Low–Frequency CISK–Rossby Wave and Stratospheric QBO in the Tropical Atmosphere①

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ABSTRACT

Dynamic study is undertaken of the tropical atmospheric CISK–Rossby wave genesis and propagation mechanisms, the vertical structure of the low–frequency wave and the basic characteristics and constraint of the vertical transport of momentum and wave energy fluxes in relation to the quasi–biennial oscillation (QBO) of the stratospheric zonal winds over the tropics in the context of a baroclinic quasi–geostrophic model. Results suggest that in the properly posed thermal conditions and zonal belt there exist two kinds of CISK–Rossby waves of low frequency (LF) and very low frequency (VLF), travelling zonally in opposite directions, which act as sources responsible for upward transferring momentum and wave energy fluxes for easterly and westerly perturbations in such a way as to provide required momentum and energy for the stratospheric QBO genesis and maintenance. The present study offers interpretations for some of the fundamental observational facts of the QBO and proposes new ideas of the QBO generation mechanism.

Key words: Thermal forcing, QBO, CISK–Rossby wave

1. Introduction

The low–frequency oscillation (LFO) is a very important weather phenomenon in the atmosphere. The 30–50–day and quasi–biweekly oscillations in the tropical atmosphere are most intensively studied, and comparatively speaking, another kind of LFO concerning the QBO of the stratospheric zonal winds is much less studied, which, although occurring in the stratosphere, bears a close relation to the tropospheric activities and the evolution of low–latitude circulations so that it is worthwhile to explore in depth the physical mechanism for the QBO occurrence and development and the mutual action and influence between the stratospheric QBO and the tropospheric synoptic systems. Many hypotheses have been developed regarding the QBO triggering mechanism, but only the theory on wave vertical propagation and momentum transfer produced by Lindzen and Holton (1968) and Holton and Lindzen (1972) is widely accepted as a more classic result, stating that the vertical propagation of the mixed Rossby gravity wave and Kelvin wave in the tropical troposphere and the vertical transport of wave momentum furnish the perturbation energy to the zonal wind QBO

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in the stratosphere and the energy absorption in the critical layer acts as a conversion mechanism of wave energy into a disturbance form, which have been supported by many other researchers from different aspects. Maruyama (1968) and Kousky and Wallace (1971), for instance, confirmed the existence of upward transferring momentum and energy in the troposphere, whose amounts are close to the needs of the QBO development through their diagnosis and data analysis. Lindzen et al. (1971) investigated the contribution to the conversion of wave energy into QBO energy made by such non–adiabatic damping as Rayleigh friction and Newtonian cooling in addition to the absorption in a critical layer. The foregoing studies have affirmed, on the whole, the contribution of vertical propagation of low–level waves and vertical transport of energy to the stratospheric QBO genesis and development. However, the easterly and westerly alternate emergence and oscillation in the stratosphere depend on the occurrence of momentum and wave energy fluxes upward transported for the winds. Following Holton and Lindzen (1972) such momentum and energy sources are offered by the westward propagating mixed Rossby gravity wave and the eastward migrating Kelvin wave. But, is the QBO associated only with the two kinds of wave? The present study shows that the tropical low frequency CISK–Rossby wave has the same effects and the ability to move east– and westward and keep symmetric stability in vertical wave propagation and momentum / energy transport; the CISK–Rossby wave under research displays LF features, very similar to those of the zonal wind QBO (and even with the quasi–biennial period) as compared to the other two kinds of wave mentioned earlier. We are thus led to assume the CISK–Rossby wave to possibly be an important factor for the genesis and maintenance of the stratospheric QBO.

2. Dynamic model

We employ the zonally–axisymmetric disturbance baroclinic linear model with the Boussinesq approximation:

$$\begin{cases} \dfrac{\partial u}{\partial t} - fv = -\dfrac{\partial \varphi}{\partial x} , \\ \dfrac{\partial v}{\partial t} + fu = 0 , \\ \dfrac{\partial u}{\partial x} + \dfrac{\partial w}{\partial z} = 0 , \\ \dfrac{\partial}{\partial t}\left(\dfrac{\partial \varphi}{\partial z}\right) + N^2 w = N^2 \eta w_B , \end{cases} \tag{1}$$

where N denotes the Brunt–Vaisala frequency, η the parameter of the argument of dimensionless convective condensation latent heat and w_B the vertical velocity at the top of the boundary layer. As we know, the tropical atmospheric LFO is of planetary scale and the motion we are dealing with is quasi–geostrophic (Li, 1985a) so that baroclinic quasi–geostrophic filtering is made of the model, leading to a system of dynamic equations containing β effect and thermal forcing in the form

$$\begin{cases} \dfrac{\partial}{\partial t}\left(\dfrac{\partial^2 \varphi}{\partial x^2}\right) + \beta\dfrac{\partial \varphi}{\partial x} - f^2\dfrac{\partial w}{\partial z} = 0 , \\ \dfrac{\partial}{\partial t}\left(\dfrac{\partial \varphi}{\partial z}\right) + N^2 w = N^2 \eta w_B . \end{cases} \tag{2}$$

In a similar way to Jiang (1996), elimination of φ in (2) results in an equation including w as the only variable

$$\frac{\partial}{\partial t}\left(\frac{\partial^2 w}{\partial x^2}+\frac{f^2}{N^2}\frac{\partial^2 w}{\partial z^2}\right)+\beta\frac{\partial w}{\partial x}=\eta\frac{\partial^3 w_{\mathrm{B}}}{\partial t\partial x^2}+\beta\eta\frac{\partial w_{\mathrm{B}}}{\partial x}\ , \tag{3}$$

to focus on longwave–related LFO, the terms of β and η on the *rhs* of (3) are retained (Jiang, 1996) with no reference to the others, and then the expression is simplified as

$$\frac{\partial}{\partial t}\left(\frac{\partial^2 w}{\partial x^2}+\frac{f^2}{N^2}\frac{\partial^2 w}{\partial z^2}\right)+\beta\frac{\partial w}{\partial x}=\beta\eta\frac{\partial w_{\mathrm{B}}}{\partial x}\ . \tag{4}$$

3. Conditions of wave generation and vertical propagation

We assume approximately the vertical velocity at the top of the boundary layer to be $w_{\mathrm{B}}=b\cdot w(b\leqslant 1$, Li, 1990), where w is set to be $W(z)\cdot e^{i(kx-\sigma t)}$, which is substituted into (4), leading to

$$\frac{d^2W}{dz^2}+QW=0\ , \tag{5}$$

where $Q=-\frac{N^2}{f^2}[k^2+\frac{k\beta}{\sigma}(1-b\eta)]$.

We find from (5) an analytical solution as $W(z)=c_1 e^{-\sqrt{-Q}\cdot z}+c_2 e^{\sqrt{-Q}\cdot z}$. Normally we assume the boundary condition in such a way that $W(z)$ is bounded for $z\to\infty$. Thus we get $c_2=0$ and so $W(z)=c_1 e^{-\sqrt{-Q}\cdot z}$. When this solution is substituted back into $w=W(z)\cdot e^{i(kx-\sigma t)}$ we have

$$w=W(z)\cdot e^{i(kx-\sigma t)}=c_1 e^{-\sqrt{-Q}\cdot z}\cdot e^{i(kx-\sigma t)}. \tag{6}$$

Likewise, let u be $U(z)\cdot e^{i(kx-\sigma t)}$ and with the aid of the third sub–equation of (1) we obtain

$$U(z)\cdot ik\cdot e^{i(kx-\sigma t)}=-\frac{\partial w}{\partial z}=c_1\cdot\sqrt{-Q}\cdot e^{-\sqrt{-Q}\cdot z}\cdot e^{i(kx-\sigma t)}\ ,$$

$$u=U(z)\cdot e^{i(kx-\sigma t)}=\frac{c_1}{ik}\sqrt{-Q}\cdot e^{-\sqrt{-Q}\cdot z}\cdot e^{i(kx-\sigma t)}\ . \tag{7}$$

By setting $v=V(z)e^{i(kx-\sigma t)}$ and $\varphi=\Phi(z)e^{i(kx-\sigma t)}$ we derive through the second and first sub–equations of (1),

$$V(z)=-\frac{fc_1}{k\sigma}\cdot\sqrt{-Q}\cdot e^{-\sqrt{-Q}\cdot z}\ ,\qquad \Phi(z)=\frac{ic_1}{\sigma k^2}(f^2-\sigma^2)\sqrt{-Q}\cdot e^{-\sqrt{-Q}\cdot z}\ ;$$

$$v=-\frac{fc_1}{k\sigma}\cdot\sqrt{-Q}\cdot e^{-\sqrt{-Q}\cdot z}\cdot e^{i(kx-\sigma t)}\ ,\qquad \varphi=\frac{ic_1}{\sigma k^2}(f^2-\sigma^2)\sqrt{-Q}\cdot e^{-\sqrt{-Q}\cdot z}\cdot e^{i(kx-\sigma t)}\ . \tag{8}$$

If Q be a slowly varying function of z, then we introduce localized vertical wavenumber m (Lu et al., 1997) and get

$$Q=m^2=-\frac{N^2}{f^2}[k^2+\frac{k\beta}{\sigma}(1-b\eta)]\ . \tag{9}$$

It is easily seen from (6) to (8) that for $Q = m^2 > 0$, we have

$$w = c_1 e^{-\sqrt{-Q}\cdot Z}\cdot e^{i(kx-\sigma t)} = c_1 e^{\pm imz}\cdot e^{i(kx-\sigma t)} = c_1 e^{i(kx\pm mz-\sigma t)},$$

$$u = \frac{c_1}{ik}\sqrt{-Q}\cdot e^{-\sqrt{-Q}\cdot Z}\cdot e^{i(kx-\sigma t)} = \frac{c_1\cdot m}{k}\cdot e^{i(kx\pm mz-\sigma t)},$$

$$\varphi = \frac{ic_1}{\sigma k^2}(f^2-\sigma^2)\sqrt{-Q}\cdot e^{-\sqrt{-Q}\cdot Z}\cdot e^{i(kx-\sigma t)} = -\frac{c_1}{\sigma k^2}(f^2-\sigma^2)\cdot m\cdot e^{i(kx\pm mz-\sigma t)}$$

In the z direction, w, u and φ are all in wavy form in both horizontal and vertical. As shown in Fig. 1, the wave amplitude from the disturbance of vertical velocity c_1 is constant and that from horizontal velocity and geopotential perturbations is associated with m and hence inconstant. With $Q = m^2 < 0$, we have $w = c_1 e^{-\sqrt{-Q}\cdot Z}\cdot e^{i(kx-\sigma t)} = c_1 e^{-|m|z}\cdot e^{i(kx-\sigma t)}$, in which vertical velocity perturbation produces waves in horizontal with that trapped in vertical, leading to a fact that the amplitude $c_1 e^{-|m|z}$ is a variable and attenuates exponentially as a function of height, such that horizontal wave is noticeable only at lower levels. And for

$$u = \frac{c_1}{ik}\sqrt{-Q}\cdot e^{-\sqrt{-Q}\cdot Z}\cdot e^{i(kx-\sigma t)} = \frac{c_1\cdot|m|}{ik}\cdot e^{-|m|Z}e^{i(kx-\sigma t)},$$

$$\varphi = \frac{ic_1}{\sigma k^2}(f^2-\sigma^2)\sqrt{-Q}\cdot e^{-\sqrt{-Q}\cdot Z}\cdot e^{i(kx-\sigma t)} = \frac{i\cdot c_1}{\sigma k^2}(f^2-\sigma^2)\cdot|m|\cdot e^{-|m|Z}e^{i(kx-\sigma t)},$$

the amplitudes from horizontal velocity and geopotential disturbances are complex-rooted, indicating that such disturbances yield no real solution of wave.

Based on the analysis, we have $m^2 > 0$ as the condition of vertical wave or its propagation. From $m^2 = Q = -\frac{N^2}{f^2}[k^2 + \frac{k\beta}{\sigma}(1-b\eta)] > 0$ we find $\frac{N^2}{f^2}[k^2 + \frac{k\beta}{\sigma}(1-b\eta)] < 0$, which suggests that the thermal forcing has to be strong enough to allow the wave to travel in

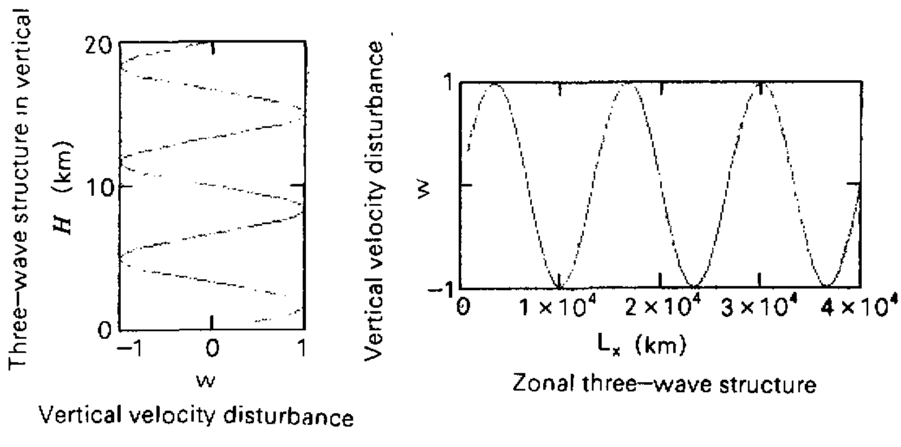

Fig. 1. Schematic of equatorial three-wave form in horizontal and vertical.

vertical when $\eta > \eta_c = \frac{1}{b}[\frac{\sigma k^2}{k\beta} + 1]$ as the critical value. It can thus be inferred that intense thermal forcing is an important condition for triggering the CISK–Rossby wave which is made to migrate in vertical.

Another form of wave vertical propagation condition from $m^2 > 0$ is $\sigma < -\frac{\beta(1-b\eta)}{k}$ or $C_x < -\frac{\beta(1-b\eta)}{k^2}$ (where c_x is the phase velocity of zonal wave), suggesting that for weak enough η to make $(1-b\eta) > 0$, only westward travelling CISK–Rossby wave is probably allowed to migrate vertically while for strong enough η to make $(1-b\eta) < 0$, west ward migrating and quasi steady CISK–Rossby wave is possibly permitted to do so, and so is the slowly eastward moving counterpart.

The condition of vertical propagation trapping is $m^2 < 0$. From $m^2 = Q = -\frac{N^2}{f^2}[k^2 + \frac{k\beta}{\sigma}(1-b\eta)] < 0$ we obtain $\frac{N^2}{f^2}[k^2 + \frac{k\beta}{\sigma}(1-b\eta)] > 0$. It is clear that the condition is satisfied only when the thermal forcing η is weak enough to cause $\eta < \eta_c = \frac{1}{b}[\frac{\sigma k}{\beta} + 1]$, meaning that a horizontal CISK–Rossby wave is excited only when the thermal forcing is smaller than its critical value.

Now we deal with the vertical transport–related critical thermal effect $\eta_c = \frac{1}{b}[\frac{\sigma k}{\beta} + 1] = \frac{1}{b}[\frac{c_x k^2}{\beta} + 1]$ with the aid of zonal wavelength $L_x = 1.0 \times 10^7$ m, $b = 0.4$, $\beta = 2.25 \times 10^{-11}$ / ms, obtaining (a) $\eta_c = 2.28$ with $C_x = -5$ m / s taken for the west–travelling, (b) $\eta_c = 2.72$ with $C_x = 5$ m / s for the east migrating and (c) $\eta_c = 2.5$ with $C_x = 0$ m / s for steady CISK–Rossby wave. These conditions show that in the first case, relatively small heating η is responsible for the vertical propagation of wave, in the second relatively strong thermal effect is necessary and in the third the heating intensity is in between. Since the thermal difference arises in the broad tropical band, suggestive of localized characteristics of cumulus convection, the η spectrum covers certain ranges of values, thus responsible for the simultaneous or alternate occurrence of east– and westward CISK–Rossby waves, which are often made to migrate vertically because of the thermal condition there.

Based on the foregoing analysis we can roughly infer that the mechanism of the vertical travel works in such a manner that for $\eta < \eta_c$ (hence $Q = m^2 < 0$), thermal forcing will produce horizontal CISK–Rossby wave with vertical velocity perturbation at lower levels only (in which the amplitude attenuates exponentially with height); as η increases, so do the amplitudes of horizontal waves in the identical stratification layers; when η reaches, or is higher than η_c $(Q = m^2 > 0)$, the amplitude of horizontal wave with the vertical velocity perturbation is maximized and maintained, followed by wave disturbance emerging in vertical, and at that time u and φ disturbances generate waves in horizontal and vertical, whose amplitudes are proportional to $|m|$. For this reason, thermal forcing is an innegligible factor responsible for the strengthening of horizontal wave and genesis of vertical wave perturbation.

4. Vertical migration of zonal wind momentum and wave energy fluxes related to the QBO occurrence

Studies show that the happening of stratospheric zonal wind QBO depends on the upward transfer of disturbance momentum and energy fluxes from the troposphere. It is generally accepted that the vertical transport is performed by the westward travelling mixed Rossby gravity wave and the eastward Kelvin wave. Here the CISK–Rossby wave under study is dispersive, suggesting that it is able not only to propagate both east– and westward and characterized by energy dispersion but to travel vertically as well. Can the wave of interest transport momentum and energy fluxes upward into the stratosphere to generate and maintain zonal wind QBO there? Thus we make investigation of the problem in the following. From the foregoing derivation we have the expressions for vertical migration of the momentum and energy fluxes for the zonal wind perturbation,

$$\overline{u'w'} = \frac{m}{k}\overline{w'^2} = \overline{uw} = \frac{m}{k}\overline{w}^2 \ , \tag{10}$$

$$\overline{\varphi' w'} = -\frac{m}{\sigma k^2}(f^2-\sigma^2)\cdot\overline{w'^2} = \overline{\varphi w} = -\frac{m}{\sigma k^2}(f^2-\sigma^2)\overline{w}^2 \ , \tag{11}$$

where the prime for disturbance quantities has been neglected and the $\overline{(\cdot)}$ denotes the mean over the integration of a zonal or a vertical section.

The solution of frequency σ of a horizontal wave is found by use of (9), whereupon we get the phase velocity C_x and group velocity C_{gx} of a horizontal wave in the form

$$\sigma = -\frac{k\beta(1-b\eta)}{k^2+\frac{m^2f^2}{N^2}} \begin{cases} C_x = \frac{\sigma}{k} = -\frac{\beta(1-b\eta)}{k^2+\frac{m^2f^2}{N^2}} \\ C_{gx} = \frac{\partial\sigma}{\partial k} = -\frac{\beta(1-b\eta)}{k^2+\frac{m^2f^2}{N^2}} + \frac{2k^2\beta(1-b\eta)}{[k^2+\frac{m^2f^2}{N^2}]^2} = \frac{\beta(1-b\eta)[k^2-\frac{m^2f^2}{N^2}]}{[k^2+\frac{m^2f^2}{N^2}]^2} \end{cases} , \tag{12}$$

and the phase velocity of a vertical wave migration is given by

$$C_z = \frac{\sigma}{m} = -\frac{\beta\cdot k(1-b\eta)}{m\cdot(k^2+\frac{m^2f^2}{N^2})} \ . \tag{13}$$

It is seen from the expression of C_x form of (12) that, as η is small enough to make $(1-b\eta)>0$, we have $C_x<0$, meaning that the CISK–Rossby wave goes westward, a result which agrees with that of the classic Rossby wave in the free atmosphere at low latitudes except for lower velocity; for η growing to $(1-b\eta)=0$, the wave has zero velocity zonally and becomes quasi–steady; with η increasing to $(1-b\eta)<0$ it is east travelling. Moreover, when $\eta>\eta_c$, the wave will propagate vertically. Regardless of the vertical structure of the wave, its zonal migration features are the same as the classic Rossby wave (Li, 1985b).

One can see from $Q=m^2=-\frac{N^2}{f^2}[k^2+\frac{k\beta}{\sigma}(1-b\eta)]$ of (9) that only when

$$-\frac{N^2}{f^2}[k^2+\frac{k\beta}{\sigma}(1-b\eta)]>0,\ \text{i.e.,}\ [k^2+\frac{k\beta}{\sigma}(1-b\eta)]=[k^2+\frac{\beta}{C_x}(1-b\eta)]<0\ , \tag{14}$$

is m a real number, implying that the vertical transport of the zonal wind disturbance momentum flux given by (10) and the wave energy flux by (11) is made possible.

With $L_x = 10^7$ m, $k=\frac{2\pi}{L_x}$ for zonal wavenumber, $H= 2.0\times 10^4$ m for the characteristic height of the lower stratosphere, $m=\frac{n\pi}{H}$ for vertical wavenumber with $n= 0,1,2,\cdots$ to represent the different wave structures in vertical, $f=0.25\times10^{-4}$ / s, $\beta=2.25\times10^{-11}$ / ms (about 10°N), $b=0.4$, and $N^2=1.0\times10^{-4}$ / s^2, we investigate the vertical transfer of momentum and wave energy fluxes in the following.

a) With $(1-b\eta)>0$ for weak thermal effect we see from (12) that the horizontal phase velocity $C_x<0$, meaning that the wave migrates westward, in which case m is a real number if only C_x is relatively small ($|C_x|<34.2$ m / s for $\eta=1$). This suggests that with weak thermal effect the slowly west travelling CISK−Rossby wave will transfer upward the momentum and energy fluxes. The expression of group velocity C_{gx} of (12) indicates that at this time $C_{gx}<0$ at $n\neq 0$, and 1, suggesting the westward dispersion of wave energy. In view of the fact that the slowly westward displacing CISK−Rossby wave has its energy dispersed during the vertical transfer of momentum and energy fluxes, we are allowed to assume that there occurs upward transport of energy of the wave, thereby providing energy for the development and maintenance of stratospheric zonal wind disturbance;

b) With $(1-b\eta)<0$ for intense heating, $C_x>0$ indicates the eastward migration, in which case m is a real number if only C_x is relatively small ($|C_x|<34.2$ m / s for $\eta=4$). This implies that with stronger thermal effect the eastward travelling CISK−Rossby wave will perform vertical transfer of the fluxes, and in this case $C_{gx}>0$ at $n\neq 0$ and 1, suggestive of the eastward dispersion of wave energy. We are led to assume that the slowly migrating CISK−Rossby wave transfers wave energy upward in its eastward course, furnishing energy to stratospheric westerly disturbance in its development and acceleration.

The above analysis shows that there always happens the vertical propagation of momentum and energy fluxes from the slowly west− and eastward migrating CISK−Rossby waves, regardless of the thermal intensity. Generally, such waves have the phase velocity at 10−20 m / s so that the vertical transfer of the fluxes requires looser constraints on phase velocity in such a way that they are thought to be frequent in tropical weather systems.

As for the direction of the fluxes transferred upward we see from (10) that in the above cases $\overline{u'w'}$ is positive, meaning that, regardless of the heating intensity, there always take place the slowly east and west travelling CISK−Rossby waves from weak and strong heating and upward propagation of the fluxes for zonal wind perturbation. The vertical transport of wave energy flux is associated with wave frequency, as shown in (11), which is investigated as follows. In case a), in all the tropics but the equator, the wave energy flux will be transferred upward provided that the phase velocity is relatively small (at 10°N, S, for example, $|C_x|<39.8$ m / s and at 5°N, S, $|C_x|<20$ m / s) and the lower the frequency, the bigger the upward transported flux (Note that vertically the phase velocity and the transport of wave energy flux are out of phase) and in case b) around the equator where $f\to 0$ and at relatively higher phase velocity ($|C_x|>20$ m / s must be at 5°N, S and $|C_x|>12$ m / s at 3°N, S) the energy flux is permitted to go upward (Note that vertically the phase velocity and the transfer of energy flux are in phase). This suggests that the slowly (fast) west– (eastward) progressing CISK–Rossby wave will transport upward easterly (westerly) wave energy outside (around) the equator. The westward wave transfer corresponds to the upward transport of easterly momentum by the mixed Rossby gravity wave (Holton and Lindzen, 1972) and the fast eastward migrating wave transfer of westerly momentum by the Kelvin wave. It is clear that both the types of the CISK–Rossby wave have ability to transport upward energy for the stratospheric easterly and westerly disturbance and their dynamic mechanisms are identical on the whole.

The study shows that for the two types the constraints on upward migration of the fluxes are quite easily satisfied at the tropics, irrespective of the heating intensity, which allows the CISK–Rossby waves to upward transport momentum and energy for easterly and westerly disturbance on a continuous basis, thereby making possible the maintenance of the stratospheric QBO. The transfer can be employed in interpreting some basic synoptic facts of the QBO and is regarded as a possible dynamic mechanism for generating and sustaining the QBO.

5. Low–frequency CISK–Rossby waves and stratospheric QBO

Observational facts show that quasi–steady extra–longwave systems are frequently present in the tropical latitudes and act as the background circulations and cause the formation and development of synoptic systems there. It can be assumed that over an appropriate range of thermal forcing, the quasi–steady extra–longwave systems are likely to be formed of the CISK mechanism–produced low frequency (LF) and very low frequency (VLF) CISK–Rossby waves which provide the QBO with momentum and energy for the need of disturbance development. Due to the fact that the tropical quasi–steady ultra–longwave systems are always available, so are the dynamic processes which are worked by LF and VLF periods that are close to those of the QBO. Compared to the faster moving mixed Rossby gravity wave and the Kelvin wave, the tropical thermal forcing–yielded LF and VLF CISK Rossby waves seem to bear a still closer relation to the stratospheric QBO.

The period of horizontal wave oscillation is obtained by wave frequency σ:

$$T=\left|\frac{2\pi}{\sigma}\right|=\left|\frac{2\pi[k^2+\frac{m^2f^2}{N^2}}{k\beta(1-b\eta)}\right| . \tag{15}$$

Assuming $L_x = 1.0\times10^7$ m (equivalent to zonal 3–4 waves around the equator) and the other parameters of the previous section and taking a three–wave structure ($n=6$) in vertical in the lower stratosphere, we have prepared a diagram of thermal effect–dependent periods of the CISK–Rossby wave (Fig. 2), where around $\eta= 2.35- 2.4$ and $\eta= 2.6- 2.65$ there occur west– and eastward CISK–Rossby waves at QBO, respectively, for which $T\approx 510- 765$ days, $C_x=-0.23 - -0.15$ m / s and 0.15–0.23 m / s and $C_{gx}=-0.20 - -0.13$ m / s and 0.13–0.20 m / s, respectively, and the discriminants of the conditions of momentum and wave energy transport in vertical meet the needs of $[k^2+\frac{\beta}{C_x}(1-b\eta)]<0$, indicating that such waves are allowed to transfer upward the momentum and energy for easterly and westerly disturbance in the stratosphere.

Taking no account of vertical waves ($m=0$), we obtain 10–20–day periods of east– and

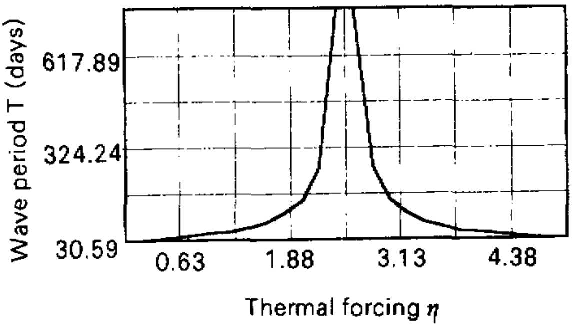

Fig. 2. Diagram of thermal forcing–varying periods of CISK–Rossby wave with 3–4 waves zonally and three waves in vertical (n = 6).

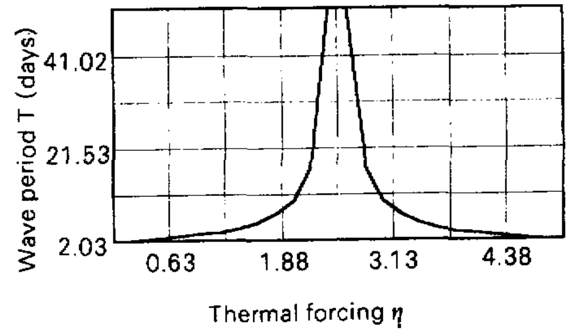

Fig. 3. Thermal effect–dependent periods of CISK–Rossby wave with 3–4 waves zonally and no waves in vertical.

westward marching waves within $\eta \in [2.1, 2.2]$ and [2.8,2.9] whilst 30–50–day periods are found in the ranges $\eta \in [2.3, 2.4]$ and [2.6,2.7], respectively, with no access to periods of VLF oscillations greater than interannual scales (see Fig. 3). And we see from (10) to (11) that both the vertical transfer of momentum and that of wave energy flux for the zonal wind perturbation are zero–valued. Evidently, the barotropical CISK–Rossby wave possesses no dynamic character necessary for the genesis and maintenance of the QBO.

In addition, over $\eta = 1.8 - 2.2$, we get from (13) the downward phase velocity $C_z = -7.06 \times 10^{-4} - -3.03 \times 10^{-4}$ m / s for the baroclinic CISK–Rossby wave, for which T = 109 – 255 days and over $\eta = 2.8 - 3.2$ the upward phase velocity $C_z = 3.03 \times 10^{-4} - 7.06 \times 10^{-4}$ m / s for which T = 255– 109 days. The velocity is equivalent to 0.78–1.83 km per month which is quite close to 1–2 km per month for the vertical propagation of easterly and westerly phases observed for the stratospheric QBO.

The above calculations show that the tropical CISK mechanism is able to excite a kind of VLF Rossby waves of a vertical wave structure and dispersive character, which is likely to be an important factor of the QBO. The production, migration and energy dispersion of such waves depend on thermal forcing, thus demonstrating that the forcing of which the heating by convective condensation from tropical cumulus clouds is dominant gives rise to low frequency oscillation in the tropical troposphere and further causes the stratospheric QBO through vertical transport . From (6) to (8) we can see that, taking no account of vertical wave ($m=0$), no horizontal waves of u and φ are possible but that of w disturbance, thus showing numerically no existence of VLF periods greater than interannual scales. Therefore, to reveal such VLF oscillations as QBO, we have to take into consideration the baroclinic CISK–Rossby wave mode. Additionally, it should be stressed that η is a crucial factor, as revealed in our study. From (12) we see that, without thermal effect involved, there occur just westward moving wave and westward energy dispersion so that the CISK–Rossby wave degrades into its general form, which does not have the full ability to transport vertically momentum and energy as does the CISK–Rossby wave for the zonal wind disturbance for lack of vertical transfer mechanism of both easterly and westerly wind momentum and wave energy flux.

6. Summary

In comparison to the mechanisms of the migration and energy transport of the classic mixed Rossby gravity wave and the Kelvin wave, the LF CISK–Rossby wave presented here displays east– and westward migration and propagation in vertical within properly posed heating ranges (which transfers upward momentum and wave energy for stratospheric zonal wind disturbance). And this can be employed to describe and explain some basic weather facts of stratospheric QBO. It is a common practice in classic literature to assume the mixed Rossby wave and the Kelvin wave to be excited by convective disturbance in the neighborhood of the equator, suggesting that they are produced by non–adiabatic forcing, and also, both the types of wave cannot relate tropospheric convective disturbance to the stratospheric QBO tropically in a direct manner because tropical observations are insufficient to do so (Hoskins and Pearce, 1987). The LF CISK–Rossby wave presented here is excited mainly by the joint action of convective heating, its feedback and β effect at tropical latitudes and the CISK–Rossby wave is almost available all the year round there because of the geography and acts as sources of momentum and energy for persistent development and maintenance of the stratospheric QBO. Due to the fact that the LF CISK Rossby wave depends, to a great extent, on the thermal factor for the east– and westward migration and vertical transport of momentum and energy fluxes, the tropospheric thermal properties will exert influence on the genesis and evolution of the stratospheric QBO through the CISK–Rossby wave, study of which will advance the understanding of the relation of tropospheric convective activities to the QBO in the tropical atmosphere.

Because of the simplicity of the model used it remains difficult to interpret problems as to the interaction of upward transfer of momentum and wave energy with the zonal wind in the stratosphere and their conversion into the zonal forms for the QBO happening and development, which await further research.

REFERENCES

Holton, J. R., and Lindzen, R. S., 1972: An updated theory for the quasi–biennial cycle of the tropical stratosphere. *J. Atmos. Sci.*, **29**, 1076–1080.

Hoskins, B., and R. Pearce, 1987: *Large–Scale Dynamics in the Atmosphere* (Chinese edition), translated by Sun

Zhaobo et al., China Meteorological Press, Beijing, 308–312 (in Chinese).

Jiang Guorong, 1996: On CISK–Rossby wave and 30–50–day oscillations in the tropical atmosphere. *Acta Meteorologica Sinica*, **6**, 745–752 (in Chinese).

Kousky, V. E., and Wallace, J. M., 1971: On the interaction between Kelvin waves and the mean zonal flow. *J. Atmos. Sci.*, **28**, 162–169.

Li Chongyin, 1985a: Characteristics of atmospheric motion in tropics. *Scientia Atmospherica Sinica*, **4**, 366–375 (in Chinese).

Li Chongyin, 1985b: South–Asian monsoon trough / ridge, tropical cyclone's activities and movable CISK wave. *Scientia Sinica* (Ser. B), 668–675 (in Chinese).

Li Chongyin, 1990: A dynamical study on 30–50–day oscillations in the tropical atmosphere outside the equator. *Scientia Atmospherica Sinica*, **1**, 83–92 (in Chinese).

Lindzen, R. S., and Holton, J. R., 1968: A theory of the quasi–biennial oscillation. *J. Atmos. Sci.*, **25**, 1095–1107.

Lu Keli et al., 1997: *Dynamical Meteorology*, Nanjing University Press, Nanjing, 330–343 (in Chinese).

Lindzen, R. S., 1971: Equatorial planetary waves in shear, Part I. *J. Atmos. Sci.*, **28**, 609–622.

Maruyama, T., 1968: Upward transport of westerly momentum due to large–scale disturbances in the equatorial lower stratosphere. *J. Meteor. Soc. Japan*, **46**, 404–417.