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Wintertime Stratospheric Anomalies—Part II: Sudden Warmings

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ABSTRACT

The process of stratospheric sudden warmings from development of planetary waves to the sudden cooling after reversal of mean zonal circulation will be studied with the primitive equations of heat and momentum balances. It will be explained that the sudden warmings may occur only in the polar regions of winter stratosphere where zonal mean temperature decreases poleward. The heating rate in the order of major warmings is produced by developed planetary waves in the stratospheric breaking layers. The particular perturbation structure characterized by large amplitude of wave 1 together with minimum of wave 2 discovered by Labitzke (1977) is crucial for initiation of major warmings. The cooling by the same mechanism can be produced in the regions with reversed mean temperature gradient.

I. INTRODUCTION

Since the first report of Scherhag (1952), the stratospheric sudden warmings have been observed every year, and provided a fascinating challenge to dynamic meteorology.These events happen at high latituds of winter stratosphere only, where zonal mean temperature decreases poleward. They are usually preceded by amplification of planetary waves. The heating rate in the major warmings, which produce reversal of mean zonal circulation, may be greater than 10K / day. The major warmings in midwinter occur less frequently than minor warmings, and has been observed only in the Northern Hemisphere. At the same time of warming, reversed temperature changes may take place in the mesosphere (Quiroz, 1969; Labitzke, 1972a,b). The simultaneous cooling may occur also at lower latitudes in the stratosphere (Fritz and Soules, 1972).

The temperature perturbations in the episode of sudden warmings manifest a typical nonlinear property characterized by the superposition of periodical pulses of variant periods on a non-periodical deviation from the time mean. A typical example is demonstrated in Fig.1,which shows the zonally averaged 30-hPa temperature differences between 80° and 50°N together with the amplitudes of waves 1 and 2 during the major warming in 1970 / 71.

According to Holton (1983), sudden warmings result from wave– mean flow interaction induced by amplification of forced planetary waves.The interaction mechanism given initially by Matsuno(1971) depends crucially on existence of a critical level, at which mean zonal velocity is identical to wave phase speed. A warming in his theoretical model takes place near the critical level. The deceleration of mean westerly is produced only by Coriolis deflexion of mean meridional flow, since no meridional eddy flux of momentum was considered.

This mechanism is not exactly the same as involved in his numerical model given in the same study, because the numerical model includes eddy flux of zonal momentum. The

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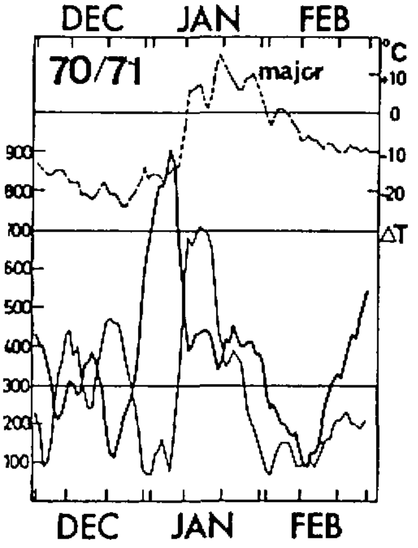

Fig. 1. The zonally averaged 30–hPa temperature differences (K) between 80°and 50°N (broken line) during the major warming in 1979 / 71. The heavy and thin lines represent the geopotential amplitudes of waves 1 and 2 respectively. (After Labitzke, 1977)

important effect of the eddy flux convergence may be found from the observational diagnoses of O'Neill and Taylor (1979) and Palmer(1981a). Moreover, simulations with nonlinear models did not show the significant effect of the critical level (e.g., O'Neill,1980). The simulated warmings could initiate at the heights far from the critical level (Grose and Haggard,1981;Koermer et al., 1983).

O'Neill and Taylor (1979) argued that the reversal of momentum flux from its normal poleward direction to an equatorward direction was crucial in effecting a zonal wind reversal in major warmings. The dynamical mechanism was explained firstly by O'Neill (1980). He proposed that changes in direction of eddy flux depended on change of phase tilt due to the differential variation of wave phase speed. He proved further that the variation of phase speed may be caused in turn by local variation in meridional gradient of zonal mean potential vorticity, when the polar night jet has jumped to higher latitudes. This phase change did occur in his numerical integration with a general circulation model.

However, the O'Neill's theory did not receive enough attentions. Dunkerton et al. (1981) suggested that equatorward momentum transport or focusing of EP flux into the polar cap is produced by the partially–reflecting of a nonlinear critical layer associated with a zero–wind line at lower latitudes. The reflection depends explicitly on a formal assumption of small wave amplitude. The limitations of using this theory were discussed by McIntyre (1980,1982). Also, observations have not provided persuasively an amount evidences of the predicted reflection in the atmosphere. Some diagnoses have revealed that the lower latitude zero–wind line or the reflection did not always exist in every event of sudden warmings (e.g., Palmer 1981a; Baldwin and Dunkerton, 1989).

Palmer (1981b) forwarded that this focusing can be produced by poleward shift of the polar night jet, which is a necessary precondition for major warmings. McIntyre (1982) developed a potential–vorticity mixing theory to illustrate the mechanism of precondition characterized by poleward shift and intensification of polar night jet. However, it has been found recently (Baldwin and Dunkerton,1989) that this preconditioning procedure is not always necessary. The major warming in early December 1987 occurred in a westerly phase of equatorial QBO, when the polar vortex was normally expected to grow in size, so the area of main vortex did not decrease markedly until the major warming was well under way.

In addition to the theories described above, Davies (1981) considered that sudden warmings are a manifestation of the reversal of zonally averaged relative potential vorticity distribution over a deep layer of the stratosphere and mesosphere. The reversals of the mean zonal flow and mean temperature gradient were regarded as the accompanying features. Also, Chao (1985) gave a different approach by using the theory of catastrophe. The study of McHall (1990) showed that the rapidity of atmospheric variations increases with irreversibility of the variations. The irreversible mixing of potential vorticity in wave breaking has been discussed by McIntyre (1982) and McIntyre and Palmer (1983).

The previous simple review shows that there is no common agreement on the mechanism of sudden warmings. A further quantitative investigation will be necessary, especially for the unusual heating rate. In particular, no any attempt has been found in explaining the apecific relation between the sudden warmings and the large amplitude ratio of waves 1 and 2 as shown in Fig.1, which was revealed by Labitzke (1977). In this study, the wave mean–flow interaction in sudden warmings will be discussed with the primitive equations of heat and momentum balances. The warmings at midwinter will be considered as the anomalous circulation pattern caused by the imbalance of heat and momentum in the breaking layers of the stratosphere, which occur only in the polar regions of winter hemsphere. The results may explain the observed heating rate and mean circulation reversal in major warmings.

II INSTABILITY MECHANISM

The sudden warmings are usually preceded by wave amplification. It was considered that the previous investigations tend to rule out the effects of baroclinic and barotropic instabilities (Schoeberl,1978), because the growth rates derived before for planetary waves were generally small.But these instabilities still attract many studies till recent time (e.g., Frederiksen,1982;Martmann, 1983; Manney et al., 1989).

The studies of Labitzke (1977,1981) and van Loon et al. (1975) showed that the wave development associated with sudden warmings occurred just after stratospheric baroclinity in the regions reached the largest intensity.This can be seen also from Fig.1. Quiroz (1975) found that at the onset of each major warming observed during 1969–74, a gradient of 15 mW(m^2 ster cm^{-1})$^{-1}$per 10°latitude was attained in the uppermost radiance channels of the SIRS and VTPR instruments. The gradient of mean stratospheric temperature (100–2 hPa)responding to the radiance gradient is 20 K per 10° latitude, and is significantly greater than the mean climatological state. This temperature gradient may be even larger at a local place.

The enhancement of baroclinity is not restricted merely in the stratosphere. It was reported (e. g., Craig and Hering, 1959;Teweles, 1963; Finger and Teweles, 1964) that sudden warmings tend to occur roughly in concurrent with blocking events and strong cyclonic activity in the troposphere below. Labitzke (1965) pointed out that sudden warmings stared after the similar synoptic conditions characterized by extremely strong cyclonic activity; occurrence of the European type warmings (see the reference) was followed by blocking actions within about ten days. An evidence of the baroclinity enhancement is poleward shift the polar night jet in winter stratosphere before sudden warmings. Moreover, the study of energetics (Julian,1965) revealed that the lower and middle troposphere and the middle stratosphere were baroclinically active regions before and during the major warming in January–February 1963. Perry (1967) concluded also that the growth of wave 2 which dominated the later phase of the major warming in 1963 was due to baroclinic development and to vertical flux of geopotential.

In this study, the development of planetary waves in the stratosphere is explained also in terms of the nonlinear wave instability discussed by McHall (1992, hereafter referred to as M). This instability takes place in the breaking layers of winter stratosphere where mean temperature decreases poleward. The temperature gradient of breaking layer may be calculated from Part I (McHall, 1992). The instability will happen if baroclinity in a breaking layer exceeds the breaking baroclinity(M) .

$$B_b = \frac{\delta^4 a C_v^3 m^4}{8(1+\delta)^3 C_p^3 R^3 \vec{T}_d^2}\left(\Phi_1^2 + \Phi_2^2\right)^2 |\sin 2\varphi|, \tag{1}$$

in which, $\vec{T}_d$ measures the zonal mean rate of temperature change produced by diabatic heating. It shows that the planetary waves may be destabilized by thermal forcing. From this point of view, the effect of anomalous diabatic heating on the warmings is pfoviding a trigger to release the latent dynamic instability as suggested by Kriester (1966). The observational evidences for the troposphere were referred to M.

The growth rate of the unstable planetary waves has been derived in M, which possesses the observed magnitudes also for the planetary waves in the stratosphere. Since the growth rate depends on baroclinity which varies as waves amplify, study of the wave development requires the consideration of variation in baroclinity. This will be discussed in another place using the recent approach of McHall (1990) in the study of available potential energy.

III ZONAL MEAN HEATING

1. *At Early Stage*

One important discovery in the sudden warmings was made by Labitzke. She revealed firstly in 1977 that a major warming happens when wave 1 in height field reaches a pronounced peak together with a pronounced minimum of wave 2. While in a minor warming, the amplitude of wave 2 may be significantly larger. This can be seen clearly from Fig.1. She stated:

> During the pre–warming phase the amplitude ratio between waves 1 and 2 (at 30 hPa) is about 10:1 in the case of a major warming, but only about 2:1 in the case of minor warming, or wave 2 may even be larger than wave 1, as during the winter1971 / 1972.

The further investigations were reported in her other studies (1978,1981). This fact has not been explained on theory since its discovery. The theoretical explanation will be given in the following discussions.

The zonal mean heating given by equation (6) in Part I may be rewritten as

$$\vec{T} = \frac{\delta m}{2fR}\left[\frac{2R\overline{T}\Delta\Gamma}{ag\overline{T}_y \sin 2\varphi}\left(\frac{k_1^2 - k_T^2}{k_1^2 + k_T^2} k_1 \Phi_1^2 + \frac{k_2^2 - k_T^2}{k_2^2 + k_T^2} k_2 \Phi_2^2\right) + \frac{C_v}{C_p}(k_1 \Phi_1^2 + k_2 \Phi_2^2)\right]$$
$$- \frac{p}{R}(\sigma_y \bar{v} - \sigma_z \overline{\omega}) + \frac{\overline{H}}{C_p}. \tag{2}$$

Here, we have used

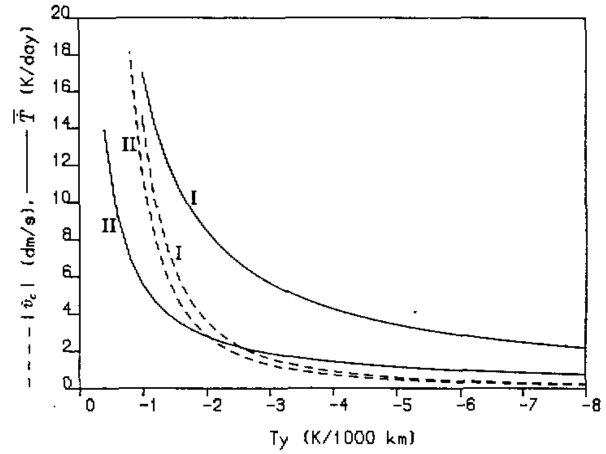

Fig. 2. The variation of zonal mean heating rate with baroclinity at 60°N (solid lines) at the early stage of sudden warmings. The dashed lines present the critical speed of mean meridional flow at the same latitude.

$$\sigma_y = \frac{R}{p}\frac{\partial \overline{T}}{\partial y}, \quad \sigma_z = \frac{R^2\overline{T}}{gp^2}\Delta\Gamma \tag{3}$$

for stratified atmosphere with temperature lapse rate Γ, in which, $\Delta\Gamma = g / C_p - \Gamma$. When heat balance is established in breaking layers, we may use the expressions of k_1 and k_2 together with equation (5) derived in M and have $\vec{T} = 0$. When the original breaking layer has been changed by wave amplification or for the waves which have propagated out of the breaking layers, these relationships cannot be used again and so there may be net heating or cooling produced by the anomaly circulations.

The first term in the right hand side of (2) is contributed by the convergence of eddy flux, which is much greater than the other terms in the events of sudden warmings. Using the relationship

$$K_T^2 = k_1 k_2 \tag{4}$$

obtained in M, we gain approximately

$$\vec{T} = \frac{\delta m}{2fR}\left(\frac{C_v}{C_p}(k_1\Phi_1^2 + k_2\Phi_2^2) - \frac{2R\overline{T}\Delta\Gamma(k_2 - k_1)}{ag\overline{T}_y(k_2 + k_1)\sin 2\varphi}(k_1\Phi_1^2 - k_2\Phi_2^2)\right). \tag{5}$$

Usually, $\overline{T}_y < 0$ in the polar regions of winter northern stratosphere, and $m > 0$ for k_1, $k_2 > 0$ in the areas with poleward eddy flux of momentum. Supposing $k_2 > k_1$, we see that more heating will be produced if amplitude of the longer wave is greater than that of the shorter wave.

To give an example, we provide that $\Gamma = 0$ and $\overline{T} = 210$ K in the stratosphere. The superposition waves of wavenumbers k_1 and k_2 possess zonal wavenumber $\overline{k} = (k_1 + k_2) / 2$ and meridional wavenumber m(Part I). Here, m is determined by $m = \overline{k}\ \mathrm{ctan}\gamma$, in which, γ measures the angle between a phase surface of the superposition wave and a latitudinal plane, and is assumed to be 85° in following examples. The calculated heating rates from the eddy flux produced by waves 1 and 2 at 60°N are depicted by the solid lines in Fig.2 for two cases, respectively. The case I is drawn for $\Phi_1 = 500$hPa and $\Phi_2 = 50$hPa, and case II is for $\Phi_1 = 400$hPa and $\Phi_2 = 200$hPa. Although the maximum amplitude of superposition wave in case II may be larger than in case I, the produced heating rate in case II is relatively small. It can be proved that cooling may be produced if the ratio of Φ_1 and Φ_2 is further less.

From (5), the heating rate is greater at higher latitudes for the same wave amplitudes. This is the usual situation of sudden warmings. The heating rate increases as baroclinity is reduced. Although it is very large when baroclinity becomes small, an infinite temperature increment may not be produced, since temperature gradient will be reversed soon by the strong heating.

The heating rate depends highly on the amplitude of long waves. Labitzke (1977,1981) noted that just before major warming, the amplitude of height wave 1 may reach 700 gpm at 30 hPa, and the amplitude at 10 hPa is even greater. In the Canadian warming defined by Labitzke (1977), the amplitude ratio of waves 1 and 2 is comparable with that in major warmings, and the wave 1 may grow to the same extent at 30 hPa. However, as wave 1 decreases with height, Canadian warming is generally weaker than major warmings. It originates through the pulsation of the Aleutian anticyclone. The possible reversal of temperature gradient is produced by the poleward displacement of the Aleutian high, and the polar vortex may not be breakdown.

2. *At Late Stage*

Usually, amplitude of wave 1 decreases but wave 2 grows after a warming started (referring to Fig.1). The heating rate estimated from (5) will be negative if waves 1 and 2 have the similar amplitudes. The further warming which takes place at the late stage is discussed in the following.

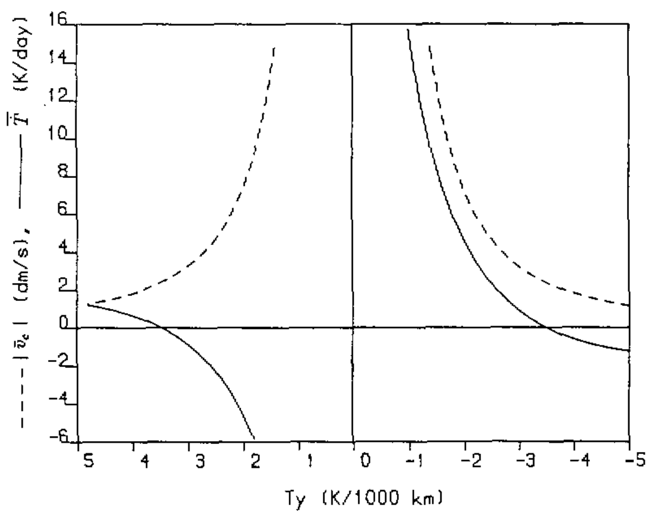

Fig.3. The same as Fig.2, but for $\Phi_1 = 100hPa$ and $\Phi_2 = 200hPa$ at the late stage.

As temperature increases abruptly in sudden warmings, heat balance in warming regions is destroyed irregularly, so that (4) can no longer be used and the original equation

$$k_T^2 = \frac{\delta^2 f^2 \bar{T}\Delta\Gamma m^2}{(1+\delta)^3 g \bar{T}_y^2} \tag{6}$$

given in Part I will be applied for evaluation of the heating rate

$$\dot{\bar{T}} = \frac{\delta m}{2fR}\left[\frac{2R\bar{T}\Delta\Gamma}{ag\bar{T}_y \sin 2\varphi}\left(\frac{k_1^2 - k_T^2}{k_1^2 + k_T^2} k_1 \Phi_1^2 + \frac{k_2^2 - k_T^2}{k_2^2 + k_T^2} k_2 \Phi_2^2\right) + \frac{C_v}{C_p}(k_1\Phi_1^2 + k_2\Phi_2^2)\right]$$

at the late stage. For a reversed amplitude ratio of waves 1 and 2, the calculated examples are displayed in Fig.3. The solid lines in Fig.3 show that warming may still be produced at the late stage although wave 1 is smaller than wave 2. A typical wave 2 warming was observed in February of 1979 (Palmer, 1981a; Labitzke, 1981). It was also preceded by amplification of wave 1, which induced the early warming discussed above. Between the waves 1 and 2 warmings at the early and late stages, there was a cooling episode noted previously and shown also in Fig.3. The cooling between a minor and a major warmings may be observed frequently, especially at a local place. An example is given by comparing the Fig.8 of Quiroz (1975) and the Fig.10 of Labitzke (1977) for the major waring in February 1973. If the late stage does not occur, the early stage may produce a minor warming. However, when the amplitude of wave 1 remains significantly larger in a long time period, the early stage may itself lead to a major warming. This is the cases of the major warming in January 1970, the final warmings in March 1978 and 1980, and the Canadian warmings in December 1973 and november 1976 (Labitzke, 1981).

When the anomaly perturbations propagate to the regions where zonal mean temperature increases poleward, the produced heating rate may be negative as shown by Fig.3. The analyses of Labitzke (1972a,b) revealed that the temperature changes of opposite sign could take place in the mesosphere and at low latitudes of the stratosphere in warming events. Since the zonal mean temperature gradient increases poleward in the summer stratosphere, the cooling may extend into the opposite stratosphere (Quiroz,1979).

After temperature gradient is reversed in the polar stratosphere, the warming may be continued by changing the horizontal phase tilt as noted by O'Neill (1980). When the waves have returned to the normal phase structure, an afterwards cooling is followed. This cooling was called particularly the sudden cooling by Palmer and Hsu (1983). If warming is started and reversal of temperature gradient is established firstly over the upper stratosphere, the sudden cooling together with warming below may cause the descendent of stratopause. This was shown by the observational survey of Labitzke (1972a,1981).

IV. REVERSAL OF ZONAL CIRCULATION

1. *Zonal Momentum Balance*

The zonal acceleration produced in breaking layers may be evaluated by inserting the superposition perturbations derived in M into the remainder equation of zonal momentum (McHall, 1991a)

$$\dot{u} = -u\frac{\partial u'}{\partial x} - v\frac{\partial u}{\partial y} - \omega'\frac{\partial u}{\partial p} + \frac{uv}{a}\tan\varphi + f(\bar{v} + v_a') + F_x .$$

In this study, we are interested only in the zonal mean acceleration

$$\dot{\bar{u}} = \overline{DU}_1 + \overline{DU}_2 + \left[f\left(1 + \frac{\bar{u}}{a^2\beta}\right) - \frac{\partial \bar{u}}{\partial y}\right]\bar{v} - \bar{\omega}\frac{\partial \bar{u}}{\partial p} + \bar{F}_x \tag{7}$$

acquired by taking zonal average for the previous equation. Here,

$$\overline{DU}_j = \frac{\delta^2 \sigma_z k_j m^3 \Phi_j^2}{f\sigma_y^2 (k_j^2 + k_T^2)}\left(\beta + \frac{f^2}{a^2\beta}\right), \quad (j = 1,2)$$

represents the contribution of geostrophic perturbations. In zonal momentum balance there is $\dot{\bar{u}} = 0$. This balance cannot be held in the events of sudden warmings.

2. *Mean Zonal Acceleration*

When external forcing and mean vertical velocity are ignored, the zonal mean acceleration in sudden warmings may be calculated from (7), showing

$$\dot{\bar{u}} \approx \frac{\delta^2 \sigma_z m^3}{f\sigma_y^2}\left(\frac{k_1 \Phi_1^2}{k_1^2 + k_T^2} + \frac{k_2 \Phi_2^2}{k_2^2 + k_T^2}\right)\left(\beta + \frac{f^2}{a^2\beta}\right) + f\bar{v}.$$

At the early stage, the eddy flux of momentum is in the normal direction toward the equator. So there is $m > 0$ for k_1, $k_2 > 0$ in the Northern Hemisphere, but $m < 0$ in the Southern Hemisphere. We find, using (4), that the mean westward acceleration may be produced only when equatorward mean circulation is sufficiently large, so that

$\bar{v} < \bar{v}_c$ (Northern Hemisphere),
$\bar{v} > \bar{v}_c$ (Southern Hemisphere),

where,

$$\bar{v}_c = -\frac{\delta^2 \overline{T} m^3 \Delta\Gamma(\Phi_1^2 + \Phi_2^2)}{ag\Omega\overline{T}_y^2 (k_1 + k_2)\sin\varphi\sin 2\varphi}$$

is the critical mean meridional velocity at which no zonal acceleration is produced. In deriving this velocity, we have used (3) for the stratified stratosphere. For the magnitudes of $\overline{T}$, γ and m provided previously, variation in the critical mean meridional velocity with baroclinity at the early stage is sketched by the dashed lines in Fig.2. According to Palmer (1981b), the mean equatorward flow may attain a speed over 1 m / s in sudden warmings. This meridional flow will produce an westward acceleration in the depicted example, until temperature gradient is greatly reduced.

When (4) cannot be used at the late stage of warmings, the critical mean meridional velocity is given by

$$\bar{v}_c = -\frac{\delta^2 \overline{T} m^3 \Delta\Gamma}{ag\Omega\overline{T}_y^2 \sin\varphi\sin 2\varphi}\left(\frac{k_1 \Phi_1^2}{k_1^2 + k_T^2} + \frac{k_2 \Phi_2^2}{k_2^2 + k_T^2}\right),$$

in which, k_T is computed from (6). The calculations for poleward eddy flux are displayed the dashed lines in Fig.3. The breakdown of zonal circulation at the transient moment when temperature gradient is in the vicinity of zero requires the reversal of eddy flux of momentum as discussed in the first section. The deceleration of mean westerlies will be more efficient in the regions with equatoward eddy flux. After mean temperature gradient has reversed and the eddy flux has returned toward the pole cap again, the mean zonal circulation may be restored by the weak meridional circulations or the poleward mean meridional flows.

V. HEAT AND MOMENTUM FLUXES

The heat flux produced by wave interference may be evaluated by inserting the superposition perturbations provided in Part I into $F_T = C_p T\mathbf{v}$, where, $\mathbf{v}$ indicates three dimensional velocity. It gives, after zonal average,

$$\overline{\mathbf{F}}_T = C_p\left(\overline{u}\overline{T} - \frac{\delta\sigma_z m^2 p}{2(1+\delta)fR\sigma_y}(\Phi_1^2+\Phi_2^2)\right)\hat{\mathbf{x}} + C_p\left(\overline{v}\overline{T} - \frac{\delta\sigma_z mp}{2fR\sigma_y}(k_1\Phi_1^2+k_2\Phi_2^2)\right)\hat{\mathbf{y}} + C_p\left(\overline{\omega}\overline{T} - \frac{\delta mp}{2fR}(k_1\Phi_1^2+k_2\Phi_2^2)\right)\hat{\mathbf{p}},$$

in which, $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$ and $\hat{\mathbf{p}}$ are unit vectors along corresponding coordinates.
Furthermore, the momentum flux is calculated from $\mathbf{F}_M = u\mathbf{v}$. The zonal mean portion shows

$$\overline{\mathbf{F}}_M = \left(\overline{u}^2 + \frac{m^2}{2f^2}(\Phi_1^2+\Phi_2^2)\right)\hat{\mathbf{x}} + \left(\overline{u}\overline{v} + \frac{m}{2f^2}(k_1\Phi_1^2+k_2\Phi_2^2)\right)\hat{\mathbf{y}} + \left(\overline{u}\overline{\omega} + (1+\delta)\frac{\sigma_y m}{2f^2\sigma_z}(k_1\Phi_1^2+k_2\Phi_2^2)\right)\hat{\mathbf{p}}.$$

The relation between zonally averaged meridional eddy fluxes of heat and momentum reads

$$\overline{F_{T_y}} = -\frac{\delta C_p}{R\sigma_y}f\sigma_z p\overline{F_{M_y}}.$$

When zonal mean temperature decreases poleward, the eddy flux of heat is in the same direction of momentum flux. In addition, the poleward momentum flux is followed by upward eddy fluxes of heat and momentum.

The convergence of mean heat flux may be calculated from

$$\nabla\cdot\overline{\mathbf{F}} = \frac{\partial\overline{F}_x}{\partial x} + \frac{\partial\overline{F}_y}{\partial y} + \frac{\partial\overline{F}_p}{\partial p} - \frac{\overline{F}_y}{a}\tan\varphi,$$

giving

$$\dot{\overline{T}} = \frac{1}{C_p}(-\nabla\cdot\overline{\mathbf{F}}_T + \overline{\alpha'\omega'} + \overline{\alpha}\overline{\omega} + \overline{\mathbf{H}}).$$

It tells that zonal mean heating is the net effect of heat flux convergence plus adiabatic and diabatic heatings. Usually, there are indirect mean meridional circulations in warming areas (Perry, 1967; Mahlman, 1969; Palmer, 1981a). Thus, heat flux of the mean flow and the adiabatic effect of the upward mean motions in the polar regions cause a cooling other than warming. The warming is then produced by three dimensional eddy fluxes of heat. This was confirmed by energetic analysis in many studies (e.g., Mahlman, 1969; Quiroz et al., 1975).

In a Lagrangian description, air trajectories are not determined by mean Eulerian circulation only, when velocity field is resolved into a mean portion and perturbation. According to Matsuno (1980), the motion of air parcels in the stratosphere is better represented by the residual circulation described by transformed Eulerian–mean equations (Andrews and McIntyre, 1976, 1978). This residual circulation includes part of the convergence of meridional heat flux as well as Eulerian mean circulation, so it may be different from the Eulerian mean circulation itself. From the studies of Palmer (1981b) and Dunkerton et al. (1981), there is a direct residual meridional circulation in warming areas with subsidence at higher latitudes. This circulation is just in the opposite direction of the Eulerian mean circulation. The warmings are produced essentially by the adiabatic down motions of air parcels.

Furthermore, the zonal mean acceleration (7) may be rewritten as

$$\dot{\bar{u}} = -\nabla \overline{\mathbf{F}}_M . \tag{8}$$

The convergence on the right hand side includes the contributions of mean flow and perturbations. As vertical motion of air parcels has a great effect on temperature changes, it may make an important contribution to momentum variations also. This may be see clearly in derivation of (7). Thus, convergence of three dimensional eddy flux cannot be approximated generally by the horizontal convergence

$$D_M = -\frac{\partial}{a\cos^2\varphi\partial\varphi}(\overline{v'u'}\cos^2\varphi),$$

and the mean acceleration is not represented precisely as the net contribution of D_M and the Coriolis effect on mean meridional flow $f\bar{v}$. This was showed by Palmer (1981b) and Dunkerton et al. (1981). In a simplified Lagrangian description, the right hand side of (8) is divided into two portions associated respectively with residual circulation and EP–flux. The mean acceleration is then measured by the torque of residual circulation and the convergence of EP–flux (Palmer, 1981b; Dunkerton et al., 1981).

VI CONCLUSIONS AND DISCUSSIONS

The stratospheric sudden warmings are produced by imbalance of heat and momentum in breaking layers, which may occur at high latitudes of winter hemisphere. The imbalance is produced by amplification of planetary perturbations in response to thermally forced baroclinic nonlinear instability. In the atmosphere where zonal mean temperature decreases poleward, developed perturbations may produce a heating of the order as in the observed major warmings. The heating is more efficient at early stage when amplitudes of longer waves are much larger than those of shorter waves. Thus, most warming events are initiated by unusually developed wave 1 together with pronounced minimum of wave 2, as revealed by Labitzke (1977). A reason that the major warming has never be recorded in the southern stratosphere is that the perturbations there do not exhibit the particular structure in midwinter (Labitzke, 1981).

According to the same mechanism, cooling other than heating may be produced by the anomaly perturbations which have propagated into the atmosphere with reversed temperature gradient. This is presumably responsible for the simultaneous coolings at low latitudes of the stratosphere and in the mesosphere above. It may also cause the sudden cooling after reversal of mean zonal circulation.

Usually, both the baroclinity intensity and mean zonal momentum decrease at the main stage of sudden warmings. So conversion of available potential energy into kinetic energy is rather inefficient in warming events. According to the study of McHall (1990), this process is highly irreversible and is characterized by sudden changes. A large amount of entropy will be produced by mixing of air parcels. The irreversible mixing process was referred to particularly as the planetary wave breaking by McIntyre and Palmer (1983). An evidence of this mixing in the stratosphere is the small gradient of vorticity at the middle latitudes (McIntyre, 1982).

The heating and acceleration mechanisms are discussed in terms of the primitive equations of heat and momentum balances in the present study. They may be changed significantly by simplifications or by adding artificially external forcings. For example, the warming events are triggered by development of planetary perturbations. It is noted in M that the development can be aroused by internal forcing in breaking layers. This internal forcing may not be reproduced in a simplified numerical model. Thus, an external forcing was usually introduced into the model atmosphere to induce the wave development. However, the obtained results do not lead to the conclusion that the added external forcing is the real and unique forcing of sudden warmings.

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