Seasonal Variation of Stationary and Low–Frequency Rossby Wave Rays

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ABSTRACT

The wave rays and their seasonal variation of stationary and low–frequency Rossby waves are studied by using the Runge–Kutta scheme. The results show that for stationary waves the rays can reach lower latitudes in winter, and are limited in higher latitudes in summer. The main differences between the stationary and low–frequency wave rays are that low–frequency waves can propagate across the equator and the easterlies will not be an obstacle on their propagation. It explained to some extent the interaction of disturbances between the Northern and Southern Hemispheres. The lower wave frequencies and the stronger easterly flow are, the more difficult low–frequency waves will be to propagate across the equator. The waves with 20–day period are easier to propagate across the equator than that with 50–day period. The winter is the most favorable season for low–frequency waves to propagate into another hemisphere.

Key words: Ray, Propagation, Teleconnection, Low–frequency wave

I. INTRODUCTION

Rossby wave is the most important in the large–scale motion of atmosphere. Its propagation has great influences on evolution and development of the general circulation. Hoskins and Karoly (1981) studied propagation of the stationary Rossby waves by using the method of slowly varying wave train in a barotropic atmosphere, and calculated the ray tracing of waves, they showed that there existed poleward and equatorward rays for the meridional wave numbers. The poleward rays were reflected at the latitude with $k = K_S$, and then propagated eastwards. The equatorward rays were trapped by easterlies which played the role of "blocking". The rays propagated toward the area with high value of refractive index and show obvious character of a great circle. It explained to some extent the planetary–scale teleconnections in the atmosphere. Wu Guoxiong (1990) utilized time–average data to calculate the space–time variation of vertical and horizontal propagation of planetary waves, and concluded that the rays could reach lower latitudes in winter, but were limited in middle–high latitudes in summer.

Atmospheric circulation exhibits obvious seasonal variation, the wind field is one of the key factors in determining the propagation of Rossby waves. One of the aims in this article is to study Rossby wave rays for different seasons in order to explain some obvious atmospheric phenomena.

Low–frequency oscillation is one of the most important phenomena in atmospheric motion. Studying the propagation of low–frequency Rossby waves is another important problem in this paper, thus we will discuss the rays with 20–day period and 50–day period.

II. MODEL

After introducing the Mercator projection of the sphere, we can write a barotropic, linear vorticity equation for the perturbation as follows:

$$\left(\frac{\partial}{\partial t}+\bar{u}_M\frac{\partial}{\partial x}+\bar{v}_M\frac{\partial}{\partial y}\right)\nabla^2\psi+\beta_M\frac{\partial\psi}{\partial x}=0 \ , \tag{1}$$

where

$$(\bar{u}_M, \bar{v}_M)=(\bar{u}/\cos\varphi, \bar{v}/\cos\varphi) \ ,$$

$$\beta_M=\frac{2\Omega}{a}\cos^2\varphi-\frac{\partial}{\partial y}\left[\frac{1}{\cos^2\varphi}\frac{\partial}{\partial y}(\cos^2\varphi\bar{u}_M)\right] ,$$

$$(\bar{u},\bar{v})=\left(-\frac{1}{a}\frac{\partial\bar{\Psi}}{\partial\varphi}, \frac{1}{a\cos\varphi}\frac{\partial\bar{\psi}}{\partial\lambda}\right) ,$$

ψ is the perturbation streamfunction, φ is latitude, λ is longitude.

In general, the space and time scale of mean flow $\bar{u}$ is much larger than that of perturbations, thus the so-called WKB approximation can be used to obtain a solution of Eq.(1). Assumed ε to be ratio of length scale of the perturbation to the mean flow, and ε is obviously much less than 1. Introducing the slowly varying coordinates X, Y, T gives

$$X=\varepsilon x \ , \qquad Y=\varepsilon y \ , \qquad T=\varepsilon t \ . \tag{2}$$

Thus, the solution of (1) takes the form

$$(x,y,t)=A(X, Y, T)e^{i\theta/\varepsilon} \ , \tag{3}$$

$$(k, l, \omega)=\left(\frac{\partial\theta}{\partial X}, \frac{\partial\theta}{\partial Y}, -\frac{\partial\theta}{\partial T}\right) , \tag{4}$$

where k, l are local wavenumbers in x and y directions respectively, ω is frequency.

Amplitude A is expressed as power series in terms of

$$A(X,Y,T)=A_0(X,Y,T)+\varepsilon A_1(X,Y,T)+\varepsilon^2 A_2(X,Y,T)+\cdots\cdots \tag{5}$$

Substituting (3)–(5) into (1), we get the zeroth-order equation

$$\omega=\bar{u}_M k+\bar{v}_M l-\frac{\beta_M k}{k^2+l^2}\equiv\Omega(k,l,X,Y,T) \ . \tag{6}$$

We only take zonally symmetric flow $\bar{u}_M$ into account and assume that $\bar{v}_M$ is zero. Thus, (6) becomes

$$\omega=\bar{u}_M k-\frac{\beta_M k}{k^2+l^2}\equiv\Omega(k,l,X,Y,T) \ . \tag{7}$$

The group velocities are obtained from the dispersion relation (7) giving

$$c_{gx}=\frac{\omega}{k}+\frac{2\beta_M k^2}{(k^2+l^2)^2} \ , \tag{8}$$

$$c_{gy}=\frac{2\beta_M kl}{(k^2+l^2)^2} \ . \tag{9}$$

Considering background fields $\bar{u}_M$ and β_M to be independent of X, Y, T with the use of the theory of wave propagation in a slowing varying medium, yields

$$\frac{d_g k}{dt} = -\left.\frac{\partial \Omega}{\partial X}\right|_{k,l,Y,T} = 0 , \tag{10}$$

$$\frac{d_g l}{dt} = -\left.\frac{\partial \Omega}{\partial Y}\right|_{k,l,X,T} = 0 , \tag{11}$$

$$\frac{d_g \omega}{dt} = \left.\frac{\partial \Omega}{\partial T}\right|_{k,l,X,Y} = 0 , \tag{12}$$

where

$$\frac{dg}{dT} = \frac{\partial}{\partial T} + c_{gx}\frac{\partial}{\partial X} + c_{gy}\frac{\partial}{\partial Y} .$$

From the definition of rays $\frac{dx}{dt} = c_{gx}$, $\frac{dy}{dt} = c_{gy}$ we have

$$\frac{dx}{dt} = \frac{\omega}{k} + \frac{2\beta_M k^2}{(k^2 + l^2)^2} \tag{13}$$

$$\frac{dy}{dt} = \frac{2\beta_M kl}{(k^2 + l^2)^2} . \tag{14}$$

Eqs.(10)–(14) are the equations for us to compute rays. Returning them to spherical coordinate gives

$$\begin{cases} \dfrac{d\lambda}{dt} = \dfrac{\omega}{ak} + \dfrac{2\beta_M k^2}{a(k^2 + l^2)^2} \\ \dfrac{d\varphi}{dt} = \dfrac{2\beta_M kl\cos\varphi}{a(k^2 + l^2)^2} \\ \dfrac{d_g l}{dt} = -\dfrac{k\cos\varphi}{a}\dfrac{\partial \bar{u}_M}{\partial \varphi} + \dfrac{\cos\varphi k}{a(k^2 + l^2)}\dfrac{\partial \beta_M}{\partial \varphi} \\ \dfrac{d_g k}{dt} = 0 \\ \dfrac{d_g \omega}{dt} = 0 \end{cases} . \tag{15}$$

The above differential equations are basic equations which are utilized to calculate the wave rays. Using a fourth–order Runge–Kutta scheme for a given initial value, we can get rays for different wavenumber and frequency.

III. THE SEASONAL VARIATION OF ROSSBY WAVE RAYS

To discuss the seasonal variation of rays, we utilize the zonal flows which were given by Tang (1973) for different seasons. To simplify the calculation, the mean flows have been expanded as Fourier series.

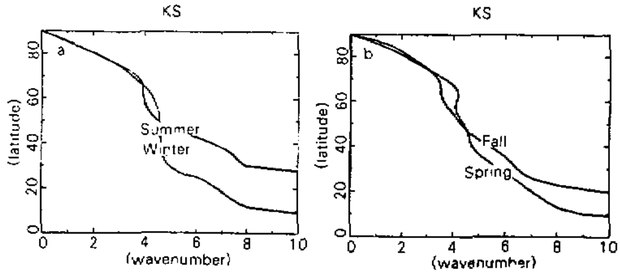

Fig. 1. K_S curves as a function of latitude. (a) winter and summer, (b) spring and fall.

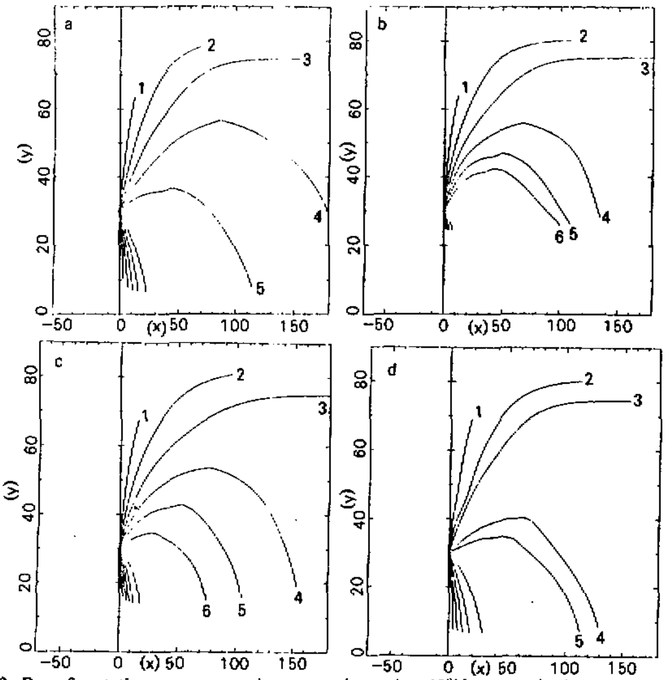

Fig. 2. Rays for stationary waves with a source located at 30°N. (a) spring(wavenumbers 1–5). (b) summer(wavenumbers 1–6). (c) fall(wavenumbers 1–6). (d) winter(wavenumbers 1–5).

1. *The Seasonal Variation of the Rays for Stationary Rossby Waves*

For stationary waves, $\omega = 0$, the critical wavenumber is $K_S = (k^2 + l^2)^{1/2} = [\frac{\beta_M \cos\varphi}{\bar{u}}]^{1/2}$. Thus we can get the variation curve of K_S as a function of latitude φ. Figure 1 gives the curve of K_S for different seasons. Rays can propagate freely in the south–north direction for $k < K_S$ but are trapped in the meridional direction for $k > K_S$. There exists a turning latitude at $k = K_S$, where the poleward rays are reflected and continue to propagate eastward. From Fig. 1 we find that critical wavenumbers decrease from equator to pole.

Thus the rays with lower wavenumbers can reach higher latitude, while the rays with higher wavenumbers are limited in lower latitude. This conclusion is consistent with the view of Wu Guoxiong et al. (1990). Making a comparison among curves of the K_S for different seasons, we easily find that: (a) The K_S curves of four seasons are almost overlapping for $K_S < 3.2$. This indicates that the long wave with wavenumber less than 3.2 can reach very high latitude in all seasons, which agrees well with the fact that the trough and ridge of long wavelength mainly exist in high latitude for all year. (b) K_S decreases with increasing latitude. This implies that the waves with longer wavelength can propagate to higher latitude, but those with shorter wavelength are confined in middle–low latitudes, which is in accord with the fact that wavelength in high latitude is longer than that in middle–low latitudes. (c) The long waves with wavenumbers more than five can reach the latitude less than 20°N in winter, but they can reach 40°N in summer. In fall and spring, they may reach more northern latitude than that in winter, but get more southern zones than in summer. It appears to be obvious seasonal variation in propagating region for longwave with shorter wavelength. (d) The fact of K_S between three and five there exists an abrupt change indicating that there is a split of the wavetrains in latitude, one has longer wavelengths and the other has shorter wavelengths. The former can propagate to higher latitude, but the latter is confined in middle–low latitude.

To verify the above inference and show the differences of rays in every season, we assume $\omega = 0$ in (15) and use a Runge–Kutta scheme to compute stationary Rossby wave rays for sources located at 15°N, 30°N and 45°N. Fig. 2 gives the rays for the four seasons with a source at 30°N. Results show that: (a) In the case of $l = \pm(k_S^2 - k^2)^{1/2}$, calculation gives two groups of rays. The poleward rays propagate initially poleward, then make a turning at $k = K_S$ and propagate equatorward ending in equatorial easterlies. The rays appear to be a great circle, which can explain teleconnection phenomenon. (b) Rays are trapped by easterlies, thus the position of equatorial easterlies has great influence on rays. As the position of easterlies is located at the most southern in winter and the most northern in summer, the rays can propagate to lower latitudes in winter and are confined in middle to high latitudes in summer, the rays in spring and fall are located at areas between them. It can explain to some extent why the teleconnection phenomenon is the most obvious in winter. (c) As K_s decreases from equator to pole, the rays for lower wavenumbers exist in higher latitudes and for higher wavenumbers in lower latitudes. Results also show that rays with wavenumbers 5 and 6 don't exist for a source at 45°N, the rays with wavenumbers 7–10 may even be produced for a source at 15°N. It explains why extra–long waves generally exist in middle to high latitudes. (d) The poleward rays are divided into two groups, one with longer wavelengths can propagate northward to very high latitude and has no obvious turning, the other with shorter wavelengths propagates northwards to a certain latitude and turns southward. The splitting of rays seems to be some connection with the abrupt change of the K_S curve. Two groups of rays correspond with two wavetrains as shown by Grose and Hoskins (1979), which are a southern wavetrain and a northern wavetrain. (e) The position of source and zonal mean flow are two important factors which have great influences on rays. Calculating results show that they may produce three different kinds of rays for sources at 15°N, 30°N and 45°N for a certain season. For a certain source the rays are also different for the different flows. From winter to spring, fall and summer, easterlies moves from south to north, it leads to the rays propagating higher latitude. In general, the location of rays is the most northern in summer

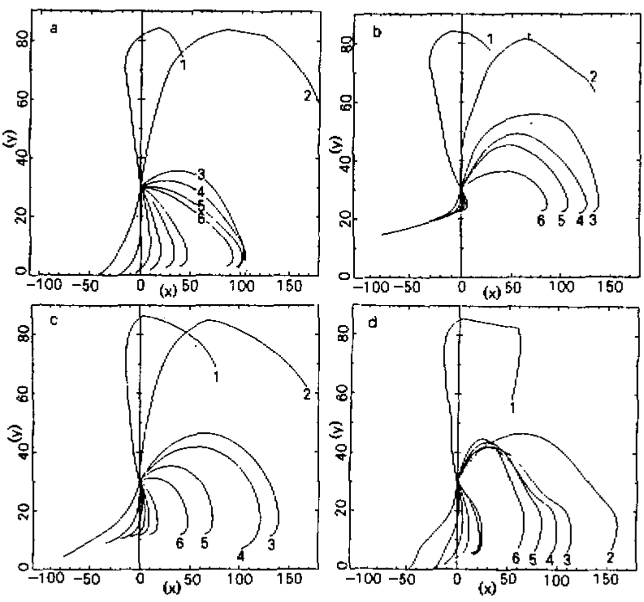

Fig.3. Low-frequency Rossby wave rays with 50-day period for a source at 30°N. (a) spring (b) summer (c) fall (d) winter.

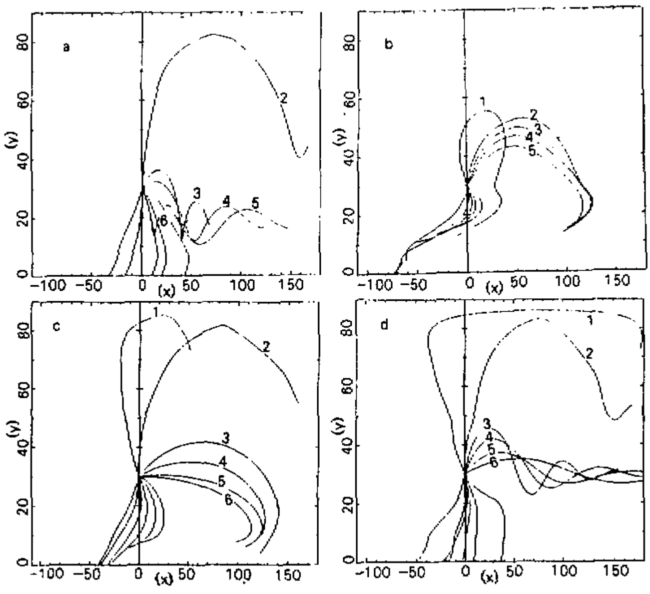

Fig. 4. Low-frequency Rossby wave rays with 20-day period for a source at 30°N. (a) spring (b) summer (c) fall (d) winter.

and the most southern in winter, and the rays in spring and fall are located at positions between of them. The above results can explain the observed facts that atmospheric circulation of middle to high latitudes has obvious response to SSTA in the tropics in winter, however, it has obvious response to SSTA in middle to high latitudes in summer. It is in accord with the fact that rays stemming from low latitude in winter can reach higher latitude and no poleward rays are produced in easterly zone in summer.

2. *The Seasonal Variation of Rays for Low–frequency Rossby Waves*

For low–frequency Rossby waves, $\omega \neq 0$, using $\omega = \frac{2\pi}{T}$ we may calculate the rays for different periods from (15). In order to discuss the propagation of low–frequency Rossby wave in different seasons, we compute the rays of low–frequency Rossby waves with 50–day and 20–day periods, then make a comparison among them and compare them with rays for stationary waves. Figs. 3 and 4 give four–season rays of Rossby waves with 50–day period and 20–day period respectively. From figures, it is easily seen that low–frequency waves and stationary waves have some resemblance. Both of them are reflected in high latitude, then turn to low–latitude. Low–frequency rays with 50–day period get closer to stationary rays than those with 20–day period, and the difference between rays for low–frequency waves and stationary waves becomes smaller with increasing wavenumbers. It is also seen that the difference between rays for wavenumbers 1–2 and 3–6 is larger. The latter gets closer to the rays for stationary waves than the former. It follows that the longer periods and the higher wavenumbers are, the closer the low–frequency wave rays will be to stationary ones. From the expression $K_S = [\beta_M / (\bar{u}_M - \frac{\omega}{R})]^{1/2}$, it is easily shown that K_S for low–frequency waves is closer to that of stationary waves with decreasing ω and increasing k.

For stationary waves, $K_S = (\beta_M / \bar{u}_M)^{1/2}$, and for low–frequency waves, $K_S = [\beta_M / (\bar{u}_M - \frac{\omega}{R})]^{1/2}$, they are obviously different, the critical latitudes for stationary and low–frequency waves are also different. The critical latitude for stationary waves exists in the joint of easterlies and westerlies in the northern hemisphere, but that for low–frequency waves lies in the equatorial easterlies which is either in the Northern Hemisphere or in the Southern Hemisphere. When the critical latitude for low–frequency waves lies in the Southern Hemisphere, it is possible for low–frequency wave rays to propagate into the Southern Hemisphere. However, the rays for stationary waves end in the joint of easterlies and westerlies, and can't propagate into the Southern Hemisphere. We can see this point clearly from Figs. 3 and 4. Whether rays can propagate into the Southern Hemisphere is also a major distinction between stationary and low–frequency waves.

From Figs. 3 and 4, we also find that it is easier to propagate into the Southern Hemisphere for low–frequency waves with 20–day period than those with 50–day period. Wave with higher frequency is easier to propagate between two hemispheres, namely, it is advantageous to the exchange of energy and to the interaction. The position of equatorial easterlies exerts an influence not only on the propagation of stationary waves, but also on that of low–frequency waves. Equatorial easterlies in summer is stronger than that in winter, thus the larger westward phase speed is necessary for rays to enter into another hemisphere. In the case of same phase speed, it is the most difficult for low–frequency waves in summer to propagate into the Southern Hemisphere, and it is much easier in winter and spring. From Figs. 3

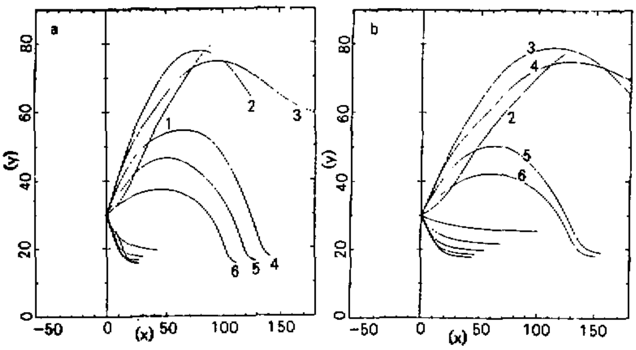

Fig. 5. Low–frequency Rossby wave rays with positive phase speed in fall. (a) 50–day period, (b) 20–day period.

and 4. it is easily shown that rays for low–frequency waves in summer are in the northeast position and in the southeast in winter. We can make a conclusion from above that Rossby waves are easier to propagate into another hemisphere in the case of higher frequency and weaker easterlies, and this may explain why low–frequency waves with 20–day period are easier into another hemisphere than those with 50–day period and why low–frequency waves propagate more easily into the Southern Hemisphere in winter than in summer.

As the critical latitudes for low–frequency waves which have positive phase speed lie in the westerlies, they have higher critical latitudes than for stationary waves. The propagation of low–frequency waves with positive phase speed is more limited in middle to high latitudes than that of stationary waves and low–frequency waves with westward phase speed. Fig.5 shows this character.

It is very interesting to note that low–frequency wave rays with 20–day period and wavenumbers 3–6 in spring and winter show damping wave patterns, which may be the phenomenon stemming from waves being reflected repeatedly at the critical latitude.

IV. CONCLUSIONS

In this paper, we compute rays for stationary and low–frequency Rossby waves by using a fourth–order Runge–Kutta scheme. Results show that rays in winter can propagate to lower latitudes, however, in summer they are confined in middle to high latitudes, in spring and fall the rays are between them. Observations show that atmospheric circulation anomaly is the most obvious in winter and atmospheric interaction between middle and high latitudes is also the strongest for this season, those are in accord with our calculating results, namely, the propagation of rays is the nearest to the equator in winter. In addition, seasonal variation of rays is consistent with the fact that atmospheric circulation in middle–high latitude has obvious response to SSTA in the tropical zone in winter and has obvious response to that in middle high latitude in summer. The results show that the major difference of rays between stationary waves and low–frequency waves is that stationary wave rays are trapped by easterlies and can't propagate into another hemisphere, but low–frequency wave rays can propagate across the equator. Thus, low–frequency Rossby waves may play an important role in the interaction between the Southern and Northern Hemispheres. Calculating results also show that stronger easterlies and lower frequency are not advantageous to propagating across the equator for low–frequency wave rays.

What is more, low–frequency wave rays in winter are easier to propagate across the equator than in summer, and the low–frequency waves with 20–day period are easier to propagate across the equator than that with 50–day period.

Rays represent energy propagation path. Rossby wave rays have close connection with atmospheric teleconnection and denote the pattern of teleconnection. The rays are mainly determined by the distribution of zonal mean flow. Thus teleconnection also has close connection with mean flow. If zonal mean flow changes, teleconnection field will change correspondingly. This indicates that teleconnection also has an obvious seasonal variation.

This paper only discusses rays in the case of zonally symmetric flow, but doesn't discuss rays in the case of zonally varying flow. When westerly wind appears in some region of the zonal mean easterlies, the waves which cannot propagate across the equator originally can propagate into another hemisphere through a westerly wind duct. This strengthens greatly the interaction between the Southern and Northern Hemispheres and causes a significant change in atmospheric circulation.

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