ADVANCES IN ATMOSPHERIC SCIENCES, VOL. 24, NO. 1, 2007, 89–100

Principle of Cross Coupling Between Vertical Heat Turbulent Transport and Vertical Velocity and Determination of Cross Coupling Coefficient

CHEN Jinbei*[1,2,3] (陈晋北), HU Yinqiao[1,2] (胡隐樵), and ZHANG Lei[1] (张 镭)

[1] *College of Atmospheric Sciences, Lanzhou University, Lanzhou* 730000

[2] *Cold and Arid Regions Environment and Engineering Institute, Chinese Academy of Sciences, Gansu Province Key Laboratory of Arid Climatic Change and Reducing Disaster, Lanzhou* 730000

[3] *Department of Environment and Resource, Gansu Agriculture University, Lanzhou* 730060

(Received 20 July 2005; revised 8 May 2006)

ABSTRACT

It has been proved that there exists a cross coupling between vertical heat turbulent transport and vertical velocity by using linear thermodynamics. This result asserts that the vertical component of heat turbulent transport flux is composed of both the transport of the vertical potential temperature gradient and the coupling transport of the vertical velocity. In this paper, the coupling effect of vertical velocity on vertical heat turbulent transportation is validated by using observed data from the atmospheric boundary layer to determine cross coupling coefficients, and a series of significant properties of turbulent transportation are opened out. These properties indicate that the cross coupling coefficient is a logarithm function of the dimensionless vertical velocity and dimensionless height, and is not only related to the friction velocity u_*, but also to the coupling roughness height z_{W0} and the coupling temperature T_{W0} of the vertical velocity. In addition, the function relations suggest that only when the vertical velocity magnitude conforms to the limitation $|W/u_*| \neq 1$, and is above the level z_{W0}, then the vertical velocity leads to the cross coupling effect on the vertical heat turbulent transport flux. The cross coupling theory and experimental results provide a challenge to the traditional turbulent K closure theory and the Monin-Obukhov similarity theory.

Key words: linear thermodynamic, turbulent transportation, atmospheric boundary layer, coupling coefficients

DOI: 10.1007/s00376-007-0089-7

1. Introduction

A cross coupling phenomenon between vertical heat turbulent transport and vertical velocity, viz. the influence of convergence movement on turbulent transportation, in the atmospheric boundary layer has been proven by using linear thermodynamics (Hu, 2003). It is beneficial to first review the history of turbulent transport to understand the cross coupling between vertical heat turbulent transport and vertical velocity. It is well known that experiments of energy and substance transportation validated that heat flux and substance flux transported by the molecular viscosity have a direct ratio to their own gradients. They are known as Fourier's Law and Fick's Law:

$$\boldsymbol{J}_{qj} = -\rho\tilde{\lambda}\frac{\partial T}{\partial x_j}\,, \quad \boldsymbol{J}_{cj} = -\rho\tilde{D}\frac{\partial c}{\partial x_j}\,. \tag{1}$$

These are two basic physical empirical laws, characterized by the phenomenological relations of molecular viscosity transportation (Hu, 2002a). In Eq. (1), $\boldsymbol{J}_{qj}$ and $\boldsymbol{J}_{cj}$ are the fluxes of heat and substance, respectively; ρ fluid density; T and c the temperature and specific component; and $\tilde{\lambda}$ and $\tilde{D}$ the Fourier conductivity and Fick diffusivity. Prandtl (1904) extended first Fourier's Law and then Fick's Law to develop a mixing length theory of turbulent transport. The mixing length theory considers that the viscosity of turbulent eddy is similar to the molecular viscos-

*E-mail: cjb@gsau.edu.cn

ity, and the mixing length of an eddy is similar to the molecular free path, and further that the turbulent transport flux has a direct ratio with the gradient of relevant transport quantity, as shown in Eq. (1). The conductivity and diffusivity are defined as the turbulent transport coefficients K of heat and substance, respectively. The mixing length theory is also known as the K closure theory of turbulent transport. Hence, Prandtl (1904) mixing length theory of turbulent transport forms the theoretical basis of atmospheric turbulent transport. However, the turbulent transport coefficient is a parameter that must be determined experimentally in order to avoid difficulties in practical application.

Monin and Obukhov (1954) developed a similarity theory of the atmospheric boundary layer (ABL) under the supposition of the stationary state and the homogeneous underlying surface to establish the theoretical basis of applying the turbulent transport theory of the ABL in practice. Businger et al. (1971) and Dyer and Bradley (1982) determined Monin-Obukhov's similarity functions and relevant turbulent transport coefficients, respectively, by using experiments in the surface layer. The similarity theory of the ABL is employed in various fields of atmospheric science, e.g. atmospheric diffusion, land surface processes etc. However, Monin-Obukhov's similarity theory was deduced by hypothesis of the homogeneous underlying surface and the constant flux layer in which any vertical movement of the system never existed, viz. the vertical velocity is always equal to zero. This hypothesis means that the application of Businger-Dyer similarity relations is limited in range of the ideal homogeneous underlying surface with non-vertical velocity. However, natural processes such as atmospheric diffusion, land surface processes etc. commonly occur over the heterogeneous underlying surface, causing great difficulty in applying the theory of the ABL. The turbulent transport K closure theory has been applied to atmospheric studies for more than a century, and the Monin-Obukhov similarity theory and Businger-Dyer similarity relations have been employed for nearly half a century. They still form the theoretical bases of turbulent transport and parameterization of the ABL in the modern climate system. However, in practical situations, the mean vertical velocity is small ($\sim$0.1 mm s^{-1}), but not exactly zero. Some scholars, such as Webb et al. (1980), van Dijk et al. (2004), pointed out the importance of a non-zero mean vertical velocity, called the Webb term, for surface fluxes and eddy-correlation measurement. Kaimal (1968) indicates that it is peculiar to notice that in recent literature some researchers still omit the Webb term. Of course, there is a basic problem, which will be called the eddy-correlation problem, caused by the immeasurability of mean vertical velocity.

With the development of linear thermodynamics (Onsager, 1931a,b; Prigogine, 1945), in theory, one proved the phenomenological relations of molecular viscosity transport, and discovered that there exists a cross coupling between the transport of energy and substance. In this cross coupling process, substance flow caused by the temperature gradient is called the Soret effect, and heat flow caused by the concentration gradient of substance the Dufour effect (Li, 1986; Hu, 2002a). The cross coupling theory between the transports of energy and substance is mainly applied to physics and chemistry, and rarely applied to atmospheric science. Clapp and Homberger (1978) first applied the cross coupling theory between the transports of energy and substance to the parameterization of soil heat and vapor transport in the land surface process model of climate. Subsequently, Sun (1987) applied the cross coupling parameterization of soil heat and vapor transport to the land surface process model of arid area to improve the existing climatic model.

Hu (2002a,b) developed the atmospheric linear thermodynamics to prove phenomenological relations of turbulent transport based on thermodynamics theory, and then went on to demonstrate the existence of linear phenomenological relations in the ABL by using large amounts of observed data from the ABL to obtain relations between the phenomenological coefficient and the turbulent transport coefficient of K theory (Hu, 2002c). Even a cross coupling effect between the mechanical dynamic process and the thermal process was proved, e.g., the relationship between the geostrophic and thermal winds is a special cross coupling phenomenon between dynamic and thermal processes in the atmospheric system (Hu, 2002c). Furthermore, Hu (2003) studied another cross coupling phenomenon between dynamic and thermal processes in the atmospheric system to prove that there exists a cross coupling between vertical movement and vertical turbulent transport. This breaks away from classical turbulent transport theory, which states that the vertical turbulent transport flux of any macroscopic quantity is equivalent to the vertical gradient transport flux of a corresponding macroscopic quantity. However, the cross coupling between vertical movement and vertical turbulent transport points out that the vertical heat turbulent flux and the vertical vapor turbulent flux must both include the vertical transport caused by the convergence or the divergence of atmosphere due to the cross coupling effect, except the transport caused by the vertical gradient of the corresponding temperature and vapor. The transport of the convergence or divergence movement always occurs for the ABL under the natural underlying surface and convection boundary layer (CBL), because that the heterogeneity of underlying surface and atmospheric convection cause the convergence or divergence movement. These studies may offer clues not only for establishing the ABL theory regarding the heterogeneous underlying surface, but also for overcoming the difficulties encountered in recent applications of the ABL theory (Hu, 2004).

Until now, the cross coupling between vertical movement and vertical turbulent transport is only a theory deduced from linear thermodynamics. Just as the turbulent transport coefficient must be determined experimentally, so as must the cross coupling coefficient between vertical movement and vertical turbulent transport. The aims of the present study are to validate the cross coupling phenomenon between vertical movement and vertical heat turbulent transport, and to determine the relevant cross coupling coefficient and its properties by using observed data.

2. Basic theory: principle of cross coupling between vertical heat turbulent transport and vertical velocity, and the determination of cross coupling coefficient

The turbulent heat flux $\boldsymbol{J}_{\theta j}$ and airflow $\rho\boldsymbol{U}$ are both defined as "generalized flow", and the force that drives the generalized flow is defined as "generalized force" in nonequilibrium thermodynamics. In the atmospheric system, the temperature gradient causes the turbulent heat flux, and Newton force drives the airflow. The generalized force $\boldsymbol{X}_{\theta j}$ of the generalized flow of heat turbulent flux and the generalized force $\boldsymbol{X}_{gi}$ of the generalized flow of airflow are, respectively (Hu, 2002a,b):

$$\boldsymbol{X}_{\theta j} = \frac{\partial}{\partial x_j}\left(\frac{1}{\theta}\right) , \tag{2}$$

$$\boldsymbol{X}_{gi} = \frac{1}{T}\left(\frac{1}{\rho}\frac{\partial p}{\partial x_j}\delta_{ij} + \boldsymbol{g}\delta_{i3} - f_c\varepsilon_{ij3}\boldsymbol{U}_j\right) . \tag{3}$$

Based on the Curier-Prigogine principle of linear thermodynamics (Prigogine, 1967), there exists a cross coupling between the airflow and the heat flux due to the fact that they are both vectors. Considering the Onsager reciprocal relation (Onsager, 1931a,b); Prigogine, 1945), the cross coupling relations between the heat flux and airflow are as follows (Hu, 2003):

$$\begin{aligned}\boldsymbol{J}_{\theta j} =& L_\theta\frac{\partial}{\partial x_j}\left(\frac{1}{\theta}\right) + \\ & L_{\theta\alpha}\frac{1}{T}\left(\frac{1}{\rho}\frac{\partial p}{\partial x_j}\delta_{ij} + \boldsymbol{g}\delta_{i3} - f_c\varepsilon_{ij3}\boldsymbol{U}_j\right) ,\end{aligned} \tag{4}$$

$$\begin{aligned}\rho\boldsymbol{U}_i =& L_\alpha\frac{1}{T}\left(\frac{1}{\rho}\frac{\partial p}{\partial x_j}\delta_{ij} + \boldsymbol{g}\delta_{i3} - f_c\varepsilon_{ij3}\boldsymbol{U}_j\right) + \\ & L_{\theta\alpha}\frac{\partial}{\partial x_j}\left(\frac{1}{\theta}\right)\delta_{ij} ,\end{aligned} \tag{5}$$

where T, θ, p, and ρ are the absolute temperature, potential temperature, air pressure and air density, respectively; $\boldsymbol{g}$ and $f_{\rm c}$ are gravity acceleration and the Coriolis coefficient; L_θ, L_α, and $L_{\theta\alpha}$ are respectively defined as the thermodynamic phenomenological coefficient, the dynamic phenomenological coefficient, and the cross coupling coefficient. Relationship (4) shows that the potential temperature gradient and any departure from the dynamic balance would all cause the turbulent heat flux. This means that the turbulent heat flux is related not only to the potential temperature gradient but also to the departure from the dynamic balance owing to the cross coupling between turbulent heat flux and airflow. Relation (5) shows that the airflow is related not only to the departure from the dynamic balance but also to the potential temperature gradient due to the same reason. Hu (2002c) demonstrated that the cross coupling between turbulent heat transport and horizontal velocity in Eq. (5) describes the well known relationship between the geostrophic and thermal winds.

The left side of Eq. (5) expresses the airflow, for which the vertical component can be written as:

$$\rho W = L_g\frac{1}{T}\left(\frac{1}{\rho}\frac{\partial p}{\partial z} + g\right) - L_{\theta p}\frac{1}{\theta^2}\frac{\partial\theta}{\partial z} , \tag{6}$$

where W is the vertical velocity, and L_g and $L_{\theta p}$ the relevant phenomenological and cross coupling coefficients. The Eq. (6) indicates that any departure from the static balance or from the neutral stratification causes vertical velocity. On the other hand, the atmosphere is assumed to be an incompressible fluid, and then the continuity equation is:

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} = 0 . \tag{7}$$

The vertical component of Eq. (4) can be written as:

$$\begin{aligned}J_{\theta z} =& H|_z = \rho c_p\overline{w'\theta'} = -\rho c_p K'_\theta\frac{\partial\theta}{\partial z} + \\ & L_{\theta\alpha}\frac{1}{T}\left(\frac{1}{\rho}\frac{\partial p}{\partial z} + g\right) , \quad K'_\theta = \frac{L_\theta}{\rho c_p\theta^2} ,\end{aligned} \tag{8}$$

in which c_p is the specific heat at constant pressure, and the turbulent transport coefficient K'_θ is a linear function of the phenomenological coefficient L_θ (Hu, 2003). Using Eq. (6), one can eliminate the second term on the right side of Eq. (8):

$$J_{\theta z} = H|_z = \rho c_p \overline{w'\theta'} = -\rho c_p K'_\theta \frac{\partial \theta}{\partial z} + \\ + \rho \frac{L_{\theta\alpha}}{L_g} W + \rho \frac{L_{\theta\alpha} L_{\theta p}}{L_g} \frac{1}{\theta^2} \frac{\partial \theta}{\partial z} . \quad (9)$$

Then, using Eq. (7), one can eliminate the vertical velocity on the right side of Eq. (9) to obtain the vertical component of turbulent heat flux:

$$\left.\begin{aligned} J_{\theta z} = H|_z = \rho c_p \overline{w'\theta'} = -\rho c_p K_\theta \frac{\partial \theta}{\partial z} + \rho c_p K_{\theta w} W \\ = -\rho c_p K_\theta \frac{\partial \theta}{\partial z} - \rho c_p K_{\theta W} \int_0^z (\nabla|_h \cdot \boldsymbol{V}) dz , \\ \nabla|_h \cdot \boldsymbol{V} = \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} . \end{aligned}\right\} \quad (10)$$

Here, the turbulent transport coefficient K_θ of vertical heat transport caused by the potential temperature gradient and the relevant cross coupling coefficient $K_{\theta W}$ are respectively defined as:

$$K_\theta = \left(L_\theta - \frac{L_{\theta\alpha} L_{\theta p}}{L_g} \right) \frac{1}{\rho c_p \theta^2} , \quad K_{\theta W} = \frac{L_{\theta\alpha}}{c_p L_g} . \quad (11)$$

In Eq. (10), $J_{\theta z}$ is the vertical component of heat turbulent flux, $H|_z$ the vertical component of heat turbulent flux observed by the eddy correlation method at level z, w' the fluctuation of vertical velocity, and θ' the fluctuation of the potential temperature. The turbulent vertical flux relation [Eq. (10)] indicates that the vertical component of heat turbulent flux is constituted by both the transports of vertical potential temperature gradient and vertical movement, viz. horizontal convergence or divergence movement.

The turbulent flux relation [Eq. (10)] of vertical heat transport is only a theoretical conclusion deduced by linear thermodynamics, which must be validated by atmospheric observational facts. Moreover, the vertical heat turbulent transport coefficient and the cross coupling coefficient must be determined through experimental observations. If the vertical component $H|_z = \rho c_p \overline{w'\theta'}$ of heat turbulent transport flux, temperature gradient $\partial\theta/\partial z$, and the average vertical velocity $W|_z$ at the level (z) in Eq. (10) have been gained through experiment, and the turbulent transport coefficient K_θ of vertical heat transport can be determined by the classical theory of the ABL, then the cross coupling coefficient $K_{\theta W}$ of vertical velocity could also be determined by experimental observations, as shown:

$$K_{\theta W} = \frac{\overline{H|_z} + \rho c_p K_\theta \partial\theta/\partial z}{\rho c_p W|_z} = \frac{\overline{w'\theta'} + K_\theta \partial\theta/\partial z}{W|_z} . \quad (12)$$

In order to determine the cross coupling coefficient $K_{\theta W}$, it is beneficial to analyze the possible form of $K_{\theta W}$ by applying the π similarity principle of the physical dimension. The dimensional analysis for the relation shown in Eq. (10) points out that $K_{\theta W}$ possesses a temperature dimension (K), and therefore a parameter T_{W0} possessing the temperature dimension (K) is given as a temperature characteristic scale of the airflow coupling. Considering vertical velocity W is an important parameter of the cross coupling between vertical turbulent transport and vertical velocity, friction velocity u_* characterizing momentum flux might be the velocity characteristic scale of airflow coupling. Furthermore, it is considered that the underlying surface as the boundary influences $K_{\theta W}$, viz. $K_{\theta W}$ is likely to relate to the height z, and it is presumed that there is a coupling roughness z_{W0}. Based on the π dimension principle, three non-dimensional parameters are defined as $\pi_1 = K_{WT}/T_{W0}, \pi_2 = z/z_{W0}$, and $\pi_3 = W/u_*$, and then the function relation $\pi_1 = \phi(\pi_2, \pi_3)$ is obtained. This non-dimensional relation can be written as:

$$\frac{K_{\theta W}}{T_{W0}} = \phi\left(\frac{z}{z_{W0}}, \frac{W}{u_*}\right) , \text{ or } K_{\theta W} = T_{W0}\phi\left(\frac{z}{z_{W0}}, \frac{W}{u_*}\right) . \quad (13)$$

Equation (13) shows the likely similarity relationship between the coupling coefficient of vertical velocity and the characteristic scales of the ABL. It is possible to determine the specific form of Eq. (13), and the characteristic scales, u_*, T_{W0} and z_{W0}, by using observed data from the ABL.

3. Properties of the cross coupling coefficient of vertical velocity

3.1 *Source of data and operated method*

Unquestionably, the first term on the right side of Eq. (10) (turbulent transport of the temperature gradient) is a dominant term; and the second term (the cross coupling of vertical velocity) is only an amendatory minor term. In order to determine the cross coupling coefficient of vertical velocity from Eq. (12) using observed data, it is necessary that such observations should include the vertical component $H|_z = \rho c_p \overline{w'\theta'}$ of heat turbulent transport flux, the temperature gradient $\partial\theta/\partial z$, and the average vertical velocity $W|_z$ at the level (z), and that these data should possess higher accuracy. We selected and used observed data of a turbulent experiment in the surface layer, which were provided by Prof. Högström of Uppsala University, Sweden. These data were observed at the Lövsta site in the south of the city of Uppsala during the period 16 May to 17 June 1966. The details of the experimental data are described in Högström (1988). The data include turbulent fluctuations of wind, temperature, and humidity at three levels, and the average wind, temperature, and humidity at six levels. Information about the experimental data is provided in Table 1. The accuracy of wind calibrated in the wind tunnel was ±0.05 m s^{-1}, and the accuracy of temperature calibrated in the water bath was 0.01°C. The accuracy of temperature in the field does not usually exceed 0.02°C, and when the average wind speed is greater than 3 m s^{-1}, the relative error of velocity fluctuates between 2%–3%. The accuracy of the data is sufficient for the purposes of this study. The selecting criteria of data is similar to Prof. Högström's in eliminating the friction velocity $u_* < 0.1$ m s^{-1} and the heat flux $H < 10$ W m^{-2} from the analysis, and the direction of the temperature gradient isn't identical to the direction of the heat turbulent flux. However, the non-stationary data (period during 0600–0800 and 1900–2100) are not restricted; they are reserved in this analysis. Considering that the heat turbulent flux, gradient transport flux, and coupling term of vertical velocity in Eq. (10) under stable stratification are all minor and their relative errors are great, then the measurement accuracy could not be satisfactory for this study, and therefore only the data under unstable stratification are selected in the analysis. In order to filter perturbation of turbulent fluctuations, the moving average for 30 mins is calculated for the original data, and in order to use adequately the data of average wind and temperature, the curve fitting of the mean wind and temperature at six corresponding levels are made using the least squares procedure. Hereafter, the turbulent fluxes caused by the temperature gradient and the average vertical velocities at three levels (3 m, 6 m, and 13.9 m) are taken from the fitting curve. Each gradient value of the potential temperature and the horizontal wind at each level (3 m, 6 m, and 13.9 m) is calculated by the corresponding values at two levels in the neighborhood of the above three levels.

When measurements began on 16 May, the height of vegetation in the experimental site was 25 cm, gradually growing to 75 cm on 13 June. The underlying surface within the sectors where the wind came from 220°–310°, or from 110°–170°, was absolutely uniform and is to be regarded as a homogeneous underlying surface. The underlying surface within the remaining sectors was complex and is to be regarded as a heterogeneous underlying surface, as in Högström (1988). In this paper, therefore, the data over the two different underlying surfaces are analyzed separately.

In order to determine the coupling coefficient $K_{\theta W}$ of vertical velocity from Eq. (12), the traditional Monin-Obukhov similarity theory of the surface layer and the Businger-Dyer similarity relation are applied to calculate the vertical turbulent transport coefficient K_θ (Businger et al., 1971). The calculated formulas are as follows:

$$K_\theta = \frac{\kappa^2(z-d-z_0)^2}{\Phi_M\Phi_H}\frac{\partial u}{\partial z}\,, \tag{14}$$

$$\begin{cases} \left.\begin{array}{l}\Phi_M = (1-\alpha_M z/L)^{-1/4}\,,\\ \Phi_H = a(1-\alpha_H z/L)^{-1/2}\,;\end{array}\right\} z/L<0,\\ \left.\begin{array}{l}\Phi_M = 1+\beta_M z/L\,,\\ \Phi_H = b+\beta_H z/L\,.\end{array}\right\} z/L>0\end{cases} \tag{15}$$

$$z/L = \begin{cases} 1.0Ri & Ri \leqslant 0\,,\\ R_i/(1.0-5.0Ri) & Ri>0\,,\end{cases} \tag{16}$$

$$u_* = \frac{k_M(z-d)}{\Phi_M}\frac{\partial u}{\partial z}\,. \tag{17}$$

where

$$L = \frac{-u_*^3\theta}{kg\overline{w'\theta'}}$$

is the Monin-Obukhov stability length; z/L is an atmospheric stability parameter;

$$Ri = \frac{g\partial\theta/\partial z}{\theta(\partial u/\partial z)^2}\,,$$

is the gradient Richardson parameter; the Kàrmàn constant is $\kappa = 0.40$; and Φ_M and Φ_H are Monin-Obukhov's similarity universal functions. The parameters in Monin-Obukhov's similarity universal functions [Eq. (15)] are $a=0.95, b=0.95, \alpha_M=19.3, \beta_M=6.0, \alpha_H=11.6$, and $\beta_H=7.8$, and were obtained by Businger et al. (1971). The roughness z_0 adopts $z_0=0.01$ m, which is obtained by the wind profile in neutral stratification, and the zero displacement height

Table 1. Measuring quantities and the relevant exact heights.

Measured property	Level (m)
Horizontal wind speed	0.76, 1.68, 3.43, 6.57, 12.45, 23.58
Temperature	0.94, 1.86, 3.61, 6.75, 12.64, 23.76
Humidity	13, 24
Wind direction	3, 6, 13.9
Turbulent flux	3, 6, 13.9
Vertical velocity	0.76, 1.68, 3.43, 6.57, 12.45, 23.58

$d = 0.75h$ is a result from Högström (1988). The zero displacement height d is calculated by the vegetation height h, measured every day.

Firstly, the Richardson parameter Ri and Monin-Obukhov's atmospheric stability parameter z/L is calculated by the gradient data of temperature and wind from Eq. (16); secondly, Monin-Obukhov's universal function is obtained using Eq. (15); and thirdly, the vertical turbulent transport coefficient K_θ of heat is determined using Eq. (14). The friction velocity u_* can be obtained from Eq. (17). Finally, all data are normalized according to Eq. (13), and the relation among the normalized quantities is analyzed further to obtain specific form and properties of the cross coupling coefficient $K_{\theta W}$ of vertical velocity.

3.2 *Properties of the cross coupling coefficient $K_{\theta W}$ of vertical velocity*

Elementary analysis for the normalized data shows that $K_{\theta W}/T_{w0} \to 0$, when $|W/u_*| \to 1$ and $z/z_{w0} \to 1$. Moreover, $K_{\theta W}/T_{W0}$ has two fluctuating branches at the point $|W/u_*| = 1$ for $W > 0$ and $W < 0$. According to these properties, one expects that $K_{\theta W}$ possesses characteristics as follows:

$$K_{\theta W} = T_W \left[\ln \left(\frac{W}{u_*} \right)^2 \right]^\beta , \tag{18}$$

$$T_W = T_{W0} \left(\ln \frac{z}{z_{W0}} \right)^\alpha . \tag{19}$$

The function relation in Eq. (18) of the coupling coefficient $K_{\theta W}$ of vertical velocity vs $\ln (W/u_*)^2$ obtained by the normalized data is shown in Fig. 1. The value of β is determined as $\beta = 4$ owing to β values of fitting curves all closing to four at each level. The correlation coefficient R and residual error S of the fitting curves for $K_{\theta W}$ are also given in Fig. 1. In Fig. 1, the left values (a) and (b) are the results for the homogeneous underlying surface; the right values (c) and (d) are the results for the heterogeneous underlying surface; the top values (a) and (c) result from $W > 0$; and the bottom values (b) and (d) result from $W < 0$. Figure 1 shows results at the level $z = 13.9$ m and is illustrated as an example.

The results of fitting curves at the other levels are analogous with the results in Fig. 1, but their parameters T_W are different to each other (Table 2). As expected from Eq. (19), the results in Table 2 indicate that there is a logarithmic relation between T_W and the height z. The function relation of T_W to $\ln (z/z_{W0})$ is shown in Fig. 2, in which the correlation coefficients R and residual errors S of the fitting curve are also given. In Fig. 2, the top values (a) and (b) are the results for the homogeneous underlying surface; the bottom values (c) and (d) are the results for the heterogeneous underlying surface; the left values (a) and (c) result from $W < 0$; and the right values (b) and (d) result from $W > 0$. Figure 2 shows that the fitted straight lines all possess a good logarithmic relation, as in Eq. (19), viz. corresponding parameters α equal one; $\alpha = 1$.

The intercept on ordinate $\ln z$ is the height characteristic scale z_{W0}, which is defined as the coupling roughness height of vertical velocity; and the slope rate of the fitted straight line is the temperature characteristic scale T_{W0}. As the scales z_{W0} and T_{W0} characterize the dynamic and thermal features of the underlying surface, their values are therefore different under the dissimilar conditions of the homogeneous or heterogeneous underlying surface, and of the upward flow $W > 0$ or downward flow $W < 0$.

To generalize the above conclusions, the cross coupling coefficient of vertical velocity for the vertical heat turbulent flux is determined as follows:

$$K_{\theta W} = T_{w0} \ln \left(\frac{z}{z_{w0}} \right) \left[\ln \left(\frac{W}{u_*} \right)^2 \right]^4 . \tag{20}$$

This is an empiric function form of the similarity relation of the cross coupling coefficient of vertical velocity for the vertical heat turbulent flux. It possesses the following important properties:

(1) The empiric function form [Eq. (20)] of the cross coupling coefficient of vertical velocity is possibly a similarity relation that possesses universality. The coupling coefficient $K_{\theta W}$ is related to the velocity characteristic scale, friction velocity u_*; the height characteristic scale, coupling roughness height z_{W0}; and the temperature characteristic scale, coupling temperature T_{W0}. The coupling roughness height and the temperature characteristic scale are all decided by the dynamic and thermal features of the underlying surface.

(2) The cross coupling coefficient $K_{\theta W}$ of the vertical velocity branches off at point $|W/u_*| = 1$, viz. $K_{\theta W} > 0$, as $W > 0$ and $K_{\theta W} < 0$, as $W < 0$. Furthermore, the ratio of the coupling coefficient to the temperature characteristic scale tends to zero as the normalized vertical velocity or the normalized height tends to 1, viz. $K_{\theta W}/T_{W0} \to 0$ as $|W/u_*| \to 1$ or $z/z_{W0} \to 1$; however $K_{\theta W} = 0$ as $|W/u_*| \to 1$ or $z/z_{W0} \leqslant 1$. This means that only when the absolute value of vertical velocity varies in a definite scope, viz. it is less than the friction velocity ($|W| < u_*$), and when the level is greater than the height characteristic scale $z > z_{W0}$, the cross coupling effect of vertical velocity for the vertical heat turbulent transport flux is valid. It is a pity that our study excludes data for $|W| > u_*$.

The characteristic scale z_{W0} of the coupling rough-

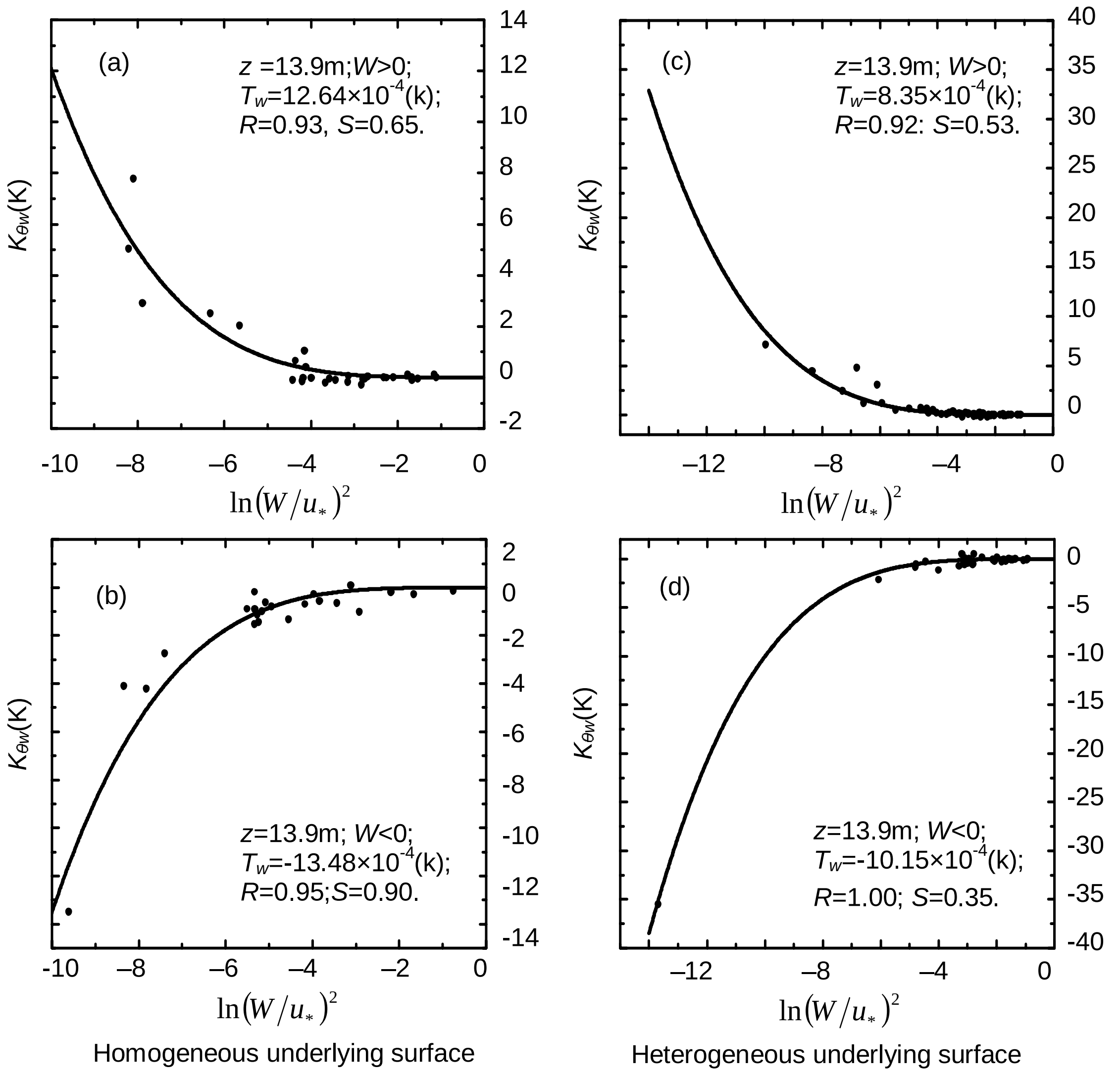

Fig. 1. The function relation of the cross coupling coefficient $K_{\theta W}$ (K) of vertical velocity vs $\ln(W/u_*)^2$.

Table 2. The relation between $T_w(\times 10^{-4}$ K) and height z.

Level	Homogeneous underlying surface		Homogeneous underlying surface	
	$W > 0$	$W < 0$	$W > 0$	$W < 0$
3 m	0.85	−4.83	2.63	−2.06
6 m	6.27	−8.66	5.54	−6.42
3 m	12.64	−13.48	8.35	−10.15

ness height and the characteristic scale T_{W0} of the coupling temperature of vertical velocity, as shown in Table 3, are determined by the dynamic and thermal features of the underlying surface. However, it is difficult to deduce whether the dynamic and thermal features of the underlying surface relate to these scales due to a limitation of data. The results in Table 3 show that these scales are related to the vertical velocity, $T_{W0} > 0$ as $W > 0$ and $T_{W0} < 0$ as $W < 0$. It is estimated from the results in Table 3 that T_{W0} is likely to be related to the horizontal temperature gradient between the observational site and the surroundings; viz. the horizontal temperature gradient causes the vertical velocity. The characteristic temperature scale T_{W0} of vertical velocity coupling characterizes the horizontal temperature gradient that causes the

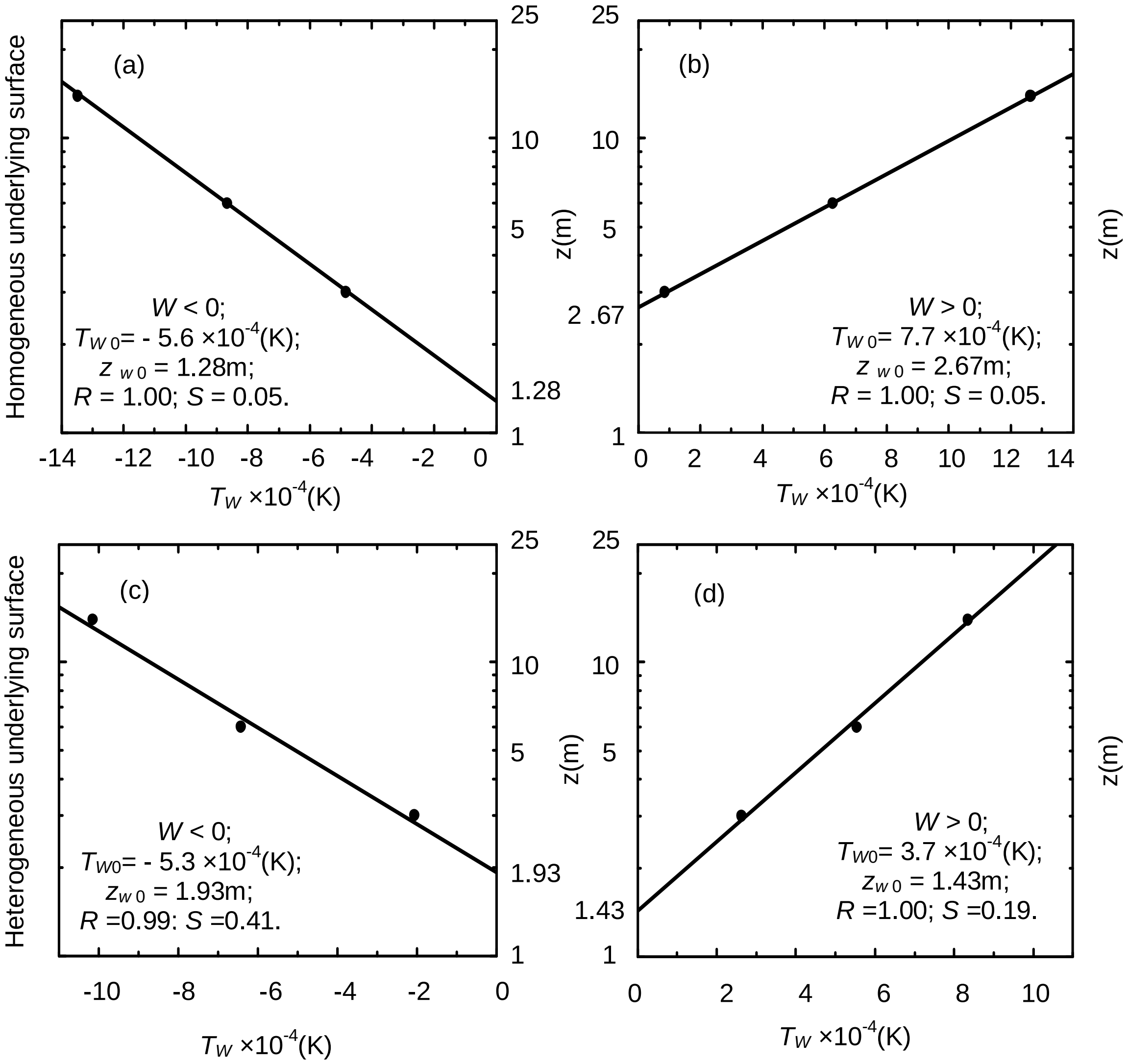

Fig. 2. The relationship of coupling coefficient $K_{\theta W}$ of vertical velocity vs $\ln(z/z_{W0})$.

convergence and divergence movement. In addition, the characteristic scale z_{W0} of the coupling roughness height is possibly a function of the height or displacement height of vegetation. These doubtful points need validation with observed facts.

The cross coupling coefficient could be decided by Eq. (20), once the characteristic scales are determined. Consequently, for a special underlying surface, as long as the data of temperature gradient, wind speed and the vertical velocity are obtained, the cross coupling coefficient of vertical velocity could be calculated, then the turbulent transport term of temperature gradient and the coupling transport term of vertical velocity can also be calculated.

4. Verifying the cross coupling effect of vertical velocity on the vertical heat turbulent transport flux

Equation (10) shows that the coupling term of vertical velocity is a revision by the temperature gradient transport term in the heat turbulent transport flux. Now we apply the cross coupling coefficient of vertical velocity from Eq. (20) to estimate this revision term, in order to verify the cross coupling effect of vertical velocity on the vertical heat turbulent transport flux and the reliability of the cross coupling coefficient K_{W0}. Each term in Eq. (10) is noted as:

Table 3. The characteristic scales, z_{W0} and T_{W0}.

Characteristic scale	Homogeneous underlying surface		Homogeneous underlying surface	
	$W > 0$	$W < 0$	$W > 0$	$W < 0$
T_{W0} (K)	7.7×10^{-4}	-5.6×10^{-4}	3.7×10^{-4}	-5.3×10^{-4}
z_{W0} (m)	2.67	1.28	1.43	1.93

$$\begin{cases} H_T = \rho c_p \overline{w'\theta'} \ , \\ H_K = -\rho c_p K_\theta \dfrac{\partial\theta}{\partial z} \ , \\ H_W = \rho c_p K_{\theta W} W \ ; \end{cases} \tag{21a}$$

There exists no cross coupling:

$$\begin{cases} H_T = \rho c_p \overline{w'\theta'} = -\rho c_p K_\theta \dfrac{\partial\theta}{\partial z} \ , \\ H_T = H_K \ ; \end{cases} \tag{21b}$$

There exists cross coupling:

$$\begin{cases} H_T = \rho c_p \overline{w'\theta'} = -\rho c_p K_\theta \dfrac{\partial\theta}{\partial z} + \rho c_p K_{\theta W} W \ , \\ H_T = H_K + H_W \ . \end{cases} \tag{21c}$$

Subsequently, the cross coupling coefficient [Eq. (20)] and each heat flux in Eq. (21) are calculated by using the observed data of above temperature gradient, and wind and vertical velocity, along with the characteristic scales of the underlying surface. The result at level $z = 13.9$ m under the homogenous underlying surface is illustrated in Fig. 3. The left values, (a) and (b), result from no revision by vertical velocity coupling; the right values, (c) and (d), result from a revision by vertical velocity coupling; the top values, (a) and (c), result from the upward flow $W > 0$; and the bottom values, (b) and (d), result from the downward flow $W < 0$. The correlation coefficient R and the residual error S of the fitted straight line are also shown. Furthermore, C_D and C_{DW} in Fig. 3, respectively, are the slope rate of the fitted straight line of heat flux, nonrevised and revised by the vertical velocity coupling to characterize the revised effect.

The traditional turbulent transport theory states that the vertical heat turbulent transport flux is equivalent to the transport flux of the temperature gradient, viz. $H_T = H_K$, and then the slope rate of the fitted straight line should be $C_D = 1$. However, values (a) and (b) in Fig. 3, which result from no revision by vertical velocity coupling, show that the slope rate is $C_D = 0.77$ for $W > 0$ and C_D=0.70 for $W < 0$, viz. $H_T \neq H_K$. This means that the heat flux calculated by the temperature gradient is greater than the turbulent transport heat flux observed directly by the eddy correlation method, viz. the heat flux calculated by the temperature gradient is greater than the turbulent transport heat flux. However, the values (c) and (d), which are the revised results, show that the slope rate is $C_{DW} = 1.01$ for $W > 0$ and $C_{DW} = 0.92$ for $W < 0$. This shows that after revision by vertical velocity coupling, the transport flux value calculated by the temperature gradient is not only more close to the directly observed value, but also the correlation coefficient R of the fitted straight line increases, and the residual error S decreases. The effect of revision by vertical velocity coupling is remarkable. The results over the heterogeneous underlying surface are shown in Fig. 4. Just as in Fig. 3, the vertical heat turbulent transport flux is not as equivalent to the vertical temperature gradient transport flux, $H_T \neq H_K$, but the transport flux calculated by the vertical temperature gradient is also greater than the directly observed value. However, the deviation of the vertical temperature gradient transport flux from the vertical heat turbulent transport flux decreases after revision by vertical velocity coupling, and the correlation coefficient R also increases, and the residual error S decreases. The fact after having been revised by vertical velocity coupling for $W > 0$ is interesting, for which the slope rate C_{DW} reaches $C_{DW} = 1.01$ over the homogeneous underlying surface (Fig. 3c) and $C_{DW} = 0.98$ over the heterogeneous underlying surface (Fig. 4c), respectively. The observed results prove almost entirely the facticity of Eqs. (10) and (20). These results validate that the vertical heat turbulent transport flux is indeed composed of both the vertical temperature gradient transport and the coupling transport of vertical velocity, viz. the coupling transport of convergence or divergence of the horizontal velocity. However, the revision of vertical velocity coupling for the downward flow $W < 0$ is not good enough comparing with the upward flow $W > 0$, the reason for which requires further research.

5. Discussion

Firstly, it is proved that there exists a cross coupling between vertical turbulent transport and vertical velocity based on linear thermodynamics. The vertical heat turbulent transport flux is composed of both the vertical temperature gradient transport and the cou-

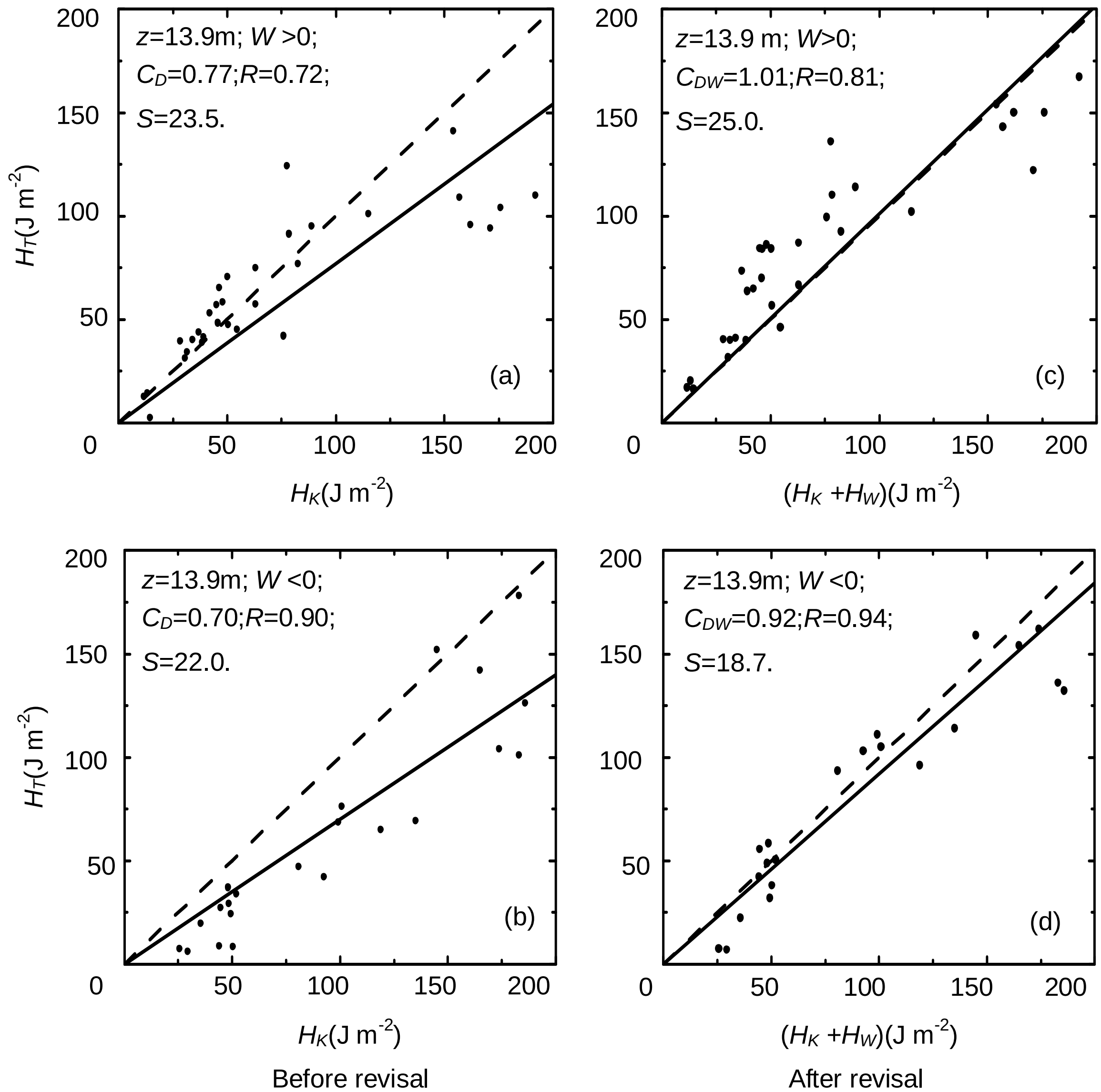

Fig. 3. The heat turbulent transport flux revised by vertical velocity coupling for the homogeneous underlying surface.

pling transport of vertical velocity. The coupling effect of vertical velocity on the vertical turbulent heat transport is validated by using the experiment data from the ABL, and further to determine concretely the relevant coupling coefficient. Even a series of properties of the cross coupling coefficient is revealed. These studies establish a foundation for the practical application of the cross coupling theory between vertical velocity and vertical turbulent transport. The results show that the cross coupling coefficient $K_{\theta W}$ of vertical velocity is a logarithmic function of the vertical velocity and height; the cross coupling coefficient $K_{\theta W}$ is also related to the friction velocity u_*, and that the coupling roughness height z_{W0} and the coupling temperature T_{W0} of vertical velocity, which are the characteristic scales caused by the feature of the underlying surface. The results also suggest that the coupling effect of vertical velocity on the vertical heat turbulent transport is suitable definite scope that the vertical velocity varies in $|W| < u_*$ and the level is higher than the height characteristic scale.

The traditional turbulent transport K theory states that the vertical turbulent transport flux of any macroscopic quantity is equivalent to the transport flux of the vertical gradient of the relevant macroscopic quantity. Monin-Obukhov's similarity theory is obtained under the hypotheses of the homogeneous underlying surface and the constant flux layer, these

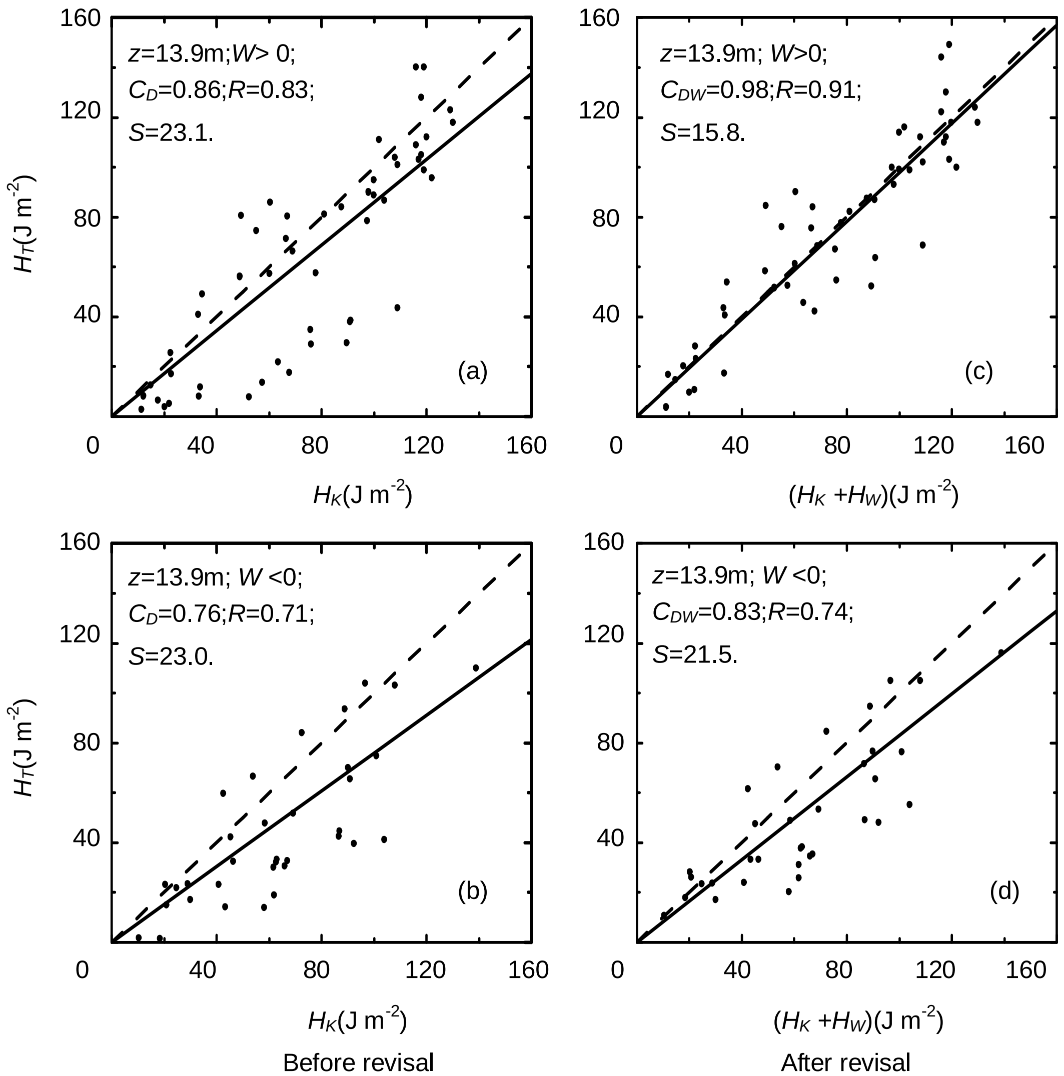

Fig. 4. The heat turbulent transport flux revised by vertical velocity coupling for the heterogeneous underlying surface.

hypotheses mean vertical velocity is equal to zero forever. However, the cross coupling theory between vertical velocity and vertical turbulent transport states that the surface layer cannot simply be presumed to be a constant flux layer owing to the fact of existence of vertical movement caused by the heterogeneity of the underlying surface or the development of convection. Thus, the vertical heat turbulent transport flux is always not entirely equivalent to the transport flux of the vertical temperature gradient, but is instead equivalent to the sum of both the transport flux of the vertical temperature gradient and the coupling transport flux of vertical velocity. Though the coupling transport flux of vertical velocity is a small revision compared with the gradient transport flux of vertical temperature, this revision could not be neglected for the heterogeneous underlying surface and convection boundary layer. Until now, the turbulent transport K theory has been employed for more than a century, and Monin-Obukhov's similarity theory has been applied to the ABL for nearly half a century, and they still form the theoretical basis for studying atmospheric turbulent transport. However, the results of theory and experiment of cross coupling between vertical velocity and vertical turbulent transport is a challenge to the traditional turbulent transport K theory and Monin-Obukhov's similarity theory. It is a demanding task to develop a cross coupling theory between vertical velocity and vertical turbulent transport derived from linear thermodynamics, and to carry out an associated experimental study. This cross coupling theory may offer the key to dealing with turbulent transport under the conditions of heterogeneous underlying surface.

The similarity relation [Eq. (20)] is obtained by only using one experiment. This study is preliminary and needs more observed data to validate the conclusions and to eliminate the limitation of localization in the possessing of data. Is the experiential similarity relation [Eq. (20)] of the coupling coefficient of vertical velocity universal? What is the scope of its application? And why does the coupling coefficient of vertical velocity possess the above mentioned properties? These questions need more experimental validation and clarification by theoretical studies.

Acknowledgements. This study has been supported by the National Natural Science Foundation of China under Grant No. 40633014. We thank Prof. Högström for offering accurate and detailed observed data to help the success of this work.

REFERENCES

Businger, J. A., J. C. Wyngaard, Y. Izumi, and E. F. Bradley, 1971: Flux profile relationship in the atmospheric surface layer. *J. Atoms. Sci.*, **28**, 181–189.

Clapp, R. B., and G. M. Homberger, 1978: Empirical equation for some soil hydraulic properties. *Water Resour. Res.*, **14**(4), 501–604.

Dyer, A. J., and E. F. Bradley, 1982: An alternative analysis of flux-gradient relationships at the 1976 ITCE. *Bound.-Layer Meteor.*, **22**, 3–19.

Högström, U., 1988: Non-dimensional wind and temperature profiles in the atmospheric surface: A reevaluation. *Bound.-Layer Meteor.*, **42**, 55–78.

Hu Yinqiao, 2002a: The linear thermodynamics of nonequilibrium state in the atmospheric system. *Atmospheric Thermodynamics and Dynamics*, Geological Publishing House, 168–235. (in Chinese)

Hu Yinqiao, 2002b: Application of the linear thermodynamics to atmosphere system (I), Linear phenomenological relation and thermodynamic property of the atmosphere. *Adv. Atmos. Sci.*, **19**(3), 448–458.

Hu Yinqiao, 2002c: Application of the linear thermodynamics to atmosphere system (II), Exemplification of the leaner phenomenological relation in the atmosphere system. *Adv. Atmos. Sci.*, **19**(5), 767–776.

Hu Yinqiao, 2003: The influence of convergence movement on turbulent transportation in the atmospheric boundary layer. *Adv. Atmos. Sci.*, **20**(5), 794–798.

Hu Yinqiao, 2004: Some aspects of the turbulent transportation in boundary layer along with atmospheric linear thermodynamics. *Plateau Meteor.*, **23**(2), 132–138. (in Chinese)

Kaimal, J., 1968: The effect of vertical line averaging on the spectra of temperature and heat-flux. *Quart. J. Roy. Meteor. Soc.*, **94**, 149–155.

Li Rusheng, 1986: *Nonequilibrium States Thermodynamics and Dissipates Structure*. Qinghua University Press, 407pp. (in Chinese)

Monin, A. S., and A. M. Obukhov, 1954: Basic laws of turbulent mixing in the atmosphere near the ground. *Trudy Geofizicheskogo Instituta, Akademiya Nauk SSSR*, **24**(151), 163–187.

Onsager, L., 1931a: Reciprocal relation in irreversible processes I. *Phys. Rev.*, **37**, 405–426.

Onsager, L., 1931b: Reciprocal relation in irreversible processes II. *Phys. Rev.*, **38**, 2265–2279.

Prandtl, L., 1904: Motion of fluids with very little viscosity. NACA TM, 452, (March 1928).

Prigogine, I., 1945: Thermodynamics of irreversible processes. *Bulletin de la Classe des Sciences Academie Royale de Belgique*, **31**, 600.

Prigogine, I., 1967: *Introduction to Thermodynamics of Irreversible Processes.* 3rd ed., Interscience Pub, New York, 119pp.

Sun Shufen, 1987: Computation of moisture and temperature profiles in the soils-coupled model, 1987. *Acta Mechanica Sinica*, **3**(1), 44–51.

van Dijk, A., A. F. Moene, H. A. R. de Bruin et al, 2004: The principles of surface flux physics: theory, practice and description of the ECPACK library. Internal report 2004/1, 96pp. [Available online from http://www.met.wau.nl/projects/jep].

Webb, E., G. Pearman, and R. Leuning, 1980: Correction of flux measurements for density effects due to heat and water vapour transfer. *Quart. J. Roy. Meteor. Soc.*, **106**, 85–100.

ERRATUM

DOI: 10.1007/s00376-007-0100-3

On Page 909 of Issue No. 6, Vol. 23, the first author of the manuscript "Impact of the Thermal State of the Tropical Western Pacific on Onset Date and Process of the South China Sea Summer Monsoon" should be HUANG Ronghui. We apologize for this mistake.