

## SIMPLIFIED DYNAMICAL ANOMALY MODEL FOR LONG-RANGE NUMERICAL FORECASTS

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### ABSTRACT

In this atmosphere-ocean-land coupled model, two basic ideas are developed. One is that the observational climate field is utilized and only the anomalous components are predicted. The other is that the transient Rossby wave is treated as the meteorological "noise" on the long-term variation that must be predicted in a climate forecasting. According to the latter, the transient Rossby wave can be filtered by omitting the partial derivative with respect to time in the atmospheric vorticity equation. But the time derivative term is still kept in the thermal equation for underlying ocean and land. With this assumption, the vorticity equation becomes time-independent, i.e. it is only a balance relationship between the anomaly geopotential height field and earth's surface heating field. This model is different from the usual GCM, so it may be called as the filtered anomaly model (FAM).

A dozen examples of one month prediction are summarized in this report.

### I. INTRODUCTION

It is well known that in the present stage the usual GCM includes many high-frequency dynamic events that are intrinsically unpredictable on longer time scales than about two weeks. It is conceivable that one of the possible numerical models for monthly and seasonal forecasts would explicitly filter out those high-frequency noise. Considering that the characteristic period of transient Rossby wave is about one week, it will certainly have more important effects on short-term numerical forecasting, however, this is in doubt whether it is still one of the important factors for monthly or seasonal climate forecasting. For instance, some successful examples of monthly and seasonal forecasting have been obtained by the synoptic-statistical method in which the running mean on the meteorological data is taken into account. It could easily be comprehended that, if the time scale of running mean is longer than the characteristic period of transient Rossby wave, say 10 days, the latter will automatically be filtered to some extent.

According to the idea mentioned above, the authors suppose audaciously that, compared to the monthly and seasonal variation, the transient Rossby wave in a dynamical model can be considered as the high-frequency meteorological noise and can be filtered out. We noted that Monin (1972), Opsteegh and Van Den Dool (1980) also described a similar idea for making long-range forecasting<sup>[1,2]</sup>. In fact the practical FAM had been developed (Group of LRF, 1977, 1979; Chao et al 1982)<sup>[3-5]</sup> and more than a dozen examples of one month prediction were given (Miyakoda and Chao 1982)<sup>[6]</sup>, particularly, the spectacular month of January 1977 was predicted in GFDL/NOAA (Chao and Caverly 1981)<sup>[7]</sup>. All these examples of one month prediction are obtained by one level model for the atmosphere, and the time-step of prediction just takes one month.

However, the skill of prediction of those examples reported in Miyakoda and Chao's paper

is lower than that of the persistence except for a few examples<sup>[6]</sup>. Recently, the predicting method of this one level model was improved and the skill of prediction clearly showed its advantages. In the following sections the new predicting results will be introduced.

## II. MODEL

One of the essential ideas is that, since the climatology is already known, why do people introduce errors in the forecast trying to predict it? On the anomaly model system, the climatological component can be removed from the total field equations by dividing all the system variables into their climatic and anomalous components and the observational climate values are utilized. Formally, this results in the separation of the total field equation into dependent climatic and anomalous system. This procedure is the same one used by Reynolds to remove the turbulent component from the time mean flow. Unlike Reynolds, however, we are only interested in the time evolution of the anomaly rather than the mean or climatological flow which is supposed well known. Since the climatic components are assumed time independent for short period, say 1 to 3 months, this equation system can be ignored leaving only the time-dependent anomalous system. Usually, the equation for the anomaly includes the Reynolds term, but these terms are not considered in the present model, and besides the current model uses the geostrophic approximations.

In  $x, y, p, t$  coordinates, the vorticity equation and the first law of thermodynamics become respectively

$$\frac{d}{dt}(\zeta + f) = f \frac{\partial \omega}{\partial p}, \quad (1)$$

$$\frac{dT}{dt} - \sigma_p \omega = \frac{\partial}{\partial p} (k_p + k_R) \frac{\partial T}{\partial p} - \frac{1}{\tau_R} (T - T_e) + \tilde{T}^* \zeta + \frac{k'}{\rho c_p} s, \quad (2)$$

where  $s$  is the solar radiation,  $k'$  is the absorption coefficient of the short wave radiation; and the term with parameter  $k_p$  denotes the sensible heat exchange; the condensational heat exchange is proportional to the vorticity, and  $\tilde{T}^*$  is a parameter with temperature dimension; the terms dealing with parameters  $k_R$  and  $\tau_R$  come from the radiative heat exchange according to Kuo's scheme (1968) and  $T_e$  is the temperature of the environment<sup>[8]</sup>.

It is assumed that the climatic monthly mean process satisfies the following equations:

$$\left( \frac{\partial}{\partial t} + \bar{u} \frac{\partial}{\partial x} + \bar{v} \frac{\partial}{\partial y} \right) (\zeta + f) = f \frac{\partial \bar{\omega}}{\partial p}, \quad (3)$$

$$\left( \frac{\partial}{\partial t} + \bar{u} \frac{\partial}{\partial x} + \bar{v} \frac{\partial}{\partial y} \right) \bar{T} - \sigma_p \bar{\omega} = \frac{\partial}{\partial p} (k_p + k_R) \frac{\partial \bar{T}}{\partial p} + \tilde{T}^* \bar{\zeta} + \frac{k'}{\rho c_p} s. \quad (4)$$

Here we assume the reference temperature  $T_e$  to be equal to the climatic monthly mean air temperature  $\bar{T}$ .

From these equations mentioned above, the anomalous vorticity equation and the first law of thermodynamics become respectively

$$\frac{d\Delta\phi'}{dt} + \bar{\beta}_y \frac{\partial \phi'}{\partial x} - \bar{\beta}_x \frac{\partial \phi'}{\partial y} = f^2 \frac{\partial \omega'}{\partial p}, \quad (5)$$

$$\left(\frac{d}{dt} - \frac{\partial}{\partial p}(k_p + k_R)\frac{\partial}{\partial p}\right)\frac{\partial \phi'}{\partial p} - \left(\frac{\partial \bar{u}}{\partial p}\frac{\partial \phi'}{\partial x} + \frac{\partial \bar{v}}{\partial p}\frac{\partial \phi'}{\partial y}\right) + \frac{1}{\tau_R}\frac{\partial \phi'}{\partial p} = -\bar{\sigma}_p \omega' - \frac{R\bar{T}^*}{pf}(\Delta \phi'). \quad (6)$$

Eliminating  $\omega'$  from Eqs. (5) and (6), we obtain the non-adiabatic vorticity equation that is one of the major equations in this model, as follows

$$\begin{aligned} \frac{d}{dt}\Delta \phi' + \bar{\beta}_y \frac{\partial \phi'}{\partial x} - \bar{\beta}_x \frac{\partial \phi'}{\partial y} = & -\frac{f^2}{\tau_R \bar{\sigma}_p} \frac{\partial^2 \phi'}{\partial p^2} + \frac{f^2}{\bar{\sigma}_p} \frac{\partial^2}{\partial p^2} (k_p + k_R) \frac{\partial^2 \phi'}{\partial p^2} \\ & - \frac{f^2}{\bar{\sigma}_p} \frac{\partial}{\partial p} \left[ \frac{d}{dt} \left( \frac{\partial \phi'}{\partial p} \right) - \left( \frac{\partial \bar{u}}{\partial p} \frac{\partial \phi'}{\partial x} + \frac{\partial \bar{v}}{\partial p} \frac{\partial \phi'}{\partial y} \right) \right] - fR \frac{\partial}{\partial p} \left( \frac{\bar{T}^*}{p \bar{\sigma}_p} \right) (\Delta \phi'), \end{aligned} \quad (7)$$

where the bar and prime quantities refer to the climatological and anomalous components respectively, in which the geostrophic approximation

$$u' = -\frac{1}{f} \frac{\partial \phi'}{\partial y}, \quad v' = \frac{1}{f} \frac{\partial \phi'}{\partial x}, \quad \zeta' = \frac{\partial v'}{\partial x} - \frac{\partial u'}{\partial y} = \frac{1}{f} \Delta \phi',$$

and the hydrostatic relation

$$T' = -\frac{p}{R} \frac{\partial \phi'}{\partial p}$$

are used. The operator  $\frac{d}{dt}$  and other symbols are

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + (\bar{u} + u') \frac{\partial}{\partial x} + (\bar{v} + v') \frac{\partial}{\partial y}, \quad (8)$$

and

$$\bar{\beta}_x = \frac{\partial(\bar{\zeta} + f)}{\partial x}, \quad \bar{\beta}_y = \frac{\partial(\bar{\zeta} + f)}{\partial y},$$

$$\bar{T}^* = \frac{L}{c_p} \gamma \frac{d \ln \bar{e}_s}{d \bar{T}} l_b \bar{q}_s.$$

The other symbols are:  $\phi'$ , the geopotential height anomaly;  $\bar{\sigma}_p$ , the static stability;  $\bar{e}_s$ , saturation vapour pressure;  $\bar{q}_s$ , saturated specific humidity;  $l_b$ , an empirical constant with the dimension of length;  $R$ , the gas constant;  $C_p$ , specific heat;  $L$ , latent heat and  $f$ , the Coriolis parameter.

The other prognostic equation in our system is the thermal equation for the underlying ocean and land

$$\frac{\partial^2 T'_s}{\partial z^2} - \frac{1}{K_s} \frac{\partial T'_s}{\partial t} = \frac{\delta}{K_s} \left[ \frac{\partial(\bar{\psi}_s, T'_s)}{\partial(x,y)} + \frac{\partial(\psi'_s, \bar{T}_s + T'_s)}{\partial(x,y)} \right] \equiv H_1, \quad (9)$$

where  $T'_s$  is the anomalous temperature of ocean or land, and  $\delta = 1$  for ocean,  $\delta = 0$  for land.  $\bar{\psi}_s$ , the climatological ocean stream function, can be calculated from the climatological monthly mean ocean current, and the anomalous part  $\psi'_s$  can be calculated using the Ekman wind-driven theory.

At the surface, the following heat balance is used as one of the boundary conditions for Eq. (9)

$$z=0, \quad \rho_s c_p K_s \frac{\partial T'_s}{\partial z} - \rho c_p K_T \frac{\partial T'}{\partial z} + \delta G_1 T'_s = -G_2 \zeta', \quad (10)$$

where  $K_T = k_p / \rho^2 g^2$ ,  $G_1$  and  $G_2$  parameterizes the evaporation and effect of cloudiness upon the radiation balance, respectively, and the explicit expressions of them are

$$G_1 = L \rho K_T \gamma \frac{\partial \ln \bar{e}_s}{\partial T} \frac{\partial \bar{q}_s}{\partial T}, \quad G_2 = \frac{R_0}{W_0} l_b,$$

where  $\gamma = -\frac{\partial \bar{T}}{\partial z}$ ,  $W_0$  is an empirical parameter and

$$R_0 = \overline{(S_0 + s_0)}(1 - a)C_s - \bar{I}C_i,$$

in which  $S_0$  and  $s_0$  are total direct and diffused radiations respectively;  $\bar{I}$  is the effective radiation;  $a$  is the earth's albedo;  $c_p$ ,  $c_i$  are the coefficients showing the effects of absorption medium on the radiation. It states that the anomalous cloudiness is proportional to the vertical velocity on the top of boundary layer, furthermore, according to the theory of boundary layer the latter is proportional to the geostrophic vorticity on the earth's surface.

Another boundary condition for Eq. (9) is

$$\begin{aligned} z = -\infty \quad (\text{or } z = -H, \text{ the depth of mixed layer}), \\ T'_s = 0. \end{aligned} \quad (11)$$

The second important idea of this model is as follows. It can be shown that there are two basic types of dynamical processes corresponding to different time scales by analysing the linear case of this atmosphere-ocean system, the fast one with a period of the order of one week, in essence, is the transient Rossby wave, the other is slow with a period of the order of several months. It is easily understood that the slow one is produced by the heating of ocean, or generally speaking, by the interaction of the atmosphere and ocean (Chao et al. 1982; Group of LRF 1977)<sup>15,31</sup>. Owing to the fact that the growth rate of the shorter time-scale waves is about one order of magnitude larger than that of the long time-scale waves, probably, this is one of the difficulties for the long-range numerical forecasts, because the evolution of the long-range process of smaller amplitude will be distorted by the short-range process with large amplitude. One way to overcome this difficulty is to filter out these high-frequency dynamic events from the numerical model of long-range or short-term climate forecasting. This is just the method that we have been using. A simple method of filtering is that we may omit the term of local time variation in vorticity equation for the atmosphere, i.e. the operator (8) becomes

$$\frac{D}{Dt} = (\bar{u} + u') \frac{\partial}{\partial x} + (\bar{v} + v') \frac{\partial}{\partial y}. \quad (12)$$

It is equivalent to replacing Eq. (7) by the relation between the field of atmospheric flow and that of the heating source. The equilibrium relation may be called the adaptation equation. Physically, it means that after the dispersing by transient Rossby wave, an adjustment relationship between the geopotential height field and the heating field can be established. It should be noted that although the

vorticity equation for the atmosphere becomes stationary, the total atmosphere-ocean-land coupled system still varies with time, because the coupled thermal equation for underlying is time-dependent.

Based on these ideas mentioned above, a predicting method of monthly mean anomalous state of earth's surface temperature as well as the geopotential height of the atmosphere is developed. At first, we may predict the earth's surface temperature field by Eq. (9) with the boundary conditions (10) and (11). Once the temperature of earth's surface anomaly temperature is obtained, the anomalous field of geopotential height can be calculated by the adaptation equation with relevant boundary conditions.

Considering that the vertical motion is zero at the sea level without the mountain influence, by applying the condition to Eq. (6) at sea level and replacing the reference temperature  $T_r$  in Eq. (2) by the earth's surface temperature  $T_s = \bar{T}_s + T'_s$  and assuming  $\bar{T} = \bar{T}_s$  and because, in addition, there is no condensation at sea level, we have

$$\text{at } p = p_0, \quad \left[ \frac{D}{Dt} - \frac{\partial}{\partial p} (k_p + k_R) \frac{\partial}{\partial p} \right] \frac{\partial \phi'}{\partial p} - \left( \frac{\partial \bar{u}}{\partial p} \frac{\partial \phi'}{\partial x} + \frac{\partial \bar{v}}{\partial p} \frac{\partial \phi'}{\partial y} \right) = -\frac{1}{\tau_R} \left( \frac{\partial \phi'}{\partial p} + \frac{R}{p_0} T'_s \right). \quad (13)$$

This is one of the boundary conditions for solving the adaptation equation. At the top of the atmosphere, we have

$$p = 0, \quad \phi' = \frac{\partial^2 \phi'}{\partial p^2} = 0. \quad (14)$$

And if the vertical motion as well as the condensation also disappear there, then from Eq. (6) we have another boundary condition

$$p = 0, \quad \left( \frac{D}{Dt} - \frac{\partial}{\partial p} (k_p + k_R) \frac{\partial}{\partial p} \right) \frac{\partial \phi'}{\partial p} - \left( \frac{\partial \bar{u}}{\partial p} \frac{\partial \phi'}{\partial x} + \frac{\partial \bar{v}}{\partial p} \frac{\partial \phi'}{\partial y} \right) = -\frac{1}{\tau_R} \frac{\partial \phi'}{\partial p}. \quad (15)$$

With these boundary conditions the multi-levels version for adaptation equation can be obtained. Particularly, the one-level version for the atmosphere is simple as (Chao et al. 1982, Group of LRF 1979)<sup>5,41</sup>

$$\text{at } p = 500 \text{ mb.} \quad \frac{D}{Dt} \Delta \phi' + \beta_y \frac{\partial \phi'}{\partial x} - \beta_x \frac{\partial \phi'}{\partial y} - K \Delta \phi' = F T'_s, \quad (16)$$

where

$$F = \overbrace{f^2 R / \bar{\sigma}_p p_0^2 \tau_R} > 0, \quad K = -\overbrace{bfR \frac{\partial}{\partial p} \left( \frac{\bar{T}^*}{p \bar{\sigma}_p} \right)} > 0.$$

### III. EXAMPLES OF ONE MONTH PREDICTION

The original predicted method is that at first, we may predict the earth's surface temperature including land and ocean both by the thermal equation with the boundary conditions and the relevant initial conditions as well as the corresponding monthly climatic data. The time step of integration just takes one month for one month prediction of earth's surface temperature. The corresponding 500 mb anomalous height of the same month is calculated by the adaptation equation in which the predicted earth's surface temperature is utilized.

Using this method, the early predicted results of 12 examples were described in the paper

of Miyakoda and Chao (1982) by the correlation coefficients between the prediction and observation over the Northern Hemisphere [6], these results are also described in the column B in Table 1. Apparently, the predicted skill is not higher, even the average skill is lower than that of the persistence.

Table 1. Correlation Coefficients of the Anomalous Fields between the Prediction and Observation over the Northern Hemisphere.

Cases		$T_2'$			$\phi'$		
		Prediction	Persist.		Prediction	Persist.	
		A*	B**		A*	B**	
Jan.—Feb.	1976	0.40	0.07	-0.01	0.11	-0.16	-0.13
"	1977	0.36	0.18	0.25	0.42	0.05	0.34
"	1978	0.50	0.29	0.58	0.48	0.22	0.30
April—May	1976	0.27	0.20	0.26	0.44	0.30	0.25
"	1977	0.40	0.33	0.38	0.33	0.21	0.07
"	1978	0.40	0.25	0.32	0.05	0.05	0.19
July—Aug.	1976	0.15	0.15	0.27	0.33	0.29	0.47
"	1977	0.47	0.26	0.46	0.43	0.17	0.42
"	1978	0.43	0.28	0.40	0.20	0.16	0.28
Oct.—Nov.	1976	0.57	0.32	0.50	0.55	0.16	0.21
"	1977	-0.09	-0.01	-0.05	-0.18	-0.29	-0.07
"	1978	0.35	-0.01	0.30	0.34	-0.06	0.32
Average		0.35	0.19	0.31	0.29	0.09	0.22

A\* two-time step.  
B\*\* one-time step.

The new improved method is that for one month prediction, the one-time step of one-month prediction is replaced by two-time step in one-month prediction, i.e. the time step takes half a month. The corresponding 12 examples are re-calculated by this method, and the results for  $T_2'$  and  $\phi'$  are much better than the early ones. The results are shown in Table 1 (column A), and the average correlation coefficients for predicting  $T_2'$  and  $\phi'$  are 0.35 and 0.29 which are higher than that of the persistence 0.31 and 0.22 respectively.

An example is provided for comparing the predicting results of the present and old methods. Figs. 1(a) and 1(b) are the prediction of  $T_2'$  and  $\phi'$  in February 1978 by the two-time step method and Figs. 2(a) and 2(b) are the prediction of  $T_2'$  and  $\phi'$  by the original method. The corresponding charts of observation are shown in Figs 3(a) and 3(b). Comparing these charts, we can see that the predicting results of present method are better than those of the original one. In this example, the correlation coefficients between the prediction and observation are respectively 0.50 and 0.48 for predicting  $T_2'$  and  $\phi'$  which are higher than those of the original one 0.29 and 0.22.

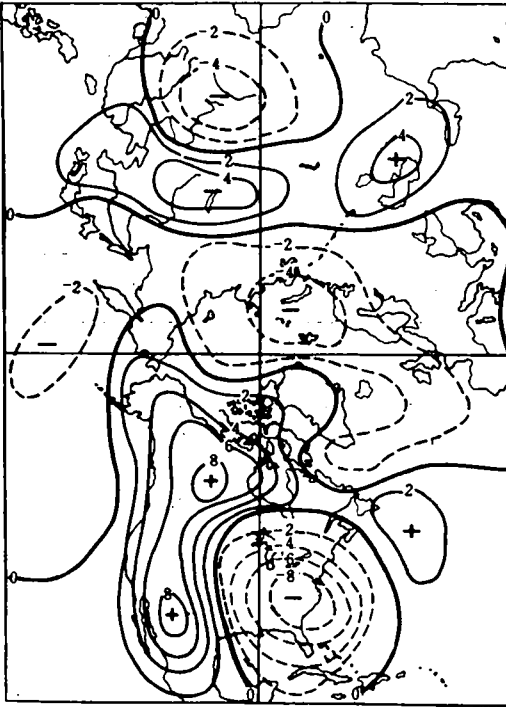


Fig 1(a). Predicted anomalous field of earth's surface temperature in February 1978 (two-time step).

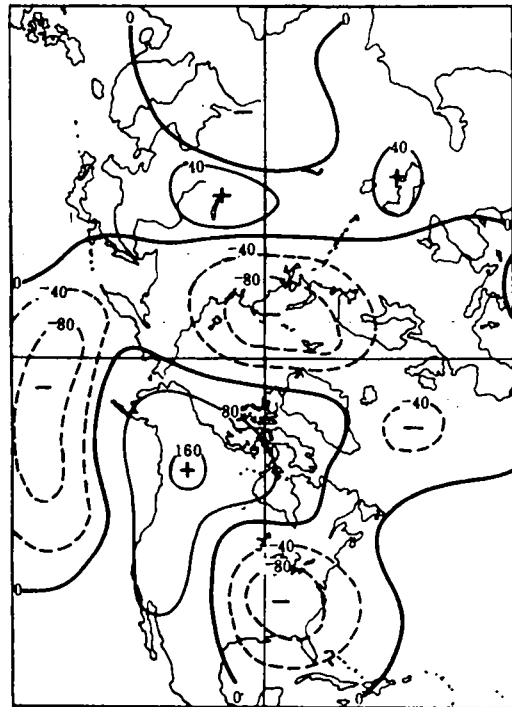


Fig. 1(b). Predicted anomalous field of 500 mb height in February 1978 (two-time step).

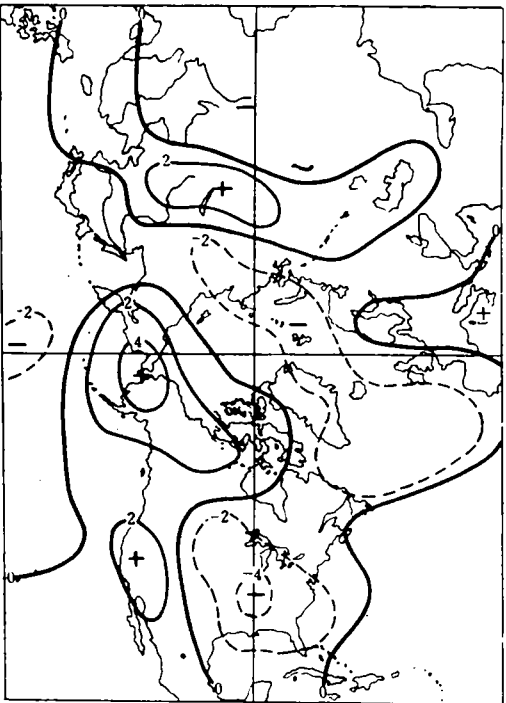


Fig. 2(a). Predicted anomalous field of earth's surface temperature in February 1978 (one-time step).

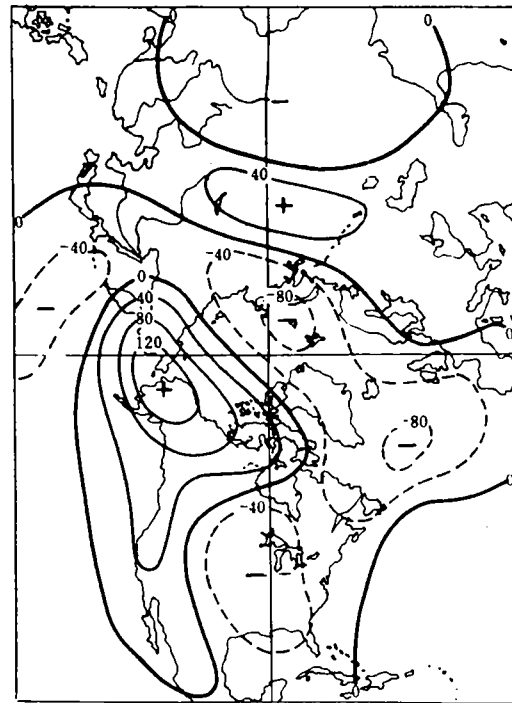


Fig. 2(b). Predicted anomalous field of 500 mb height in February 1978 (one-time step).

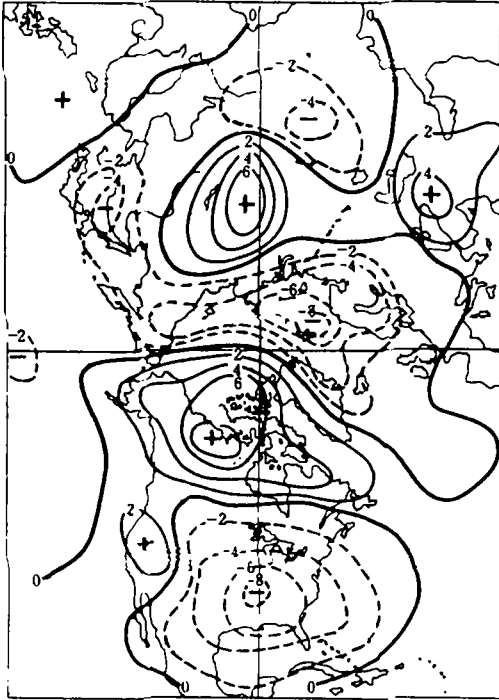


Fig. 3(a). Observed anomalous field of earth's surface temperature in February 1978

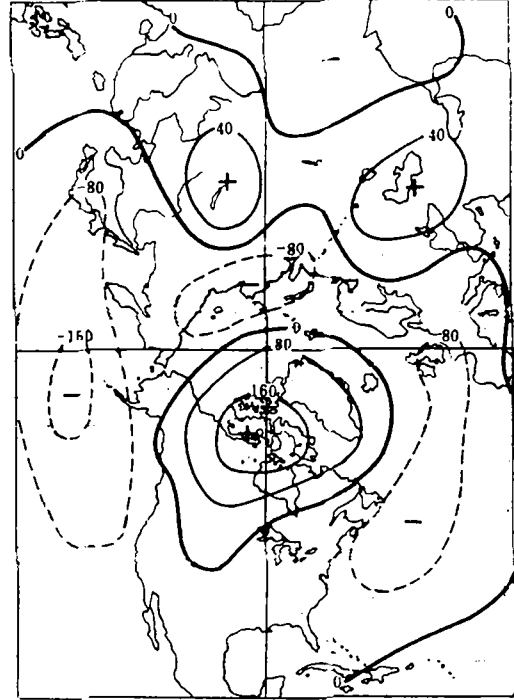


Fig. 3(b). Observed anomalous field of 500 mb height in February 1978.

In order to test the ability for predicting the blocking event, some results are given. A spectacular blocking event over the west coast of America lasted for four months, November 1976 to February 1977. The best developing month was January 1977 which had been predicted by GCM of GFDL as well as by the FAM in GFDL, Princeton University (Chao and Caverly 1982, Miyakoda and Chao 1982) <sup>(7,6)</sup>. An interesting and important point is that the GCM and the FAM, in essence, are quite different, yet the predicting results in both models are similar to each other in the map of monthly anomaly geopotential height of 500 mb. Can we predict the formation of this blocking process by the FAM yet? Fig.4(a), 4(b) and 5(a), 5(b) are the predicted charts as well as the corresponding observations in November 1976. The comparison shows that the general situation of predictions is in agreement with the observations in the blocking region. The correlation coefficients of the predicted anomaly fields for  $T'_s$  and  $\phi'$  are 0.57 and 0.55 respectively, and those of the persistences are 0.50 and 0.21 respectively.



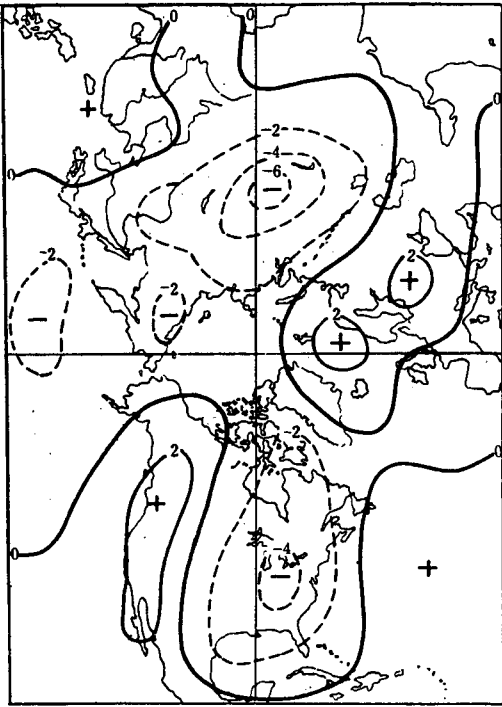


Fig. 4(a). Predicted anomalous field of earth's surface temperature in November 1976.

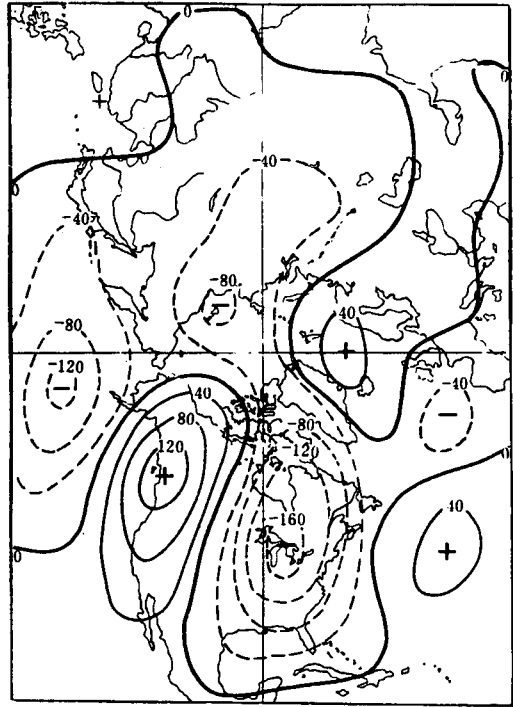


Fig. 4(b). Predicted anomalous field of 500 mb height in November 1976.

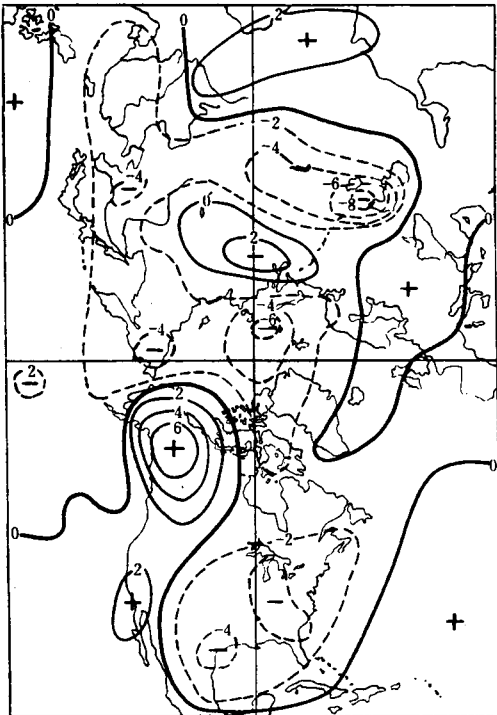


Fig. 5(a). Observed anomalous field of earth's surface temperature in November 1976.

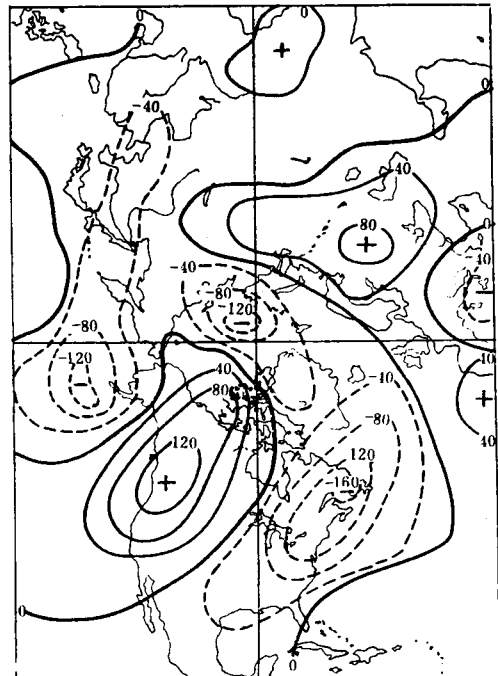


Fig. 5(b). Observed anomalous field of 500 mb height in November 1976.

#### IV. CONCLUSIONS

According to the results obtained above, we believe that the FAM has the potential ability for long-range forecasts. Particularly, this can save time in computing. For example, 60 hours (Advanced Scientific Computer in Princeton) are needed to obtain the solution of the case of January 1977 by GCM, but only 15 seconds are needed in the same case by FAM. The main disadvantage of this anomaly model is that the solutions depend in certain degree on the parameterization and the values of physical parameters of energy sources, surface boundary forcing, and internal forcing effects. On the other hand, the influences of plateau as well as tropical ocean should be considered. All these problems can be improved in future.

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