CHARACTERISTICS OF ATMOSPHERIC EXTINCTION-TO-BACKSCATTERING RATIO IN RUBY LIDAR MEASUREMENTS

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Received October 19, 1983

ABSTRACT

In this paper the dependence of the ratio k (the atmospheric extinction-to-backscattering ratio) upon the aerosol refractive index and size distribution is theoretically studied in ruby lidar measurements. An empirical expression for the ratio k is then established. Moreover the effect of molecular scattering on the ratio k is discussed.

The conventional lidar equation for the atmospheric sounding can be usually written as

$$V(R)=\frac{C}{R^2}\beta(R)\exp\left[-2\int_o^R\sigma(R')dR'\right], \tag{1}$$

where, $V(R)$is the lidar return signal from the atmosphere at the distance R: $\beta(R)$ and $\sigma(R)$ are the atmospheric backscattering coefficient and the atmospheric extinction coefficient respectively; C is a constant of lidar system.

Both $\sigma(R)$ and $\beta(R)$ in Eq. (1) are two unknowns. Therefore, a new variable, the ratio $k(R)=\frac{\sigma(R)}{\beta(R)}$ is generally introduced for obtaining the formal solution[1]

$$\sigma(R)=\frac{k(R)V(R)R^2}{\left[C-2\int_0^R V(R')k(R')R'^2dR'\right]}. \tag{2}$$

In a turbid atmosphere without clouds, $k(R)$ is obviously independent of the total concentration of aeresols and it mainly depends on the size distribution of aerosols and their refractive index[2-3].

In case of the strong mixing in the atmosphere, the spacial distribution of atmospheric aerosols is fairly homogeneous. As a result, $k(R)$ approximately remains constant for one shoot of lidar [1] and Eq. (2) can be reduced to

$$\sigma(R)=\frac{V(R)R^2}{\left[C/k-2\int_0^R V(R')R'^2dR'\right]}. \tag{3}$$

As a matter of fact, the assumption of homogeneous distribution in the real atmosphere is difficult to meet. Our further analyses for variations of the ratio k with the aerosol size distribution and refractive index indicate that, the sensitivity of the ratio k to the size distribution of aerosols is closely associated with their refractive index. Based on this fact, an empirical expression for the ratio k is formulated. All the results obtained in this paper can be applied to ruby lidar measurements.

I. VARIATIONS OF THE RATIO k WITH SIZE DISTRIBUTION OF AEROSOLS AND THEIR REFRACTIVE INDEX

Without consideration of molecular scattering, the ratio k for atmospheric aerosols can be written as

$$k_A = \sigma_A / \beta_A$$

and the extinction coefficient σ_A and the backscattering coefficient β_A can be respectively expressed as

$$\sigma_A = \int_0^\infty \pi r^2 Q_{ex}(r,\lambda,m) n(r) dr,$$

$$\beta_A = \int_0^\infty \pi r^2 Q_b(r,\lambda,m) n(r) dr ,$$

where, $n(r)$ is the size distribution of aerosols; $Q_{ex}(r,\lambda,m)$ the extinction efficiency; $Q_b(r,\lambda,m)$ the backscattering efficiency; λ laser wavelength; and m the complex refractive index of aerosols with m_R and m_I being its real and imaginary parts respectively.

In order to examine the sensitivity of the ratio k_A to size distributions, altogether seven various size distributions listed in Table 1 are accepted for computation. An averaged ratio $\bar{k}_A$ over N size distributions and its standard deviation are introduced:

$$\bar{k}_A = \frac{1}{N} \sum_{i=1}^{N} k_{Ai} , \tag{4}$$

$$\delta_A = \sqrt{\sum_{i=1}^{N} (k_{Ai} - \bar{k}_A)^2 / N} \Big/ \bar{k}_A . \tag{5}$$

Here, the ratio k_{Ai} is respectively computed for the ith size distribution and $N = 7$. The size distribution is taken as the Junge distribution for $i = 1, 2, 3, 4, 5$; the Deirmendjian continental haze distribution for $i = 6$; and the atmospheric column aerosol distribution obtained by remote sensing measurements[4] in Beijing for $i = 7$. The four curves in Fig. 1 show the results computed from the above seven size distributions with different refractive indices at the operating wavelength of ruby lidar $\lambda = 0.6943\mu$m. In consideration of facts that the imaginary part of refractive index of atmospheric aerosols ranges from 0.001 to 0.1 as pointed out by Gerber et al.[5] and that it generally falls in the range 0.005—0.02 with the exception of some special constituents as shown by Paltridge et al.[6], the complex refractive index of atmospheric aerosols is taken to be the real part with 1.33, 1.40, 1.50 and 1.60 and the imaginary part within 0- 0.1.

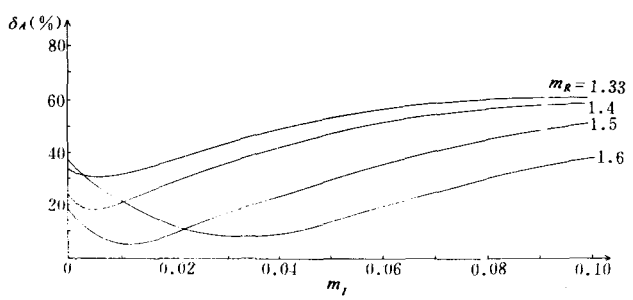

Fig. 1. The relationship between standard deviation and refractive index.

Table 1. Various Aerosol Particle Size Distributions

i	$n(r)$
1	cr^{-3}
2	$cr^{-3.5}$
3	cr^{-4}
4	$cr^{-4.5}$
5	cr^{-5}
6	$cr^2\exp(-15.12\,r^{0.5})$
7	$1.24\times10^{14}/\left[1+\left(\frac{r}{0.023}\right)^{4.2}\right]+7.2\times10^4\exp\left[-1.5\left(\ln\frac{r}{2.1}\right)^2\right]$

It is found from Fig. 1 that the sensitivity of the ratio k_A to size distribution is closely associated with the refractive index. When the imaginary part of refractive index either vanishes effectively or has greater magnitudes, the standard deviation δ_A is correspondingly greater, i.e. the ratio k_A is more sensitive to the size distribution. However, δ_A does not exceed 70% in case of the refractive index having the above-mentioned values. When $m_R = 1.5$ and $0.005 \leqslant m_I \leqslant 0.02$, then $\delta_A \leqslant 9\%$ and the variation of k_A with size distribution is very small. While the relative humidity of air increases, the refractive index of aerosol particles would gradually approach to that of water $m = 1.33 - 0i$ owing to the hygroscopic effect of particles. In such a case it is expected that the ratio k_A becomes sensitive to variations of the aerosol size distribution.

Shown in Fig. 2 is the dependence of the averaged $\bar{k}_A$ on the real and imaginary parts of refractive index. It is clearly seen that, $\bar{k}_A$ is very sensitive to variations of both parts of aerosol refractive index in such a way that k_A decreases with increase in real part of refractive index and increases with increase in imaginary part. This variation is as high as one order of magnitude, and much more than the variation caused by the size distribution. Therefore, it is suggested that more information on the refractive index of atmospheric aerosols is involved in the ratio k_A.

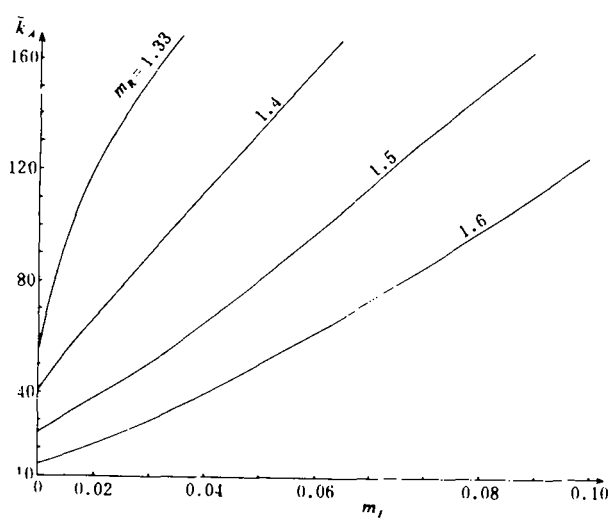

Fig. 2. The relationship between averaged $\bar{k}_A$ and refractive index of aerosols

II. AN EMPIRICAL EXPRESSION OF THE RATIO k_A FOR JUNGE DISTRIBUTION OF AEROSOLS

It is clearly seen from the above results that, the dependence of the ratio k_A on m_R, m_I and the power of Junge distribution v has an evident regularity. Using numerical computation and curve fitting, we have established an empirical expression for the ratio k_A of Junge aerosol distribution as follows

$$k_A^* = g_1(m_R,v) + g_2(m_R, v)m_I + g_3(m_R, v)m_I^2, \tag{6}$$

where

$$g_1 = (-24 + 25.2v - 2.8v^2)\left(\frac{1.5}{m_R}\right)^{-13.5+19.5v-3.8v^2} \times \left(\frac{1.54}{m_R}\right)^{2v^2-12.6v+19}$$

$$g_2 = (12590 - 7500m_R)[1 + (3-v)f^*(m_R)];$$

$$g_3 = 104815 - 63036v + 9196v^2 - 5.624(m_R - 1.5)(616430 - 538600v + 101300v^2)/m_R^4$$
$$+ 4300(v^2 - 5.2v + 6.72)(m_R^6 - 1.57m_R^5);$$

and

$$f^*(m_R) = \begin{cases} 12.55 - 8.1m_R, & \text{for } m_R \leqslant 1.55; \\ 0, & \text{for } m_R > 1.55. \end{cases}$$

If the true value of the ratio is k_A, then the accuracy of expression (6) can be measured in such a ER as

$$ER = \frac{|k_A - k_A^*|}{k_A} \tag{7}$$

The values of ER shown in Fig. 3 are calculated under conditions of various powers of Junge distribution and various refractive indices of aerosols. The results in Fig. 3 indicate that expression (6) is suitable for most of cases. For example, when $1.46 \leqslant m_R \leqslant 1.6$, $0.001 \leqslant m_I \leqslant 0.04$ and $2 \leqslant v \leqslant 4$, then $ER < 10\%$ and when $1.46 \leqslant m_R \leqslant 1.56$, $0.001 \leqslant m_I \leqslant 0.05$ and $v = 3$, then $ER < 3.2\%$. Only when $m_I > 0.05$, the accuracy of expression (6) becomes unsatisfactory.

III THE EFFECT OF MOLECULAR SCATTERING

By taking account of molecular scattering, the ratio k would have the form

$$k = \frac{\sigma_m + \sigma_A}{\beta_m + \beta_A}, \tag{8}$$

where σ_m and β_m are the molecular extinction coefficient and the molecular backscattering coefficient respectively. It is easy to prove that Eq. (8) can be rewritten as

$$k = k_m f(\alpha, x), \tag{9}$$

where

$$\alpha = k_m/k_A, \quad x = \frac{\sigma_A}{\sigma_m} \quad \text{and} \quad k_m = \frac{\sigma_m}{\beta_m} \tag{10}$$

According to the Rayleigh scattering theory, k_m turns out to be a constant, i.e. $k_m = \frac{\sigma_m}{\beta_m} = \frac{8\pi}{3}$.

The function $f(\alpha,x)$ in Eq. (9) has the form

$$f(\alpha,x) = \frac{1+x}{1+\alpha x}, \tag{11}$$

Obviously, x is a physical parameter characterizing the atmospheric turbidity. Values of α are computed for various Junge distributions and the results are listed in Table 2.

Table 2. Values of v for Various Junge Distributions ($\lambda = 0.6943\mu m$, $0.05 \leqslant r \leqslant 6\mu m$)

v \ m	$1.33-0i$	$1.5-0i$	$1.5-0.01i$	$1.5-0.05i$	$1.5-0.1i$	$1.6-0i$	$1.6-0.001i$	$1.6-0.05i$	$1.7-0i$
2	0.248	0.494	0.229	0.063	0.029	1.234	1.046	0.110	2.87
3	0.160	0.327	0.213	0.094	0.062	0.625	0.573	0.135	1.00
4	0.021	0.287	0.234	0.146	0.107	0.378	0.365	0.173	0.461

It is found from Table 2 that in the range of aerosol refractive index shown by Paltridge et al.[6] the values of α are generally less than unit. In most cases α varies within 0.1—1. When the imaginary part of refractive index m_i is far less than unit, the relationship among these factors indicates that, the less the v is and the wider the size distribution is, the less the ratio k_A is and the more the values of α are. On the other hand, when m_i is greater, the contrary is the case, viz. the less the v is, the more the ratio k_A is and the less the values of α are.

Derivating $f(\alpha,x)$ with respect to x, we have obtained

$$\frac{df}{dx} = \frac{1-\alpha}{(1+\alpha x)^2}. \tag{12}$$

It easily leads to $\frac{df}{dx} > 0$ when $\alpha < 1$. Thus, the function $f(\alpha,x)$ or the ratio k decreases with the decrease in the atmospheric turbidity. However, when $\alpha > 1$, the contrary is the case, i.e. $f(\alpha,x)$ or k increases with the decrease in the atmospheric turbidity. The remained is $\alpha = 1$ leading to $\frac{df}{dx} = 0$. That means the function f is independent of the atmospheric turbidity and remains unit. Therefore, $k \equiv k_m$.

The values of the function $f(\alpha,x)$ are given in Fig. 4. Under common conditions, the decrease of aerosol concentration with increasing heights is faster than that of molecular density. Consequently, the values of x decrease with increasing heights. If σ_A is calculated by use of horizontal visibility (V_i) relationship $\sigma_A = 3.4/V_i$ at the earth's surface, then the values of x vary within 15—755 corresponding to a range of $V = 1$—50 km. It can be seen from Fig. 4 that, when $0.01 \leqslant \alpha \leqslant 5$, the function $f(\alpha,x)$ approaches to a constant for $x \geqslant 100$. On the contrary, when α has greater values, $f(\alpha,x)$ approaches to a constant at smaller values of x. This implies that only if it is at higher altitudes, the effect of molecular scattering is taken into consideration. However, when $\alpha = 0.025$, the function $f(\alpha,x)$ remarkably varies with x even though $x > 50$. This indicates that the effect of molecular scattering must be taken into consideration at lower altitudes. From the above mentioned it follows that the effect of molecular scattering on the ratio k associates not only with the concentration of atmospheric aerosols, but also with the values of α which depend on the aerosol size distribution and refractive index.

Below we deal with the effect of molecular scattering on the retrieval of extinction coefficient profile on condition that the ratio k_A remains unvariable with increasing heights (h). If the molecular scattering is taken into account and the ratio k varies with increasing heights, then a solution, denoted as $\sigma(R)$, can be obtained through Eq. (2). If the effect of molecular scattering on the ratio k is neglected and the ratio k remains constant, then a solution, denoted as $\sigma^*(R)$, can be obtained by

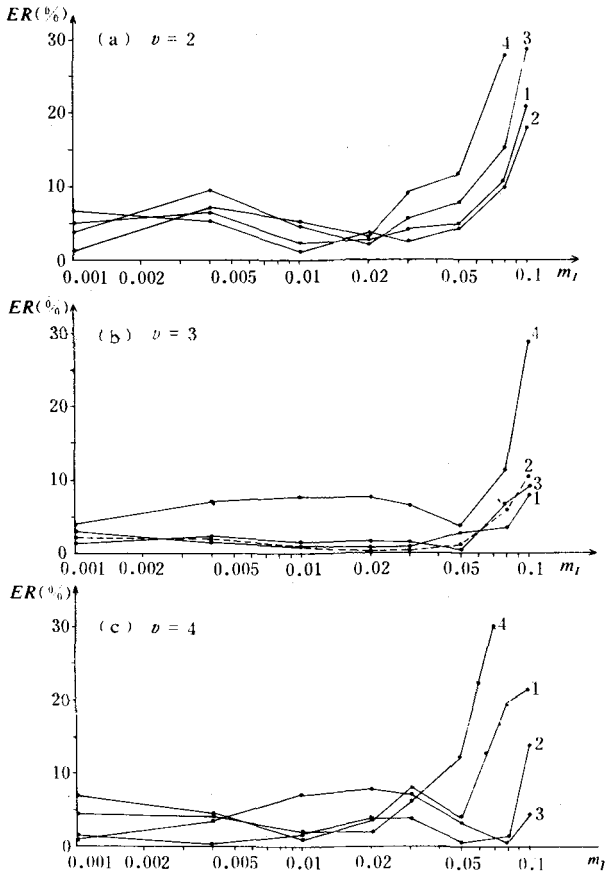

Fig. 3. Errors of empirical expression (6). The curves denoted by 1, 2, 3 and 4 respectively represent the errors for m_R= 1.46,1.50,1.56 and 1.60.

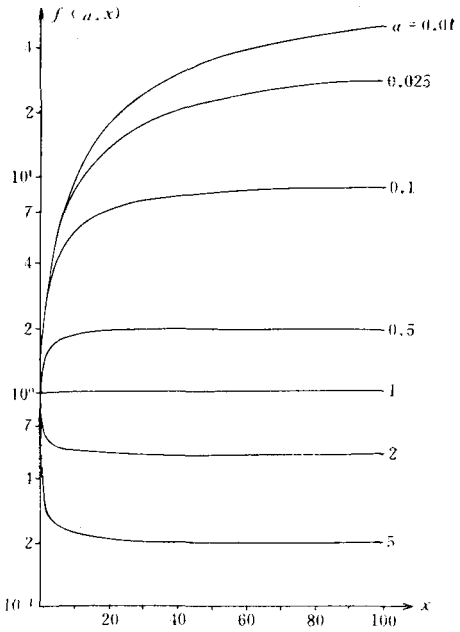

Fig. 4. The curves of function $f(\alpha,x)$.

substituting the ratio k on the ground (written as k_0) into Eq. (3). Under normal conditions that $0<\alpha<1$ and $\frac{dx}{dh}<0$, owing to $k(h)\leqslant k_0$ it is easily inferred that

$$T=\frac{\sigma^*(R)}{\sigma(R)}=\frac{V(R)R^2k_0}{V(R)R^2k(R)}\times\frac{C-2\int_0^R V(R')k(R')R'^2dR'}{C-2k_0\int_0^R V(R')R'^2dR'}$$

$$\geqslant\frac{k_0}{k(R)}=f^{(0)}(x,\alpha)/f^{(R)}(x,\alpha)=Rf\geqslant 1. \tag{13}$$

Similarly, for $\alpha>1$ and $\frac{dx}{dh}<0$, we infer that

$$T=\frac{\sigma^*(R)}{\sigma(R)}\leqslant\frac{k_0}{k(R)}=Rf\leqslant 1. \tag{14}$$

In Eqs. (13) and (14), Rf is the ratio of k on the ground to at the range R and T is the ratio of

$\sigma^*(R)$ to its exact solution. Under conditions of $0<\alpha<1$ and $\frac{dx}{dh}<0$, we infer $T\geqslant Rf\geqslant 1$. This implies that the solution $\sigma^*(R)$ is systematically overestimated in view of neglecting the molecular scattering. On the other hand, for $\alpha>1$ and $\frac{dx}{dh}<0$, we have $T\leqslant Rf\leqslant 1$ with the result that the solution $\sigma^*(R)$ is systematically underestimated. These properties can be clearly seen from Tables 3 and 4, in which E distribution represents the Elterman model [7] and AP distribution the averaged profile of extinction coefficient during fine weather of summer in Beijing[1].

Table 3. The Values of T and Rf for E Distribution

α	h(km)	0	1	2	3	4	5	6	7
0.213	Rf	1.00	1.10	1.28	1.62	2.08	2.51	3.41	3.84
	T	1.00	1.10	1.31	1.70	2.23	2.90	3.78	4.36
0.094	Rf	1.00	1.22	1.65	2.44	3.50	4.83	6.53	7.57
	T	1.00	1.24	1.75	2.71	4.13	6.00	8.53	10.41
1.0002	Rf	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	T	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2.87	Rf	1.00	0.980	0.944	0.877	0.787	0.672	0.526	0.438
	T	1.00	0.979	0.937	0.842	0.720	0.590	0.504	0.401

Table 4. The Values of T and Rf for AP Distribution

α	h(km)	0	1	2	3	4	5	6	7
0.213	Rf	1.00	1.19	1.27	1.37	1.55	1.68	1.47	1.39
	T	1.00	1.21	1.31	1.46	1.71	1.83	1.69	1.63
0.094	Rf	1.00	1.46	1.65	1.87	2.36	2.52	2.14	1.94
	T	1.00	1.53	1.83	2.24	3.02	3.44	3.12	3.04
1.002	Rf	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	T	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2.87	Rf	1.00	0.963	0.949	0.931	0.893	0.881	0.911	0.926
	T	1.00	0.960	0.942	0.920	0.878	0.863	0.889	0.900

It can be seen from Tables 3 and 4 that, when $\alpha<1$, then $T\geqslant 1$ with the result that the neglect of molecular scattering will lead to the systematically overestimated solution σ^*. For $\alpha=0.213$ and $h\leqslant 2$km, the error of solution σ^* is less than 31% which is basically reasonable. However, when $h>2$ km, the error of solution σ^* obviously increases with decreasing values of k. For example, we have $Rf=3.84$ at $h=7$ km in the E distribution. In this case, the ratio k approximately decreases by a factor of 4, thus $T=4.36$ and the solution σ^* is overestimated by 336%. At the same height in the AP distribution, the solution $\sigma^*$is also overestimated by 63%. When $\alpha=0.094$ and $0\leqslant h\leqslant 7$ km, the solution σ^* can be overestimated by one order of magnitude (the ratio k correspondingly decreases by a

factor of 4 and more) for the E distribution and by a facror of 3.44 for the AP distribution, in which the solution σ^* also has an error of 53% even though at $h=1$ km. In one word, when the absorption of aerosols is stronger and the value of α is smaller, the effect of molecular scattering will become considerable.

Shown in Fig. 5 are four curves, which respectively depict the E distribution, AP distribution, and two solutions σ^* for $\alpha=0.213$ and 0.094. From Fig. 5 the following features can be seen. The AP distribution is well consistent with the E distribution within 2 km, but above 3 km the former is systematically greater than the latter. On the other hand, when $\alpha=0.094$, the solution σ^* above 3 km in E distribution is systematically greater than the AP distribution. However, when $\alpha=0.213$, the solution σ^* in E distribution is in good agreement with the AP distribution. From here we see that neglecting the effect of molecular scattering on the ratio k enables the retrieved profile to have greater errors even up to one order of magnitude.

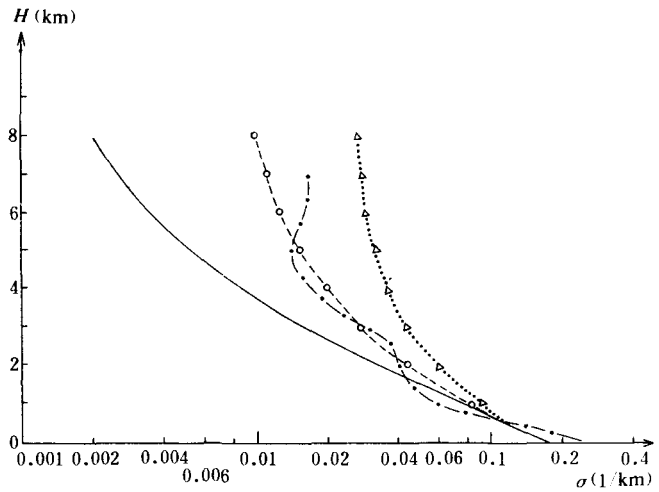

Fig. 5 The solutions σ^* at different values of α.
E distribution; AP distribution; , $\alpha=0.213$ and E distribution, Δ, $\alpha=0.094$ and E distribution.

If we use an average k over several sublayers instead of the horizontal k_0 on the ground for solving the lidar equation under the assumption that the atmosphere is stratified homogeneously[18], we will find that taking no account of molecular scattering also produces certain errors.

IV. CONCLUSIONS

The conclusion can be drawn from all the above-mentioned facts as follows. The atmospheric extinction-to-backscattering ratio k at the wavelength of ruby lidar 0.6943 μm is associated with the aerosol size distribution and refractive index. An empirical expression for the ratio k of Junge aerosol distribution has been derived from theoretical computations. The further analyses indicate that the sensitivity of the aerosol ratio k_A to its size distribution is closely dependent on the aerosol refrative index. Futhermore, the dependence of the ratio k_A on the aerosol refractive index is much more sensitive than that on the areosol size distribution. The effect of molecular scattering on the atmospheric ratio k_A depends on not only the aerosol concentration, but also the size distribution and refractive index.

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