THE APPLICATION OF CHEBYSHEV POLYNOMIALS IN IRREGULAR GRIDS IN THE FORECAST OF PRECIPITATION DISTRIBUTION

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ABSTRACT

In this paper, the Chebyshev polynomials in irregular grids are used in the forecast of precipitation distribution over the middle and lower reaches of the Changjiang River. This forecast method is available because it could be used in various kinds of original data.

I. INTRODUCTION

Previous studies on the application of Chebyshev polynomials in meteorology [1,2] dealt with the expansion at the equidistant grids only. And the availability of the polynomials is limited. The Chebyshev polynomials were generalized into irregular grids by the author[3] and the availability of the polynomials is extended. Based on this generalization, a forecast method for the horizontal distribution

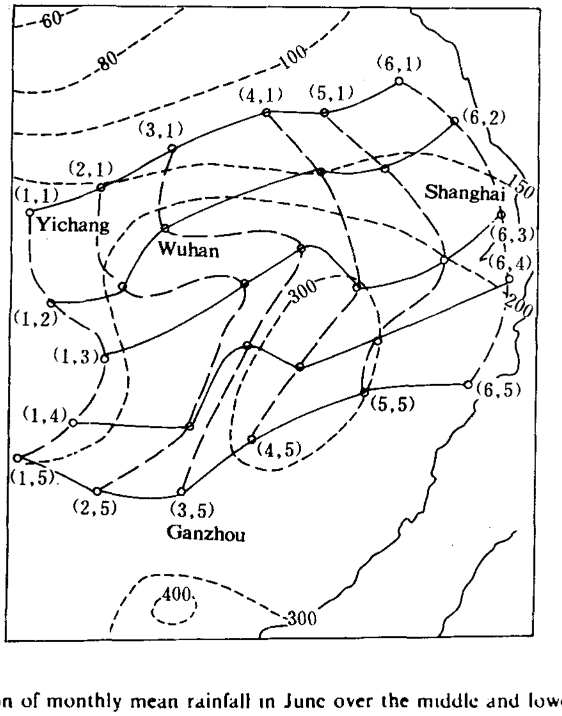


Fig. 1. Distribution of monthly mean rainfall in June over the middle and lower reaches of the Changjiang River and the relationship between grids and their ordinals (for rectangular equidistant grids[4]).

of meteorological elements was proposed[4]. For instance, two-dimensional ordinals are assigned to 30 stations over the middle and lower reaches of the Changjiang River shown in Fig. 1. The grids in ordinal space may be represented as $i=1,2,...I_0$, $j=1,2,...J_0$. Here $I_0=6$, $J_0=5$. And then the precipitation at grids in ordinal space, i.e. at stations, can be forecast by some kind of method[4]. This method interests meteorological stations and has been adopted by some meteorological services in China.

There is, however, a confine in the above study, that is, the grids in ordinal space should be rectagular equidistant, i.e. I_0 and J_0 must be constants. Sometimes, in practice, it is difficult to meet this requirement owing to the effects of topography, shoreline, administrative division, data sources and so on. The lines connecting with the grids may be curved, which can be seen from Fig. 1.

In further study on the expansion of Chebyshev polynomials in irregular grids, the above confine for ordinals was eliminated by Zhou and Li[1)]. Thus the theory on the expansion becomes perfect and more convenient for use.

In this paper, the above result is used in the forecast of precipitation distribution over the middle and lower reaches of the Changjiang River.

II. OUTLINE OF FORECAST METHOD

The forecast method described here is similar to that proposed previously by the author[4], but it possesses some new characteristics in operation. Now the operational procedures are briefly presented as follows:

1. *The Expansion of Precipitation at Stations in Terms of Chebyshev Polynomials*

Fig. 2 shows the distribution of precipitation station over the middle and lower reaches of the Changjiang River used in this paper, and the ordinals assigned for each station as well. The grid

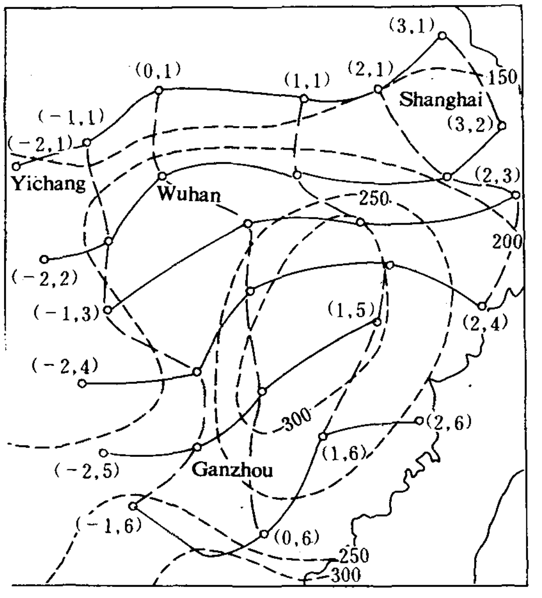


Fig. 2. As in Fig. 1 except for arbitrary irregular grids.

1) Zhou Jiabin and Li Huang, More on the Expansion of Chebyshev Polynomials in Irregular Grids, paper presented at 1983 conference on hydrological and meteorological forecasting over the Changjiang Valley.

distribution in ordinal space is illustrated in Fig. 3. The starting and terminal points of ordinal i for each row corresponding to those in Fig. 3 are given in Table 1.

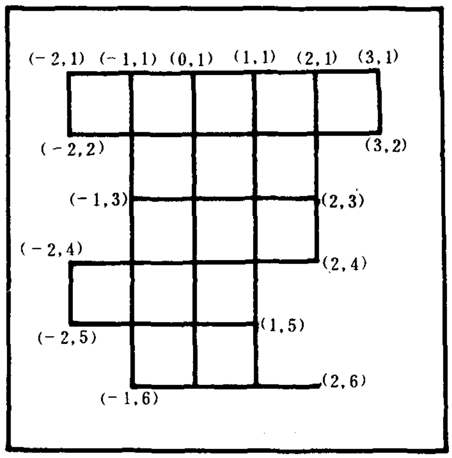


Fig. 3. The grid distribution in ordinal space corresponding to Fig. 2.

Table 1. The Starting Points ($I_A(j)$) and Terminal Points ($I_B(j)$) of Grids for Each Row in Fig. 2

j	$I_A(j)$	$I_B(j)$
1	−2	3
2	−2	3
3	−1	2
4	−2	2
5	−2	1
6	−1	2

It is necessary to assign the grid ordinals deliberately in order to interpret the characteristics of precipitation distribution well, to speed up the convergence and to raise the representativeness of Chebyshev coefficients. We made every effort in that study to assign the ordinals in a way that the lines connecting with the points which have equal second ordinals run parallel to the isohyets in order to interpret the precipitation distribution and to raise the representativeness of Chebyshev coefficients well. However, it can be seen from Fig. 1 that the ordinals assigned are not ideal because of the confine, the grids in ordinal space must be rectangular equidistant, and the lines connecting with the grids which have equal ,ordinals are curved to some extent. However, it may be known from Fig. 2 that the arrangement of ordinals is quite ideal due to the elimination of the above confinement.

The precipitation distribution at stations shown in Fig .2 is expanded in terms of Chebyshev polynomials with the following expression,

$$\tilde{R}_{ij} = \sum_{k=0}^{K_R} \sum_{s=0}^{S_R} A_{ks}(R)\varphi_{k,j}(i)\psi_s(j) \tag{1}$$

$$(j = 1,2,...J_R;\ i = I_A(j),\ I_A(j)+1,\ I_A(j)+2,...I_B(j)),$$

where A_{ks} can be evaluated from the expression,

$$A_{ks}(R) = \sum_{j=1}^{J_R} \sum_{i=I_A(j)}^{I_B(j)} R_{ij}\varphi_{k,j}(i)\varphi_s(j). \tag{2}$$

$\tilde{R}_{ij}$ in the above two expressions refers to the fitting of precipitation at the station with ordinal (i,j). R_{ij} is the value of its observation. $\varphi_{k,j}(i)$ is the value of normalized Chebyshev polynomial at grid i, subscript k indicates the order of the polynomial, j represents the number of row. $\psi_s(j)$ is the value of normalized Chebyshev polynomial with order s at grid j. $A_{ks}(R)$ is the Chebyshev coefficient of precipitation. J_R is the terminal point of ordinals j and $I_A(j)$ and $I_B(j)$ are starting point and terminal point of ordinal i for row j respectively. K_R and S_R are the truncation orders of the Exp. (1).

2. *The Expansion of Sea Surface Temperature (SST) in Terms of Chebyshev Polynomials*

Fig. 4 shows three kinds of used grid distributions for *SST* which represent the northwestern Pacific, the northern Pacific and the equatorial region respectively and refer to areas I, II and III. The starting points and terminal points of ordinal i for each row in these areas are shown in Table 2.

A kind of grid distribution of *SST* mentioned previously by the author[4] is reproduced in Fig. 5 (the figure is excluded in that paper). It is easily seen that its extent is the same basically as that displayed in Fig. 4(b). Some grids are discarded in Fig. 5 in order to arrange the grids in rectangular net. This will

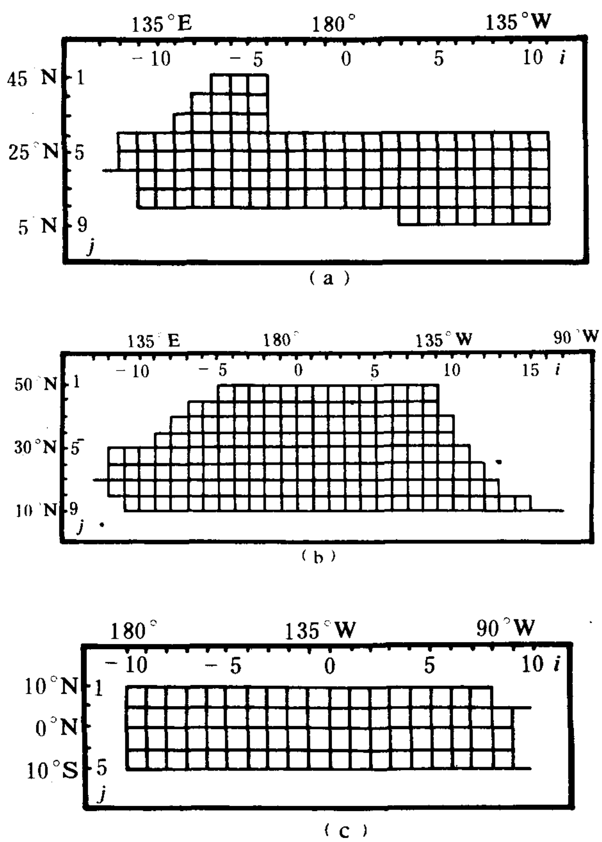


Fig. 4 The grid distribution and ordinals of monthly mean *SST* over the Pacific. (a) Area I— the northeastern Pacific, (b) Area II—the northern Pacific, (c) Area III the equatorial region

Table 2. The Starting Points and Terminal Points of Ordinal i for Each Row for SST Data in Different Areas

j	Area I		Area II		Area III	
	$I_C(j)$	$I_D(j)$	$I_E(j)$	$I_F(j)$	$I_G(j)$	$I_H(j)$
1	−7	−4	−5	7	−10	8
2	−8	−4	−7	9	−10	10
3	−9	−4	−8	10	−10	9
4	−12	11	−9	10	−10	9
5	−12	11	−12	11	−10	10
6	−13	11	−12	12		
7	−11	11	−13	13		
8	−11	11	−12	15		
9	3	11	−11	17		

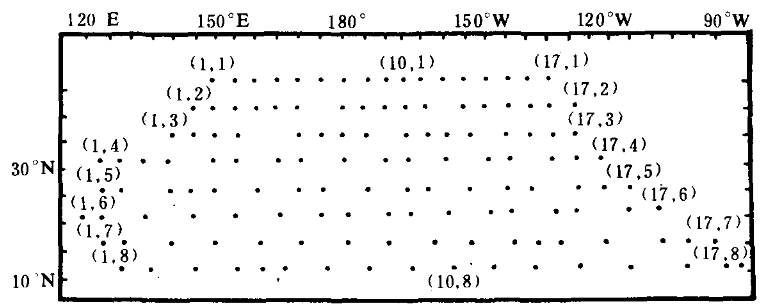


Fig. 5. A kind of grid distribution of SST and corresponding ordinals (From Zhou[3]).

make some information lost. However, there are not any losses by using grid distribution in Fig. 4. Therefore the new scheme proposed by Zhou and Li is suitable for the data from different sources.

In this condition the related formulae for expanding SST are as follows,

$$\widetilde{SST}_{ij}=\sum_{k=0}^{K_T}\sum_{s=0}^{S_T} A_{ks}(SST)\varphi_{k,j}(i)\psi_s(j), \tag{3}$$

$$(j=1,2,\cdots J_T, \quad i=I_M(j),\ I_M(j)+1, I_M(j)+2,\cdots I_N(j)),$$

$$A_{ks}(SST)=\sum_{j=1}^{J_T}\sum_{i=I_M(j)}^{I_N(j)} SST_{ij}\ \varphi_{k,j}(i)\psi_s(j), \tag{4}$$

where $\widetilde{SST}_{ij}$ refers to the fitting of SST at grid (i,j) and SST_{ij} is its value of observation. A_{ks} (SST) is the Chebyshev coefficient of SST, K_T and S_T are the truncation orders of Exp. (3). $I_M(j)$ and $I_N(j)$ represent the starting points and terminal points of ordinal i for row j in different areas, that is,

$$I_M(j)=\begin{cases} I_C(j) & \textit{for area I,} \\ I_E(j) & \textit{for area II,} \\ I_G(j) & \textit{for area III;} \end{cases}$$

$$I_N(j)=\begin{cases} I_D(j) & \textit{for area I,} \\ I_F(j) & \textit{for area II,} \\ I_G(j) & \textit{for area III.} \end{cases}$$

The rest of symbols are of the same meaning as described previously.

3. *The Formulation of Predictive Equations of Chebyshev Coefficients of Precipitation, $A_{ks}(R)$*

By using Chebyshev coefficients for *SST* as the predictors, a series of predictive equations of Chebyshev coefficient for precipitation, $A_{ks}(R)$, are formed. Here successive screening regression is used to obtain the predictive equations. For example, the forecast equation for $A_{10}(R)$ is

$$\hat{A}_{10}(R) = 55.74 - 2.601A_{02}\ (9) + 3.816A_{31}\ (11) + 6.451A_{22}\ (1) - 5.940A_{31}(2), \tag{5}$$

where A_{ks} on the right side hand of the equation denotes the Chebyshev coefficient of *SST* for area II, the numbers in parentheses represent months. We use the data of *SST* from Sept. of the preceding year to Feb. of the predicted year to predict the precipitation distribution in June. Therefore, the evolution of *SST* distribution in six months of winter has been considered in predicting the precipitation distribution in rainy season.

4. *The Fitting of Precipitation Distribution*

By substituting the Chebyshev coefficients of precipitation predicted into Exp. (1), the rainfall distribution is fitted.

III. RESULTS

By using the above method, the precipitation distribution in each June from 1951 to 1976 (excluding 1957 and 1967) is fitted and the distribution of forecast chart is obtained.

The RMS error of forecast is calculated from the Exp.

$$\sigma_R = \sqrt{\frac{\sum_{j=1}^{J_R} \sum_{i=I_A(j)}^{I_B(j)} \Delta R_{ij}}{\sum_{j=1}^{J_R} [I_B(j) - I_A(j) + 1] - 1}}, \tag{6}$$

where ΔR_{ij} is the fitting error at station (i, j), i.e. the difference between fitting and observation.

Table 3 gives the RMS errors of precipitation distribution in June predicted from the Chebyshev coefficients of *SST* in different areas.

It is seen that the mean RMS error is about 75 mm. To illustrate the availability of fitting further, two examples are given as follows.

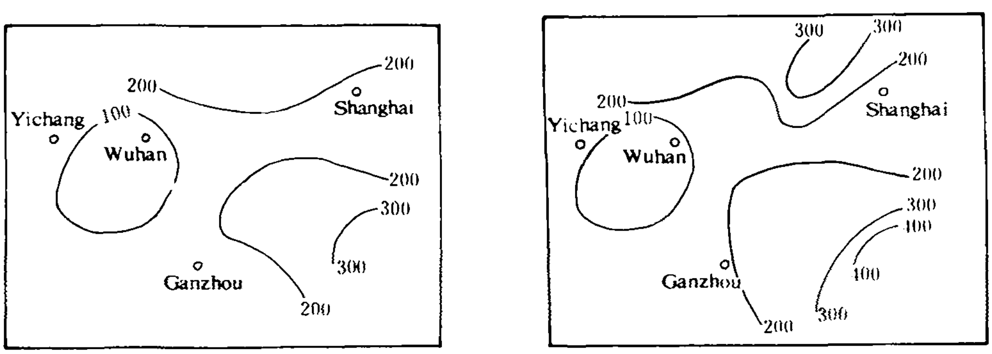


Fig. 6. Fitting of precipitation distribution in June 1972.
(a) Fitting; (b) Observation.

Table 3. RMS Errors for precipitation Distribution (in mm)

Year	Area I	Area II	Area III
1951	60.1	55.8	62.0
1952	68.0	86.6	63.4
1953	84.3	73.2	77.3
1954	186.2	147.1	138.5
1955	78.5	91.3	89.5
1956	79.2	98.2	94.4
1958	105.4	86.8	91.3
1959	99.9	99.5	84.9
1960	59.9	58.1	83.2
1961	75.3	104.7	90.9
1962	116.3	117.9	129.8
1963	69.1	81.7	66.5
1964	83.0	77.1	80.3
1965	60.6	61.5	53.3
1966	58.0	63.6	65.9
1968	54.2	61.2	57.5
1969	46.0	73.5	51.5
1970	51.5	54.4	57.2
1971	53.9	66.2	71.8
1972	51.3	60.3	43.2
1973	105.9	111.8	108.3
1974	66.2	65.1	70.5
1975	45.4	54.4	49.8
1976	51.5	54.9	50.3
Mean	75.4	79.4	76.3

Example 1. Fitting for June 1972

This is the case with the minimum RMS error (see Table 3) fitted by using *SST* data in area III. The fitting and the observation for this month are plotted in Fig. 6. It can be easily seen that the fitting is quite reasonable. The locations of heavy rainfall and low rainfall region are fitted satisfactorily and the magnitude of precipitation fitted is close to the observation

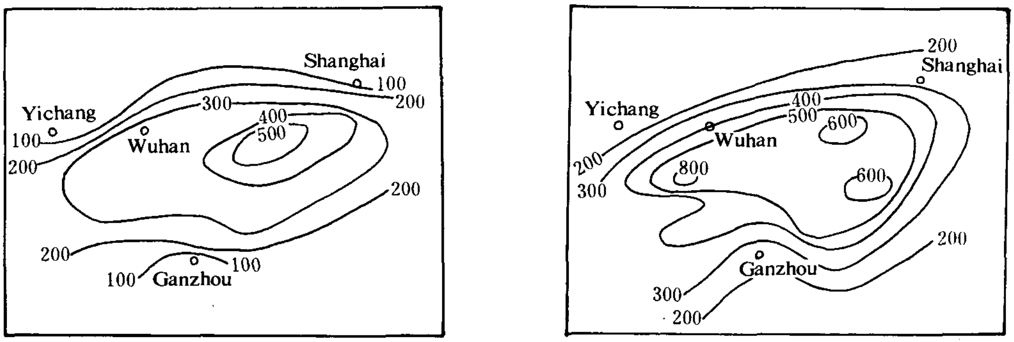
Example 2. Fitting for June 1954

Illustrated in Fig. 7 are fittings and observations. This is the case with the maximum RMS error (see Table 3) fitted by using the *SST* data in area I.

The comparison of Fig. 7(a) and 7(b) reveals that the differences between the magnitude of precipitation fitted and that observed here are larger than that in Fig. 6, but the broad characteristics of precipitation distribution have been fitted to a certain extent.

IV. SUMMARY

The forecast method of precipitation distribution illustrated in this paper, like the method proposed previously by the author[4], possesses the following advantages: (A) The precipitation distribution in meteorological station net could be calculated with this method. (B) The predictors and predictands used in this method are the main characteristics of the data in plane expressed by Chebyshev coefficients. And the representativeness of predictors and predictands may be improved and



(a) (b)

Fig. 7 Fitting of precipitation distribution in June 1954.
(a) Fitting; (b) Observation.

the statistical relation may tend to stability. (C) The method possesses physical basis because the evolution of *SST* distribution is taken into account. (D) The values of meteorological elements at irregular grids may be forecast and the forecasting error induced by instationary eigenfunction in the prediction method, in which natural orthogonal function is used, may be avoided.

The method in this paper is more convenient for predictors and predictands of all kinds. The grid distribution in Fig. 2, for example, is more satisfactory for the climatological distribution of precipitation than that in Fig. 1. Moreover, the grid distribution in Fig. 3 is more convenient for data from different sources than that in Fig. 4. Therefore, this paper is an application of the new computation scheme proposed recently by Zhou and Li and an improvement of the forecast method proposed previously by Zhou[4].

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