The Variational Assimilation Experiment of GPS Bending Angle

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ABSTRACT

More and more new types of observational data provide many new opportunities for improving numerical weather forecasts. Among these, the GPS (Global Positioning System) bending angle is undoubtedly very important. There are many advantages of the GPS bending angle, such as high resolution, availability in all weather conditions, and global data coverage. Thus it is very valuable to assimilate GPS bending angle data into numerical weather models. This paper introduces how to obtain and assimilate the GPS bending angle. There are two methods of assimilation: the indirect method and direct method, and they are both introduced in this paper. During the minimizing process of variational assimilation, calculation efficiency is very important and the optimal step size greatly influences the algorithm efficiency. Based on the characteristics of the minimizing algorithm, we obtain an adaptive method for calculating the optimizing step suitable for all kinds of minimization algorithms through mathematical deduction. Finally, a numerical variational assimilation experiment is performed using the GPS bending angle data of 11 October 1995. The numerical results indicate the validity of the variational assimilation method and the adaptive method introduced here.

Key words: GPS bending angle, variational assimilation, optimal step size

1. Introduction

In developing of a numerical weather forecast model, it is very difficult for the dynamical frame to be improved further. We can improve the numerical weather forecasting in two ways: one is to improve the physical parameterization and the other is to assimilate more data. Sometimes, it seems that the physics process is considered more complete, if the numerical model is made to be more complicated, but in fact, the numerical forecast ability is not truly improved. The main reason is that the factors influencing the atmospheric movement are very complex, while the introduction of many parameters and formulas during the physical parameterization process is experiential in some sense. Therefore, only depending on the model physical parameterization to improve the numerical forecasting is quite inadequate. For the lack of data, we often meet many difficulties in tropical hurricane cyclone intensity and track forecasts, and the mesoscale strong convective systems quantitative precipitation forecasts. But on the other hand, with the application of many new probing tools and probing means (such as satellite probing, radar probing, etc.), people can almost observe the global atmospheric and oceanic environment in real time and continuously. This greatly remedies the shortfall of the traditional observational system and provides many new opportunities for improving the initial fields of the numerical weather forecast model. This is why data assimilation is receiving more attention than model physical parameterization in recent years. The change is also attributed to the increased availability of new types of observations, especially those data provided by satellites and radars at a much higher resolution in space and time than conventional rawinsonde data.

Among all new types of observing systems, the satellite system is doubtlessly the most important one. Since the first weather satellite was launched in 1960, much of data has already been sent back to the ground. Now atmospheric scientists can hardly finish the global weather forecast without the meteorology satellite. GPS Meteorology is one type of satellite data, which uses the radio occultation technique for sounding the Earth's atmosphere from space with high accuracy in all weather conditions. The details are introduced in section 2. There are two main advantages of GPS:

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high resolution and global data coverage. First, if 100 low orbital satellites are launched to facilitate the GPS service, the vertical resolution may reach 1 km in the high layer and 100 m in the low layer, while the horizontal resolution may reach 500 km in the high layer and 200 km in the low layer. Furthermore, the atmospheric parameters provided by GPS are functions of height in form. And to the second point, GPS can probe the atmosphere remotely in all weather conditions and in all-directions. In weather systems or regions that are hard to observe directly, GPS can obtain the important information of the Earth's atmosphere. Therefore, if the GPS bending angle can be assimilated into numerical models successfully, it will be a very important and valuable thing.

Zou et al. (1995) obtained the temperature and vapor fields by inverting the GPS bending angle, and successfully assimilated them into a numerical model. Utilizing the MM5 model and its adjoint model, Kuo et al. (1996) also did some experiments on assimilating the retrieved physical fields from satellite data. They discovered that the assimilation method could readily repair the vertical structure of the vapor fields and improve the initial vapor fields, especially in the low layer. In fact, not only the retrieved physical fields can be assimilated, but also the GPS bending angle itself can be directly assimilated (Wang et al., 2000a; Zou et al., 1999, 2000). But no matter what kind of assimilation method is used, an enormous amount of computation is needed, and the computation will dramatically increase if GPS bending angle is directly assimilated. Furthermore, more computing resources are required in assimilating the GPS bending angle directly for integrating the ray equations. Therefore, various methods and skills should be applied in improving the algorithmic efficiency to accelerate the iterative convergent rate of the minimizing methods. As for minimization, there are many methods, such as the steepest descend method, the Newton method, the limited memory quasi-Newton method, and so on. But no matter which kind of optimizing algorithms is used, it can always be written as follows

$$X_0^{\nu+1} = X_0^{\nu} + \rho^{\nu} d^{\nu}, \tag{1}$$

where X_0 is the model control variable, ν is the iterative number during the minimizing procedure, d is the optimal direction, and ρ is the optimal step size. The optimal direction d can be obtained by integrating the adjoint model. How do we calculate the optimal step size? Wang (2000) pointed out that the optimal step size is very important and directly relates to the computational efficiency and iterative convergent rate. How to solve the optimal step is as important as looking for the optimal direction to some extent. It becomes a key factor in controlling the algorithm efficiency. More research work in this field is still required in the future.

In section 2, the principle for obtaining GPS bending angle data is introduced. The principles and methods of assimilating the GPS bending angle are introduced in section 3. The method of calculating the optimal step size is introduced in section 4. In section 5, the numerical results are introduced. And the conclusion is drawn in the last section.

2. The principle for obtaining the GPS bending angle

The GPS satellite is equipped with the ray emitter and the low earth orbit satellite (LEO) is equipped with the ray receiver. For the stratified atmosphere, the atmospheric refraction in each layer is different, which makes rays emitted from the GPS satellite refract when they pass through the atmosphere. Usually, the rays curve to perigee. If the rays successfully pass through the atmosphere and do not hit the Earth's surface, the emission direction always deflects the original direction. The bending angle between the rays in and out is called the refraction angle $\epsilon_{\rm obs}$. Figure 1 shows the rays from the GPS satellite passing through the earth's atmosphere and being refracted, then being received by by LEO satellite at last, where ϵ is the refraction angle and p is the influence parameter.

The magnitude of the refraction angle depends on the gradient of the refractive index of the path that the ray travels in the atmosphere. By the Doppler frequency shift of rays emitted from the GPS satellite and received by the LEO satellite through the Earth's atmosphere, the refraction angle of the GPS signal can be easily obtained. The following is the detail of this method.

Utilizing the GPS and LEO, what we actually measure is the phase delay of the rays when they reach the LEO. If the GPS shoots rays at two regular frequencies f_i (f_1=1227.6 MHz, f_2=1575.42 MHz), two phase delays can be obtained: $S_1(t)$ and $S_2(t)$. The Doppler-shifted frequency can be estimated by the following formula:

$$f_{\rm d} = -f_i c^{-1} \frac{dS}{dt}, \tag{2}$$

where, c represents the velocity of light in a vacuum.

On the other hand, taking advantage of the satellite geometry, $f_{\rm d}$ can also be expressed as:

$$f_d = -f_i\left[\frac{c - n_{\mathrm{LEO}}(v^r_{\mathrm{LEO}}\cos\phi_{\mathrm{LEO}} - v^t_{\mathrm{LEO}}\sin\phi_{\mathrm{LEO}})}{c - n_{\mathrm{GPS}}(v^r_{\mathrm{GPS}}\cos\phi_{\mathrm{GPS}} - v^t_{\mathrm{GPS}}\sin\phi_{\mathrm{GPS}})} - 1\right], \tag{3}$$

where indices LEO and GPS respectively indicate the GPS and the LEO satellites, c is the velocity of light in a vacuum, v is the magnitude of the satellite velocity, indices r and t indicate the radial and tangential components of the satellite velocity, n is the refractivity index at the satellites' locations, ϕ is the angle between the satellites' radius r and the wave vector $\boldsymbol{v}$. In this equation, c and f_i are known, v^r_{LEO}, v^t_{LEO}, v^r_{GPS}, and v^t_{GPS} can be measured by the other satellites, and f_d can be obtained by Equation (2).

Suppose the Earth's surface is symmetric and the impact parameter $p = rn\sin\phi$ at GPS is equal to that at the LEO satellite, then:

$$p = r_{\mathrm{GPS}}n_{\mathrm{GPS}}\sin\phi_{\mathrm{GPS}} = r_{\mathrm{LEO}}n_{\mathrm{LEO}}\sin\phi_2. \tag{4}$$

And suppose the satellites are high enough so that the refractivity is not affected by the atmosphere: $n_{\mathrm{GPS}} = n_{\mathrm{LEO}} = 1$. Then the two equations (3) and (4) about the variables ϕ_{LEO} and ϕ_{GPS} are closed, and can be solved iteratively by numerical methods.

According to the geometry, the bending angle can be expressed by the following formula:

$$\epsilon = \phi_{\mathrm{GPS}} + \phi_{\mathrm{LEO}} + \arccos\frac{r_{\mathrm{GPS}} \cdot r_{\mathrm{LEO}}}{r_{\mathrm{GPS}}r_{\mathrm{LEO}}}. \tag{5}$$

Because the ionosphere in Earth's atmosphere can affect the transmittal of the rays, some necessary correction must be done (Gorbunov et al., 1995). If ϵ_1 and ϵ_2 are the bending angles calculated by the two regular frequencies respectively, the correcting formula can be expressed as:

$$\epsilon = \frac{\epsilon_1 f_1^2 - \epsilon_2 f_2^2}{f_1^2 - f_2^2}. \tag{6}$$

3. The principle of variational assimilation of the bending angle

If there are M rays in a detection (for example, if the GPS signal is emitted during 60 s at a sampling frequency of 50 Hz, then M=3000) and N detections of the GPS signal in a certain period, then the total number of rays are $N \times M$. The bending angles between the away-direction and the toward-direction of these rays are the data that will be assimilated. Because the refractive index of the atmosphere is directly related to the basic physical characteristics of the atmosphere, then the bending angles are certainly related to the basic physical characteristics of the atmosphere. If the basic physical characteristics of the atmosphere do not change during a short period, all detections in this period can be regarded as occurring at the same time, and the variational assimilation can also be regarded as a three-dimensional variational assimilation.

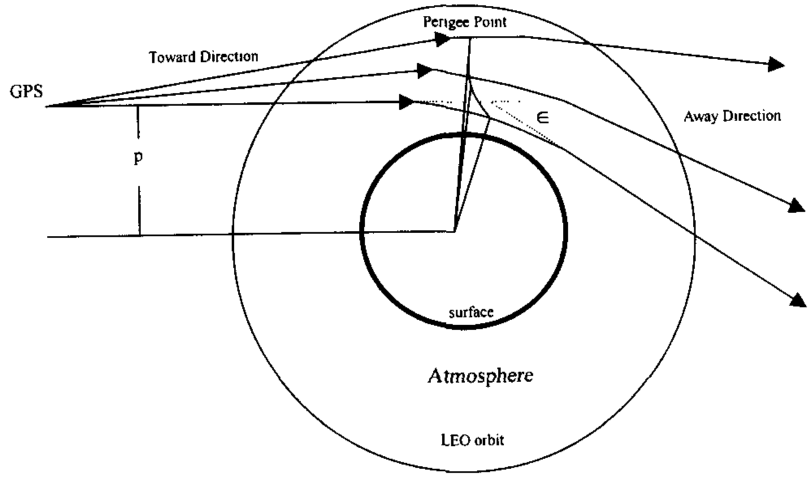

Fig. 1. The schematic representation of the bending ray paths.

3.1 *The indirect assimilation method*

An important method of variational assimilation on GPS bending angle is indirect assimilation, which is usually used in early research work. For example, Zou et al. (1995) did the retrieving research work by using the bending angle and obtained pressure fields and temperature fields first. Then they assimilated these retrieving data into the MM5 model and its adjoint model.

The retrieving methods are introduced as follows:

Under the assumption of spherical symmetry, the refraction index of the atmosphere can be calculated by Abel's formula:

$$n(p) = e^{\frac{1}{\pi}\int_p^{p_{top}} \frac{\epsilon(x)}{(x^2-p^2)^{1/2}}dx}, \tag{7}$$

where p is the impact parameter and p_{top} =90 km. Another refraction index can be written as:

$$N = (n-1) \times 10^6. \tag{8}$$

It can also be expressed as a function of pressure P, temperature T and specific humidity q:

$$N = 77.6\frac{P}{T} + 3.73 \times 10^5 \frac{Pq}{T^2\,(0.622 + 0.378q)}. \tag{9}$$

If the air is dry, P and T are retrieved. Under the condition of $q = 0$, Equation (6) can be written as

$$N = 77.6\frac{P}{T}. \tag{10}$$

Making use of the state equation $P = \rho RT$ and hydrostatic balance equation $dP = -g\rho dz$, we obtain

$$dP = -\frac{gN}{77.6R}dz \tag{11}$$

and

$$T = \frac{77.6P}{N}, \tag{12}$$

where R= 287 $J\ K^{-1}$ kg^{-1} is a constant physical parameter, and the other variables are expressed by the usual international unit. Integrating Equation (11), we get the pressure of dry air. And we also get the temperature from Equation (12). But there is still a problem in the above method. Maybe we can suppose the air in high levels is dry, and under this assumption the bias between the numerical value and the true state is small. But actually, the air is not dry in low levels, so the pressure and temperature are not accurate and the specific humidity cannot be obtained by this method. From Equation (9) we can find that the specific humidity has a great influence on the refraction index. So it is very necessary to find an effective method to accurately retrieve the specific humidity, pressure, and the temperature.

If we insert the pressure and the temperature of the dry air into Equation (9) again, a specific humidity value can been obtained. The virtual temperature can be calculated from the following equation:

$$T_v = T\,(1 + 0.608q)\,, \tag{13}$$

and the pressure profile of the wet air can be obtained by integrating

$$d\ln P = -\frac{g}{RT_v}dz. \tag{14}$$

The specific humidity profile can be obtained from the following

$$P_w = \frac{1}{3.75 \times 10^5}T_v^2\left(N - 77.6\frac{P}{T_v}\right) \tag{15}$$

$$q = 0.622P_w/\,(P - 0.378P_w)\,. \tag{16}$$

After the specific humidity has been obtained, we return to Equation (13) and solve equations (13)–(16) repeatedly until we obtain the iterative solutions. And finally, all the retrieved physical fields of the atmosphere can be assimilated into the numerical model.

3.2 *The direct assimilation method*

In the indirection assimilation method, many assumptions are introduced which may cause large errors. The spherical symmetry assumption is a simple example. Recently, more and more attention has been paid to the direct assimilation method. First, we let X_0 denote all model initial control variables, such as pressure, temperature, and specific humidity, which need to be assimilated and are provided by NECP data in our numerical experiments; then the direct assimilation method only uses these model control variables to obtain the ray tracks by the integrating ray equation instead of doing the retrieving calculation. Furthermore, the bending angles (denoting as $\epsilon_{\rm cal}$) are calculated. Then the distance between the calculated bending angles and the observed bending angles $\epsilon_{\rm obs}$ are obtained. Finally, the adjoint model is integrated and the observated bending angles are assimilated. Wang et al. (2000) and Zou et al. (2000) have done the three dimensional variational assimilation of GPS data and applied this method in an operational variational assimilation system.

The most important concept of the variational assimilation of GPS data is how to solve the ray track equation and its adjoint equation, and obtain the cost function and the gradient. The ray track equation can be expressed as:

$$\frac{\partial^2}{\partial \tau^2}\boldsymbol{x} = n\nabla n, \tag{17}$$

where $\boldsymbol{x}(\tau)$ is the ray track and it can be expressed as a function of parameter τ, τ satisfies

$$d\tau = \frac{ds}{n},$$

and s is the length of the ray. If the initial position $\boldsymbol{x}_s = \boldsymbol{x}_p - \lambda \boldsymbol{u}_p^t$ of the GPS satellite and the initial direction $\boldsymbol{u}_s^t = -\boldsymbol{u}_p^t$ of the ray are given, then the ray track and the bending angle can be obtained by integrating equation (17), where $\boldsymbol{x}_p$ is the position vector of the ray track at perigee, $\boldsymbol{u}_p^t$ is the tangent direction at the ray track at perigee, and $\lambda = 20000$. Both $\boldsymbol{x}_p$ and $\boldsymbol{u}_p^t$ can be expressed as functions of the parameter p.

After the bending angle has been obtained, we define a cost function $J = F(\epsilon_{\rm cal} - \epsilon_{\rm obs})$ to show the difference between the model variables and the observational variables. Because $\epsilon_{\rm cal}$ is calculated by integrating the model from the initial model variable X_0, the cost function can also be expressed as a function of X_0, namely as $J = F(X_0)$. If X_0 varies, the cost function also varies. We can minimize the cost function and get the optimal X_0 with a mathematical minimizing method, which is the last step of the direct assimilation method.

4. Calculation of the optimal step size—A new method

The optimization of X_0 can be written as $X_0^{\nu+1} = X_0^\nu + \rho^\nu d^\nu$. If $d = -[\nabla J]^\nu$, the minimizing method is the steepest descend method; if d is the conjugate gradient direction, it is the conjugate gradient method; if d is the Newton gradient direction, it is the Newton method; if d is the quasi-Newton gradient, it is the quasi-Newton method. But no matter which method is taken, how to calculate the optimal initial value X_0 is transferred to how to calculate the optimal direction d and the optimal step ρ. The optimal direction can be calculated by integrating the adjoint equation, so the key problem is how to calculate the step size ρ. Actually, the iterative convergent rate is not only related to the optimal direction, but also to the proper optimal step size.

In this section, based on mathematical deduction, an adaptive method for calculating the optimal step size is introduced. Wang et al. (2000b) have done some research work in these fields, but their work was mainly aimed at assimilating the data of model variables with the steepest descend method. In this paper, however, the work is aimed at the assimilation of data of all kinds of variables (for example, non-conventional observational data) in a general minimizing method.

Assume that after the ν times iteration, the optimal direction has been obtained by integrating the adjoint model. If the optimal step size ρ^ν has also been prepared, then the ν+1 times initial fields can be obtained by equation (1). In order to calculate ρ^ν, a guessed optimal step size ρ_*^ν is given firstly ($\rho_*^1 = 1/J$ for the first iteration, and $\rho_*^\nu = \rho^{\nu-1}$ for the later iteration), and the initial fields are calculated by the following equation:

$$X_{0*}^{\nu+1} = X_0^\nu + \rho_*^\nu d^\nu \tag{18}$$

The ray equation was regarded as an arithmetic operator K, linear or nonlinear, when it was transferred onto the dispersed space. When it acts on the initial fields, a bending angle can be calculated, marked as $\epsilon_{\rm cal}$.

For the ν+1 times iteration, there are:

$$\epsilon_{\rm cal}^{\nu+1} = K\left(X_0^{\nu+1}\right) \tag{19}$$

and

$$\epsilon_{{\rm cal}*}^{\nu+1} = K\left(X_{0*}^{\nu+1}\right). \tag{20}$$

If the arithmetic operator acts on equations (1) and (8) respectively, there are:

$$K\left[X_0^{\nu+1}\right] = K\left[X_0^\nu + \rho^\nu d^\nu\right] \tag{21}$$

and

$$K\left[X_{0*}^{\nu+1}\right] = K\left[X_0^\nu + \rho_*^\nu d^\nu\right]. \tag{22}$$

At every iteration, when ρ^ν and ρ_*^ν have been obtained and used to calculate the optimal initial fields, they are constant values. The optimal step size is estimated supposing that the forecast solutions vary linearly with a very small initial perturbation in the search direction. (Derber, 1987; Wang et al., 2000). By Equations (19) and (20), Equation (21) and (22) can be expressed respectively as:

$$\epsilon_{\rm cal}^{\nu} - \epsilon_{\rm cal}^{\nu+1} = -\rho^{\nu} K_L d^{\nu}, \quad (23)$$

and

$$\epsilon_{\rm cal}^{\nu} - \epsilon_{\rm cal*}^{\nu+1} = -\rho_*^{\nu} K_L d^{\nu}. \quad (24)$$

Here K_L is the tangent arithmetic operator of K. If multiplied by the same value on both sides of Equations (23) and (24), then:

$$\begin{aligned}&\left[\epsilon_{\rm cal}^{\nu} - \epsilon_{\rm cal}^{\nu+1}\right]\left[\epsilon_{\rm cal}^{\nu} - \epsilon_{\rm cal*}^{\nu+1}\right]^{\rm T}\\ &= -\rho^{\nu} K_L d^{\nu}\left[\epsilon_{\rm cal}^{\nu} - \epsilon_{\rm cal*}^{\nu+1}\right]^{\rm T},\end{aligned} \quad (25)$$

and

$$\begin{aligned}&\left[\epsilon_{\rm cal}^{\nu} - \epsilon_{cal*}^{\nu+1}\right]\left[\epsilon_{\rm cal}^{\nu} - \epsilon_{\rm cal*}^{\nu+1}\right]^{\rm T}\\ &= -\rho_*^{\nu} K_L d^{\nu}\left[\epsilon_{\rm cal}^{\nu} - \epsilon_{\rm cal*}^{\nu+1}\right]^{\rm T},\end{aligned} \quad (26)$$

where superscript T denotes transform. And after the ν+1 iteration, the perfect results satisfy:

$$\epsilon_{\rm cal}^{\nu+1} = \epsilon_{\rm obs}. \quad (27)$$

From Equations (26), (25), and (27), the following can be obtained:

$$\frac{\rho^{\nu}}{\rho_*^{\nu}} = \frac{[\epsilon_{\rm cal}^{\nu} - \epsilon_{\rm obs}][\epsilon_{\rm cal}^{\nu} - \epsilon_{\rm cal*}^{\nu+1}]^{\rm T}}{[\epsilon_{\rm cal}^{\nu} - \epsilon_{\rm cal*}^{\nu+1}][\epsilon_{\rm cal}^{\nu} - \epsilon_{\rm cal*}^{\nu+1}]^{\rm T}}, \quad (28)$$

or written as:

$$\rho^{\nu} = \rho_*^{\nu} \frac{<\hat{\Phi}, \Phi'>}{<\Phi', \Phi'>}, \quad (29)$$

where $\hat{\Phi} = \epsilon_{\rm cal}^{\nu} - \epsilon_{\rm obs}$ is the difference between the calculated bending angle $\epsilon_{\rm cal}^{\nu}$ and the observed bending angle $\epsilon_{\rm obs}$, $\Phi' = \epsilon_{\rm cal}^{\nu} - \epsilon_{\rm cal*}^{\nu+1}$ is the difference between the calculated bending angle $\epsilon_{\rm cal}^{\nu}$ and the guessed bending angle $\epsilon_{\rm cal*}^{\nu+1}$, and "$<,>$" denotes the inner product.

Considering all the observed data in different times, ρ^{ν} can be written in the following form:

$$\rho^{\nu} = \rho_*^{\nu} \sum_{i=1}^{N} W_i \frac{<\hat{\Phi}, \Phi'>}{<\Phi', \Phi'>}. \quad (30)$$

where W_i is the weighting coefficient, which can be obtained by some optimal methods (Hamming, 1989). In its simplest form, W_i can be written as $1/N$. That is to say, all weighting coefficients in different times are the same.

Equation (30) is a self-adaptive method using the observational data to calculate the optimal step size. It is very easy to perform and does not require much computation. Besides the above method, the one-dimensional-searching method can also be used to calculate the optimal step size. But it needs to integrate the numerical model one time for each search. So computation will be huge if many searches are needed. The comparison between the self-adaptive method and the one-dimensional-searching method can be found in Wang et al. (2000b). Wang et al. (2000b) had proved that the method of calculating the optimal step size by Equation (30) is more effective than the one-dimensional-searching methods. So in this paper we directly calculate the optimal step size by equation (30), without again comparing it with the one-dimensional-searching method.

5. Numerical experiments

The numerical experiments adapt the bending angle data at 1200 UTC 11 October 1995. Considering the huge computation required, we only use one sounding instance with 297 rays in this paper. The steepest descent method and L-BFGS are respectively used in the variational data assimilation experiments.

Figure 2 gives the numerical results of the steepest descent method, and the optimal step size is calculated by Equation (30). Panels (a) and (b) show the varying of the cost function J and the gradient norm $||\nabla J||$ with the iterations respectively. From the figure, we can find that the cost function and the gradient norm are both decreasing with the iterative times. The cost function decreases from 240 at the initial time to 30 at the final time. At the 13th iteration, it has almost reached its minimum value. At the same time, the gradient norm decreases from 800 to 50. Though there is an oscillation phenomenon, the basic trend is decreasing. It almost reaches its minimum value at the 27th iteration. In Fig. 1, at the beginning of the iterations, the cost function and the gradient norm both decrease very fast. But later, the decreasing trend becomes less and less. In fact, this is the character of the steepest descent method.

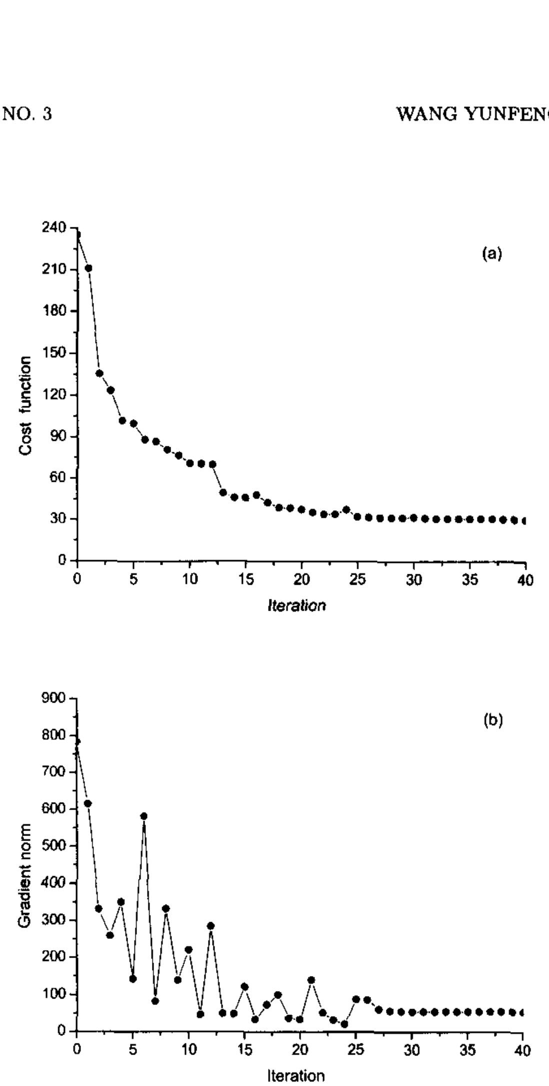

Fig. 2. The results the optimal step size calculated by Equation (30); (a) the cost function; (b) the gradient norm with the iterations.

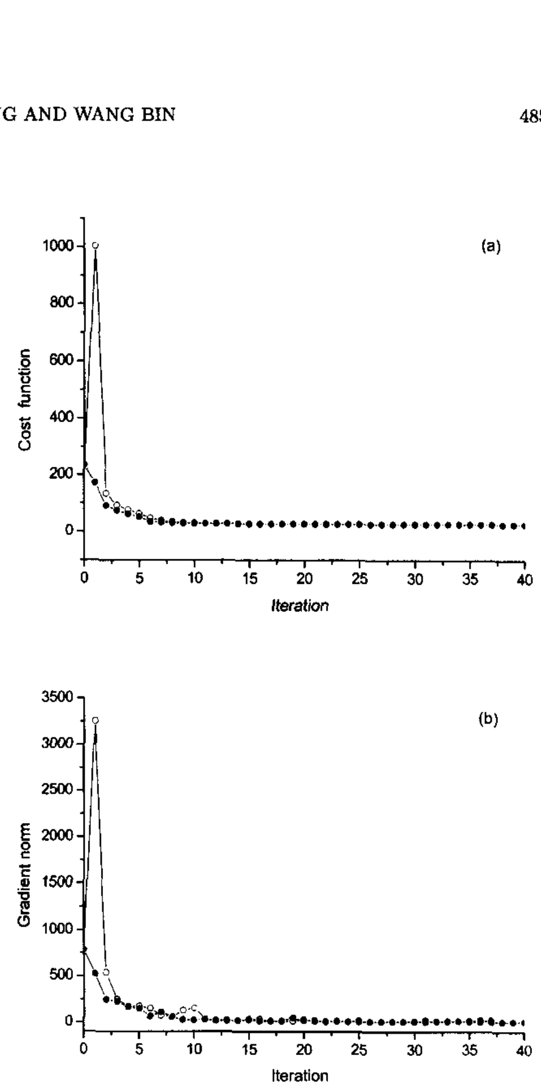

Fig. 3. The results of the optimal step of the first iteration of the limit-memory half Newton Method at iterative time calculated by Equation (30) (closed circles) and the results of the first optimal step calculated by $1/\parallel \nabla J \parallel$ (open circles); (a) the cost function; (b) the gradient norm.

Figure 3 shows the results with L-BFGS. Panels (a) and (b) give the variation of the cost function and the gradient norm with the iteration. For L-BFGS, there are two ways to calculate the optimal step size at the first iteration. One is directly by $1\,||\nabla J||$ (open circles), the other is by Equation (30) (closed circles). The figure shows that the optimal step size at the first iteration is very important. If it is not given correctly, the cost function will increase (from 240 to 1000) at the first iteration. And the gradient norm also increases from 800 to 3300. But for the correctly specified optimal step size (closed circles), the two graphs will rapidly decrease at the first iteration and keep the decreasing trend for later iteration times. However, the final results by the two ways are almost similar, because there is only one minimum point converged upon. Many other numerical experiments show that if Equation (30) is used to calculate the optimal step, ρ is almost uniform whatever initial guess is given for ρ^*. This proves the method introduced in this paper is stable and credible.

Comparing Fig. 2 with Fig. 3 shows that the convergence rate of the cost function with the L-BFGS method is faster than with the steepest descent method. With the L-BFGS method, there is no large oscillation of the gradient norm during the iterations. But by the 15th iteration, the numerical results of the two methods are the same. The cost function decreases to 30 or so and the gradient norm decreases to 50 or so. Figure 2 and Figure 3 both show that Equation (30) is valuable in the steepest descent method and in the L-BFGS. Thus it can be used successfully in the variational assimilation of GPS bending angle data. Sometimes, the steepest descent method is regarded as an unpractical method and is seldom used in practice. But the variational assimilation of GPS bending angle data shows that if the optimal step size is given by Equation (30), the steepest descent method has better convergence, and the final numerical results are same as those of L-BFGS. That is to say, the method for calculating the optimal step size given in this paper can bring a big advantage to the steepest descent method.

6. Conclusion

Through the numerical experiments, the following conclusions are drawn:

(1) The new method for calculating the optimal step size given in this paper is used successfully in the variational data assimilation of the GPS bending angle, irrespective of the use of the steepest descent method or the L-BFGS method;

(2) If the optimal step size is calculated by Equation (30), it can bring a big advantage to the convergence rate of the steepest descent method. The final numerical results are almost similar to those of the L-BFGS method;

(3) For the L-BFGS method, the optimal step size of the first iteration is very important. Given properly, it will accelerate the convergence rate at the first iteration.

There are, however, many deficiencies in our research work, for example, we assume $\epsilon_{\rm cal}^{\nu+1} = \epsilon_{\rm obs}$ at $\nu + 1$ iteration times. But the most absolutely accurate assumption would be $\epsilon_{\rm cal}^{\nu+1} = \epsilon_{\rm true}$, where $\epsilon_{\rm true}$ denotes the true GPS bending angle. Finding a solution to this problem needs more research work.

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REFERENCES

Derber, J. C., 1987: Variational four-dimensional analysis using quasi-geostrophic constraints. *Mon. Wea. Rev.*, **115**, 998–1008.

Hamming, R. W., 1989: *Digital Filters*, Third Edition. Prentice-Hall, Englewood Cliffs, New Jersey, 284 pp.

Kuo, Y.-H., X. Zou., and Y.-R. Guo, 1996: Variational assimilation of precipitable water using a nonhydrostatic mesoscale adjoint model. Part I: Moisture retrieval and sensitivity experiments. *Mon. Wea. Rev.*, **124**, 122–147.

Wang Bin, Xiaolei Zou, and Jiang Zhu, 2000a: Data assimilation and its applications. *Proc. Natl. Acad. Sci.* USA, **97**(21), 11143–11144.

Wang Yunfeng, et al., 2000b: A simple method of calculating the optimal step size in 4DVAR technique. *Advances in Atmospheric Sciences*, **17**(3), 433–444.

Zou X., B. Wang, H. Liu, R. A. Anthes, T. Matsumura, and Y. J. Zhu, 2000: Use of GPS/MET refraction angles in 3D-Var analysis. *Quart. J. Roy. Meteor. Soc.*, **126**, 3013–3040.

Zou X., and Coauthors, 1999: A ray-tracing operator and its Adjoint for the use of GPS/MET refraction angle measurements. *J. Geophys. Res.*, **104**, 22301–22318.

Zou, X., I. M. Navon, and J. Sela, 1993: Control of gravitational oscillations in variational data assimilation. *Mon. Wea. Rev.*, **121**, 272–289.

Zou, X., Y.-H. Kuo, and Y.-R. Guo, 1995: Assimilation of atmospheric radio refractivity using a nonhydrostatic mesoscale models. *Mon. Wea. Rev.*, **123**, 2229–2249.

GPS折射角资料的变分同化试验

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摘　要

越来越多的新型观测资料为数值天气预报水平的进一步提高提供了许多新的机会。在各种新型的观测资料中，GPS（全球定位卫星系统）折射角资料无疑是非常重要的。GPS折射角资料具有分辨率高、全天候探测、覆盖全球等优点，实现对GPS折射角资料的变分同化，将具有非常重要的意义。文中介绍了如何获得及同化GPS折射角资料的原理。对GPS折射角资料的变分同化可以分为两种：间接同化和直接同化，文中对这两种方法都作了具体介绍。在变分同化的最小化过程中，计算效率无疑是最重要的，而优化步长的计算又直接关系到算法效率的成败。根据最小化算法的特点，通过数学推导，得出一种适合于各种最小化算法的计算优化步长的自适应方法。最后，还利用1995年10月11日的GPS折射角资料进行了数值试验，结果表明了变分同化方法和计算优化步长方法的有效性。

关键词: GPS折射角, 变分同化, 优化步长