INTEGRATION METHOD AND RATIO METHOD FOR RETRIEVING EXTINCTION COEFFICIENT FROM LIDAR SIGNALS

Zhao Yanzeng (赵燕曾), *Hu Yuliang* (胡裕良) and *Zhao Hongjie* (赵红接)
Institute of Atmospheric Physics, Academia Sinica, Beijing

Received August 11, 1983

ABSTRACT

New solution techniques to improve the accuracy of quantitatively determining the atmospheric extinction coefficient and the backscattering-to-extinction ratio from lidar signals are developed. The integration method is proposed to analytically retrieve the extinction coefficient at ground level, which has the advantage of eliminating the effect of backscattering fluctuations on the inversion results. The ratio method, on the other hand, deals with the inversion of the vertical distribution of the extinction coefficient. The main idea of this method is to begin with a calculation of the transmittance by eliminating the backscattering through ratioing lidar signals at two elevations, and subsequently derive the extinction coefficient from the transmittance, thus avert from ambiguous results caused by inappropriate assumptions on the backscattering-to-extinction ratio. Observational investigations have demonstrated that the integration method is superior to the slope method in terms of accuracy and stability, and the ratio method is reasonable and feasible as well.

I. INTRODUCTION

Lidar as an active remote sensing instrument can provide a sensitive and three dimensional measure of the aerosol and molecule scattering in the clear atmosphere. Thus continuous efforts have been devoted to derive atmospheric optical characteristics from the lidar return signals[1-3] and subsequently convert the optical parameters into other physical parameters. While the techniques for quantitatively solving lidar equation were under investigation in the last two decades, the problem has not been fully resolved. This is attributed to the inherent difficulty that the lidar signal depends both on the backscattering and extinction features, resulting in two unknowns appearing in a single equation. In fact, the lidar equation for single-scattering is written as

$$S(r)=C_A\,\beta_\pi(r)\exp\left[-2\int_0^r\sigma(r')\,dr'\right] \tag{1}$$

where $\beta_\pi(r)$ and $\sigma(r)$ are the backscattering and extinction coefficient of the atmosphere, respectively, C_A is the instrument constant of the lidar system, and $S(r)$ is the range square compensated signal defined as

$$S(r)=V(r)\,r^2/G(r),$$

where $V(r)$ is the lidar return signal and $G(r)$ is the range-dependent relative gain of the photomultiplier when a gain modulation circuit is used.

It is clearly seen that for quantitatively deriving $\beta_\pi(r)$ and $\sigma(r)$ from Eq. (1), additional assumptions have to be made. Even though, these two quantities are not easily determined without ambiguity. The reliability and stability of the solution rely mainly on the feasibility of the assumption adopted and the signal detection and processing procedure.

Having led to a wide variety of observations providing useful information, the previous methods for the evaluations of $\beta_\pi(r)$ and $\sigma(r)$, however, are not adequate for those items in which higher accuracy is required. For example, the horizontal and slant visibility measurements often subject to large uncertainties caused by the instability or invalidity of the solution techniques under certain circumstances. The backscattering-to-extinction ratio, $K(r)=\beta_\pi(r)/\sigma(r)$, which is a sensitive indicator of the refractive indicies and size distributions of the aerosols, was assumed to be independent of height in most cases. This assumption is valid when the boundary layer is well-mixed, but may cause large errors when the vertical exchange is weak or when there is a dust storm from remote source, in those cases considerable changes of the aerosol properties in the vertical direction may occur.

In this study, we propose new solution techniques to improve the evaluation accuracy of $\sigma(r)$ and find a way to estimate the vertical distribution of $K(r)$. Section II will describe the integral method for deriving surface values of σ and K, while section III will present the principle of ratio method for deriving veritical distributions of σ and K. Observational demonstration and discussions are included in Section IV.

II. INTEGRATION METHOD FOR DERIVING σ_0 AND K_0

In the case of horizontal detection, the assumption of horizontal homogeneity is always adopted, the lidar equation thus takes the from

$$S(r)=C_A\,\beta_{\pi_0}\exp(-2\sigma_0 r), \tag{2}$$

where β_{π_0} and σ_0 are constants.

As a natural and logical consequence, Eq. (2) will lead to an idea to linearize the equation by taking logarithm of the two sides and subsequently derive β_{π_0} and σ_0 by linear regression. This is referred to as "slope method".

Although it seems straightforward and resonable to do so, observational investigations have shown that slope method often suffers instability and large uncertainties caused by small scale fluctuations in the atmosphere. In urban areas, it remains a frustration that large spikes due to smoke plumes or dust clouds superimpose on mean lidar signal, causing remarkable deviation from the regression line and consequently a unreliable value of the slope. It is not surprising to find that the calculated σ_0 varies considerably with different starting and ending ranges for the regression. Sometimes even negative values of σ_0 are attainable.

In fact, horizontally homogeneous condition, by no means implies a constant value of β_π or σ in a horizontal plane. From a statistical point of view, it only means the statistical characteristics of these two random quantities remain unchanged in horizontal directions. Therefore we have

$$\sigma=\sigma_0+\sigma', \qquad \beta_\pi=\beta_{\pi_0}+\beta_\pi',$$

where β_π' and σ' denote the fluctuation components.

Eq. (1) and (2) yield

$$S(r)=C_A\,\beta_{\pi_0}(1+\beta_\pi'/\beta_{\pi_0})\exp\left(-2\sigma_0 r-2\int_0^r\sigma' dr'\right). \tag{3}$$

If σ' is integrated over a fairly long distance so that $\int_0^r\sigma' dr'\ll\sigma_0$ and thus can be neglected, the linearized equation will take the form

$$\ln S(r)=\ln(C_A\,\beta_{\pi_0})-2\sigma_0 r+\ln(1+\beta_\pi'/\beta_{\pi_0}). \tag{4}$$

It is obvious that the uncertainty of the slope $-2\sigma_0$ is mainly attributed to the last term in (4). To reduce the uncertainty needs to increase the fire number in lidar detection, but this is time-consuming and uneconomic.

It follows from the above discussions that the slope method is not a satisfactory course. Based on the statistical explanation of "horizontal homogeneity" we have proposed an "integration method", the main idea of which is to smooth out the fluctuation by integrating the lider signals over r.

Substituting $\beta_\pi(r)=K_0\sigma(r)$ into (1), integrating the equation over r and taking K_0 as a constant, we have

$$\int_{r0}^{r} S(r')dr' = \frac{C_A K_0}{2}\left\{\exp\left[-2\int_0^{r_0}(r')dr'\right] - \exp\left[-2\int_0^{r}\sigma(r')\,dr'\right]\right\} \tag{5}$$
$$= \frac{C_A K_0}{2}\left[T^2(r_0) - T^2(r)\right],$$

where $T(r)$ denotes the transmittance between 0 and r, r_0 is a range slightly beyond the non-overlapping-zone of the lidar attena. By choosing an appropriate upper limit r_m, the final solutions of $T(r)$, σ_0 and $C_A\ K_0$ are given as follows

$$. \ T^2(r) = \alpha T^2(r_o) + (-\alpha)T^2(r_m), \tag{6}$$

$$\sigma_0 = -\frac{\ln T^2(r)}{2r}, \tag{7}$$

$$C_A K_0 = 2\int_{r_0}^{r_m} S(r)\,dr \Big/ \left[T^2(r_0) - T^2(r_m)\right], \tag{8}$$

where
$$\alpha = 2\int_{r}^{r_m} S(r)\,dr \Big/ \int_{r_0}^{r_m} S(r)\,dr.$$

For a given r_0, r_m and r, $T^2(r)$ is a function of α, and can be calculated by iteration procedure using (6) and (7).

In the integration scheme, σ_0 is related to $T^2(r)$. Since $T^2(r)$ relies on α, the ratio of two integrals of the lidar signals over long ranges, it can hardly be infuenced by small scale fluctuation. In terms of eliminating the effect of random fluctuation, the spatial integration is much more efficient than simply increasing the fire number over which the average is taken. One shot is sufficient for applying integration method to obtain stable and reliable results of $T^2(r)$ and σ_0. Moreover, K_0 can be derived independently from (8) with good accuracy.

In order to analyze the sensitivity of σ_0 to the error of α and its dependence on r_0, r_m and r, $\frac{d\ln\sigma_0}{d\ln\alpha}$ is calculated. The percentage changes of σ_0 corresponding to 1% change of α at different values of r, r_m are plotted against σ_0 in Fig. 1. It can be seen that for the same r_m, $\Delta\sigma/\sigma_0$ decreases with increasing r; while for the same r, $\Delta\sigma/\sigma_0$ increases with increasing r_m. This, however, does not mean that one should extend r and reduce r_m as much as possible. In fact, decreasing r_m and increasing r lead to an increase of random deviation of $\int_r^{r_m} S(r)\,dr$, resulting in an increase of percentage error of α.

The lidar measurements of horizontal visibility were carried out successively at the top of the institute building and at an airport in the south of Beijing from February to March, 1983. Observations have demonstrated that the integration method can provide stable and reliable results. The relative deviations of σ_0 derived at different r for the same lidar signal never exceed 10%, and concentrate below 5%. The stability of the solution remains the same even when σ_0 is as low as 0.1 km^{-1}, in spite of the large fluctuations or sharp spikes in the lidar signals. Paralell calculations using slope method were

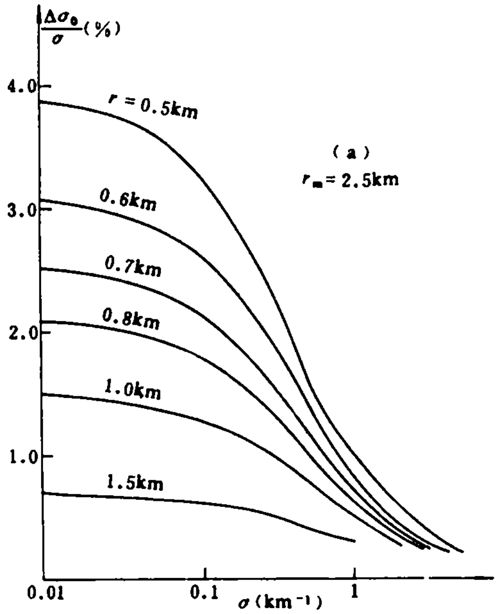

Fig. 1(a). Percentage change of σ_0 corresponding to 1% change of α ($r_m = 2.5$ km).

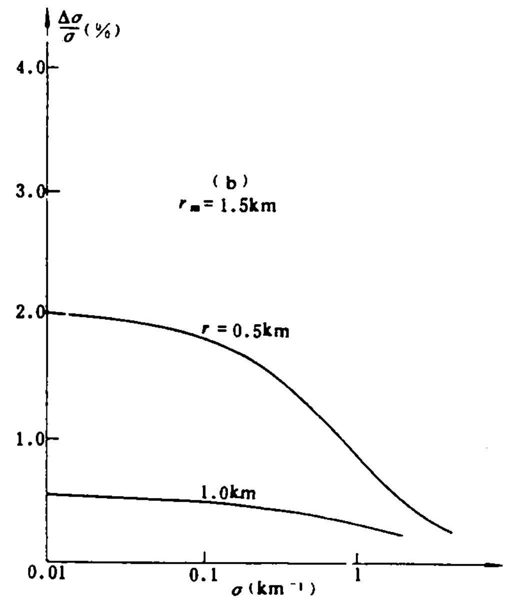

Fig. 1(b). Percentage change of σ_0 corresponding to 1% change of α ($r_m = 1.5$ km).

conducted. The comparison showed that the integration method achieved much higher stability and accuracy than the slope method , particularly in fine weather conditions and in complicated surroundings. Employing slope method for the same data often led to doubtful or even absurd results.

It is worth pointing out that the integration method can be applied to obtain gaseous species concentration No from DIAL measurements in horizontal direction. Because of the available accuracy, it is possible to calculate the total attenuation and K at each wavelength and derive No by substraction.

In horizontal case, the lidar equations for DIAL signals can be written as

$$S_1(r) = C_A K_1 \sigma_1 \exp(-2\sigma_1 r),$$
$$S_2(r) = C_A K_2 \sigma_2 \exp(-2\sigma_2 r),$$

where the subscripts 1 and 2 stand for the on-line and off-line wavelength, respectively; σ and K are respectively the total attenuation and backscattering-to-attenuation ratio defined as

$$\sigma_1 = \sigma_0 + \alpha_1 N_0, \qquad \sigma_2 = \sigma_0 + \alpha_2 N_0,$$
$$K_1 = \frac{\sigma_0}{\sigma_0 + \alpha_1 N_0} K_0, \qquad K_2 = \frac{\sigma_0}{\sigma_0 + \alpha_2 N_0},$$

where α is the absorption cross-section, N_0 is the number density of the gaseous species under investigation.

From the total attenuation one has

$$N_0 = \frac{\sigma_1 - \sigma_2}{\alpha_1 - \alpha_2} \cong \frac{\sigma_1 - \sigma_2}{\alpha_1}. \tag{9}$$

As a supplement, N_0 can also be calculated in terms of K_1 and K_2

$$N_0=\frac{1-K_1/K_2}{K_1/K_2-\alpha_2/\alpha_1}\frac{\sigma_0}{\alpha_1}\cong(K_2/K_1-1)\frac{\sigma_0}{\alpha_1}. \tag{10}$$

The signal processing of water vapor DIAL measurements has shown that this method provides more reliable results compared with the ordinary procedure (see, e.g.[2]) as the humidity is very low.

III. RATIO METHOD FOR EVALUATING VERTICAL DISTRIBUTION OF σ AND K

The formal solution of $\sigma(r)$ is easily obtained by using (1) and (5)

$$\sigma(r)=\frac{S(r)}{C_A K(r)T^2(r)}, \tag{11}$$

where $T^2(r)=T^2(r_0)-\frac{2}{C_A}\int_{r_0}^{r}\frac{S(r')}{K(r')}dr'$. This expression, however, is of no practical use since $K(r)$ is unknown. The natural and simplest way to solve this problem is to set $K(r)=K_0$, as is suggested in [1]. Nevertheless, though the basic features of extinction profile can be obtained by this approximation, large errors in σ and optical depth may occur. Moreover, useful information about the vertical changes of K may be lost. An experimental investigation by Zhou Shijianetal.(1981)[4] has indicated that the mean values of K averaged over a height range of 3 km differ from K_0 by a factor up to 2. 5.

In the authors opinion, it is much better to explore a converse procedure, i.e., instead of treating $\alpha(r)$ as a derivative quantity of an unknown quantity $K(r)$, one should begin with a calculation of $T^2(r)$ directly from lidar signals without regard to $K(r)$, and subsequently derive $\sigma(r)$ and then $K(r)$ from $T^2(r)$.

By using horizontally homogeneous assumption, which is valid in most cases, and measuring lidar signals at two elevations θ_1 and θ_2, the ratio of the corresponding lidar signals $S_1(r)$ and $S_2(r)$ at the same altitude h can be obtained

$$R(h)=\frac{S_1(h\alpha_1)}{S_2(h\alpha_2)}=T(h)^{2(\alpha_1-\alpha_2)}, \tag{12}$$

where $\alpha_1=\csc\theta_1$, $\alpha_2=\csc\theta_2$.

The optical depth is given by

$$\tau(h)=\frac{1}{2(\alpha_1-\alpha_2)}\ln T(h). \tag{13}$$

It is clearly seen that $\beta_\pi(h)$ and $K(h)$ are eliminated by the ratioing. However, the optical depth $\tau(h)$ thus determined is not stable due to the fluctuation of β_π. If the standard deviation of β_π is about 20%, the standard deviation of $R(h)$ may reach 30%, so that the relatively error of $R(h)$ may be as high as $0.6/2(\alpha_1-\alpha_2)$.

The situation can be greatly improved by integrating $S_1(r)$ and $S_2(r)$ over a thin vertical layer before the ratio is taken. In this case, the ratio of the two integrals is related to the optical depth, i.e.

$$\overline{R}(h)=\frac{\int_{r_{11}}^{r_{12}}S_1(r)dr}{\int_{r_{21}}^{r_{22}}S_2(r)dr}=CT(h)^{2(\alpha_1-\alpha_2)}, \tag{14}$$

where

$$C=\frac{\exp[2\alpha_1\tau(h-\Delta h,h)]-\exp[-2\alpha_1\tau(h,h+\Delta h)]}{\exp[2\alpha_2\tau(h-\Delta h,h)]-\exp[-2\alpha_2\tau(h,h+\Delta h)]}. \tag{15}$$

The relation of $r_{i,j}$ $(i,j=1,2)$ and h and Δh are schematically illustrated in Fig. 2.

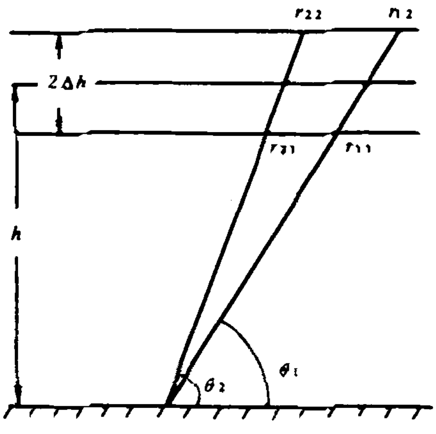

Fig. 2. Slant ranges at two elevations on the boundary of a thin layer over which the integration is taken in the improved ratio method.

From (14) we obtain the expressions for vertical transmitance $T(h)$ and optical depth $\tau(h)$ as follows

$$T(h)=\left[\frac{\overline{R(h)}}{C}\right]^{\frac{1}{2(\alpha_1-\alpha_2)}}, \tag{16}$$

$$\tau(h)=\frac{1}{2(\alpha_1-\alpha_2)}\ln\left[\frac{\overline{R(h)}}{C}\right]. \tag{17}$$

If Δh is within several tens of meters, C equals to α_1/α_2 in a good approximation. By using α_1/α_2 as the initial value of C, $\tau(h)$ and C are determined through an iteration process.

Taking the derivative of smoothed $\tau(h)$ with respect to h, we have

$$\sigma(h)=\frac{\Delta\tau}{\Delta h}. \tag{18}$$

According to (1),

$$K(h)=\int_{r_{11}}^{r_{12}}S_1(r)dr\Big/C_A\alpha_1\sigma(h)\Delta h=\int_{r_{11}}^{r_{12}}S_1(r)dr\Big/C_A\alpha_1\Delta\tau\,. \tag{19}$$

The above method is not suitable for the evaluation of σ within non-overlapping zone, where the overlapping coefficient $\eta(r)$ has to be taken into account. Due to the large uncertainties in the calibration of $\eta(r)$ and the non-linear effect of the photomultiplier in near ranges, another ratio method capable of eliminating $\eta(r)$ and $G(r)$ is used.

Let $S_1(r)$ and $S_0(r)$ be the lidar signals detected at certain elevation angle and in horizontal direction, respectively, and assume that $K(r)=K_0$ within a thin layer near the surface, the ratio of $S_1(r)$ to $S_2(r)$ is

$$R_s(r)=\frac{\sigma(r)}{\sigma_0}\frac{T_1^2(r)}{T_0^2(r)}\,. \tag{20}$$

Integrating (20) over r, we have

$$T_1^2(r)=T_0^2(r)\left\{\frac{T_1^2(r_0)}{T_0^2(r_0)}-2\sigma_0\int_{r_0}^{r}\left[R_s(r')-\frac{T_1^2(r')}{T_0^2(r')}\right]dr'\right\}, \tag{21}$$

$$\sigma(r)=\sigma_0 R_s(r)\left\{\frac{T_1^2(r_0)}{T_0^2(r_0)}-2\sigma_0\int_{r_0}^{r}\left[R_s(r')-\frac{T_1^2(r')}{T_0^2(r')}\right]dr'\right\}. \tag{22}$$

According to (16), the relative error of vertical transmittance is

$$\frac{\delta T}{T}=\frac{1}{2(\alpha_1-\alpha_2)}\left(\frac{\delta J_1}{J_1}+\frac{\delta J_2}{J_2}\right), \tag{23}$$

where

$$J_1=\int_{r_{11}}^{r_{12}}S_1(r)dr,\qquad J_2=\int_{r_{21}}^{r_{22}}S_2(r)dr\,.$$

The contributions of random fluctuation of $S_1(r)$ and $S_2(r)$ *to* J_1 and J_2 can be eliminated to a high degree by the integration, the residual is no more than 1%. The errors of J_1 and J_2 are primarily the systematic errors caused by the non-linear effect of photodetector when the near-range signals are too strong, or by d.c. level variation due to the receiver electronics or the background noise. These errors can be greatly reduced if the photodetector system is carefully treated and calibrated and the d.c. shift is substracted from the data.

IV. OBSERVATION AND DISCUSSION

An analysis of the lidar observation carried out on March 11, 1983 offers a demonstration of the

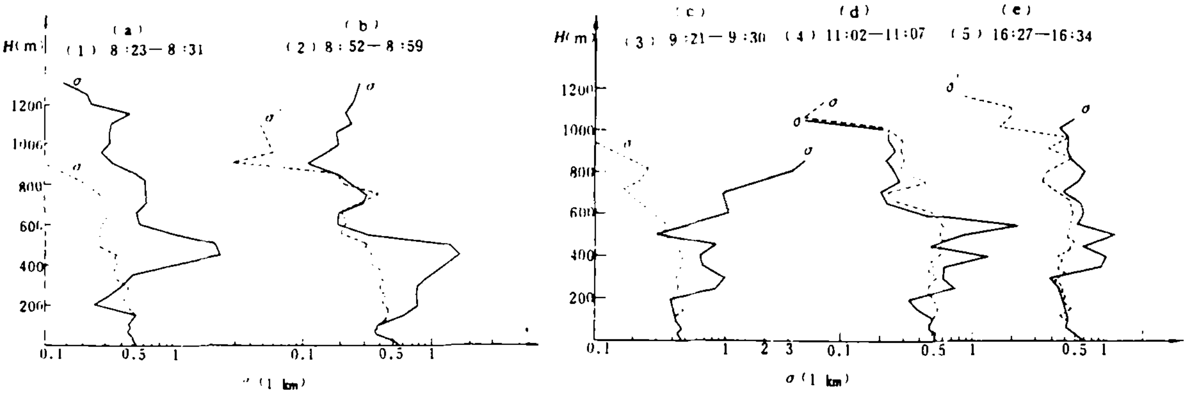

Fig 3. Extinction coefficient distributions calculated by ratio method (——) compared with those calculated under the assumption of $K=K_0$ (- - -).

present methods. The lidar measurements were made at three elevations, i.e., 0^0, 30^0, and 19.5^0 successively. An on-line TRS-80 microcomputer was programmed to calculate $\sigma(r)$ and $K(r)$ by using integration method and ratio method. Because of the non-linear effect of the photomultiplier, the aperture of the receiver has to be significantly reduced, the detection range is thus restricted to below 2.5 km, and the altitute below which the ratio method is applied is accordingly limited to 0.6 km. Above 0.6 km, $K(h)=K(0.6)$ is assumed and (7) is used for further calculation of $\sigma(r)$.

The extinction coefficient profiles derived are plotted in Fig. 3 (a)—(e). The solid lines represent the results of the ratio method, while the dashed lines are the corresponding ones when $K(r)=K_0$ is assumed.

These figures clearly show that different methods yield remarkably different results. The extinction coefficient $\sigma(r)$ derived with ratio method are much higher than that calculated under the assumption of $K=K_0$, which is denoted as $\sigma'(r)$; the vertical changes of $\sigma(r)$ are more pronounced as well. Visual observations and the routine meteorological data show that the former is more reasonable. It is seen in synoptic charts that Beijing is under the influence of a weak "back flow" weather, above a layer of cold air near the surface there is the humid and warm air from the Bohai Sea, the top of the inversion lies at 900 mb. Visual observations show that it is very turbid throughout the morning. The vertical distributions of σ exhibit a high-peak around 400—500 m in 08:23 and 08:52, it begins to break since 09:21. In the afternoon the extinction distribution becomes quite uniform, it is very likely a result of mixing. Although the temporal variations of σ' profiles have similar tendency, no distinct peaks exist. All these features are probably the results of setting $K=K_0$.

Interesting results of K distributions are illustrated in Fig. 4. It should be pointed out that March 11 is a very special day. Before then the value of $C_A K_0$ wandered around 400—500 from January to early March; after then it jumped to 1×10^3—4×10^3, nearly increased one order of magnitude. The jumping can clearly be seen in Fig. 4. This sudden change is very likely due to the ending of the heating season, resulting in a sudden reduction of the coal particles. According to the calculation by Qiu Jinhuan et al. (1983) (private communication), K is very sensitive to the imaginary part of the refractive indices (n_I) of the aerolols. Therefore the deep valley of K and high peak of σ near 400 m probably imply a high n_I of the aerosols near there. The vertical gradient of K tends to decrease and almost vanishes in the afternoon. It indicates that the assumption $K=K_0$ is valid if there is a strong vertical exchange.

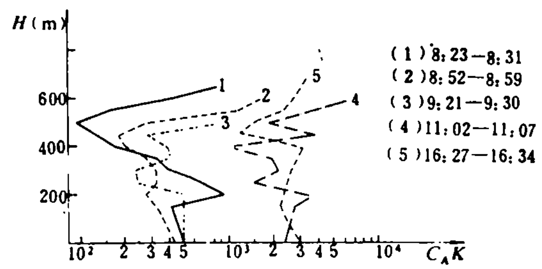

Fig. 4. Temporal evolution of vertical distribution of $C_A K$.

V. SUMMARY AND CONCLUSION

(1) The integration method has been proved to be an accurate and reliable method to retrieve σ_0 and K_0 from horizontal lidar measurement, it is therefore superior to the slope

method normally used.

(2) The main idea of the ratio method is more reasonable than the previous ones, it can provide useful information about the vertical change of K which was often ignored before. Although there is no essential problem in the principle as long as the horizontal homogeneous condition is met, it is still necessary to continue observational investigation to explore which limitations this method is restricted to.

The authors are indebted to Asistant Prof. Wu Shaoming and Mr. Huang Chirong for their efforts in preparing and maintaining the lidar system.

REFERENCES

[1] 中国科学院大气物理研究所, 激光测定大气消光系数及其在能见度探测上的应用,(集刊第1号,《激光在气象探测上的应用》),科学出版社,1973年.

[2] Hinkley, E. D., *Laser Monitoring of the Atmosphere*, Springer-Verlag, New York, 1976, 71—151.

[3] Klett, J.D., *Applied Optics*, **20** (1981), 221.

[4] 周诗健,陶丽君,朱文琴,大气科学,5 (1981) 4, 444.