The Variability of the Interannual Oscillations of the Indian Summer Monsoon Rainfall

R. Vijayakumar and *J. R. Kulkarni*
Indian Institute of Tropical Meteorology, Pune—411008, India
Received April 19, 1994; revised August 3, 1994

ABSTRACT

A new method of analysis namely, Singular Spectrum Analysis (SSA) is applied to the Indian Summer Monsoon (June—September) Rainfall (ISMR) series. The method is efficient in extracting the statistically significant oscillations with periods 2.8 and 2.3 year from the white noise of the ISMR series. The study shows that 2.8 / 2.3 year cycle captures the variability of the ISMR related to Southern Oscillation / Quasi Biennial Oscillation. The temporal structure of these oscillations show that these are in phase in extreme (excess and drought) monsoon conditions as well as in El Nino Southern Oscillation (ENSO) years. Both these oscillations show minimum variability during the period 1920—1940 and there is an increasing trend in the variability of these oscillations in the recent decades. The study enables to obtain pure signal consisting of reconstructed time series using these two oscillations, from the original white noise series.

Key words: Singular Spectrum Analysis, Monsoon Variability.

I. INTRODUCTION

The devastating droughts over India in the later period of nineteenth century prompted the necessity of the long range forecast of the Indian Summer Monsoon (June—September) Rainfall (ISMR) to mitigate the drought conditions. Walker (1923) initiated the studies of the interannual variability of ISMR (hereafter called "the variability") in relation to the antecedent meteorological parameters to identify the predictors for forecasting ISMR. As the predictors loose statistical significance with the passage of time, there is always search for new predictors and therefore the work initiated by Walker is still in continuance (for review see Jagannathan, 1960; Shukla, 1986).

Another approach, which was thought to be important from the forecasting point of view, was to study the variability and obtain the prominent periodicities and rhythms. The rhythm of solar cycle for 11 and 22 years is well established and known for a long time. Ananthakrishnan and Parthasarathy (1984) observed that the excess rainfall years were significantly more during the ascending phase of sunspot cycle. Bhalme and Jadhav (1984 a) found that there is a strong tendency for occurrence of more frequent floods in the major sunspot cycle than in the minor sunspot cycle. This information, though important, found less use in forecasting the ISMR because of the nature of the information itself. Bhalme and Jadhav (1984 b), Mooley and Parthasarathy (1984) applied the power spectrum technique given by Blackman and Tukey (1958), to the ISMR series to obtain the oscillations in the ISMR. They observed that the power spectrum for ISMR series was characteristic of white noise. There was accumulation of the spectral power in the wavelength bands between 2 to 3 years. The spectral peaks in the period ranging from 2.6—2.9 years were significant at 95% confidence level which was close to the first modal peak of the Southern Oscillation (SO).

There was another prominent spectral peak corresponding to the period of about 2.3 years significant at 90% confidence level, which was compatible with Quasi Biennial Oscillation (QBO). These findings proved useful in the sense that SO and QBO are considered in the 16 parameter Gowariker et al.,(1989) model for ISMR forecasting. Going one step further, we consider that it would be very useful to forecasters, if it is possible to isolate the variability of ISMR related to those two oscillations. The time series thus formed may be considered as pure signal in ISMR. At present, the authors are not aware of any other technique which extracts signal from white noise series, so efficiently.

The main objective of this paper is to extract signal from the ISMR series and study its temporal structure. It is possible to investigate this aspect because of the development of the new technique called Singular Spectrum Analysis (SSA). SSA is algorithmically equivalent to the application of extended empirical orthogonal functions to a univariate time series. The method is also known as Karhunen–Loeve expansion (Pike et al.,1984) in digital signal processing. Fraedrich (1986) introduced this technique to atmospheric fields. The technique was further improved and given strong footing by Vautard and Ghil (1989). Rasmusson(1990) applied it to ENSO variability and observed a rather inactive period of biennial variability during most of the 1950's. Keppen and Ghil (1993) applied this technique for successful prediction of SOI.

SSA technique, which is found very useful in variety of fields such as signal processing, nonlinear dynamics and paleoclimatology, has been discussed briefly in Section 3. For comprehensive treatment of the subject, reader is referred to Vautard and Ghil (1989) and Vautard et al.(1993). Data used in this study are given in Section 2. Results are discussed in Section 4 and Section 5 gives conclusions.

II. DATA

The ISMR series for 121 years from 1871 to 1991 of Parthasarathy et al. (1992) have been utilized in this study. The series has mean X=85.2 cm and standard deviation $\sigma=8.3$ cm. The series is normalized with mean 85.2 cm and standard deviation 8.3 cm. Figure 1 shows the normalized time series of ISMR. The curves shows the variability is of a year to year nature. ISMR denoted by R is classified as excess / normal / deficient according to

$$R>(X+\sigma);\quad (X-\sigma)<R<(X+\sigma);\quad R<(X-\sigma)$$

There are 17 excess years marked by $R>(X+\sigma)$ and 21 deficient years marked by $R<(X-\sigma)$. Also seen from the figure that there is no long-term trend. Figure 2 shows the conventional power spectrum of it. The SOI data (Taihiti–Darwin for December, January and February) for the period 1972 to 1991 (from Climate Diagnostic Bulletin, Table 3, March (1986) and updated up to 1992) are used in this study. The data of zonal winds in QBO for the period 1957 to 1985 are interpolated from Fig.3 of Bhalme et al., (1987).

III. SINGULAR SPECTRUM ANALYSIS

SSA is based on Karhunen–Loeve expansion theorem. It is the univariate application of Principal Component Analysis in the time domain. The theoretical aspects of SSA have been discussed comprehensively by Vautard and Ghil (1989) and Vautard et al. (1993). The technique can be explained briefly as follows.

The given normalized series X can be resolved into M orthogonal components such that

$$X_{i+j} \approx \sum_{k=1}^{M} a(k,i)E(k,j) \ ,$$

where $a(k,i)$ represents the ith element of kth principal component, $E(k,j)$ represents the jth element of kth eigenvector, and M gives the embedding dimension or window length. The eigenvectors $E(k,j)$ are computed by constructing the Toeplitz matrix such that the element in ith row and jth column is given by the covariance at lag $i-j$. Because of the peculiar structure of the Toeplitz matrix, the eigenvectors are either symmetric or antisymmetric with respect to M / 2. A pure oscillation whose time scale is shorter than the embedding dimension M, appears as an even and odd eigenvector pair in quadrature with each other. If the time scale of the oscillation is much lager than the embedding dimension M, the eigenvector pair appears as a running mean (even) and a trend (odd). In contrast with standard spectral analysis in which the basis functions are given a priori (e.g. the sines and cosines of Fourier analyses), in SSA they are determined from the data themselves to form an optimal orthogonal basis in the statistical sense (Keppen and Ghil, 1993). The eigenvectors reveal the dominant modes of variability and the associated principal components provide the information relating to intermittency and variations in amplitude that are not obtainable from conventional spectrum analysis (Rasmusson, 1990).

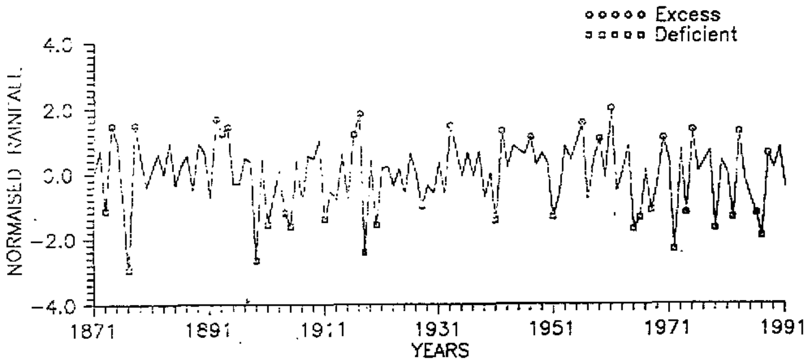

Fig. 1. Normalized Indian summer monsoon rainfall for the period 1871–1991.

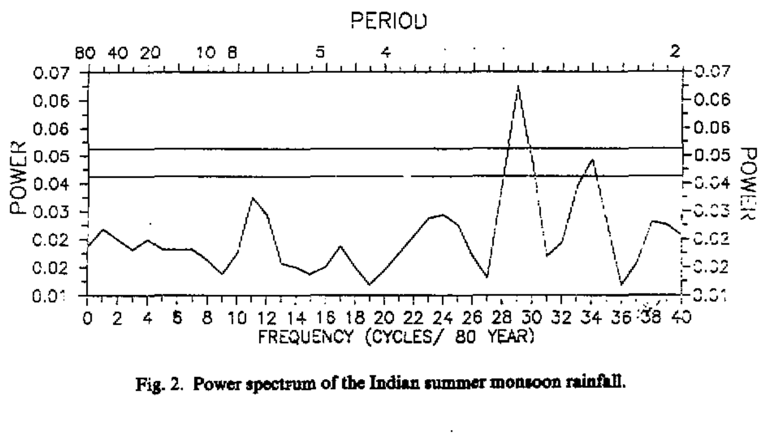

Fig. 2. Power spectrum of the Indian summer monsoon rainfall.

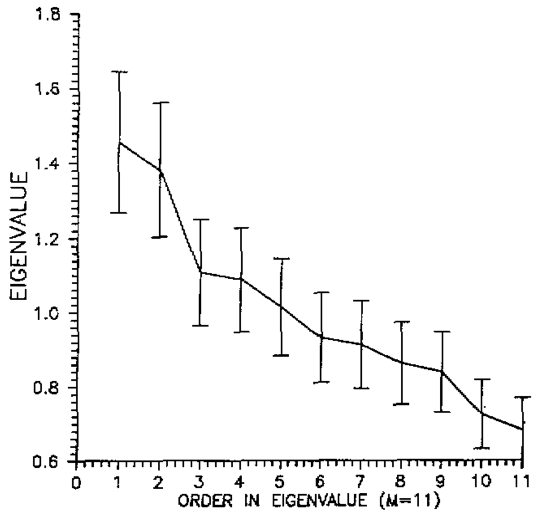

Fig. 3. Eleven eigenvalues of the singular spectrum of the rainfall.

IV. RESULTS

The first step in SSA is the proper selection of the value of M. The choice of M is a compromise between the amount of information one hopes to retain the larger M the better and statistical confidence one needs to achieve, the smaller M the better. If M is too small, the course resolution may mix together several neighboring peaks in the spectrum of X. On the contrary, if M is too large, the high resolution may split the peak into several components with neighboring frequencies. SSA is typically successful in analyzing periods in the range $M/5$ to M, (Vautard et al., 1993). Figure 2 shows there are two oscillations of period 2.8 years and 2.3 years in ISMR series. Therefore we selected $M=11$. Hence this technique can analyze oscillations of periods 2.2 years to 11 years. With $M=11$, Toeplitz matrix is generated as discussed in Section 3. Figure 3 shows 11 eigenvalues with standard error calculated using North et al. (1982) formula which is

$$\sigma \approx \lambda \sqrt{\frac{2}{N}} ,$$

where σ is standard error, λ is eigenvalue and N is total number of observations. From the figure it is seen that eigenvalues 1 and 2 are nearly equal and according to Vautard and Ghil (1990) terminology, they form plateau. Then there is sudden decrease in eigenvalues which may be considered as forming a slope. Eigenvalues 3 and 4 are again nearly equal and may be considered as forming another plateau. Similar is the case for eigenvalues 6, 7; 8, 9; 10, 11 which form plateau and slope alternately. The spacing between eigenvalues 2 and 3 is more than the standard error and therefore they may be considered above the noise floor.

Figure 4 shows 11 eigenvectors (EV) with variance explained on the right hand corner. EV1 and EV2 are in quadrature with each other, EV1 is even and EV2 is odd. Again EV3

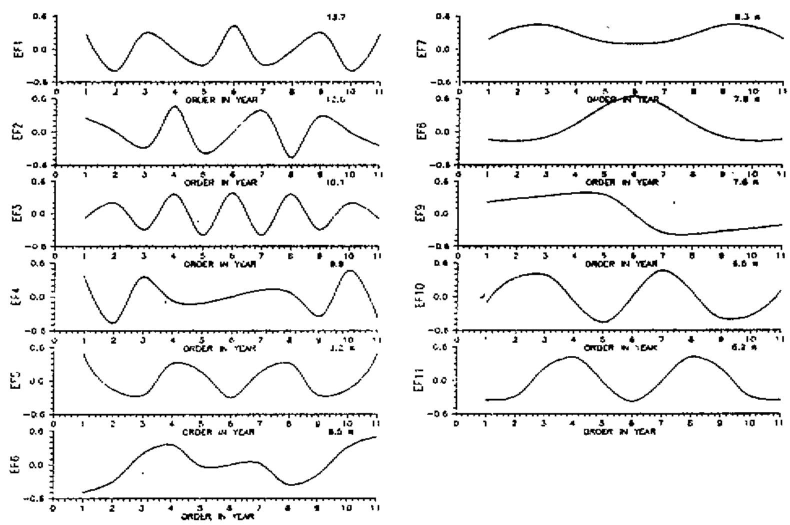

Fig. 4. Eleven Singular Spectrum Analysis (SSA) eigenvectors for the rainfall.

and EV4 show a pair of even and odd eigenvector pair in quadrature with each other. Similar is the case for EV5, EV6 and EV7, EV8. In the last group there are three eigenvectors EV9, EV10, EV11 in quadrature with each other. There are two odd eigenvectors EV8, EV9 and one even eigenvector EV11.

Figure 5 shows the power spectrum of the Principal Components (PC) corresponding to these eigenvectors. PC1, PC3 and PC7 show the peak for the period 2.8 years and PC2 shows the peak for 2.3 years. As noted earlier, eigenvalues 1 and 2 are above the noise floor, therefore, the signal is constructed using PC1 and PC2 only.

Figure 6 shows the time series reconstructed using the method justified rigorously by Vautard et al. (1992) for the signal given by PC1 and PC2 respectively. The correlation coefficient (CC) between ISMR and SOI was found to be 0.43 (for the period 1972–1991) whereas the CC between the reconstructed series by PC1 and SOI for the same period was fornd to be 0.51. SOI has variability over different frequencies, prominences among them are about 2 to 3 and 5 year cycle. The reconstructed series with PC1 shows spectral peak at 2.8 year cycle. Therefore it may be inferred that the variability in ISMR which is associated with the southern oscillation of 2.8 year period, has been reflected in the time series reconstructed using PC1.

The CC between the reconstructed series by PC2 and QBO is 0.60 whereas CC between QBO and original series is 0.50. Thus again it may be seen that the reconstructed series with PC2 extracts the variability of ISMR related to QBO. Eigenvalues 1 and 2 together explain 24% variance in ISMR. Thus signal to noise ratio in ISMR is 0.32.

Interannual and decadal variability of the time series related to SO (PC1) and QBO (PC2) is summarized in Table 1. It may be seen that there are is a period of 30 years from 1909 to 1939, in where both the oscillations are in phase. On the average there are 7 years in a decade when both the oscillations are in phase and in three years they are in opposite phase. However the decade 1960–1969 shows five years when these are in opposite phase which is the maximum. Out of total 17 excess–years 2.8 and 2.3 year oscillation are in phase in 16

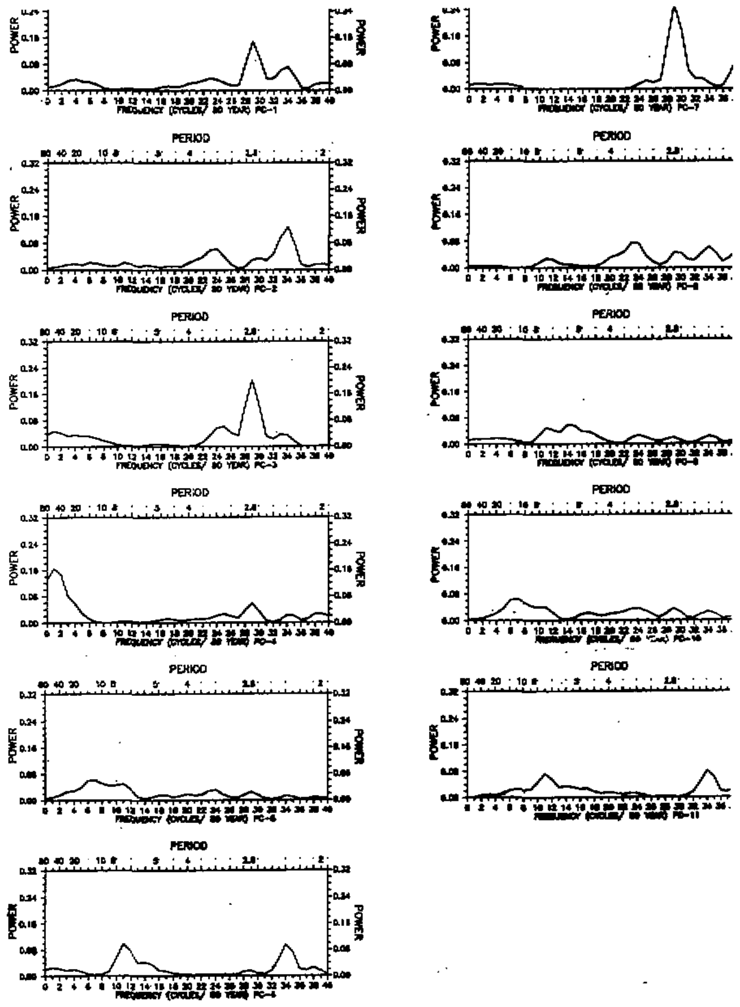

Fig. 5. Spectra of the eleven PC's for the rainfall.

years. Similarly out of 21 deficient years, these are in phase in 18 years. out of 25 El Nino years, these are in phase for 21 years. Further if these El Nino events are classified as strong and moderate (Mooley & Parthasarathy, 1984), in 11 out of 12 strong El Nino events, SO and QBO are in phase.

Figure 7 shows the decadal variability. It may be seen from the figure that during the period 1871 to 1910 amplitude of SO related cycle was more variable than the amplitude of QBO related cycle. Both the oscillations show less variability during the period 1920–1940, and there is increasing trend in the variability in the recent decades.

In most of the present methods of ISMR forecasting, ISMR is taken as basis which contains signal as well as noise. SSA technique discussed in this paper is able to extract signal

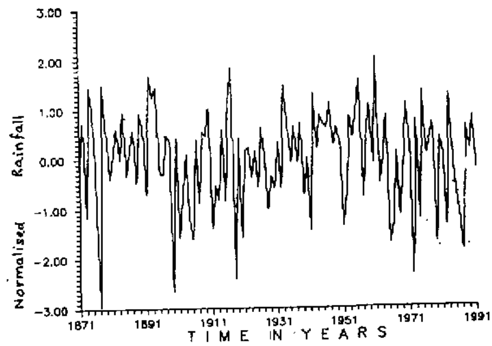

Fig. 6. Normalized rainfall series reconstructed using PC1 and PC2.

from the noise. This information is particularly important to forecasters because they can treat ISMR series separately in signal domain and in noise domain. There are large amount of work which go to predict SOI. Therefore a part of ISMR which is associated with SOI, thus can be predicted which will be useful for total ISMR prediction.

Table 1. Interannual and Decadal Variability of the Time Series Related to SO (PC1) and QBO (PC2)

Decade	In phase years	Out of phase years
1871–1879	1871*,73,75,76,77**,78^{+},79	72,74^{+}
1880–1889	80*,82,83,86,87*,88,89	81,84**, 85
1890–1899	90,91**,92^{+},93^{+},94^{+},97,99^{**-}	95,96*,98
1900–1909	1900,01^{-},02*,03,04,05^{-*},09	06,07,08
1910–1919	10,11^{**-},12,13,14*,15,16^{+},17^{+},18^{**-},19	NIL
1920–1929	20^{-},21,23,24,25**,26,27,28^{-},29*	22
1930–1939	30,31,33^{+},34,35,36,37,38,39*	32
1940–1949	41^{**-}, 42^{+},44,45,47^{+},48,49	40,43,46
1950–1959	51^{-},52,53*,55,56^{+}57**,59^{+}	50,54,58
1960–1969	61^{+},62,64,65^{-*},69	60,63,66^{-},67,68^{-}
1970–1979	70^{+},72^{**-},73,74^{-},75^{+},78,79^{*-}	71,76*,77**
1980–1989	80,81,82^{**-},83^{+},84,87^{**-},88^{+}	85,86^{-},89

+ Excess rainfall, − Deficient rainfall, * Moderate El–Nino, * * Strong El–Nino

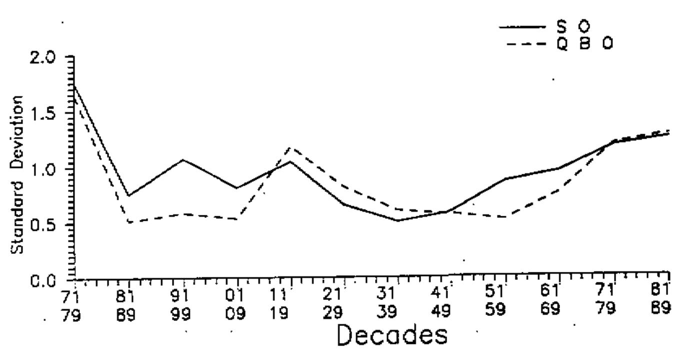

Fig. 7. Decadal variability of ISMR related to SO and QBO.

V. CONCLUSIONS

The newly developed technique of Singular Spectrum Analysis has been applied to ISMR series to extract the variability related to statistically significant oscillations in ISMR which are 2.8 years cycle and 2.3 years cycle. The temporal structure of these two oscillations has been investigated. These are in phase in excess, deficient monsoon years and in ENSO years. The period 1920–1940 is the period of reduced variability of these two oscillations. The recent decades show increased variability of these two oscillations.

The time series constructed using PC1 and PC2 represents the signal in ISMR. This separation of signal and noise in ISMR may be useful for prediction of the ISMR.

The authors are thankful to Prof. R. N. Keshavamurty, Director for his constant encouragement. They wish to express their sincere thanks to Dr. A. S. R. Murty, Deputy Director for the guidance and valuable suggestions.

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