

## OBSERVATIONAL AND THEORETICAL STUDIES OF THE MOIST BAROCLINIC ATMOSPHERE

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### ABSTRACT

In this paper, the main scientific conclusions of a national wide project of heavy summer rainstorms are presented. The active role of the moisture in the large scale motion of the atmosphere is stressed when the water vapour is saturated. The concept of moist baroclinity is introduced, and moist baroclinic processes are studied. Theoretical results, i. e., moist solenoid, moist available potential energy, moist jet, moist baroclinic instability, etc. are presented. Some observational and numerically experimental results are also shown.

In this paper, a brief summary of the achievements made through a project for the purpose of solving the difficult problem of severe summer rainstorm forecast and flood control is presented.

The project started at the midst of 1950s after the severe Changjiang River flood, proposed by Central Meteorological Bureau (now State Meteorological Administration). It was interrupted several times for the reason known to everybody. However, on its way of fighting forward, certain characteristics of the structure of the heavy summer rainstorms and their relation to flow patterns at the low latitude were clarified, certain conclusions of the disturbances at the latitude were obtained earlier than other countries<sup>[1,2]</sup>.

After the disastrous flood of 1975, a national wide effort was renewed to study this disaster-bringing meteorological process. A group of meteorologists led by lei Yushun designed a method of analysing and forecasting the severe summer rainstorms. It is an excellent blend of synoptic and static stability analysis called in our country as static energy principle<sup>[3]</sup>.

It is under the pressure of public necessity and stimulated by the results of the static energy principle, a more or less systematic conclusions concerning the dynamics of moist air are drawn. It emerges as a new branch of dynamic meteorology despite some easily-overcome objections.

The scientific overview is that moisture plays an active role in the large scale motion of the atmosphere, and should not be considered as some constituent passively transported by air current<sup>[4]</sup>.

### 1. THE LAW OF CONSERVATION OF TOTAL ENERGY OF MOIST AIR AND ITS APPLICATIONS

Considering the process being reversible moist adiabatic, from the equations of motion and thermodynamics, the law of conservation of the total energy of moist air may be written as follows,

$$E_t = C_p T + gz + L q_s + \frac{V^2}{2} = \text{const.} \quad (1)$$

or

$$T_t = T + \frac{gz}{C_p} + \frac{L q_s}{C_p} + \frac{V^2}{2C_p} = \text{const.}, \quad (2)$$

where  $E_t$  and  $T_t$  are the total energy and total temperature of the moist air respectively.

Eqs. (1) and (2) may be written as

$$E_t = E_\sigma + \frac{V^2}{2} = \text{const.}, \quad (3)$$

$$T_t = T_\sigma + \frac{V^2}{2C_p} = \text{const.}, \quad (4)$$

$E_\sigma$  and  $T_\sigma$  may be named as generalized moist potential energy and generalized moist temperature respectively.

The geostrophic equations on the constant  $\theta_{se}$  surface may be written as

$$\begin{aligned} fu &= -\left(\frac{\partial}{\partial y}\right)_{\theta_{se}} (C_p T + gz + Lq_s), \\ fv &= \left(\frac{\partial}{\partial x}\right)_{\theta_{se}} (C_p T + gz + Lq_s), \end{aligned} \quad (5)$$

where  $C_p T + gz + Lq_s$  may be named as moist isentropic function.

In summer, the temperature contrast across the rain belt usually is weak, whereas the contrast of  $\theta_{se}$  and  $T_\sigma$  are quite large. The vertical wind shear computed by thermal wind equation is much smaller than the observed wind shear. A set of equations is designed as

$$\begin{aligned} f \frac{\partial v}{\partial p} &\approx -\frac{R}{p_{00}^k} p^{k-1} \left(\frac{\partial \theta_{se}}{\partial x}\right)_p, \\ f \frac{\partial u}{\partial p} &\approx \frac{R}{p_{00}^k} p^{k-1} \left(\frac{\partial \theta_{se}}{\partial y}\right)_p. \end{aligned} \quad (6)$$

The results computed by Eq. (6) are in quite good agreement with the observations. They are named as moist baroclinic balance equations, while the well-known thermal wind equations are considered as dry adiabatic balance equations. (see Fig. 1)

## II. MOIST SOLENOID FIELD AND MOIST BAROCLINITY

The  $T_\sigma$  field on isobaric surface indicates that the surfaces of constant pressure and constant  $T_\sigma$  intersect one another, and divide the space into many tubes. They may be named moist solenoids. The atmosphere where moist solenoids exist is considered as moist baroclinic.

Taking volume integral of the Eq. (1), we have

$$[E_t] = \frac{C_p}{g} \int_s \int_0^{p_0} T_\sigma dp dS + \frac{C_p}{g} \int_s \int_0^{p_0} \frac{V^2}{2C_p} dp ds, \quad (7)$$

where  $dS$  is the area element. The first and the second terms on the left side of the Eq. (7) are the total potential energy and total kinetic energy of the atmosphere within the space where the volume integral is taken.

When the atmosphere is changed moist adiabatically from state 1 to state 2, we have

$$\frac{C_p}{g} \int_s \int_0^{p_0} (T_{\sigma_1} - T_{\sigma_2}) dp ds = \frac{C_p}{g} \int_s \int_0^{p_0} \frac{V_2^2 - V_1^2}{2C_p} dp ds. \quad (8)$$

Eq. (8) states that within the space of saturation, the total kinetic energy of the atmosphere

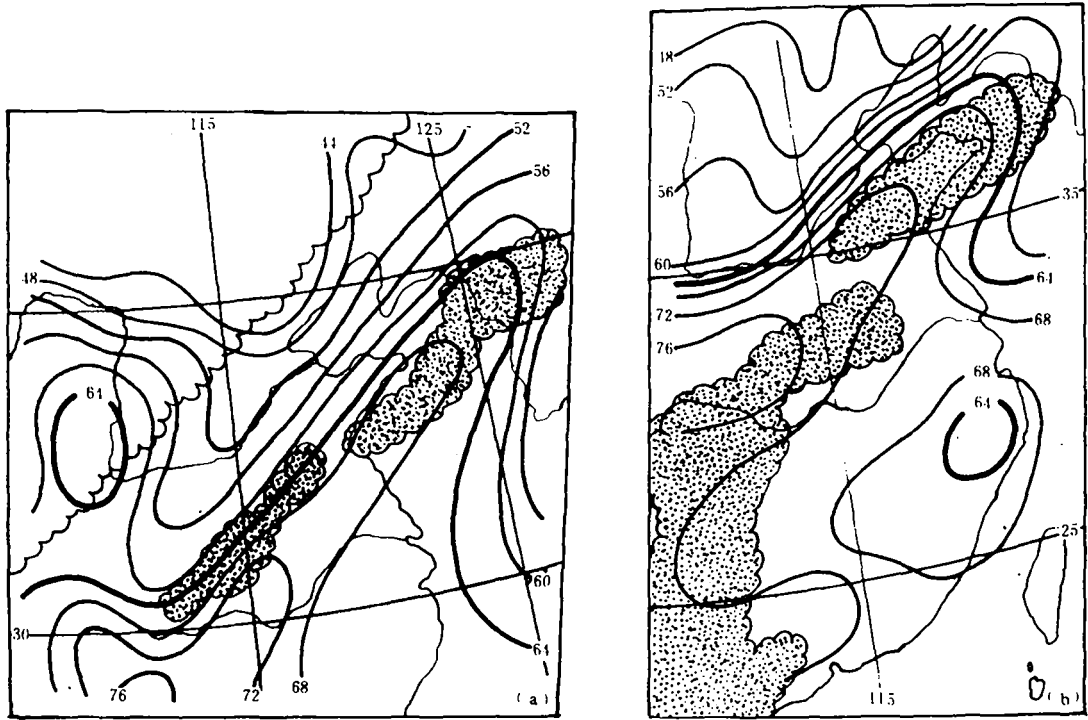


Fig. 1. The static energy situation on 850 mb and the cloud system for stationary heavy rain case. The shaded areas are the thick bright cloud belts on the satellite picture.  
 (a) On 10 August 1978, as in Fig. 1 (a). The cloud belt is on the front of energy front zone;  
 (b) As in Fig. 2, except for 30 August 1978. The cloud belt is on the axis of high static energy tongue.

may increase at the expense of the strength of the moist solenoid field, and vice versa. This characteristic of the saturated atmosphere is called moist baroclinity.

The total moist potential energy within a unit air column may be written as

$$\frac{C_p}{gP_{00}^k} \int_{\theta_{se_0}}^{\infty} p^{1+k} e^{-Lq_3/c_p T} d\theta_{se} \approx \frac{C_p}{gP_{00}^k} \int_{\theta_{se_0}}^{\infty} p^{1+k} d\theta_{se}. \tag{9}$$

Here only the first term of the series expansion of the exponent function is taken. It is permissible due to the inaccuracy in the calculation. The available moist potential energy may be written as

$$A_m \approx \frac{C_p}{gP_{00}^k} \int_s \int_{\theta_{se_0}}^{\infty} (p^{1+k} - \bar{p}^{1+k}) d\theta_{se} dS. \tag{10}$$

### III. MOIST JET

In operational and research work, moisture is generally considered as transported by large scale air currents. According to the law of conservation of total energy, the moisture may be considered as playing an active role in the formation and maintenance of the jet stream, at least to certain extent. When a moist air parcel rises and turns in the space from state 1 to state 2 moist adiabatically, and if the

moist air parcel starts from rest, we have

$$C_p T_1 + g z_1 + L q_{s1} = C_p T_2 + g z_2 + L q_{s2} + \frac{V_2^2}{2} \quad (11)$$

or

$$C_p T_{\sigma_1} = C_p T_{\sigma_2} + \frac{V_2^2}{2} \quad (12)$$

The above expression means that as the moist air parcel rises and turns in the space by observing the law of conservation of the total energy, its kinetic energy may increase at expense of its total potential energy. The acceleration of the moist air parcel on the constant  $\theta_{se}$  surface has been observed over years. English scientists have already the idea with referring to the so-called "extended parcel method"<sup>[5]</sup>. They considered some fictitious height  $z$ . We obtained it from the law of conservation of total potential energy. Although there are moist jets every day as shown on the worldwide satellite maps, it is probably much easy to study the formation and the structure of the moist jet in our country by use of the observations of the dense radiosonde network. Investigations made by synoptic analysis and numerical modelling brought fruitful results. Fig. 2 gives the structure of the moist jet over Eastern China<sup>[6]</sup>. Fig. 3 shows the development of the moist jet. Fig. 4 gives the result of a comparative study of the formation of the moist jet with five-level numerical models<sup>[7]</sup>. In spite of the crudeness of the numerical models, the important active role of the moisture in the formation of the jet is clearly shown. It is more than that we expected, as the moist jet rose and injected into the upper westerly jet, the upper westerly jet strengthened by more than 15 per cent. It means that the moisture plays an active role not only in the synoptic scale motion but also in the planetary scale motion. Future study along this direction is being undertaken by Chen and other in European Center for Medium Range Forecasts by much refined numerical models.

It may be concluded that it is not the air currents transporting the moisture but it is moist air bringing wind and moisture to where the moist air arrives.

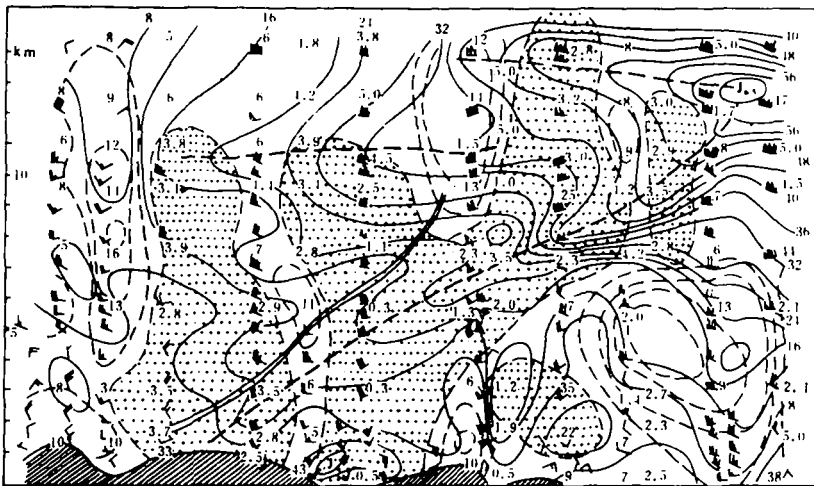


Fig. 2. Velocity field on the cross-section along the axis of the moist jet. Horizontal extent of the cross-section is about 2500 km.

Heavy dashed lines are the axes of jet streams. Double line shows the trough. Shaded areas show the space of condensation and location of precipitation.

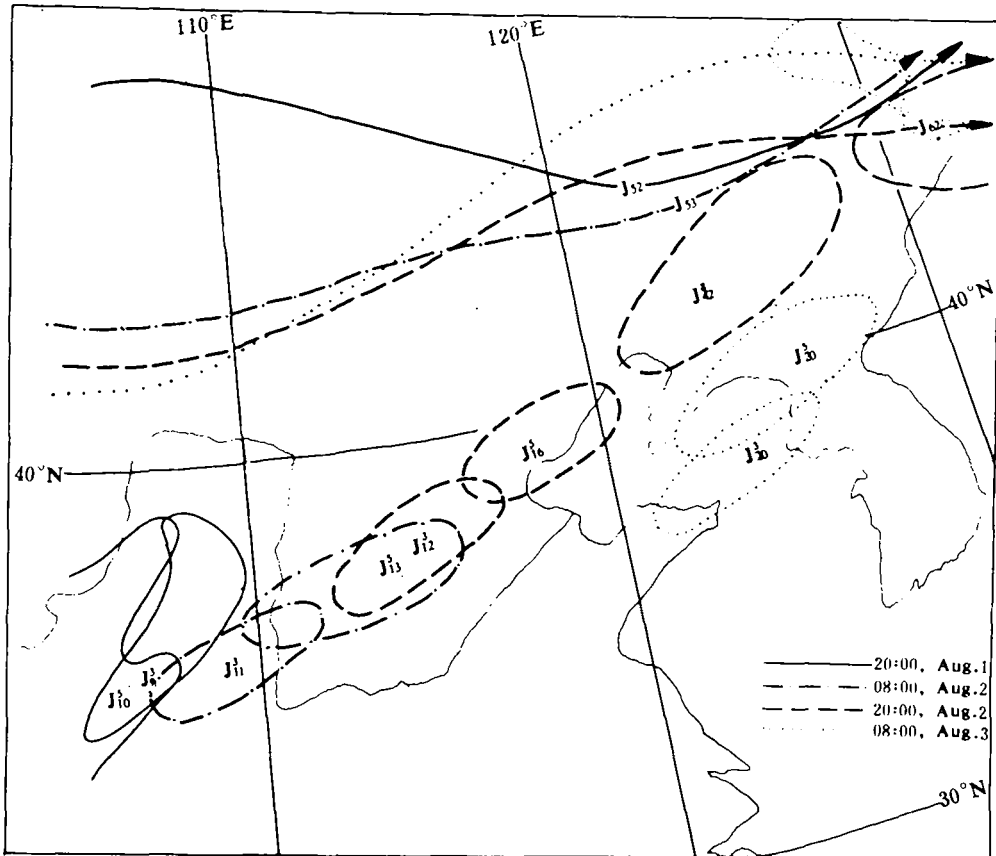


Fig. 3. Consecutive positions of the jet stream. Lower index gives the maximum wind speed in m/s. Upper index gives the height of the jet in km. Lines with arrow are the westerly jet at 12 km.

IV. MOIST BAROCLINIC INSTABILITY

According to the classical theory of baroclinic instability, the wavelength of maximum instability is about 4000 km. It is ordinarily considered as an acceptable explanation of the development of the planetary waves, although some objections have been raised by the author recently<sup>[8]</sup>.

Within the rain belt, the air may be considered as saturated and the process moist adiabatic; the thermodynamic equation may be written as

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + \omega \frac{\partial T}{\partial p} - \frac{\alpha}{C_p} \omega = -\frac{L}{C_p} \frac{dq_s}{dt} \cong -\frac{L}{C_p} \frac{\partial q_s}{\partial p} \omega \tag{13}$$

or

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = -\left( T \frac{\partial \theta}{\partial p} + \frac{L q_s}{C_p} \right) \omega = -T \left( \frac{1}{\theta_{se}} \frac{\partial \theta_{se}}{\partial p} \right) \omega = -\frac{\partial T_o}{\partial p} \omega.$$

By combination Eq. (13) with the vorticity equations on two levels, we have the same set of equations as in the classical theory of baroclinic instability, except the static stability coefficient is changed from

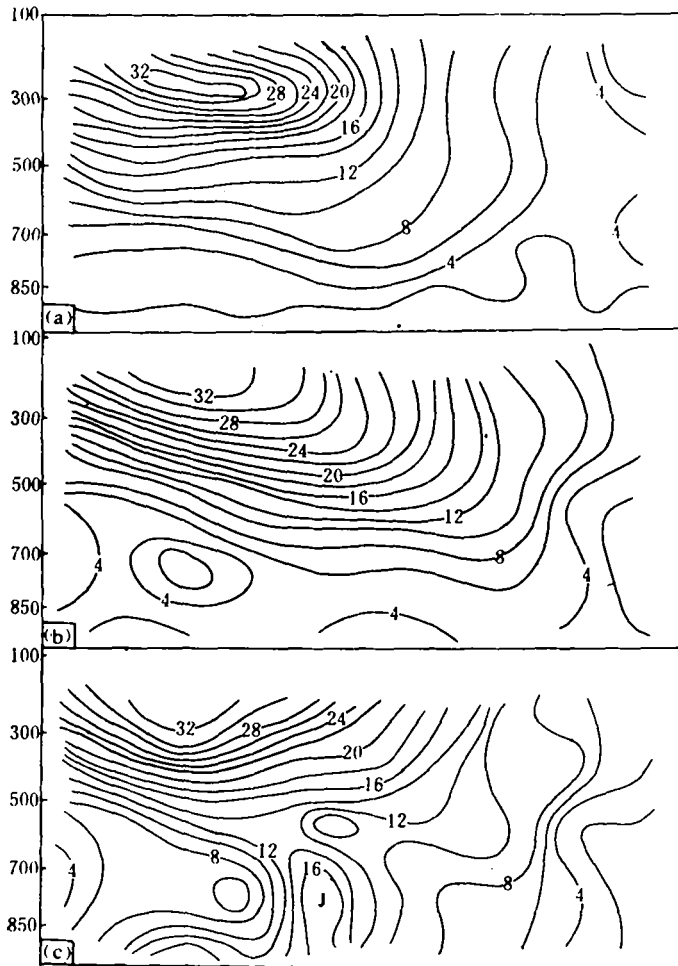


Fig. 4. Wind field on the cross section perpendicular to the precipitation belt.

(a) Observed wind field at 12Z, Aug. 2,

(b) Computed wind field by dry model at 00Z, Aug. 3,

(c) Computed wind field by moist model at 00Z, Aug. 3.

$$\sigma = -\frac{\alpha}{\theta} \frac{\partial \theta}{\partial p}$$

to

$$\sigma_m = -\frac{\alpha}{\theta_{se}} \frac{\partial \theta_{se}}{\partial p} = -\frac{\alpha}{T} \frac{\partial T}{\partial p}$$

In the space along the belt of heavy precipitation,  $\sigma_m$  is much smaller than  $\sigma$ . Sometimes,  $\sigma_m$  approaches to zero. If  $\sigma_m = 0.26$ , the wavelength of maximum instability will be less than half of that deduced from classical theory of baroclinic instability.

This conclusion is considered as the theory of moist baroclinic instability. It gives an explanation of the fast developing waves with wavelength of about 1000 km, and the small cyclones along the rain belt. It might be considered as a revised theory of Norwegian front cyclones. A comparative study between dry and moist numerical models gives affirmative results<sup>[9]</sup>. (See Fig. 5)

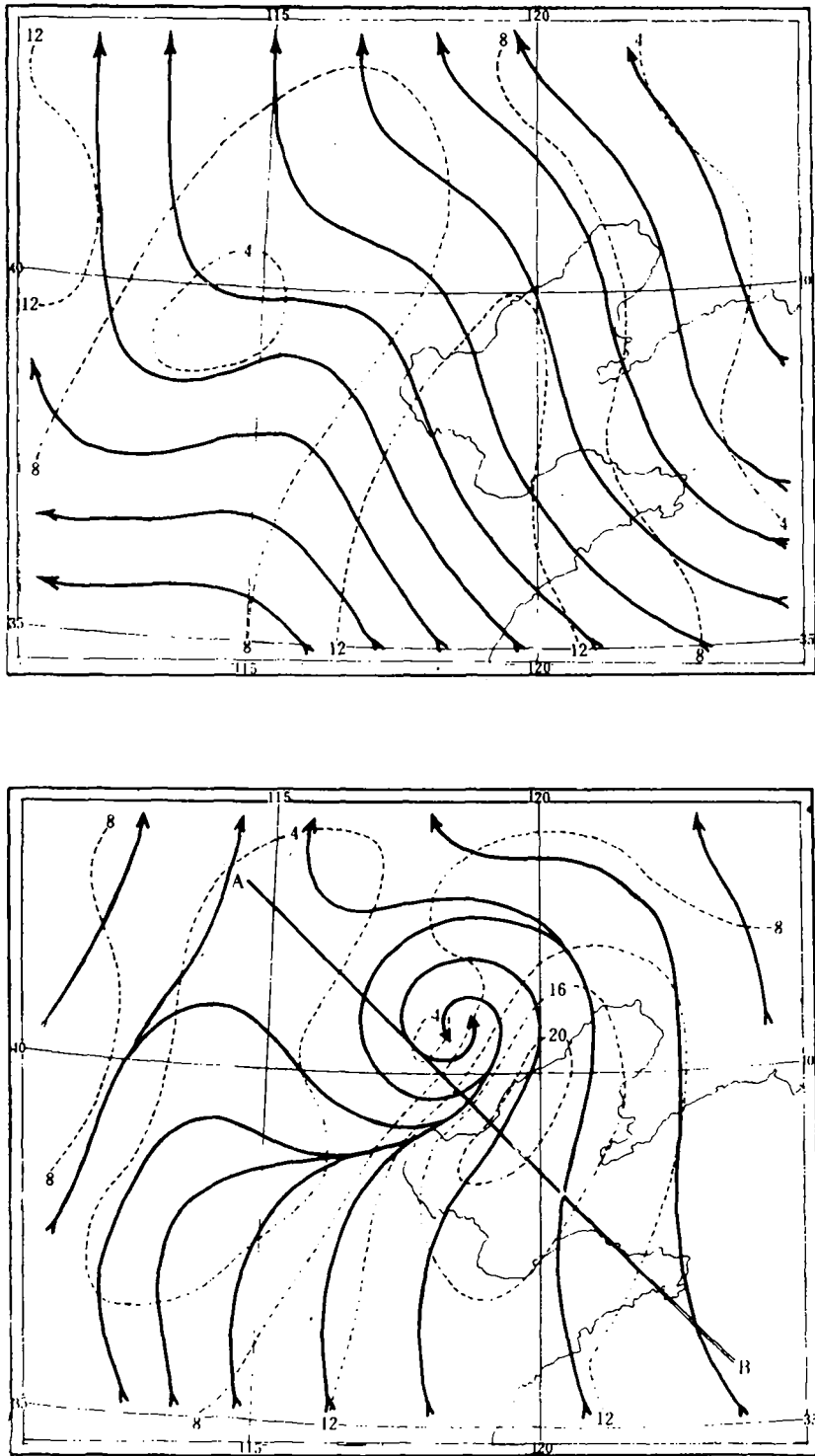


Fig. 5. Computed wind field by dry (a) and moist model (b) on surface  $\sigma=0.91$ , 12 hours from same initial conditions. Heavy lines are stream lines, dashed lines give the wind speed in m/s.

## V. MOIST TENDENCY EQUATION AND MOIST $\omega$ EQUATION

If the process is moist adiabatic, the law of conservation of moist potential vorticity may be written as follows,

$$\left(\frac{\partial}{\partial t} + \mathbf{V}_\psi \cdot \nabla\right)q = 0, \quad (14)$$

$$q = \nabla^2 \psi + f + f_0^2 \frac{\partial}{\partial p} \left( \frac{1}{\sigma_m} \frac{\partial \psi}{\partial p} \right),$$

where  $\psi$  is the geostrophic stream function.

The moist tendency equations may be written as

$$\frac{\partial}{\partial t} (\nabla^2 \psi_1 + \nabla^2 \psi_3) = -\mathbf{k} \times \nabla \psi_1 \cdot \nabla (\nabla^2 \psi_1 + f) - \mathbf{k} \times \nabla \psi_3 \cdot \nabla (\nabla^2 \psi_3 + f), \quad (15)$$

$$\frac{\partial}{\partial t} (\nabla^2 - 2\lambda^2)(\psi_1 - \psi_3) = -\mathbf{k} \times \nabla \psi_1 \cdot \nabla (\nabla^2 \psi_1 + f) + \mathbf{k} \times \nabla \psi_3 \cdot \nabla (\nabla^2 \psi_3 + f) + 2\lambda^2 \mathbf{k} \times \nabla \psi_2 \cdot \nabla (\psi_1 - \psi_3), \quad (16)$$

where  $\lambda_m^2 = f_0^2 / \sigma_m (\Delta p)^2$ .

The moist  $\omega$  equation may be written as

$$(\nabla^2 - 2\lambda^2)\omega_2 = \frac{f_0}{\Delta p \sigma_m} \{ \nabla^2 [\mathbf{k} \times \nabla \psi_2 \cdot \nabla (\psi_1 - \psi_3)] - \mathbf{k} \times \nabla \psi_1 \cdot \nabla (\nabla \psi_1 + f) + \mathbf{k} \times \nabla \psi_3 \cdot \nabla (\nabla^2 \psi_3 + f) \}. \quad (17)$$

## VI. THE PLANETARY WAVES IN MOIST BAROCLINIC ATMOSPHERE

From the well known set of equations in the classical theory of baroclinic instability, two sets of equations may be obtained,

$$A_1'' + \left( \frac{\beta}{U_1 - C} - \frac{U_3 - C}{U_1 - c} \lambda^2 - k_1^2 \right) A_1 = -\lambda^2 B_1, \quad (18)$$

$$B_1'' + \left( \frac{\beta}{U_3 - c} - \frac{U_1 - c}{U_3 - c} \lambda^2 - k_1^2 \right) B_1 = -\lambda^2 A_1,$$

and

$$A_{11}'' + \left( \frac{\beta}{U_1} - \frac{U_3}{U_1} \lambda^2 - k_{11}^2 \right) A_{11} = -\lambda^2 B_{11}, \quad (19)$$

$$B_{11}'' + \left( \frac{\beta}{U_3} - \frac{U_1}{U_3} \lambda^2 - k_{11}^2 \right) B_{11} = -\lambda^2 A_{11},$$

where  $A_1, B_1$  are the amplitudes of the upper-level moving and stationary disturbances respectively,  $A_{11}, B_{11}$  are the amplitudes of the low-level moving and stationary disturbances respectively,  $A_1', B_1', A_{11}', B_{11}'$  are their secondary derivatives with respect to  $y$ , and

$$\lambda^2 = f_0^2 / \sigma (\Delta p)^2$$

may be named as mutual forcing parameter of the upper and lower level disturbances. Eqs. (18) and



(19) may be named as the equations of mutual forcing of the upper and lower level disturbances. If the air is saturated,  $\lambda_m$  should be used instead of  $\lambda$ .

The solution and discussion of the set of Eq. (18) leads to certain worth-noting results including a revised and generalized theory of baroclinic instability<sup>[9]</sup>.

Since the disturbance introduced is not particularly identified, it may also show the dynamic influence of the high land as Tibet Plateau to the general circulation.

If  $\lambda_m$  is used instead of  $\lambda$ , it might give some dynamic explanation of the fast developing and heavy rain-bearing synoptic systems over Eastern China in summer.

The study of the dynamics of moist baroclinic atmosphere is just at the early stage of development and we expect much fruitful results in the coming years.

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