A Further Inquiry on the Mechanism of 30–60 Day Oscillation in the Tropical Atmosphere

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Received February 13, 1992; revised July 17, 1992

ABSTRACT

In a simple semi–geostropic model on the equatorial β–plane, the theoretical analysis on the 30–60 day oscillation in the tropical atmosphere is further discussed based on the wave–CISK mechanism. The convection heating can excite the CISK–Kelvin wave and CISK–Rossby wave in the tropical atmosphere and they are all the low–frequency modes which drive the activities of 30–60 day oscillation in the tropics. The most favorable conditions to excite the CISK–Kelvin wave and CISK–Rossby wave are indicated: There is convection heating but not very strong in the atmosphere and there is weaker disturbance in the lower troposphere.

The influences of vertical shearing of basic flow in the troposphere on the 30–60 day oscillation in the tropics are also discussed.

I. INTRODUCTION

In recent years, the more attentions have been paid to the research on the 30–60 day oscillation in the atmosphere, because it is considered directly as a cause of the short–term climate anomaly and the occurrence of El Nino event (Lau and Chan, 1986a; Li and Wu, 1990). A series of studies have not only indicated the action regularity and the structure feature of 30–60 day oscillation in the tropical atmosphere (Murakami, et al., 1984; Lau and Chan, 1985, 1986b; Lau and Lau, 1986; Knutson and Weickmann, 1987; and many others), but also exposed the existence and activities of 30–60 day atmospheric oscillation in the middle–high latitudes (Anderson and Rosen, 1983; Li, 1990, 1991; Li and Zhou, 1991).

In relation to the dynamical theory of 30–60 day oscillation in the tropical atmosphere, the feedback of the cumulus convection heating has been considered as an important mechanism. In order to explain theoretically the 30–50 day periodic variation of the monsoon troughs (ridges) in South Asia described by Krishnamurti and Subrahmanyam (1982), the CISK theory was introduced into the investigation on the dynamics of 30–60 day oscillation in the tropical atmosphere and the moving CISK mode was suggested as a mechanism to drive the 30–50 day oscillation of the monsoon troughs (ridges) (Li, 1985a). Lau and Peng (1987) conducted the mobile wave–CISK mechanism to originate the 30–60 day oscillation in the tropical atmosphere and then Takahashi (1987), Chang and Lim (1988) studied this CISK–Kelvin wave theory further. In which, the slow eastward propagation of 30–60 day oscillation along the equator was explained successfully.

The observations still show some characteristics of 30–60 day atmospheric oscillation in the tropics, such as it also propagates westwards sometimes, especially in the tropics outside the equator (Chen and Xie, 1988), and the 30–60 day oscillation has the wavetrain patterns even though in the tropics (Li, 1991). These properties of 30–60 day oscillation in the tropical atmosphere are hardly explained only by the CISK–Kelvin wave theory. Li (1988) and Liu and Wang (1990) indicated that the CISK–Rossby wave is also a mechanism to dirve 30–60 day oscillation in the tropical atmosphere, especially in the tropics outside the equator. This CISK–Rossby wave is able to propagate westwards or eastwards depending on the heating and it has the energy dispersion feature. But, in order to have an analytic solution, Li (1988) took tropical β–plane approximation while Liu and Wang (1990) assumed vertical velocity in the lower troposphere is independent with y–direction in their study. Therefore, those results have a certain limitation.

In the present paper, we will make a further inquiry on the mechanism of 30–60 day oscillation in the tropical atmosphere at the equatorial β–plane and understand the features of the CISK–Kelvin wave and CISK–Rossby wave further. The influences of vertical wind shearing on the CISK–Kelvin wave and CISK–Rossby wave in the tropics are also discussed.

II. BASICAL THEORY

Some studies have indicated that the intraseasonal oscillation has the characteristics of planetary scale motion in the tropical atmosphere. A dynamic study in relation to the atmospheric motion in the tropics has shown that the planetary scale motion in the tropical atmosphere is quasi–geostropic (Li, 1985b). Therefore, the basical equations with the cumulus heating feedback based on the Boussinesq fluid and the equatorial β–plane can be written as follows:

$$\frac{\partial u}{\partial t}+w\frac{\partial \overline{u}}{\partial z}-\beta y v=-\frac{\partial \varphi}{\partial x} \tag{1}$$

$$\beta y u=-\frac{\partial \varphi}{\partial y} \tag{2}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{3}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial \varphi}{\partial z}\right)+N^2 w=N^2\eta w_B \ . \tag{4}$$

There, the influences of the wave–CISK mechanism and the vertical shearing of basic flow are introduced. Generally, the basic flow is smaller in the tropical atmosphere while the vertical wind shearing is quite prominent. So the vertical shearing of basic flow here is just introduced. The Brunt–Vaisala frequency N, Rossby parameter β and vertical wind shearing $\frac{\partial \overline{u}}{\partial z}$ are assumed to be constants for convenience. The term $N^2\eta w_B$ on the right side in Eq.(4) represents the cumulus convection heating (Takahashi, 1987) in which w_B is the vertical velocity at the top of the atmospheric boundary layer and η the nondimensional heating parameter.

Eliminated u from Eqs.(1)–(3), we obtain

$$\frac{\partial}{\partial t}\frac{\partial^2 \varphi}{\partial y^2}-\beta y\frac{\partial \overline{u}}{\partial z}\frac{\partial w}{\partial y}-\beta w\frac{\partial \overline{u}}{\partial z}+2\beta^2 y v-\beta^2 y^2\frac{\partial w}{\partial z}=\beta\frac{\partial \varphi}{\partial x} \ . \tag{5}$$

Then, based on Eqs.(1) and (2), the variable v is represented by φ and we obtain

$$\frac{\partial}{\partial t}\left(\frac{\partial}{\partial y}-\frac{2}{y}\right)\frac{\partial \varphi}{\partial y}+\beta\frac{\partial \varphi}{\partial x}-\beta y\frac{\partial \overline{u}}{\partial z}\frac{\partial w}{\partial y}+\beta^2 y^2\frac{\partial w}{\partial z}+\beta w\frac{\partial \overline{u}}{\partial z}=0 \ . \tag{6}$$

Applying normal mode method, the solutions of the Eqs.(4) and (6) can be written as follows:

$$(\varphi,w)=(\Phi,W)e^{i(kx-\sigma t)} \quad . \tag{7}$$

Then, we can obtain

$$-i\sigma\frac{\partial\Phi}{\partial z}+N^2W=N^2\eta W_B \tag{8}$$

$$i\sigma\left(\frac{2}{y}-\frac{\partial}{\partial y}\right)\frac{\partial\Phi}{\partial y}-\beta yU\frac{\partial W}{\partial y}+\beta UW-\beta^2y^2\frac{\partial W}{\partial z}+ik\beta\Phi=0 \tag{9}$$

respectively, where $U\equiv\frac{\partial\bar{u}}{\partial z}$.

Taking simple 2–level model shown in Fig.1, we have the equations:

$$-i\sigma\frac{\Phi_1-\Phi_2}{\triangle}+N^2W_2=N^2\eta_2W_B \tag{10}$$

$$i\sigma\left(\frac{2}{y}-\frac{d}{dy}\right)\frac{d\Phi_1}{dy}-\beta yU\frac{dW_1}{dy}+\beta UW_1+\beta^2y^2\frac{W_2}{\triangle}+ik\beta\Phi_1=0 \tag{11a}$$

$$i\sigma\left(\frac{2}{y}-\frac{d}{dy}\right)\frac{d\Phi_3}{dy}-\beta yU\frac{dW_3}{dy}+\beta UW_3-\beta^2y^2\frac{W_2}{\triangle}+ik\beta\Phi_3=0 \tag{11b}$$

where we have assumed $w=0$ at both the top and bottom of the model, $\triangle$ is the height interval and $\triangle=7$km in this 2–level model.

From the Eqs.(11a) and (11b), we obtain

$$\begin{aligned}&i\sigma\left(\frac{2}{y}-\frac{d}{dy}\right)\frac{d(\Phi_1-\Phi_3)}{dy}-\beta yU\frac{d(W_1-W_3)}{dy}\\&+\beta U(W_1-W_3)+2\beta^2y^2\frac{W_2}{\triangle}+ik\beta(\Phi_1-\Phi_3)=0\end{aligned} \tag{12}$$

and the $(\Phi_1-\Phi_3)$ can be represented by using

$$\Phi_1-\Phi_3=-\frac{i\triangle}{\sigma}N^2W_2+\frac{i\triangle}{\sigma}N^2\eta_2W_B \quad . \tag{13}$$

According to the observational distribution of vertical velocity in the tropical atmosphere, the vertical velocity has identical sign and the maximum is at about 300–400hPa for general convection system in the tropics. Therefore, for the convenience and losing no fundamental feature of the tropical atmosphere, we would have approximation $W_1-W_3=a_1W_2$, where $a_1\approx0.5$—0.8 in general. Identically the vertical velocity at the top of the atmospheric boundary layer can also be simplified as

$$W_B=bW_2 \quad , \tag{14}$$

where b describes the intensity of the disturbance (convergence) in the lower troposphere. Thus, from Eqs.(12) and (13), we can obtain an equation in relation to the variable W_2 as follows:

$$
\begin{aligned}
&y^2\frac{d^2W_2}{dy^2}-\left[2y-\frac{\triangle\beta Ua_1}{c_1^2(1-b\eta_2)}y^3\right]\frac{dW_2}{dy} \\
&\quad -\left[\frac{2\beta^2}{c_1^2(1-b\eta_2)}y^4+\left(\frac{\beta k}{\sigma}+\frac{a_1\triangle\beta U}{c_1^2(1-b\eta_2)}\right)y^2\right]W_2=0
\end{aligned}
\tag{15}
$$

where $c_1^2=N^2\triangle^2$, the phase speed of the gravity wave in the atmosphere.

The Eq.(15) is the basic equation in the 2-level model, based on which, the cumulus heating feedback and the effect of vertical wind shearing can be discussed. For the Eq.(15), the boundary condition in y-direction can be written as follows:

$$W_2\equiv 0,\qquad y\to\pm\infty \ . \tag{16}$$

III. ANALYTIC SOLUTION (I)

At first, we discuss the case without vertical shearing of basic flow, i.e., $U=0$ in the Eq.(15). Thus, the Eq.(15) is reduced as follows:

$$y^2\frac{d^2W_2}{dy^2}-2y\frac{dW_2}{dy}-\left[\frac{2\beta^2}{c_1^2(1-b\eta_2)}y^4+\frac{\beta k}{\sigma}y^2\right]W_2=0 \ . \tag{17}$$

For convenience to solve the Eq.(17), it is necessary to alternate the variables. We take

$$\left.\begin{aligned}\zeta&=\frac{\sqrt{2}\beta}{c_2}y^2\\ W_2&=\zeta^{1/4}\omega_2\end{aligned}\right\}, \tag{18}$$

where $c_2=\sqrt{(1-b\eta_2)}\,c_1$.

Since there are some formulations as follows:

$$\left.\begin{aligned}\frac{dW_2}{dy}&=\frac{dW_2}{d\zeta}\left(\frac{2\sqrt{2}\beta}{c_2}y\right)\\ \frac{d^2W_2}{dy^2}&=\frac{4(\sqrt{2}\beta)^2}{c_2^2}y^2\frac{d^2W_2}{d\zeta^2}+\frac{2\sqrt{2}\beta}{c_2}\frac{dW_2}{d\zeta}\end{aligned}\right\} \tag{19}$$

and

$$\left.\begin{aligned}\frac{dW_2}{d\zeta}&=\zeta^{1/4}\frac{d\omega_2}{d\zeta}+\frac{1}{4}\zeta^{-3/4}\omega_2\\ \frac{d^2W_2}{d\zeta^2}&=\zeta^{1/4}\frac{d^2\omega_2}{d\zeta^2}+\frac{1}{2}\zeta^{-3/4}\frac{d\omega_2}{d\zeta}-\frac{3}{16}\zeta^{-7/4}\omega_2\end{aligned}\right\}. \tag{20}$$

The Eq.(17) can be reduced as:

$$\frac{d^2\omega_2}{d\zeta^2}-\left(\frac{5}{16}\zeta^{-2}+\frac{kc_2}{4\sqrt{2}\sigma}\zeta^{-1}+\frac{1}{4}\right)\omega_2=0 \ , \tag{21}$$

and the boundary condition (16) can be written as

$$|\omega_2|_{\zeta\to\infty}=0 \ . \tag{22}$$

It can be said that the problem becomes to solve the eigenvalue of the Whittaketer equation (Wang and Kou, 1979; Liu and Wang, 1990):

$$\begin{cases} \dfrac{d^2\omega_2}{d\zeta^2} + \left(-\dfrac{1}{4} + \dfrac{l}{\zeta} + \dfrac{\frac{1}{4}-\mu^2}{\zeta^2}\right)\omega_2 = 0 \ , \\ \omega_2\big|_{\zeta\to\infty} = 0 \end{cases} \tag{23}$$

where

$$l = -\frac{kc_2}{4\sqrt{2}\,\sigma}, \qquad \mu^2 = \frac{9}{16} \ .$$

Making

$$\omega_2 = e^{-\zeta/2}\zeta^{\mu+\frac{1}{2}}P \ . \tag{24}$$

Eq. (23) can be reduced to solve the eigenvalue of the Kummer equation as follows:

$$\begin{cases} \zeta\dfrac{d^2P}{d\zeta^2} + (2\mu+1-\zeta)\dfrac{dP}{d\zeta} - \left(\mu+\dfrac{1}{2}-l\right)P = 0 \\ P\big|_{\zeta\to\infty} = 0(\zeta^m) \end{cases} \tag{25}$$

The eigenvalues of Eq. (25) are

$$\mu + \frac{1}{2} - l = -m \qquad (m = 0,1,2,\cdots) \ , \tag{26}$$

and the corresponding eigenfunctions can be written as

$$P \equiv A_m S_m^{2\mu}(\zeta) = A_m \cdot \frac{(2\mu+1)_m}{m!}K(-m,2\mu+1,\zeta) \ , \tag{27}$$

where A_m is an arbitrary constant and $(2\mu+1)_m$ is the Gauss symbol which is defined as

$$(2\mu+1)_m = (2\mu+1)(2\mu+2)\cdots(2\mu+m) = \frac{\Gamma(2\mu+m)}{\Gamma(2\mu+1)} \tag{28}$$

and

$$(2\mu+1)_0 = 1;$$

while $S_m^{2\mu}$ and $K(-m,2\mu+1,\zeta)$ are known as the Sonine polynomial and Kummer function respectively.

Taking $\mu = -\frac{3}{4}$ as general case, from (26), it can be obtained

$$-\frac{1}{4} + \frac{kc_2}{4\sqrt{2}\,\sigma} = -m \tag{29}$$

and the angular frequency can be represented

$$\sigma = \frac{kc_2}{\sqrt{2}(1-4m)} \ , \qquad (m = 0,1,2,\cdots) \ . \tag{30}$$

And from (27) we have

$$P \equiv A_m S_{\tilde{m}}^{-3/2}(\zeta) = A_m \frac{(-1/2)_m}{m!} K\left(-m, -\frac{1}{2}, \zeta\right), \tag{31}$$

then, according to (18) and (24), it can be obtained

$$W_2^m(y) = A_m \frac{(-1/2)_m}{m!} e^{-y^2\sqrt{2}\beta/2c_2} K\left(-m, -\frac{1}{2}, \frac{\sqrt{2}\beta}{c_2} y^2\right). \tag{32}$$

When $m=0$, it is clearly shown in (30) that

$$\sigma = \frac{\sqrt{2}}{2} k c_2 , \tag{33}$$

and it is the Kelvin wave propagating eastwards. But when $m \neq 0$, then $\sigma < 0$, it represents the westward Rossby wave. Since c_2 has included the cumulus heating, the Kelvin wave and Rossby wave mentioned here are the one with the cumulus convection feedback.

Making $L_0 = (\sqrt{2}c_2 / 2\beta)^{1/2}$, (32) can be written as:

$$W_2^m(y) = A_m \frac{(-1/2)_m}{m!} e^{-\frac{1}{2}(y/L_0)^2} K\left(-m, -\frac{1}{2}, (y/L_0)^2\right). \tag{34}$$

The latitudinal structures for $m=0$ Kelvin mode and $m=1$ Rossby mode are shown in Fig.2. Obviously, the latitudinal structures of these modes with the cumulus convection feedback (the heating is not quite strong) are similar to ordinary modes without the convection heating in the tropical atmosphere, i.e., the Kelvin mode has a maximum at the equator ($y=0$) and it is symmetrical about y; the Rossby mode has extreme values at $y=0$ and $y = \pm\sqrt{3/2}L_0$ and it is also symmetrical about y. The convection heating is unable to change the fundamental features of latitudinal structures.

1. *CISK–Kelvin Wave*

When $m=0$, the angular frequency (33) can be written as

$$\sigma = \sqrt{(1-b\eta_2)} k c_1 / \sqrt{2} , \tag{35}$$

this is equatorial Kelvin wave with condensational heating, i.e., the so-called CISK–Kelvin wave. Because when $\eta_2 = 0$ (no heating), (35) becomes $\sigma = kc_1/\sqrt{2}$, it is the eastward equatorial Kelvin wave with the phase speed:

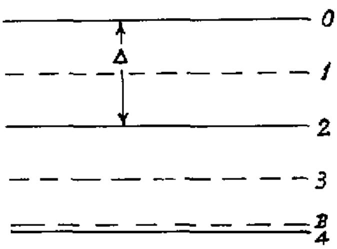

Fig.1. Schematic illustration of a 2-level model.

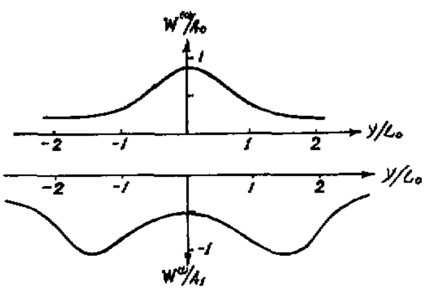

Fig.2. Meridional structure of $W(y)$ for the CISK–Kelvin mode $m=0$ (a) and CISK–Rossby mode $m=1$ (b).

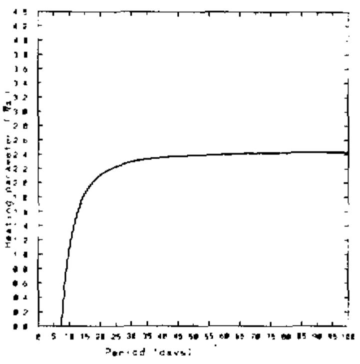

Fig.3. Variation of the periods of the CISK–Kelvin wave ($m=0$) with the heating parameter η_2.

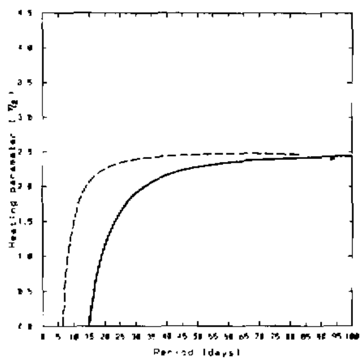

Fig.4. Variation of the periods of the CISK–Rossby wave $m=1$ (solid line) and $m=2$ (dashed line) with the heating parameter η_2.

$$c_{xo} \equiv \frac{\sigma}{k} = \frac{\sqrt{2}}{2} c_1 \quad . \tag{36}$$

In general, $N = 10^{-2} s^{-1}$, we then can obtain $c_{xo} = 49.5\text{m} / \text{s}$ which is consistent with one in the past.

When the cumulus heating is existent but weaker, then $1 - b\eta_2 > 0$, (35) should indicate $\sigma = \sigma_r > 0$, the CISK–Kelvin wave is stable and propagates eastwards with the phase speed

$$c_x = \frac{\sqrt{2(1 - b\eta_2)}}{2} c_1 < c_{xo} \quad . \tag{37}$$

Taking $b = 0.4$ and $\eta_2 = 2.0$, we then obtain $c_x = 15.7\text{m} / \text{s}$. It is consistent with the 30–60 day oscillation in the tropical atmosphere.

The period of the CISK–Kelvin wave can be described as

$$T \equiv \frac{2\pi}{\sigma_r} = \frac{2\pi}{kc_1} \frac{2}{\sqrt{2(1 - b\eta_2)}} = \frac{\sqrt{2} L}{\sqrt{1 - b\eta_2}\, c_1} \quad , \tag{38}$$

where L is the zonal scale of the CISK–Kelvin wave. Obviously, the period of the CISK–Kelvin wave with a certain zonal scale depends on the intensity of the convection heating (η_2). Taking $L = 3.2 \times 10^7$m, the periods associated with η_2 can be obtained from (38) and they are shown as in Fig.3. It is clear that the periods of the CISK–Kelvin wave are about 15–70 days for general convection heating $\eta_2 \approx 1.8 - 2.4$ (Hayashi, 1970).

When the cumulus convection heating is strong, then $1 - b\eta_2 < 0$, (35) should indicate $\sigma = i\sigma_i$. The CISK–Kelvin wave is unstable and stationary since $c_x = 0$. These are consistent with the Takahashi's results (1987) even though the model in this study is different from the one used by Takahashi.

2. *CISK–Rossby Wave*

If $m \neq 0$, the angular frequency can be written as

$$\sigma = -kc_2 / \sqrt{2}(4m - 1) = -\sqrt{1 - b\eta_2}\, kc_1 / \sqrt{2}(4m - 1), \; (m = 1,2,\cdots) \quad . \tag{39}$$

When $\eta_2 = 0$, (39) shows a westward equatorial Rossby wave with the phase speed

$$c_{xo} \equiv \frac{\sigma}{k} = -c_1 / \sqrt{2}(4m-1) \; . \tag{40}$$

Therefore, (39) represents the CISK–Rossby wave.

If the cumulus convection heating is not quite strong, then $1 > 1 - b\eta_2 > 0$, we can obtain the phase speed and the period of the CISK–Rossby wave as follows:

$$c_x = \frac{\sigma}{k} = -\sqrt{1 - b\eta_2} / \sqrt{2}(4m-1) \tag{41}$$

$$T = \left| \frac{2\pi}{\sigma} \right| = \sqrt{2}(4m-1)L / \sqrt{(1 - b\eta_2)}\, c_1 \, . \tag{42}$$

Eq. (41) shows that the westward phase speed of the CISK–Rossby wave is slower than thst of the equatorial Rossby wave. Taking $L = 9.0 \times 10^6$m, for $m = 1$ and $m = 2$, the periods associated with η_2 can be obtained from (42) and they are shown in Fig.4. For general convection heating ($\eta_2 \doteq 1.8 - 2.4$), the periods of the CISK–Rossby wave are about 20–70 days.

When the cumulus convection heating is strong, then $1 - b\eta_2 < 0$, it is shown in (39) that the CISK–Rossby wave is reductive and stationary.

Above–mentioned analyses show that there are eastward CISK–Kelvin wave and westward CISK–Rossby wave resulted from the exciting of the cumulus convection heating. They are all the low–frequency modes and the CISK–Rossby wave is not only able to partly explain westward propagation of 30–60 day oscillation in the tropical atmosphere, but also can decrease eastward speed of the CISK–Kelvin wave which is faster than the propagation of 30–60 day oscillation. Therefore, it is obvious that the CISK–Kelvin wave and CISK–Rossby wave are an important mechanism to drive the activity of 30–60 day oscillation in the tropical atmosphere even though the coupled moist Kelvin–Rossby wave (Wang and Rui, 1990) is not obtained in this simple model.

It should be pointed out that the CISK–Kelvin wave is unstable growing but stationary while the CISK–Rossby wave is reductive and stationary when the cumulus convection heating is strong (such as $\eta_2 > 2.5$). This is different from the slower propagation of 30–60 day oscillation and implies that the strong convection heating is unfavourable to excite the 30–60 day oscillation in the tropical atmosphere. The general convection heating, it is not quite strong in the large area, can excite the CISK–Kelvin wave and CISK–Rossby wave which drive the activity of 30–60 day oscillation in the tropics. This result is consistent with the observation, that is, the strong convection heating in the tropical atmosphere will lead to the occurrence of the disturbance like tropical cyclone. Recently, a linear theory study shows that the disturbance caused by wave–CISK in the tropical atmosphere has a tendency to generate smaller scale feature as the convection heating increased (Frederiksen and Frederiksen, 1991). It also means that the strong convection heating is unfavourable to produce intraseasonal oscillation which is planetary scale system in the tropical atmosphere.

In this paper, the parameter b is introduced for analysing conveniently. The greater (smaller) value of b represents the stronger (weaker) disturbance in the lower troposphere. An interesting result shows that the existence of stronger disturbance in the lower troposphere is unfavourable to excite the 30–60 day oscillation in the tropical atmosphere for general convection heating but favorable to produce stationary unstable growth of the disturbance. The occurrence of 30–60 day oscillation is more favourable in general convection heating when there is only weaker disturbance in the lower troposphere. Of course, if there is stronger disturbance in the lower troposphere, the 30–60 day oscillation in the tropical atmosphere can

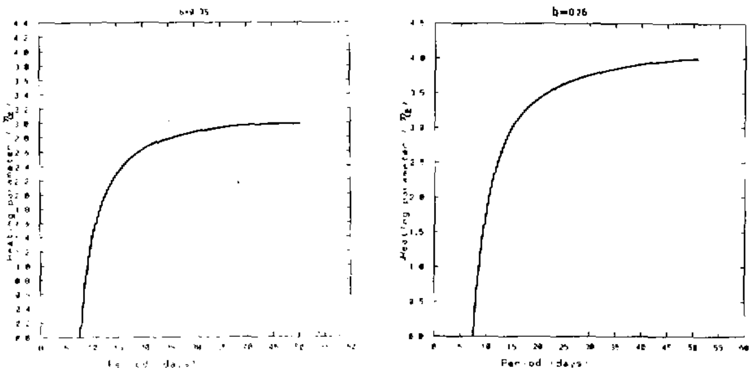

Fig.5. The influence of the disturbance intensity in the lower troposphere on the CISK–Kelvin wave. Above for $b=0.35$ and bottom for $b=0.25$.

be still excited by weaker convection heating. In Fig.5, the relationships between the heating parameter η_2 and the periods of the CISK–Kelvin wave are shown for $b=0.35$ and $b=0.25$, respectively. Obviously, the stronger heating is still able to excite the 30–60 day oscillation in the tropical atmosphere as the disturbance is weaker in the lower troposphere.

IV. ANALYTIC SOLUTION (2)

The case with vertical shearing of basic flow will be discussed in this section. Since Eq.(15) is complicated and difficult to solve analytically, the simplification is necessary. According to the distribution of the vertical velocity $W(y)$ shown in Fig.2, the following approximate formulation can be used:

$$y\frac{d(W_1-W_3)}{dy} \approx \frac{(W_1-W_3)_{j+1}-(W_1-W_3)_{j-1}}{y_{j+1}-y_{j-1}}y_j \approx -a_2(W_1-W_3) , \tag{43}$$

where a_2 is a smaller constant, we can take $a_2=0.2$. Thus, Eq.(15) can be written as

$$y^2\frac{d^2W_2}{dy^2}-2y\frac{dW_2}{dy}-\left[\frac{2\beta^2}{(1-b\eta_2)c_1^2}y^4+\left(\frac{\beta k}{\sigma}+\frac{a\beta U\triangle}{(1-b\eta_2)c_1^2}\right)y^2\right]W_2=0 , \tag{44}$$

where $a=(1+a_2)a_1$. If we assume the variation of the difference of vertical velocities at level 1 and level 3 with y being ignored (i.e., $\frac{d(W_1-W_3)}{dy}=0$) to replace the formulation (43), and Eq. (44) is still the same except $a=a_1$.

Using the similar methods to above analyses and taking

$$\zeta=\frac{\sqrt{2}\beta}{c_2}y^2, \qquad W_2=\zeta^{1/4}\omega_2 , \tag{45}$$

then Eq.(44) can be written as

$$\frac{d^2\omega_2}{d\zeta^2}-\left[\frac{5}{16}\zeta^{-2}+\left(\frac{kc_2}{4\sqrt{2}\sigma}+\frac{aU\triangle}{4\sqrt{2}c_2}\right)\zeta^{-1}+\frac{1}{4}\right]\omega_2=0 , \tag{46}$$

The boundary condition is still

$$\omega_2 \big|_{\zeta \to 0} = 0 \ . \tag{47}$$

There is still the following Whittaker equation

$$\begin{cases} \dfrac{d^2 \omega_2}{d\zeta^2} + \left(-\dfrac{1}{4} + \dfrac{l'}{\zeta} + \dfrac{\frac{1}{4} - \mu^2}{\zeta^2} \right) \omega_2 = 0 \ , \\ \omega_2 \big|_{\zeta \to 0} = 0 \end{cases} \tag{48}$$

but where

$$\mu^2 = 9/16, \qquad l' = -(kc_2 / 4\sqrt{2}\sigma + aU\triangle / 4\sqrt{2}c_2) \ .$$

Corresponding to Eq.(48), the Kummer equation is

$$\begin{cases} \zeta \dfrac{d^2 P}{d\zeta^2} + (2\mu + 1 - \zeta)\dfrac{dP}{d\zeta} - \left(\mu + \dfrac{1}{2} - l' \right) P = 0 \\ P\big|_{\zeta \to \infty} = O(\zeta^m) \end{cases} . \tag{49}$$

The eigenvalues of Eq. (49) are

$$\mu + \frac{1}{2} - l' = -m, \qquad (m = 0,1,2,\cdots) \ . \tag{50}$$

Taking $\mu = -\frac{3}{4}$, from (50) we have

$$-\frac{1}{4} + \frac{kc_2}{4\sqrt{2}\sigma} + \frac{aU\triangle}{4\sqrt{2}c_2} = -m, \quad (m = 0,1,2,\cdots) \ . \tag{51}$$

Then, the angular frequency can be written as

$$\sigma = \frac{kc_2}{(1-4m)\sqrt{2} - aU\triangle / c_2} \ . \tag{52}$$

The corresponding eigenfunction can be still represented by (31) or (32).

In the following, we will discuss the CISK–Kelvin wave and CISK–Rossby wave with vertical shearing of basic flow respectively.

1. *The CISK–Kelvin Wave Case*

For the CISK–Kelvin wave, $m = 0$, Eq. (52) can be written

$$\sigma = \frac{kc_1^2(1 - b\eta_2)(aU\triangle + \sqrt{2(1 - b\eta_2)}\, c_1)}{2(1 - b\eta_2)c_1^2 - a^2 U^2 \triangle^2} \ . \tag{53}$$

It is obvious in (53) that the vertical westerly shearing $(U = \frac{d\bar{u}}{dy} > 0)$ will accelerate the eastward propagation of the CISK–Kelvin wave, but the vertical easterly shearing $(U < 0)$ will slow down the eastward propagation; And all the vertical shearing should decrease the unstable growth rate of the CISK–Kelvin wave. It is also shown in (53) that the propagation property of the stable CISK–Kelvin wave will be changed when the vertical shearing of basic flow is strong enough and it will propagates westwards.

2. The CISK–Rossby Wave Case

For the CISK–Rossby, $m \neq 0$, taking $m=1$ to analyze simply. Then, the angular frequency is

$$\sigma = -\frac{k(1-b\eta_2)c_1^2}{3c_1\sqrt{2(1-b\eta_2)}+aU\triangle} \tag{54}$$

We can see from (54) that the westerly (easterly) shearing will slow down (speed up) the westward propagation of the CISK–Rossby wave. When the vertical easterly shearing is very strong (it is difficult to occur), the propagation property of the CISK–Rossby wave will be changed and propagates eastwards.

In general, we can take $a_1 = 0.6$, then $a = 0.72$. Based on Eqs.(53) and (54), the relationships between the vertical shearing of basic flow and the phase speeds of the CISK–Kelvin wave and CISK–Rossby wave for certain convection heating can be obtained. For the CISK–Kelvin wave, the phase speed increases with the decrease of the easterly shearing and the increase of the westerly shearing (Fig.6). But, when the westerly shearing is strong enough, the CISK–Kelvin wave propagates westwards. The very strong easterly or westerly shearing will all lead the CISK–Kelvin wave to quasi–stationary. For the CISK–Rossby wave, the westward phase speed decreases with the decrease of easterly shearing and the increase of the westerly shearing (Fig.7). The easterly and westerly shearing of basic flow will all decrease the reduction of the CISK–Rossby wave.

V. CONCLUSION REMARKS

Throughout this paper, we have discussed further the dynamical mechanism of the 30–60 day oscillation in the tropical atmosphere. Especially, the more favourable conditions to excite the CISK–Kelvin wave and the CISK–Rossby wave which drive the activities of 30–60 day oscillation in the tropical atmosphere and the effects of vertical shearing of basic flow are indicated.

1). The cumulus convection heating, through the CISK mechanism, can excite the CISK–Kelvin wave and CISK–Rossby wave in the tropical atmosphere. These waves are the low–frequency modes and they are able to drive the activities of 30–60 day oscillation in the tropics. The obvious eastward propagation of 30–60 day oscillation near by the equator is

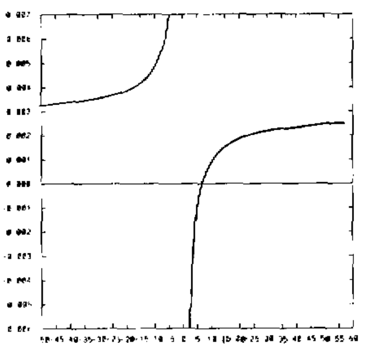

Fig.6. The influence of vertical shearing of basic flow on the phase speed of the CISK–Kelvin wave.

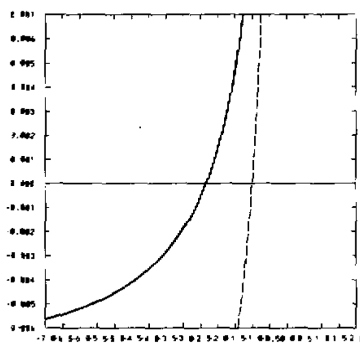

Fig.7. The influence of vertical shearing of basic flow on the phase speed of the CISK–Rossby wave $m=1$ (solid line) and $m=2$ (dashed line).

dominated by the CISK–Kelvin wave as pointed out in many studies. The obvious westward propagation of 30–60 day oscillation in the tropical atmosphere outside the equator is probably dominated by the CISK–Rossby wave; and the CISK–Rossby wave is able to decrease eastward speed of the CISK–Kelvin wave which is greater than the observation of 30–60 day oscillation.

2). The more favourable conditions to excite the CISK–Kelvin wave and CISK–Rossby wave are:

(a). There is convection heating in the large area but it is not very strong, otherwise the quite strong convection heating is just favourable to produce the disturbance like tropical cyclone.

(b). There is only weaker disturbance in the lower troposphere, otherwise it is unfavourable to excite the 30–60 day oscillation in the tropical atmosphere.

3). The vertical shearing of basic flow in the troposphere is obviously able to affect the propagation of the CISK–Kelvin wave and CISK–Rossby wave. Therefore, the vertical shearing of basic flow will also affect the propagation of 30–60 day oscillation in the tropical atmosphere. The westerly (easterly) shearing will speed up (slow down) the eastward propagation of the CISK–Kelvin wave. But, the CISK–Kelvin wave will propagate westwards when the westerly shearing is strong enough. The westward propagation of the CISK–Rossby wave will be accelerated with the decrease of the westerly shearing (or the increase of the easterly shearing).

4). The vertical shearing of basic flow will weaken the instability of the CISK–Kelvin wave and the reduction of the CISK–Rossby wave.

This research was completed when the author was visiting in CSIRO Division of Atmospheric Research in Australia. The author is grateful to Dr. J. Frederiksen for reading the manuscript and for his helpful discussions.

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