}Z^2=\frac{\alpha^2-1}{2}X_0^2-\frac{1}{2(1+\alpha^2)}Z_0^2, \tag{5}$$

where X_0, Y_0, Z_0 are the initial values. The X, Y, Z are regarded as a point of the space. This point $(X(t), Y(t), Z(t))$ draws a curve in the space. It equals to determine a direction fields in the space for solving the ordinary differential equations of (1)–(3). There exists a unique solution at given initial values. It means that any point in space is always passed by one and only one curve. How are these curves? We can easily see that in $\alpha>1$ case, they satisfy Eqs. (4) and (5). Eq.(4) represents a sphere, and its center is located at the origin of the coordinates. When $X_0=Z_0=0$, Eq.(5) represents two planes which intersect at Y–axis, i.e., $Z=\sqrt{\alpha^2-1}\,X$, $Z=-\sqrt{\alpha^2-1}\,X$, they divide the space into four parts which contain positive Z–axis, negative Z–axis, positive X–axis, negative X–axis, separately. When both X and Z are not zero, Eq.(5) represents an one sheet hyperboloid located in certain region of four parts. The integral curve is the intersection line of hyperboloid and sphere. No matter how close two points (which are situated at two sides of a plane) may be, (for example, the precision of modern computer is 10^{-15}, the initial location difference is ε), if these two points of $\varepsilon \ll 10^{-15}$ are in different regions, then the integral curve of one point is a closed curve around Z–axis and another is around X–axis. Their distance will be sufficiently far sooner or later. It is seen directly that a deterministic system changes to an undeterministic one at a certain time, due to the fact that the initial values are not known exactly. In addition, one can assume that the initial values are known in the way of absolutely accurate and are near the plane, but the parameter α in the equation cannot be known absolutely accurately and has an extremely small error. This will lead to the change of intersection angle between the planes and the region which initial value belongs to, and also the change of a deterministic system to an undeterministic one at a certain time. This characteristic is different from the former, the reason of the difference is caused either by initial error (equation is accurate), or by the parameter error in equation (initial values are accurate). In fact, both errors exist at the same time. For the real world, we have nothing that can be known absolutely exactly.

Nevertheless, the above situation solely occurs near particular points, i.e., near the points on plane. All of the points on the whole plane are set zero about their measure. The atmospheric sensitivity to initial values is much more common. It is concerned with non–periodic solution of dissipative system rather than periodic solution of conservative system.

2. *The Sensitive System to Initial Values: Chaos and Turbulence*

When Lorenz (1963) studied finite–amplitude convection, he proposed 3–order ordinary differential equations and gave the first example of the strange attractor which is the strictly non–periodic special solution of the equations. The motions outside attractor are all close to it. The whole is stable. When the orbits go into attractor, they are separated exponentially and become a sensitive system to initial values. Just because of the sensitivity to initial values, it makes the mean values on strange attractor of the physical variable be contrarily no longer sensitive to initial values. The motions on the strange attractor are not only ergodic but also mixing. One can introduce a stable distribution function and make statistical description. Ruelle and Takens (1971) connected the strange attractor with turbulence behaviors, thus, the predictability is physically the problem of turbulence out–growth.

There are conditions for the production of predictability. For a forced dissipative nonlinear system, the conditions leading to its sensitivity to initial value are:

1. There is a strange attractor in the system and the initial values are on the attractor.
2. There are many attractors in the system and initial values on dividing line of domains of attraction.
3. The system parameters are of small errors and lead to instability.

If initial values are not on attractors or there is no strange attractor, the initial errors do not increase rapidly.

All mentioned above, of course, are idealized, the real atmosphere is close to one of these only in some time and some aspects.

Since the above discussions are based on a low order model it is naturally to ask whether they are tenable for real complex atmospheric system. In recent years, the author and his collaborator did a series of studies, and showed that the above results of low order model are of universal significance. The summary is given as follows.

3. *The Global Asymptotic Behaviors of the Partial Differential Equations for the Atmospheric System*

After introducing Hibert space which consists of the vector functions, Chou (1983) proved that the large–scale atmospheric dynamic equations can be written as the following operator equation.

$$B\frac{\partial\varphi}{\partial t} + N(\varphi)\varphi + L\varphi = \xi \tag{6}$$

$$B\varphi = B\varphi_0, \quad \text{when} \quad t = t_0, \tag{7}$$

where B, L are self–adjoint and positive definite operators; N, anti–adjoint operator, i.e.,

$$(\varphi_1, B\varphi_2) = (\varphi_2, B\varphi_1)$$
$$(\varphi_1, L\varphi_2) = (\varphi_2, B\varphi_1)$$
$$(\varphi, B\varphi) \geqslant 0, \quad (\varphi, L\varphi) \geqslant 0$$
$$(\varphi_1, N(\varphi)\varphi_2) = -(\varphi_2, N(\varphi)\varphi_1).$$

By using the above–mentioned properties of the operators, we have discussed the characteristic of the global asymptotic behaviors of the partial differential equations of the atmospheric system. Wang, Huang and the author (1988) have proved that the large–scale atmospheric dynamic equations exist in a bounded global absorbing set B_K. In addition, they estimated a definite critical time t_0, and proved that if

$$t > t_0 = \frac{1}{\tilde{c}_1} \ln \frac{|B\varphi_0|^2 - \frac{1}{\tilde{c}_1 c_1}|\xi|^2}{K - \frac{1}{\tilde{c}_1 c_1}\|\xi\|^2}, \tag{8}$$

the orbits which start from φ_0 must go into B_K, and remain in B_K. Based on this proving they further proved the existence of invariant set in B_K and revealed that the system is nonlinearly adjusted to exterior sources.

On the other hand, Folas and Temem (1979) first proved that the attractor for the Navier–Stokes equations is finite–dimensional, while Constantin et al. (1985) have derived sharp estimates of the fractional dimension for nonconvection turbulent flow. More recently, Ghidaglia (1986) proved that attractors to various equations of viscous incompressible fluid flows, such as thermo–hydraulic equations and magnetohydrodynamic equations, have finite fractional dimension and lie in the set of C^∞. Folas et al.(1987) have obtained bounds for the dimension of attractors which have physical interest (i.e., in terms of nondimensional physical numbers) in case of convection equations

$$\rho_0\left(\frac{\partial u}{\partial t} + (u \cdot \nabla)u\right) - \rho_0 v\Delta u + \nabla p = \rho_0 g\left[1 + \alpha(T_1 - T_0)\right], \tag{9}$$

$$\rho_0 C_v\left(\frac{\partial T}{\partial t} + (u \cdot \nabla)T\right) - \rho_0 C_v K\nabla T = 0, \tag{10}$$

where $u=(u_1, u_2)$ or $u=(u_1, u_2, u_3)$ is the velocity of the fluid, p is the pressure, g is the gravity, $\rho_0 > 0$ is the constant mean density, α is the volume expansion coefficient of the fluid, C_v is the specific heat at constant volume and v and k are the (constant) coefficients of kinematic viscosity and thermometric conductivity, respectively. T_0 and T_1 are the temperature at lower and upper plates. We introduce the usual nondimensional numbers, i.e. Grashof (G_r), Prandtl (P_r) and Rayleigh (R_a):

$$G_r = (v')^{-2}, \quad P_r = \frac{v'}{k'}, \quad R_a = (v'k')^{-1},$$

where

$$v' = \frac{v}{\left(h^3 g\alpha(T_0 - T_1)\right)^{\frac{1}{2}}}, \quad k' = \frac{k}{h^3 g\alpha(T_0 - T_1)^{\frac{1}{2}}},$$

They proved the existence of functional invariant sets in dimensions 2 and 3. The fractional dimension of the attractor is found by 2 m. In the two–dimensional case,

$$m \leqslant C_1\left\{G_r^{\frac{1}{2}}(1 + P_r) + G_r(1 + P_r)^2\right\}.$$

In the three–dimensional case,

$$m \leqslant C_2\left\{(R_a + G_r^{\frac{1}{2}})^{\frac{3}{2}} + \left(\frac{l_0}{l_d}\right)^3\right\}\left(1 + P_r\right)^{\frac{3}{2}},$$

where

$$l_d = (v^3 / \varepsilon)^{1/4}.$$

ε is the Kolmogoroff dissipation length (the dissipation rate of the energy per unit mass and time averaged on the attractor). l_0 is a (dimensional) typical macroscopic length.

Based on the works mentioned above, Wang, Huang and the author (1988) made use of the similar method under suitable widespread conditions, and proved that large–scale atmospheric system of equations has finite fractional dimension, and obtained bounds for the dimension of attractors. They showed that the global asymptotic behaviors of the partial differential equations of atmospheric dynamics which have infinite freedoms can be described by finite ordinary differential equations in ideal stable surrounding conditions.

4. *In Space R^n*

The replacement of partial differential equations by ordinary differential equations is equivalent to the change from state space H to state space R^n. The operator equation in H space must also be changed to that in space R^n, correspondingly. It is suggested that the properties of the equation operators must be kept unchanged during such transformation (Chou, 1983), Chou (1986) proved that almost all curves asymptotically approach to a special set having zero volume—the attractor by only using the properties of the operator like Eq. (6) in R^n space. A few attractors can exist at the same time and have respective region of attraction. Lyapunov's characteristic exponent is an important numerical figure which describes the stability and randomness of the motion orbits. If at least one exponent is positive, then the attractor is "strange", and consists of an infinite complex of manifolds of degree less than n. The motion on strange attractor is locally unstable. This is the source from deterministic to undeterministic. The existence of attractors (including strange attractor) reflects the global stability of the orbits in phase space. This is the source from undeterministic to deterministic.

The system tends to an attractor state spontaneously if the environmental conditions do not change with time. It is a limitation of idealization. In reality, it is unavoidable that the environmental conditions change with time, and it leads to the change of the attractor structure. Especially, when the external parameter passes a bifurcation value, the attractor will be runaway. It leads to fully undeterministic of another kind of motion. The predictability study with respect to this kind of motion is still not satisfactory.

III. APPLICATION

1. *The Prediction of Predictive Skill*

The key of the classical predictability is the variation of initial errors with time. If the system has only an attractor and the attractor is a constant solution, initial errors decrease with time. If the attractor is a period solution, initial errors may not change with time. If the attractor is a strange attractor and initial values are on the attractor, initial errors increase rapidly.

The real atmosphere is among these three situations. Under different initial fields, the evolution of initial errors with time is different.

Zeng (1979) showed that the evolution of some flow fields is stable, and the effect of initial errors is not strong, so the limit of predictability for these processes may be well above the mean value. Identification of these processes is very useful for actual forecast.

Palmer (1987) found from an assessment of a small set of extended range forecasts which are from two centres, and from a much large set of medium range forecasts which are from one centre, that the variability in predictive skill is strongly related to fluctuations in the Pacific / North American (PNA) mode of low frequency variability. A physical hypothesis is put forward that the growth of analysis errors or short range forecast errors depends on the barotropic stability of the forecast flow. The hypothesis is tested in a barotropic model, using basic states, composite skillful and unskillful forecasts from a set of 500 wintertime medium range forecasts. It is found that the degree of instability strongly depends on the amplitude of the PNA mode.

Discussing of the evolution of initial errors is only limited in theory, the stability of the forecast flow. Hydrodynamic instability including the instability of atmospheric motions is a classical but difficult problem. It is noteworthy that Zeng (1987) developed a generalized variational method which is universal for obtaining criteria of instability in all models with all possible basic flows. i.e., the model can be barotropic or baroclinic, quasi-geostrophic or nongeostrophic; and the basic flow can be zonal or nonzonal, steady or unsteady. To analyse actual flow by using this theory and to compare it with the time evolution of numerical weather forecast accuracy will help us in getting a deeper understanding on the mechanism of the time evolution of predictability. This is the work which needs to be done.

2. *A Possible Physical Mechanism about a Few Very Bad Predictive Skill*

As the improving of our understanding of physical processes in the atmosphere and the developing of computer technique in treating data, numerical weather forecasts have become more and more skillful. Despite this, forecasts still show considerable variability in predictive skill. Analyses of the skill of numerical weather forecast in the main forecasting centres showed that although the average scores of predictions were very good, some unsatisfactory episodes i.e. a few very bad forecasts still took place quite often (Lange and Hellsten, 1984; Bengtsson, 1985; Wash and Bogle, 1986). What caused these extremely unsuccessful forecast? How can we improve the forecast in these cases? Qiu and the author (1987) discussed these problems in detail. The foundamental idea is that the errors of the parameters in the models exist inevitably and give rise to forecast errors which depend on the initial values. Generally the forecasts are not sensitive to small errors of the model parameters, but for certain particular initial values the forecasts are highly sensitive. This is probably one of the reasons to bring about serious failures to the forecast. An example of numerical experiments shows that for certain particular initial fields, the errors in the parameters, though they are very small, may bring about serious consequence. It is suggested that drawing support from the invertion method one can modify the parameters in the models with the aid of the information provided by the observational data of the recent atmospheric evolution. The simulation experiments with the simple barotropic model show that the improvement over the forecast by this method is obvious. It is feasible that this method can be applied to the realistic operational models.

3. *Predictability of Mesoscale Circulation*

The increasing of initial errors is the source of producing inherent predictability. In theory, the rate of error increasing of three–dimensional turbulence is larger than that of two–dimensional turbulence. Since the mesoscale spans scales of motion from the synoptic scale, which behaves as two–dimensional turbulence (for the microscale, which behaves as three–dimensional turbulence) one would expect that mesoscale atmospheric systems, especially at the smaller scales, would have considerably less inherent predictability than synoptic scale system (Tennekes, 1978). An obvious difficulty is the lower resolution of the routine meteorological observational network, so the errors of initial fields are very large. However, it is quite expensive and impossible at least today to maintain a high resolution observational network similar to operational forecast ones. This leads to pessimistic conclusions concerning mesoscale predictability. However lower boundary forcing can strongly affect the behaviors of many atmospheric mesoscale phenomena and make them differ from those in turbulence model. Boundary forcing on the synoptic and planetary scale associated with land–sea contrasts and orography appears to be the reason of the more predictable of these scales of motion in the Northern Hemisphere than in the Southern Hemisphere (Shukla, 1984). In mesoscale problems, surface inhomogeneities including elevation and surface characteristics (albedo, heat capacity, moisture availability) generate many phenomena (such as mountain waves, sea breezes, convection, orographic precipitation, coastal fronts) and modulate their behaviors. Such surface inhomogeneities, if incorporated properly with numerical models, are likely to increase the predictability of motions they force.

Anthes (1984) classified the development of mesoscale weather systems into two types: (1) those resulting from forcing by surface inhomogeneities and (2) those resulting from internal modifications of large–scale flow patterns. On his opinion, a subset of the second class of phenomena are those mesoscale features that develop in regions of instability produced by large–scale flows; an example is the isolated thunderstorm which develops in a region of large–scale convective instability. Such individual phenomena are likely to have little predictability, even though the development of the large–scale area of instability may have significant predictability. An optimistic hypothesis is that many significant mesoscale atmospheric phenomena evolve from an interaction between large–scale flow and known or predictable surface inhomogeneities. In that case there is hope for skillful forecasts over period of 1–3 days using deterministic methods, and provided the synoptic–scale motions are predicted correctly.

Anthes et al. (1985), in some cases studies with a regional–scale numerical model, indicate that 72h simulations are not sensitive to random uncertainty errors in initial wind, temperature, and moisture fields. It is not like the behavior of global models. In contrast, the simulations are more sensitive to large variations in lateral boundary conditions. The most important practical result suggested by these experiments is that meso–α–scale models depend critically on accurate specification of the large–scale atmospheric variables at lateral boundaries.

It is possible that the above–mentioned mathematical theory provided a key for understanding the difference between global models and regional–scale models in the growth of initial errors. In global models initial values are always on the attractor and the attractor is a strange one, but in regional–scale models initial values are not on attractor and the attractor may not be a strange.

4. *Predictability of Monthly Mean Ocean / Atmosphere Variables*

It is inevitable to make some approximations and simplifications in the atmospheric governing equations in developing a numerical weather prediction model, because of two demands: (1) the governing equation must be closed; (2) the variables required for the definite conditions of solving the equations must be known. Different kinds of approximations and simplifications will result in different models and bring about different forecasting accuracies. In fact, the different definite conditions may be employed for the same predictand, and the different forecast schemes may also be developed by using the same definite conditions. That is often the case in developing a long–range weather forecasting model. Is it possible to compare the potential predictability among different schemes in advance?

There are two basic factors determining the forecast accuracy of a dynamical model, i.e. how much information about predictands is included in the difinite conditions and how efficiently the model picks up the information. In order to improve the forecasting accuracy, first we must try to select some definite conditions which include the information as much as possible, and then to develop an available model.

As well known, it is difficult to extend the daily forecasting up to two or three weeks. Therefore, the long–range numerical forecast turns into predicting the mean values of the variables in a definite period, such as the monthly averaged anomalies of geopotential height and temperature. In recent years, a few forecast schemes have been presented, which can be divided into two kinds. One is getting the monthly averaged values of the daily forecasts computed by the GCM (General Circulation Model) and the other is making out monthly averaged forecast directly by using the governing equations for the mean variables. The definite conditions required in the above two kinds of models are different, although their eventual objects are the same. The former requires the instantaneous values of the atmosphere / ocean variables at a certain time but the latter requires the monthly averaged ones.

Qiu and the author (1987) studied the potential predictability levels of forecasting the monthly mean ocean / atmosphere variables, which are only based on the monthly averaged data of sea surface temperature and geopotential height. Using the analysis of naturally occurring analogues, which is quite alike the method used to study the atmospheric predictability of daily weather forecast by Lorenz (1969), they found that in the ocean–atmosphere system the forecast of geopotential height may be more difficult than SST, and that the predictability level of monthly mean geopotential height anomaly calculated from the corresponding monthly mean SST appears relatively poor, but it can be improved by using the past observational data of monthly mean SST / geopotential field.

5. *Change over to New Ways*

Assuming that the atmosphere is expressed in n real parameters and written by $X=(X_1, X_2,\cdots\cdots X_n)$, the space R^n is its state space. The state $X_{t+\Delta t}$ at $t+\Delta t$ moment can be solely determined by the state X_t, at t moment. This is a deterministic model which numerical weather forecast is seemingly like. From mathematical viewpoint, this is equal to determining a point mapping in R^n space

$$X_{n+1}=G(X_n) \qquad n=0,1,2,\ldots\ldots$$

where X_n is the value of X in $t+n\Delta t$ moment. So, we can demonstrate the X_n value based on initial value, X_0. This is the deterministic forecast. The problem is that initial observational errors are not completely negligible, even if one assumes that the point mapping has no errors (in reality unlike this). It is inevitable to be limited by computer word length when we do the numerical computation by using computer. Assuming that h is the observational error (or rounding error of computer), the initial states between $a-h/2<x<a+h/2$ are expressed by a and in turn the state a might be the state between $a-h/2<x<a+h/2$. Thus the state space is not really continuous, and is discretized. Through a process of discretization the point mapping in R^n should be replaced by a cell–to–cell mapping (Chou, 1987). The point mapping is that a point maps a point; the cell mapping is the point mapping of all points in cell, its result is usually scattered in a few cells and is not correspondence one by one. Hsu (1981) pointed out such a cell mapping can be identified with a Markov chain and the well–developed mathematical theory can be immedi-

ately applied. So, for an ordinary deterministic model, it becomes that the probability is 1 at only one cell and is 0 at all the others at the initial moment, with the evolution of time, the probability at a group of cells is not 0. When the number of these cells is increased to certain degree, it happens that the instantaneous state cannot be determined in reality. However, the states display on a certain group of cells based on certain probability distribution; there are some deterministics, i.e., the weather is not determinant and the climate is determinant. The characteristics of underlying surface (for examples, sea–surface temperature, soil moisture, ice cap and snow cover and so on) changed more slowly than the atmosphere. Assuming that this external forcing is idealized to be stable, we know, because of the turbulent properties of atmosphere, the asymptotic behaviour is non–periodic, we can determine the probability distribution only for some states in phase space. When we further consider the evolution of underlying surface, the atmosphere is changed to a random input (the atmospheric exact state cannot be determined, only a probability distribution is determined). It is better to change from deterministic forecast to probability forecast for long–term forecast or short–term climatic forecast.

IV. DISCUSSION

The predictability problem refers to those sources of uncertainty as "model uncertainty" and "initial uncertainty", respectively. Assuming that the model is accurate, to discuss "initial uncertainty" is much easier than its opposite. So, most theoretical studies of atmospheric predictability tend to focus on the initial uncertainty and its propagation forwards in time through the integration of an otherwise deterministic flow model. The atmospheric phenomena studied by us are numerous and varied, such as, mesoscale and small–scale phenomena, synoptic and planetary scale phenomena, monthly and seasonal mean fields, and so on. Models built for different phenomena are different, so the predictability of the different mathematical models are different. But objectively existing physical systems have the determinant predictability. Initial error has its certain characteristic with respect to time, including rapid growth of small errors and the disappearance of prominent differences with time. Naturally, it should be required to keep these characteristics agreement between mathematical model and the real system described by mathematical model. Notice that nonlinear effects make the predictability decrease and the forcing and dissipative effects make the predictability increase. Therefore, it is suggested that the relative intensity of nonlinearity, forcing and dissipation in a mathematical model must be kept in agreement with those in real situation. How available are models in these respects? Discussions on these respects are not sufficient and further studies are required.

As to "model uncertainty", it is not enough that only discuss the possible effect of the small difference in physical parameters in a model (whether this effect really exists in reality is still unclear). This requires further study as well.

REFERENCES

Anthes, R.A. (1984a), The general question of predictability, lecture for AMS intensive course on mesoscale meteorology and forecasting, 9–20 July 1984, Boulder, Colorado.

———, (1984b), Predictability of mesoscale meteorological phenomena. In: *Predictability of Fluid Motions* (Lajolla Institute–1983), Greg Holloway and Bruce J. West, Editors, American Institute of Physics, New York, 247–270.

———, Y.H., Kuo, D.P., Baumhefnes, R.M., Errico, and T.W., Bettge (1985), Predictability of mesoscale atmospheric motions, *Advances in Geophysics*, **28b**: 159–202.

Bengtsson, L. (1985), Medium–range forecasting—— The experience of ECMWF. *Bull. Amer. Meteor. Soc.*, **66**: 1133–1146.

Bennett, A.F., and P.E., Kloeden (1981), The quasi-geostrophic equations: approximation, predictability an equilibrium spectra of solutions, *Q.J.R.M.*, **107**: 121–136.

Charney, J.G., R.G., Fteagle, H., Riehl, V.E., Lally, and D.Q., Wark (1966), The feasibility of a global observation and analysis experiment, *Bull. Amer. Meteorol. Soc.*, **47**: 200–220.

Charney, J.Q. (1969), Predictability, plan for U.S. participation in the global atmospheric research program, National Academy of Sciences, Washington, 8–14.

Chou, J.F. (1983), The decay of the influences of initial fields and some properties of their operations, *Acta Meteor. Sinica*, **41**: 385–392.

Chou, J.F. (1986), Long–range numerical weather prediction, Meteorological Press, Beijing, pp. 329 (in Chinese).

Chou, J.F.(1987), Some general properties of the atmospheric model in H space, R space, point mapping, cell maping, *Proceedings of International Summer Colloquium on Nonlinear Dynamics of the Atmosphere*, Sciences Press, 10–20 Aug., 1986, 187–189.

Constanin, P., C., Folas, and R., Temam (1985), Attractors representing turbulent flows, *Mem. Am. Math. Soc.*, **53**: No. 314.

Folas, C., and R., Temam (1979), Some analytic and geometric Properties of the solutions of the Navier–Stokes equations, *J. Math. Pures Appl.*, **58**: 339–368.

Ghidaglia, J.M (1986), On the fractional dimension of attractors for viscous incompressible fluid flows, *Siam. J. Math. Anal.*, **17**: 1139–1157.

Hsu, C.S. (1981), A generalized theory of cell–to–cell mapping for nonlinear dynamical system, *ASME Journal of Applied Mechanics*, **48**: 634–642.

Lange, A., and E., Hellsten (1984), Results of the WMO / CAS NWP data suidy and intercomparison project for forecasts for the Northern Hemisphere in 1984, WMO PSMP Report Series, No. 16, 25.

Leith, C.E., and R.H., Kraichnan (1972), Predictability of turbulent flows, *J.A.S.*, **29**: 1041–1058.

Leith, C.E. (1978), Predictability of climate, *Nature*, **276**: 352–355.

Leith, C.E. (1983), Predictability in theory and practice, Large Scale Dynamics Process in the Atmosphere, 365–383.

Lorenz, E.N. (1960), Maximum simplification of dynamic equation, *Tellus*, **12**: 243–254.

Lorenz, E.N. (1963), Deterministic nonperiodic flow, *J.A.S., 23*: 130–141.

Lorenz, E.N. (1965), A study of the predictability of a 28 variable atmospheric model, *Tellus*, **17**: 321–333.

Lorenz, E.N. (1969a), Three approaches to atmospheric predictability, *Bull. Amer. Meteor. Soc.*, **50**: 345–349.

Lorenz, E.N. (1969b), The predictability of a flow which possesses many scales of motion, *Tellus*, **21**: 289–307.

Lorenz, E.N. (1969c), Atmospheric predictability as revealed by naturally occurring analogues, *J.A.S.*, **26**: 636–646.

Lorenz, E.N. (1975), Climatic Predictability, GARP Pub. Ser., No. 16, the physical basis of climate and climate modelling, 132–136.

Lorenz, E.N. (1981), Some aspects of atmospheric predictability, Seminar 1981, problem and prospects in long and medium range weather forecasting 14–18 Sept.

Lorenz, E.N. (1982), Atmospheric predictability experiments with a large numerical model, *Tellus*, **34**: 505–513.

Madden, R.A., and D.J., Shea (1982), Potential long–range predictability of precipitation over North America, *Proceedings of the Seventh Annual Climate Diagnostics Workshop*, 18–22 Oct. 1982, 423–426.

Mansfield, D.A. (1986), The skill of dynamical long–range forecasts, including the effect of sea surface temperature anomalies, *Q.J. Roy. Met. Soc.*, **112**: 1145–1176.

Marchuk, G.I. (1974), Numerical solution of problems related to atmospheric and oceanic dynamics, Gidrometeoizdat, 303 (in Russian).

Miyakoda, K., J., Sirutis, and J. Ploshay (1987), One month forecast experiments without anomaly boundary forcings, *Mon. Wea. Rev.*, **114:** 2363–2401.

Monin, A.S. (1972). Weather forecasting as a problem in physics, MIT Press, Cambridge, Mass. and London, England, pp. 199.

North, G.R., and R.F., Cahalan (1981). Predictability in a solvable stochastic climate model. *J.A.S.* **38:** 504–513.

Palmer, T.N. (1987), Medium and extended range predictability, stability of the PNA model, and atmospheric response to sea surface temperature anomalies.

Qiu, C.J., and J.F., Chou (1987). A new approach to improve the numerical weather prediction, *Sci. Sinica (B)*, 903–910.

Qiu, C.J., and J.F., Chou (1987). Predictability levels of monthly forecast based on time–averaged ocean / atmosphere variables———a naturally occurring analogue study, *Acta Meteor. Sin.*, **1:** 34–42.

Robinson, G.D. (1967), Some current projects for global meteorological observation and experiment, *Q.J. Roy. Meteor. Soc.*, **93:** 409–418.

Ruelle, D., and F., Takens (1971). On the nature of turbulence, *Comm. Math. Phys.*, **20:** 167–192.

Shukla, J. (1981), Dynamical prediction of monthly means, *J.A.S.*, **38:** 2547–2572.

Shukla, J. (1982), Predictability of monthly means Part I: Dynamical predictability, Seminar 1981, problems and prospects in long and medium range weather forecasting, *14–18 Sept., 185–260.*

Shukla, J. (1982), Predictability of monthly means Part II: Influence of the boundary forcings, Seminar 1981, problems and prospects in long and medium range weather forecasting, 14–18 Sept., 261–312.

Shukla, J. (1983), Comments on "Natural variability and predictability", *Mon. Wea. Rev.*, **111:** 581–585.

Shukla, J. (1984), Predictability of a large atmospheric model, In: *Predictability of fluid motions* (Lajolla *Institute–1983), Greg Holloway and Bsuce. J. West, Editors*, American Institute of Physics, New York, 449–456.

Shukla, J. (1986). Physical basis for monthly and seasonal prediction, 549, *Proceedings of the first WMO workshop on the diagnosis and prediction of monthly and seasonal atmospheric variations over the globe,* 29 July——2, August, 1985.

Smagorinsky, J. (1969), Problems and promises of deterministic extended range forecasting, *Bull. Amer. Meteor. Soc.*, **50:** 286–311.

Somerville, R.C.J. (1980), Tropical influences on the predictability of ultralong waves, *J.A.S.*, **37:** 1141–1156.

Tennekes, H. (1978), Turbulent flow in two and three dimensions, *Bull. Amer. Meteor. Soc.*, **59:** 22–28.

Thompson, P.D. (1957), Uncertainty of initial state as a factor in the predictability of large–scale atmospheric flow patterns, *Tellus*, **9:** 275–295.

Vallis, G.K. (1983), On the predictability of quasi–geostrophic flow: the effects of Bata and baroclinicity, *J.A.S.*, **40:** 10–27.

Wash, C.H., and J.S., Boyle (1986). WMO PSMP Report Series, No. 19, 543.

Wang, S.H., J.P., Huang and J.F., Chou (1988), Some peoperties for the solutions of the large–scale equations of atmosphere, submitted to Sci. Sinica.

Zeng, Q.C. (1979), Mathematical and physical basis for numerical weather prediction, Science Press, Beijing, pp. 543 (in Chinese).

Zeng, Q.C. (1987). Variational principle of instability of atmospheric motion, Proceedings of international summer colloquium on nonlinear dynamics of the atmosphere, Science Press, 1987.