REMOTE SENSING OF RAINFALL PARAMETERS BY LASER SCINTILLATION CORRELATION METHOD-COMPLETE EQUATION AND NUMERICAL SIMULATION

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ABSTRACT

Remote sensing of rainfall parameters by using laser scintillation effect, originally proposed by Wang et al. is a unique approach for getting rainfall rate and raindrop size distribution with excellent temporal and spatial representativeness. In this paper, we review Wang's work, point out the weakness of their basic equations, then establish a complete remote sensing equation in which the observable quantity, the scintillation of light intensity is used. The relationships between the rainfall parameters and the spatial-temporal correlation function of light scintillation are systematically discussed. Numerical investigations show that this equation gives at least four different ways to obtain the rainfall rate, and the kernel functions for raindrop size distribution are of excellent resolution. At last, the retrieval scheme of the drop size distribution is discussed.

I. INTRODUCTION

The measurements of rainfall rate and raindrop size distribution (DSD) are significant in many researches and operational fields, such as meteorology, hydrology, radio wave propagation, etc. The rainfall rate measured by rain gauges in many weather stations is less representative because of the very small sampling area of rain gauges. The instantaneous rainfall rate is hardly measured. The average rainfall rate can be measured by a meteorological radar, but the accuracy is severely affected by changing DSD.[1] The higher accuracy can be expected by combined use of meteorological radar and radiometer[2], in this case the measured rainfall distribution is still weakly dependent on the variation of DSD. Recently, methods of observations of DSD are not satisfactory either. It is difficult to obtain accurate, representative data in heavy rain by using filter-paper sampling technique which is widely used in China, because the sampling area is too small, the sampling time can not be controlled precisely, and there are superimposing and splashing of raindrops. The sampling area of distrometer is even smaller. As a result, longer sampling time is needed to obtain enough rain drop samples. So it is not suitable to the condition where the rainfall rate and DSD change rapidly. New techniques of measuring rainfall rate and DSD are expected, particularly in cases of mesoscale severe storms.

In 1975, Wang and Clifford[3] proposed a scheme of measuring rain parameters by using light scintillation effect induced by raindrops passing through a horizontal laser beam . Since then , they have made further theoretical analyses and field observations. Both theoretical analyses and primary experiments showed that this would be a unique technique for rainfall and rain DSD measurements with excellent time resolution and representativeness. In addition, as a remote sensing technique, measurements do not interfere the falling raindrops. Recently, Wang et al. have been developing a "laser weather identifier" for unmanned operational use of differentiating rain of different intensities, snow, hail, etc[4-6]. Also, rainfall rates can be measured rather easily. The physics of this technique can be simply described as follows [3,4]. A raindrop intensely scatters visible light in the forward directions. A raindrop falling through a horizontal (laser) beam will result in a transient scattered light pulse (raindrop-induced light scintillation) on the light detector placed in front of the light beam (see Fig.1). This light pulse contains the information of that raindrop. Naturally falling raindrops distributed randomly within the laser beam will cause statistical light scintillations. Furthermore, as the falling velocities of raindrops of different size are different, two light detectors separated vertically at a distance of z_0 will receive different patterns of light scintillations. As these fluctuations contain information of falling raindrops, it is expected to obtain rain parameters from certain statistical characteristics of light fluctuations. Theoretical relationships between the spatial-temporal correlation function of scintillations and rainfall parameters, i. e. the rainfall rate and DSD were established by Wang et al. as the following two equations

$$c_\chi(z_0,\tau)=9.66\times 10^{-23} z_0^{-9} l^{-1} \tau^{-10} \overline{N_v(a')} \tag{1}$$

$$\int_0^\infty c_\chi(z_0,\tau)d\tau=0.82\times 10^{-11} l^{-1} L\bar{h}, \tag{2}$$

where

$$c_\chi(z_0,\tau)=\langle\chi_1(t)\chi_2(t+\tau)\rangle, \qquad \chi=\left|\frac{\vec{E}_0+\vec{E}_s}{\vec{E}_0}\right|-1,$$

$$\overline{N_v(a')}=\frac{1}{L}\int_0^L N_v(a',x)\,dx, \qquad \bar{h}=\frac{1}{L}\int_0^L h(x)\,dx,$$

$$a'=2.5\times 10^{-5} z_0^2 \tau^{-2},$$

where $\vec{E}_0$ and $\vec{E}_s$ are the incident and scattered light amplitudes respectively. It is reasonable to assume $|\vec{E}_s/\vec{E}_0| \ll 1$. z_0, l and L are shown in Fig. 1. x_1 and x_2 are the falling raindrop-induced amplitude scintillations at the upper and lower light detectors respectively. $N_v(a,x)$ is DSD at x, and $h(x)$ is the rainfall rate at x. In deriving Eqs. (1) and (2), single scattering and independent scattering of raindrops are also reasonably assumed.

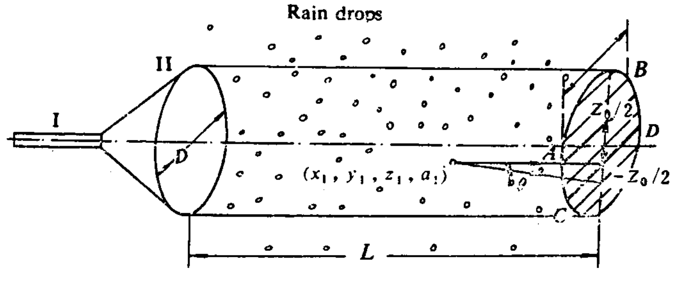

Fig. 1. The geometrical configuration of the experiment of remote sensing rainfall parameters by laser scintillation. I—a laser; II —the aperture of the transmitter. $\overline{AB}=\overline{CO}=l$.

By applying these Eqs. to field experiments and numerical simulations, although positive results have been obtained semiquantitatively, it has been found by authors themselves that for obtaining quantitative results there still exist some problems to be solved, such as "calibration", "baseline correction", etc. From analyses of the basic equations and the deriving procedures, we found that some weaknesses exist in Eqs. (1) and (2), which influence quantitative use of this technique. They are:

(1) In Eqs. (1) and (2), the scattered light amplitude is used as a measurable quantity, but in fact only the scattered light intensity F can be received by photo-electrical detectors used in experiments. Therefore, the deduced statistical function should be the spatial-temporal correlation function of light intensity scintillation $B_F(z_0,\tau)$ instead of $c_x(z_0,\tau)$. The relationship between these two kinds of spatial-temporal correlation functions (for light amplitude and intensity) depends on the probability density distribution function of scattered light amplitude which is not known very well and possibly varies with different kinds of rainfall. Thus it seems difficult to get an accurate and stable proportional coefficient relating these two functions. It results in the problem of "calibration" of the quantitative use of Eqs. (1) and (2). It is better to build up an equation that relates the measurable function $B_F(z_0,\tau)$ with rain parameters directly.

(2) In order to make use of the method of stationary phase in deriving Eqs. (1) and (2) several restrictions are imposed on the environment and raindrops (e.g. $l \gg (\lambda L)^{\frac{1}{2}}$, $a \ll (\lambda L)^{\frac{1}{2}}$, $\tau \gg (2L/k)^{\frac{1}{2}} \sigma_v^{-1}$, $\tau \gg 0.01\ (L/K)^{\frac{1}{2}}$, etc.[4]) Some of those are not necessary to establish the new remote sensing equation.

(3) The $N_v(a')$ in Eq. (1) is a weighting-average value of path-averaged DSD at a', but not the real value of DSD at a'. Weighting functions have not been discussed in detail and are varied for different a'.

(4) In Eqs. (1) and (2), $C_x(z_0,\tau)$ do not contain the component of background correlation which appears in numerical and field experiments and is called "baseline" by Wang. As the value of "baseline" is usually much larger than $C_x(z_0,\tau)$, it is important to determine and physically interpret the "baseline".

The objective of this paper is (a) to re-establish a remote sensing equation by using the spatial-temporal correlation function of scattered light intensity $B_F(z_0,\tau)$ and interpret it physically; (b) to investigate the characteristics of the kernel functions for DSD by numerical simulation; (c) to get several quantitative relationships for rainfall rate measurements; and finally, (d) to discuss the retrieval method of DSD.

II. REMOTE SENSING EQUATION

According to Van de Hulst[7], when $\eta = ka \gg 1$, the amplitude of the scattered light is

$$S(\theta) = \eta^2 J_1(\eta\theta)/\eta\theta, \tag{3}$$

where η is a size parameter, a is the radius of a particle, λ is the wavelength which is 6328A for He−Ne laser beam, $k = 2\pi/\lambda$ is the wave number, θ is the scattering angle. The scattered light intensity can be found from $S(\theta)$

$$I_{sca} = I_0 |S(\theta)|^2/k^2 r^2 = I_0 a^2 J_1^2(\eta\theta)/\theta^2 r^2, \tag{4}$$

where I_0 is the incident light intensity, r is the distance from the raindrop to the light detector (as shown in Fig.1). J_1 is the Bessel function of first kind of order one. For a raindrop of radius a at (x, y, z) and two light detectors (the length of each detector is l) at $x_1' = x_2' = L$, $z_1' = z_0/2$, $z_2' = -z_0/2$ respectively, the light intensities received by the upper and lower detectors with the length of l are the integrations of I_{sca} over l

$$f_{1,2}(x,y,z,a)=\int_{-l/2}^{l/2} I(x,y,z,a;\ L,y'\pm z_0/2)\,dy', \tag{5}$$

where $f_{1,2}$ represent the received line-integrated light intensity by the upper and lower detectors respectively. r and θ in (4) can be easily determined by geometrical relations of $(x,y,z,)$ and $(L,y',\pm z_0/2)$.

Assuming stationary rain falling through the laser beam, at time t, the raindrop of radius a_i within the laser beam is located at (x_i, y_i, z_i) $i=1,2,\cdots,N$. N is the total number of raindrops within the laser beam at time t. At this time, the line-integrated light intensity received by the upper detector is

$$F_1(t)=\sum_{i=1}^{N} f_1(x_i,\ y_i,\ z_i;\ a_i). \tag{6}$$

After a time interval of τ, each raindrop passes a distance of $v(a_i)\tau$, where $v(a_i)$ is the terminal velocity of a raindrop with radius of a_i. At time $t+\tau$, the line-integrated light intensity received by the lower detector is

$$F_2(t+\tau)=\sum_{i=1}^{N} f_2(x_i,y_i,z_i-v\,(a_i)\tau:a_i)\ . \tag{7}$$

Thus the spatial-temporal correlation function of the received line-integrated light intensity is

$$B_F(z_0,t,\tau)=\langle F_1(t)F_2(t+\tau)\rangle, \tag{8}$$

where $\langle\ \ \rangle$ represents the ensemble average. When rain is stationary, that is to say the rainfall rate and DSD are not changed (it is easily satisfied when we take just a short period of data flow), $B_F(z_0,t,\tau)$ will be independent of t. Substituting (6) and (7) into (8) and rearranging it, we have

$$B_F(z_0,\tau)=B_1(z_0,\tau)+B_2(z_0,\ \tau), \tag{9}$$

where

$$B_1(z_0,\tau)=\langle\sum_{i=1}^{N}\{f_1(x_i,y_i,z_i;a_i)f_2(x_i,y_i,z_i-v\,(a_i)\tau;a_i)\}\rangle, \tag{10}$$

$$B_2(z_0,\tau)=\langle\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N}\{f_1(x_i,y_i,z_i;a_i)f_2(x_j,y_j,z_j-v\,(a_j)\tau;a_j)\}\rangle. \tag{11}$$

It can be seen that B_1 is the sum of the contribution of scattering to the upper and lower detectors at a time interval τ by the same individual falling raindrop. It can be recognized as the coherent component. In contrast, B_2 is the sum of the contribution of scattering to the upper and lower detectors at a time interval τ by two different falling raindrops. So it can be recognized as the incoherent component. We will show that B_1 contains the information of raindrop parameters but we will analyse B_2 first.

For natural rain we can assume that raindrops fall randomly and independently. Correspondingly, the scattered light, f_1 $(x_i,y_i,z_i;a_i)$ and f_2 $[x_j,y_j,z_j-v\,(a_j)\tau;a_j]$, induced by different raindrops are independent each other. For stationary rain, the ensemble average of (11) can be performed by laser beam space-(drop size) spectrum average. Furthermore, as B_2 is independent of τ for independent scattering the correlation can be performed by the space-spectrum average of f_1 and f_2 respectively. Let N_0 be the number density of the raindrops, $n_V(a)$ be the normalized DSD. Then DSD

$N_V(a)=N_0 n_v(a)$. The total number of raindrops within laser beam is $\bar{N}=\pi D^2 L N_0/4$. By using above relations, B_2 can be expressed in terms of the averaged scattered intensity of one raindrop:

$$B_2(z_0,\tau)=B_2(z_0)=N(N-1)f_1f_2, \tag{12}$$

$$\langle f_{1,2}\rangle=\frac{1}{\frac{1}{4}\pi D^2 L}\int_0^\infty n_v(a)da\int_0^L dx\int_{-D/2}^{D/2}dy\int_{-\sqrt{D^2/4-y^2}}^{\sqrt{D^2/4-y^2}} dz f_{1,2}(x,y,z;a), \tag{13}$$

where the subscripts 1 and 2 represent the upper and lower detectors respectively. As being seen from (12) and (13), B_2 is a DC component of $B_F(z_0,\tau)$ relating to the total scattering of falling raindrops. Although it is difficult to get information of DSD from B_2, it is hopeful to get the information about the rainfall rate and the liquid water content of raindrops. Numerical simulation of this will be shown later.

Similar to the discussion of B_2, for stationary and randomly-distributed rain, we can also use the space-spectrum average to perform the ensemble average of $B_1(z_0,\tau)$ in Eq. (10). The final equation for $B_1(z_0,\tau)$ can be expressed as

$$B_1(z_0,\tau)=\int_0^\infty N_v(a)\,W(z_0,\tau,a)\,I_0^2\,da \tag{14}$$

where
$$W(z_0,\tau,a)=\frac{1}{I_0^2}\int_0^L dx\int_{-D/2}^{D/2}dy\int_{-\sqrt{D^2/4-y^2}}^{\sqrt{D^2/4-y^2}} dz\, f_1(x,y,z;a)f_2(x,y,z-v(a)\tau;a). \tag{15}$$

It is obvious that Eq. (14) is a typical remote sensing equation of DSD. $W(z_0,\tau,a)$ is the kernel function which in principle determines the quality of remote sensed DSD by this technique and will be numerically discussed later.

Up to now, we have established the complete equations [Eqs. (9)—(15)] of remote sensing of rain parameters by the laser scintillation effect. $B_F(z_0,\tau)$ is a measurable quantity, $B_2(z_0)$ contains the information of rainfall rate and/or rain liquid water content, $B_1(z_0,\tau)$ contains the information of DSD and also the rainfall rate etc. For the application of these equations, the last theoretical step is to separate B_2 and B_1. Several different approaches can be used to solve this problem. The theoretical basis of these approaches is that $B_2(z_0,\tau)$ will be independent of τ, because it represents the correlation of scatterings of different falling raindrops. Since values of $B_1(z_0,\tau)$ fall rapidly from its peak value $B_1(z_0,\tau_p)$ as $|\tau-\tau_p|$ increases, value of $B_1(z_0,\tau)$, when τ is far from τ_p, is negligibly small. For $\tau<0$, physically, no peak of $B_1(z_0,\tau)$ can exist, thus for any $\tau>0$,

$$B_F(z_0,-\tau)=B_1(z_0,-\tau)+B_2(z_0,-\tau)\doteq B_2(z_0,-\tau)=B_2(z_0)\,. \tag{16}$$

Therefore one approach of separating B_1 and B_2 is

$$B_1(z_0,\tau)\doteq B_F(z_0,\tau)-B_F(z_0,-\tau) \qquad (\tau>0). \tag{17}$$

Wang et al. have applied this method to reduce the so-called "baseline"component in their numerical simulation but did not give the physical interpretation as above. A more accurate approach is to take the average of $B_F(z_0,\tau)$ in (τ_1,τ_2)

$$B_2(z_0,\tau)=B_2(z_0)=\frac{1}{\tau_2-\tau_1}\int_{\tau_1}^{\tau_2}B_F(z_0,\tau)\,d\tau, \tag{18}$$

where $\tau_2>\tau_1\geq\tau_M$, τ_M is the time interval in which the minimal raindrop falls vertically through the laser beam, and equal to $D/v\ (a_m)$. Thus

$$B_1(z_0,\tau)=B_F(z_0,\tau)-\frac{1}{\tau_2-\tau_1}\int_{\tau_1}^{\tau_2}B_F(z_0,\tau)\,d\tau. \tag{19}$$

Now we can practically use above equations to actual field experiments or numerical simulation. When all experimental parameters D, L, z_0, and l are determined, the kernel functions $W(z_0,\tau,a)$ can be obtained accurately by computations. Then we can investigate the kernel functions and find an appropriate retrieval method for remote sensing of DSD.

III. NUMERICAL INVESTIGATION

As $J_1^2(x)$ is highly vibrant when $x\gg 1$, we use a Gaussian function to approximate $J_1^2(\eta\,\theta)$ for narrow forward scattering angles

$$I_{sca}/I_0=a^2J_1^2(\eta\,\theta)/r^2\theta^2\doteq\frac{0.922\times0.27a^2\eta^2}{r^2}\exp(-\alpha_p\theta^2), \tag{20}$$

where $\alpha_p=0.27\eta^2$. Comparisons of $J_1^2(\eta\,\theta)$ with approximate Gaussian function are listed in Fig. 2. It is seen that in very narrow forward angles, the relative errors are less than 2%. For relatively wider angles, the relative error increases, but the absolute value decreases rapidly to negligibly small. Substituting Eq. (20) into Eq. (5) and by using the integral formulae

$$\int_{-A}^{A}\exp[-b(z-c)^2]dz=\frac{1}{2}\sqrt{\frac{\pi}{b}}\{\mathrm{erf}[\sqrt{b}(A-C)]+\mathrm{erf}[\sqrt{b}(A+C)]\}, \tag{21}$$

we can calculate $f_1(x,y,z,a)f_2[x,y,z-v(a)\tau;a]/I_0^2$ and finally complete the integrations of $B_1(z_0,\tau)$ and $B_2(z_0)$. In terms of the normalized quantities, we have

$$B_1^*(z_0,\tau)=\frac{B_1(z_0,\tau)}{2I_0^2}=0.1087L\int_0^\infty N_v(a)a^4da\int_0^1dx^*\int_0^{D/l}dy^*Q\exp\{-0.135Q^2(\frac{z_0}{l}-\frac{v\tau}{l})^2\}$$
$$\times\{\mathrm{erf}[0.2598Q(1-y^*)]+\mathrm{erf}[0.2598Q(1+y^*)]\}^2$$
$$\times\{\mathrm{erf}[0.3674Q(\sqrt{1+\frac{z_0^2}{l^2}-y^{*2}}-\frac{v\tau}{l})]+\mathrm{erf}[0.3674Q(\sqrt{1+\frac{z_0^2}{l^2}-y^{*2}}+\frac{v\tau}{l})]\}, \tag{22}$$

$$B_2^*(z_0)=\frac{B_2(z_0)}{2I_0^2}=\frac{0.922^2N(N-1)l^2}{2D^4}[\int_0^\infty n_v(a)a^2da\int_0^1dx^*\int_0^{D/l}dy^*I(x^*,y^*,z_0,a)]^2$$
$$\simeq\frac{0.922^2\pi^2L^2l^2}{2\times4^2}[\int_0^\infty da\,a^2N_v(a)\int_0^1dx^*\int_0^{D/l}dy^*\,I(x^*,y^*,z_0,a)]^2, \tag{23}$$

$$I(x^*,y^*,z_0,a)=\{\mathrm{erf}[0.2598Q(\sqrt{1+\frac{z_0^2}{l^2}-y^{*2}}-\frac{z_0}{l})]+\mathrm{erf}[0.2598Q(\sqrt{1+\frac{z_0^2}{l^2}-y^{*2}}+\frac{z_0}{l})]\}$$

$$\times\{\mathrm{erf}[0.2598Q(1-y^*)]+\mathrm{erf}[0.2598Q(1+y^*)]\}, \tag{24}$$

where $x^*=x/L,\quad y^*=2y/l,\quad z^*=2z/l,\quad Q=\frac{ka}{(1-x^*)}\cdot\frac{l}{L}.$

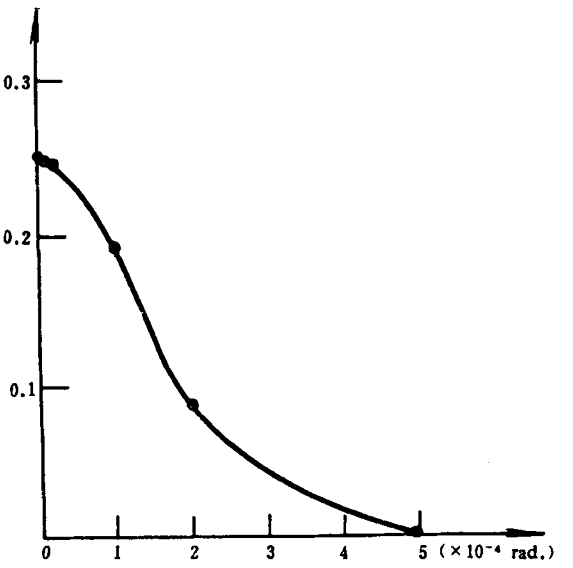

Fig. 2. The comparison between the Gaussian function and $J_1^2(\eta\theta)/\eta^2\theta^2$.
0.249 exp $(-0.27\,\eta^2\theta^2)$; —— $J_1^2(\eta\theta)/(\eta^2\theta^2)$.

The kernel function $W(z_0, \tau, a)$ can be obtained by performing the integrations with respect to x^* and y^* in Eq. (22).

To investigate some numerical relationships between $B_1^*(z_0, \tau)$, $B_2^*(z_0)$ and h, $N_v(a)$, the Marshall-Palmer's raindrop size distribution is used in the calculations

$$N_v(a)=N_p(a,h)=N_0\exp(-8200\,h^{-0.21}a), \tag{25}$$

where $N_0=16\times10^6m^{-4}$, a is the raindrop radius (m), h is the rainfall intensity (mm/hr). For a series of values of h, we can obtain a series of $B_1^*(z_0,\tau)$ and $B_2^*(z_0)$ which are shown in Fig.3 and 7 respectively. As N_p (a) changes several orders of magnitude with a, it is more useful to calculate $N_p(a,h)\,W(z_0,\tau,a)$. $N_p(a, h)\,W(z_0, \tau, a)\,da$ represents the partial contribution to $B_1^*(z_0,\tau)$ by raindrops with radius a and radius interval da. Similar to Wang's integration $\int_0^\infty C_X(z_0,\tau)\,d\tau$, we also calculate the integration $\int_0^\infty B_1^*(z_0,\tau)\,d\tau$. In calculations, following parameters are chosen $l/L=0.004$, $z_0/l=0.1$, $l=0.2$ m, $D/l=1$, $k=9.929\times10^6m^{-1}$.

IV. RESULTS AND DISCUSSIONS

Primary numerical investigations show that there are at least four ways to measure rainfall rate by this technique, i. e:

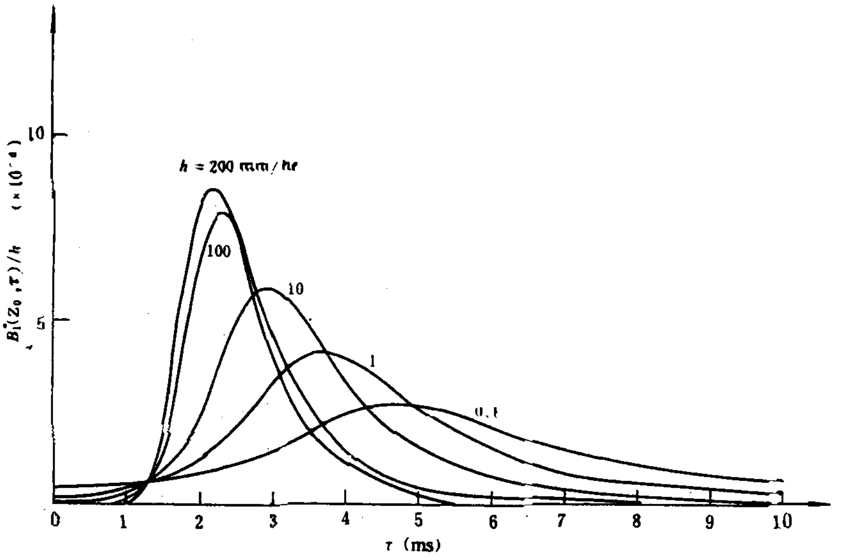

Fig. 3. The coherent components for various rainfall rates.

(1) There exists a very good correlation between rainfall rate h and time τ_M when $B_1^*(z_0, \tau_M)$ reaches its maximum (as shown in Fig.3). The linear regression shows

$$\log h(\text{mm/hr}) = 4.7644 - 7.8466 \log \tau_M \text{ (ms)}.$$

The correlation coefficient is -0.9814. Fig. 4 shows this result.

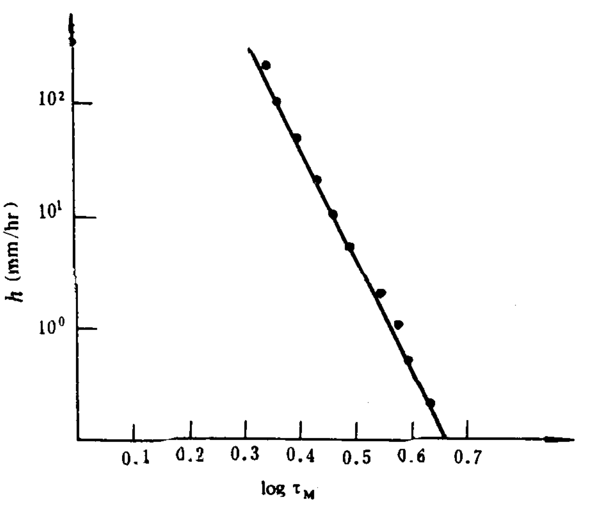

Fig. 4. The relation between the rainfall rate h and the peak time τ_M. The line is an approximating line.

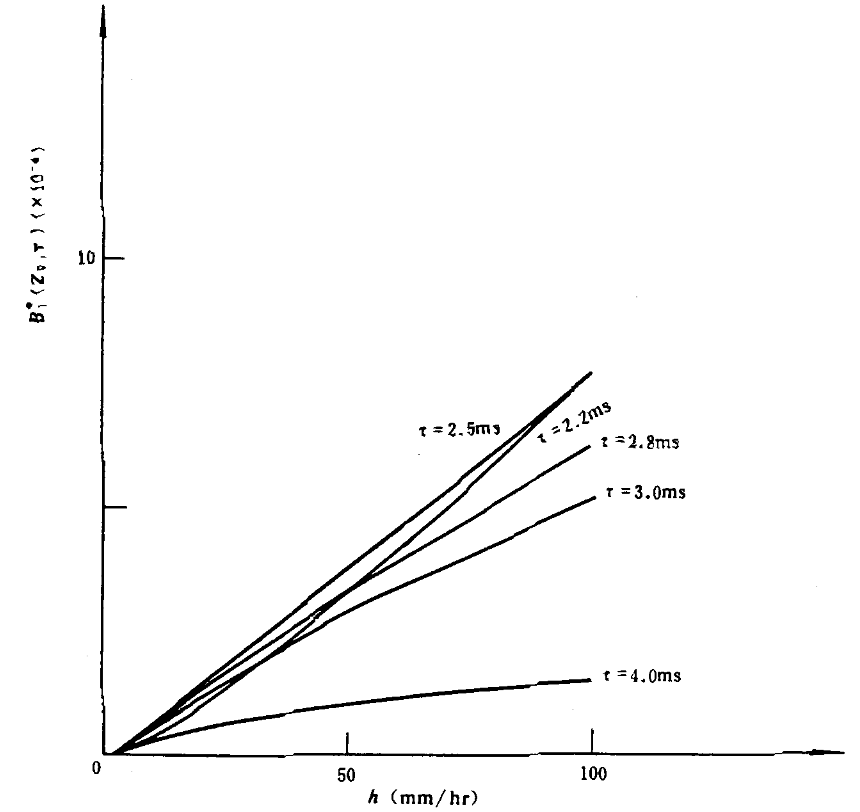

Fig. 5. The coherent component at a certain τ_i as a function of h.

(2) For certain time interval τ_i, there exists a good correlation between h and $B_1^*(z_0, \tau_i)$.Fig. 5 shows these numerical relations for different τ_i.
(3) Similar to Wang's result, the integration of $B_1^*(z_0, \tau)$ with respect to τ is linearly related to h. Fig. 6 shows the computation results which are normalized to the integration for $h=200$ mm/hr. Wang's results (numerical calculation repeated by present authors) are also shown there.
(4) We find that $B_2^*(z_0)$ is related to h with very high correlation. The linear regression gives

$$\log B_2^* (z_0) = 1.2865 \log h - 6.2289.$$

The correlation coefficient is as high as 0.999986. (see Fig. 7)

At a first sight, these four relations are all good for rainfall intensity measurements. Further analysis shows that the last relation (B_2-h relation) would be the best. The main reasons are 1) the coefficient of B_2-h relation is the highest; 2) As $B_F = B_1 + B_2$, the absolute errors of B_1 and B_2 induced by the separation are the same, but $B_2(z_0) \gg B_1(z_0, \tau)$ for any τ, so that the relative error of $B_1(z_0, \tau)$ is much higher than that of B_2.

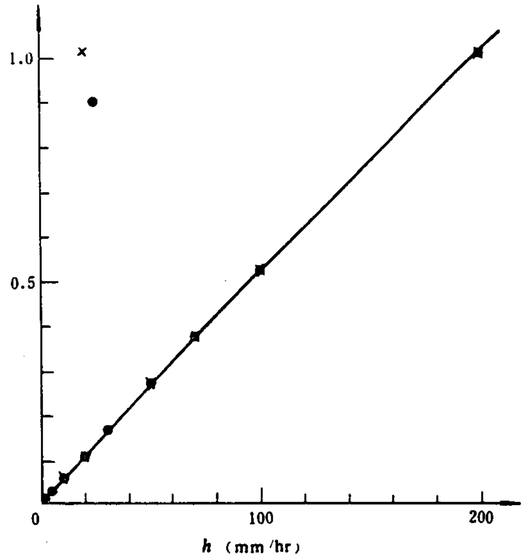

Fig. 6. The comparison between the integrals for intensity and amplitude.

$\times - \int_0^\infty C_\chi(z_0,\tau)\, d\tau / [\int_0^\infty C_\chi(z_0,\tau)\, d\tau]_{h=200}$;

$\cdot - \int_0^{\tau^*} B_1/I_0^2\, d\tau / [\int_0^{\tau^*} B_1/I_0^2\, d\tau]_{h=200}$.

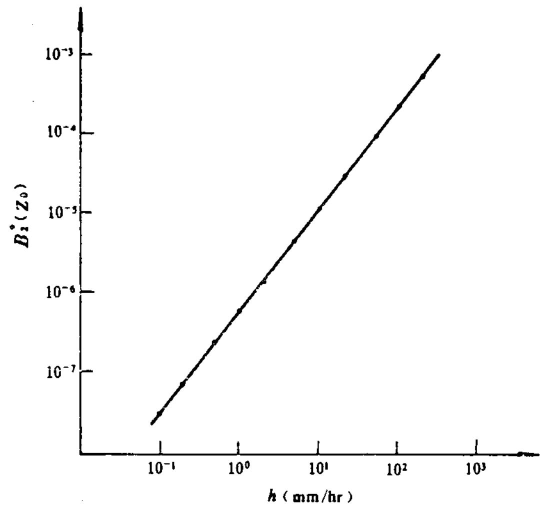

Fig. 7. The incoherent components for various rainfall rates.

As for the remote sensing of DSD, $W(z_0, \tau, a)$ is calculated. In Fig. 8, $\tilde{W}=1/2\ N_p\ (a,h)\ W\ (z_0, \tau, a)$ is shown for the M-P DSD of different rainfall rates h. It is shown from Fig. 8 that the kernel functions are well peaked and separated in these lagged-time between $\tau=1$—10 ms. Therefore, if h is obtained by one of above relations, we can choose the optimum τ_i of $W(z_0, \tau, a)$ for retrieving DSD. Similar to the remote sensing of aerosol size distribution[8], we can apply the similar retrieving method by using a rapidly-varying function $N^*(a, h)$ and a slowly-varying function $f(a)$. Let us assume that $N_V(a) = N^*(a,h)f(a)$. When the first guess of $N^*(a,h)$ is made, $N^*(a,h)\ W(z_0, \tau, a)$ is the kernel function for retrieving $f(a)$. An iterative procedure for retrieving rain DSD can be the same as that for retrieving aerosol size distribution. The numerical simulation of remote sensing of DSD is being made.

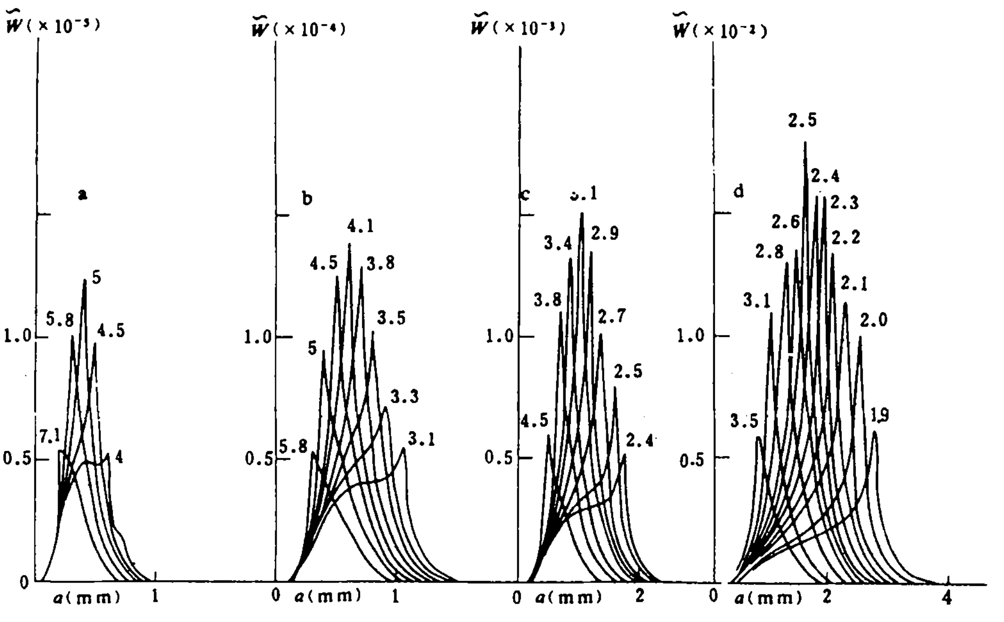

Fig. 8. Kernel functions. The number with each curve is the time interval τ.

V. SUMMARY

In this paper we have established the complete remote sensing equation by using the practically measurable quantity, scattering light intensity, analysed the incoherent component $B_2(z_0)$ and coherent component $B_1(z_0, \tau)$ of the spatial-temporal correlation function of scattering light intensity $B_F(z_0, \tau)$, found the way to separate them. Numerical simulation shows there are at least four ways to obtain the rainfall rate. Among them, $B_2(z_0)$ -h relation might be the best one. The kernel functions for remote sensing of rain DSD are very favorable to retrieving DSD by using the method that has been used in the remote sensing of aerosol size distribution. At present, we are preparing the field observation.

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