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DYNAMICS OF LATERAL BOUNDARY MESO-SCALE JET IN THE OCEAN AND ATMOSPHERE

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ABSTRACT

By using an ageostrophic shallow water model, it is pointed out that a kind of lateral boundary meso-scales jet can be established near the plateau or coast. The characteristic width of this kind of jet is proportional to the scale of $L_c = L_0(C_0/V_g)$, where $L_0 = C_0/f$ is the radius of Rossby deformation, $C_0 = (g^*H)^{\frac{1}{2}}$ the speed of gravity wave and g* the reduced gravity. In general, L_c is of the order of one hundred kilometers and tens of kilometers in the atmosphere and in the ocean respectively. The large-scale geostrophic current is an important background condition for forming this kind of jet. From this view point it seems that this kind of atmospheric meso-scale jet only occurs in late spring and summer in the eastern part of Asia, because there is a large-scale south monsoon over there. For the ocean, this kind of meso-scale jet seems to be a semi-persistant system and not to show a significant seasonal variation, and it can be established on both sides of the ocean.

I. INTRODUCTION

Jet is one of the spectacular phenomena in the atmosphere and ocean, and the dynamics of its development is also one of the most interesting problems in geophysical fluid dynamics. Many kinds of jets with various time-scale as well as space-scale have been revealed in both mediums. The Kuroshio and Gulf streams in the ocean and the jet of westerlies in the upper atmosphere are well known. Another sort of jet in the atmosphere is the lower-level jet associated with the southeast or southwest monsoon which is an important system for the rainfall as well as the rainbelt in late spring and summer in the eastern part of China[1]. This jet usually occurs on the eastern side of mountains and plateaus, particularly, in Asia continent. All of these jets are large-scale systems with several thousands of kilometers in the axial direction of stronger current, and the scales of width are thousand kilometers and hundred kilometers in the atmosphere and in the ocean respectively. Many scientists made efforts in investigating the formation and development of these large-scale jets in the past thirty years. For oceanic jet stream, Munk[2], Stommel[3] and Charney[4] have suggested theories, in which, probably, Charney's inertial boundary theory is quite attractive. On the other hand, all of these large-scale jets can be simulated by the atmospheric and oceanic GCM[5,6]. It is not the aim, therefore, to study these large-scale jets in this paper. But the large-scale boundary jet in the ocean and the lower-level jet in the atmosphere are considered as the important background conditions for forming the meso-scale lateral boundary jet investigated here.

In general, the time and space scales of meso-scale jets are respectively one order smaller than those of the large-scale jets both in the atmosphere and ocean. The atmospheric meso-scale jet usually connects with the severe convective activity[7,8], therefore, it is worth studying the problem in the practical sense. According to the characters and the conditions of formation, various types of meso-scale jets are distinguishable in the atmosphere. There is one kind of interesting meso-scale jet that frequently occurs in the eastern parts of continents. It may be called as lateral boundary meso-scale jet.

This kind of jet has an oscillating character with a period of several hours [9], and the space-scales are several hundreds of kilometers in the axial direction and about one hundred kilometers in width. Another interesting fact of this jet is that, in general, it is located under the temperature inversion with a thickness of one to two kilometers. In the ocean, the observational data about the meso-scale jet stream are very rare compared with the atmospheric cases. However, it is well-known that the wind-driven coastal jet only has the radius of Rossby deformation in width, i.e., several ten to one hundred kilometers[10]. The interesting problem is whether this kind of coastal narrow current still exists without the wind-driven action. In other words, is there any intrinsic mechanism for forming the lateral boundary jet?

In this paper, the dynamics of this kind of lateral boundary meso-scale jet both in the atmosphere and ocean has been investigated by a linear shallow water model with the large-scale jet stream considered as an important background condition. The result shows that a sort of lateral boundary meso-scale jet with the characteristic width in proportion to the radius of Rossby deformation and in inverse proportion to the large-scale geostrophic current comes into existence closing to the edge of plateau or coast.

II. MODEL

The y-axis is taken along the edge of plateau or coast with the northward direction as positive. This lateral boundary is vertical both in the atmospheric and oceanic models, i.e., there are no mountain slope and continental shelf slope to take into account. For the oceanic model the bottom under consideration is the top of the thermocline, for the atmospheric model the upper boundary is the top of lower temperature inversion.

The depth in model is $H(x)$ and the large-scale background stream is geostrophic current

$$V_g=\frac{g^*}{f}\frac{\partial H}{\partial x}, \tag{1}$$

where $g^*=(\delta\rho/\rho)g$ is the reduced gravity, $\delta\rho$ is the density difference across the inversion interface in the atmospheric model and the density difference across the mixed layer and thermocline in the oceanic model, f is the Coriolis parameter. Considering the fact that the characteristic scale in x-direction of V_g is, at least, one order larger than that of the horizontal scale of meso-scale jet in question, for the sake of simplicity in mathematical treatment a uniform V_g in x-direction is assumed, the speed of gravity wave is given by

$$C_0=(g^*\overline{H})^{\frac{1}{2}}, \tag{2}$$

where $\overline{H}$ is the mean value of H.

The ageostrophic velocity in the atmospheric meso-scale jet usually reaches about 5 m/s, but it is much smaller than the velocities of large-scale jet, 15-20 m/s. As a preliminary study, the linearization of the equations of shallow water model is used. Putting $u=u'$, $v=V_g+v'$ and $h=H+h'$, we obtain the basic equations

$$\frac{\mathrm{D}u'}{\mathrm{D}t}=-g^*\frac{\partial h'}{\partial x}+fv', \tag{3}$$

$$\frac{\mathrm{D}v'}{\mathrm{D}t}=-g^*\frac{\partial h'}{\partial y}-fu', \tag{4}$$

and

$$\frac{\mathrm{D}h'}{\mathrm{D}t}+H\left(\frac{\partial u'}{\partial x}+\frac{\partial v'}{\partial y}\right)+\frac{f}{g^*}V_g u'=0, \tag{5}$$

where

$$\frac{\mathrm{D}}{\mathrm{D}t}=\frac{\partial}{\partial t}+V_g\frac{\partial}{\partial y}\,, \tag{6}$$

and u' and v' are the perturbing velocities in the x- and y-directions respectively. Elimilating variables v' and h' ,we have a single equation for u', i.e.,

$$\left[\frac{\mathrm{D}^2}{\mathrm{D}t^2}-C_0^2\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)-2fv_g\frac{\partial}{\partial x}+f^2\right]\frac{\mathrm{D}^2u'}{\mathrm{D}t^2}-f^2V_g^2\frac{\partial^2u'}{\partial y^2}=0. \tag{7}$$

III. EIGENVALUE PROBLEM

Writing the solution of Eq. (7) in the form of

$$u'=\mathrm{Re}\ U(x)\exp\left(-\frac{fV_g}{C_0^2}\ x\right)\exp[in\ (y-ct)], \tag{8}$$

we obtain the equation for $U(x)$, i.e.

$$\frac{d^2U}{dx^2}+\left[\left(n^2C_\pm^{*2}+\left(\frac{fV_g}{C_0^2}\right)^2\frac{1}{C_\pm^{*2}}\right)-\left(n^2-\frac{f^2}{C_0^2}+\left(\frac{fV_g}{C_0^2}\right)^2\right)\right]U=0, \tag{9}$$

where

$$C_\pm^*=\frac{V_g-c}{C_0}\,. \tag{10}$$

Here the subscripts "+" and "−" express $V_g>c$ and $V_g<c$ respectively.

Under the circumstance that the height of atmospheric temperature inversion is lower than that of mountains or plateaus, the lateral boundary conditions for both mediums are as follows:

$$x=0, \qquad U=0, \tag{11}$$

and

$$x=x_0, \qquad u=0, \tag{12}$$

where x_0 is the distance from the plateau or coast. Thus, the problem is to solve the eigenvalue problem of the homogeneous Eq. (9) under the homogeneous boundary conditions (11) and (12). Obviously, if the value in the bracket of Eq. (9) satisfies

$$m^2\equiv\left(\frac{M\pi}{x_0}\right)^2=n^2\left[\left(C_\pm^{*2}+L^{*2}\frac{V_g^{*2}}{C_\pm^{*2}}\right)-(1+L^{*2}+V_g^{*2}L^{*2})\right]>0, \tag{13}$$

the problem has the non-trivial solutions. In Eq. (13)

$$V_g^*=V_g/C_0, \qquad L^*=L/L_0, \tag{14}$$

and $L=1/n$ is the wavelength in y-direction, $L_0=C_0/f$ the Rossby deformation.

Condition (13) is satisfied in the following cases:

(a) If $C_\pm^{*2}$ is very large, then we have

$$C_\pm^{*(1)}=(1+L^{*2}+L^{*2}V_g^{*2})^{\frac{1}{2}}. \tag{15}$$

Clearly, $C_\pm^{*(1)}$ is the modified inertial-gravitational wave.

(b) If $C_{\pm}^{*2}$ is very small, then we have

$$C_{\pm}^{*(2)}=\frac{L^{*}V_{g}^{*}}{(1+L^{*2}+V_{g}^{*2}L^{*2})^{\frac{1}{2}}}. \tag{16}$$

This wave forms due to the effect of large-scale geostrophic current on the perturbation.

(c) In general case, we have

$$|C_{\pm}^{*(1)}|\geqslant\{\frac{1}{2}[(1+L^{*2}+V_{g}^{*2}L^{*2})+((1+L^{*2}+V_{g}^{*2}L^{*2})^{2}-4L^{*2}V_{g}^{*2})^{\frac{1}{2}}]\}^{\frac{1}{2}}, \tag{17}$$

and

$$|C_{\pm}^{*(2)}|\leqslant\{\frac{1}{2}[(1+L^{*2}+V_{g}^{*2}L^{*2})-((1+L^{*2}+V_{g}^{*2}L^{*2})^{2}-4L^{*2}V_{g}^{*2})^{\frac{1}{2}}]\}^{\frac{1}{2}}. \tag{18}$$

Under the assumption of $(1+L^{*2}+V_{g}^{*2}L^{*2})^{2}\gg 4L^{*2}V_{g}^{*2}$ and by using the binomial expansion, the asymptotic formulae of (17) and (18) clearly become (15) and (16) respectively.

If conditions (17) and (18) are satisfied, the eigenvalue of the problem is m defined by (13), and the solution of Eq. (7) is

$$u'=u_{0}\exp\left(-\frac{V_{g}}{C_{0}L_{0}}x\right)\sin(mx)\cos(n(y-ct)). \tag{19}$$

The domains in which the solution is established are given in Fig. 1, in which V_{g}^{*} is taken to be 1.

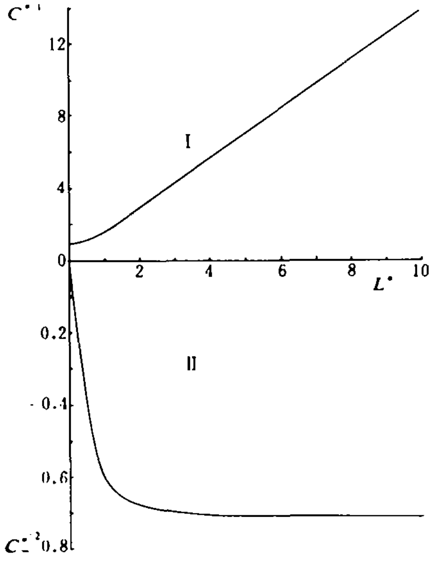

Fig. 1. The domain of solution (8).

IV. THE DYNAMICAL BOUNDARY LAYER AND MESO-SCALE JET

After u' is obtained, the perturbing velocity in the meridional direction can be determined by

$$\frac{D^{2}v'}{Dt^{2}}-C_{0}^{2}\frac{\partial^{2}v'}{\partial y^{2}}=C_{0}^{2}\frac{\partial^{2}u'}{\partial x\partial y}+fV_{g}\frac{\partial u'}{\partial y}-f\frac{Du'}{Dt}. \tag{20}$$

The solution with harmonic form in the y-direction is

$$v' < \frac{u_0}{C_\pm^{*2}-1}\left[\frac{C^*}{L_0 n}\sin(mx)+\frac{m}{n}\cos(mx)\right]\exp(-\frac{V_g}{C_0 L_0}x)\sin(n(y-ct)). \tag{21}$$

The physical meaning of this solution may be discussed as follows:

(a) The solution having a term of $\exp(-\frac{V_g}{C_0 L_0}x)$ in the x-direction indicates that V_g must be the northward geostrophic current in the region of $x \geqslant 0$, otherwise, the amplitude of perturbation will increase with distance x. On the contrary, V_g must be the southward geostrophic current in the region $x \leqslant 0$. Under this situation, the stream forms a dynamical boundary layer near the edge of plateau or coast where $x=0$.

(b) The characteristic width of this dynamical boundary layer is

$$L_c = L_0(\frac{C_0}{V_g}) \tag{22}$$

It is in direct proportion to the radius of Rossby deformation and in inverse proportion to the geostrophic current. If C_0 has the order of 10 m/s and 10 cm/s for the atmosphere and ocean respectively, and the order of V_g is the same as that of C_0, then L_c will be about 100 km and 10 km for the atmosphere and ocean respectively. It indicates that this dynamical boundary layer is very narrow. It is noted that the lateral boundary layer obtained here is different from the coastal jet formd by wind-driven action. In the latter case, the width of boundary layer is in proportion to the radius of Rossby deformation.

(c) In association with the lateral boundary layer, solution (21) indicates that a narrow meso-scale jet is formed in the large-scale jet stream closing to the edge of plateau or coast. This sort of jet differs from the Charney's[10] and Yoshida's[11] wind-driven coastal jet. It is produced by the interaction between large-scale current of background and perturbation. Clearly, this narrow jet will disappear if the geostrophic current does not exist, i. e. $L_c \to \infty$ when $V_g \to 0$. Besides, the width of this kind of jet is more narrow than that of the coastal jet produced by wind-driven action under the situation of $V_g > C_0$, so that a stronger geostrophic current is one of the necessary conditions for establishing this kind of meso-scale lateral boundary jet.

(d) The appearance of the atmospheric meso-scale jet has different dependences of season in different geographic regions. For instance, in the eastern part of Asia, this kind of jet can be found in late spring and summer, because there is a large-scale southern monsoon over there. But, in winter a north wind prevails there, so that it is impossible to form this kind of meso-scale jet in the eastern part of Asia. However, in the western part of America, for example, on the western side of the Rocky mountains where $x<0$, the speed of wind is so weak and the lateral boundary layer is so wide that a narrow jet cannot be formed though there exists a north wind coming from the eastern part of subtropic high.

(e) It seems that this kind of meso-scale lateral boundary jet will be able to exist in regions adjacent to both coasts of the ocean all the year round, because there are the south Kuroshio and Gulf currents as well as the north California and Canary currents both in the Pacific and Atlantic oceans near the weatern and eastern boundaries. And all of those large-scale oceanic currents are semi-persistent systems, i.e. their seasonal variation is not significant.

(f) For the atmospheric meso-scale jet, the axial lengths of stronger wind are about 400 km[7], the propagating speed almost coincides with the wind velocity, and the period is about 6—8 hours for velocity fluctuations in the jet axis[9]. On the basis of these observations, it seems that this kind of meso-scale jet does not belong to the variety of the inertial-gravity waves discussed above because the propagating speed of inertial-gravity waves is much higher than that of the observational jet with a wavelength of 400 km. It probably belongs to the variety of another slower waves determined by (16) and (18). In other words, the domain of eigenvalue or the solution is valid in region II in Fig. 1. If so, the fluctuating periods of wind in jet axis will be about eight hours, when the wavelength is 400 km, $V_g \sim C_0$ and $C_0 = 50$ m/s. This estimation is in agreement with the observations at first appearance.

V. TRANSVERSE CIRCULATION OF JET NEAR THE BOUNDARY

After u' is obtained, the perturbing height can be given by

$$\frac{D^2 h'}{Dt^2} - C_0^2 \frac{\partial^2 h'}{\partial y^2} = -\frac{1}{g^*}\left(C_0^2 \frac{\partial}{\partial x} + f v_g\right)\frac{Du'}{Dt}. \tag{23}$$

The solution in harmonic form in the y-direction is

$$h' = h_0 \frac{u_0}{C_0}\left(\frac{C_+^{*2}}{1 - C_\pm^{*2}}\right)\frac{m}{n} \exp\left(-\frac{V_g}{C_0 L_0} x\right) \cos(mx)\sin(n(y - ct)), \tag{24}$$

where $h_0 = C_0^2/g^*$ is the scaling height of the atmosphere and ocean.

For the shallow water model, the vertical velocity can be defined as

$$w' = \frac{Dh'}{Dt} \tag{25}$$

and from (24) we have

$$w' = u'_0 h_0 \left(\frac{C_\pm^{*2}}{1 - C_+^{*2}}\right) m \cos(mx) \exp\left(-\frac{V_q}{C_0 L_0} x\right) \cos(n(y - ct)). \tag{26}$$

The structure of transverse circulation in the jet can be discussed based on (26). Firstly, the phase of w' is the same as that of v', thus, for the significant roots $C_\pm^{*2} < 1$ discussed before, w' is directly proportional to u_0 when $x \to 0$, i.e. near the boundary of plateau or coast. Thus, it can be seen that the upward velocity or the upwelling is associated with the off-boundary or off-coast current on the eastern side of plateau or on the western coast. On the contrary, the upwelling is associated with the on-coast current on the eastern coast for the case of $C_\pm^{*2} < 1$.

VI. SUMMARY

By means of the ageostrophic shallow water model, three main results are obtained in this paper. First, there exists a dynamical lateral boundary layer near the edge of mountains and plateaus in the atmosphere and near the coast in the ocean; its characteristic width is proportional to the scale of $L_c = L_0(C_0/V_g)$ where L_0 is the radius of Rossby deformation, C_0 the speed of gravity wave and V_g the large-scale geostrophic current in background. Second, a narrow meso-scale jet associated with this dynamical boundary layer is formed. This kind of jet has the character of wave with a wavelength of several hundred kilometers and a period of several hours and one month in the atmosphere and ocean respectively. The atmospheric jet usually occurs in late spring and summer in the eastern parts of the continent, particularly, in the eastern part of Asia, because there is a large-scale south monsoon over there; for the oceanic jet, it seems to be a semi-persistant system without significant seasonal variation, and it can be established in both sides of ocean. Third, there are two varieties of waves in this model, one is the faster modified inertial-gravity wave, the other is slower wave formed due to the effect of large-scale geostrophic current on the perturbing flow in the Coriolis force field; this sort of slower wave is more important for forming the meso-scale lateral boundary jet.

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