

AN OPERATIONAL 5-LAYER PRIMITIVE EQUATION MODEL FOR NORTHERN HEMISPHERE PREDICTION

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ABSTRACT

A 5-layer primitive equation Northern Hemisphere operational model in a modified σ -coordinate system is developed in BMC, NMB. Finite difference schemes are constructed to conserve the total energy without imposing any constraints on the difference scheme of hydrostatic equation and pressure gradient term.

The physical factors of orography, friction, horizontal diffusion and various non-adiabatic heatings are included.

The model has been under development since the beginning of 1980, and became operational in September 1981. Preliminary results for selected series of 40 prognosis are summarized and the verifications are encouraging.

I. INTRODUCTION

A 3-layer hemispheric primitive equation numerical model including orography and non-adiabatic heating was designed and tested by Zhu et al^[1] in 1980. This model has been developed as an operational 5-layer prediction model in Beijing Meteorological Centre of National Meteorological Bureau by the authors.

The main features of the model are as follows:

(1) Adoption of the modified sigma-coordinate

The model atmosphere is divided into five layers in the modified σ -coordinate^[2]. The lower part of the model atmosphere retains the properties of σ -coordinates. The upper part is equivalent to p -coordinates.

(2) Construction of the finite difference scheme

The thermodynamic equation is rewritten in the form which is favourable to conserving a finite difference scheme based on the energy conserving without imposing any constraints on the difference scheme of hydrostatic equation and pressure gradient term.

(3) The physical factors of orography, friction, horizontal diffusion and non-adiabatic heatings such as radiation, sensible heating, evaporation, large-scale and cumulus scale condensation are included.

II. BASIC EQUATIONS AND FINITE DIFFERENCE APPROXIMATION

1. The Definition of Modified σ -coordinate

Define a modified vertical coordinate as follows:

$$\begin{aligned} \text{if } p_m < p \leq p_*, \quad \sigma &= \frac{p - p_m}{\pi} + \sigma_m, & \text{where } \pi &\equiv \pi_L = \frac{p_* - p_m}{1 - \sigma_m}; \\ \text{if } 0 \leq p < p_m, \quad \sigma &= \frac{p}{\pi}, & \text{where } \pi &\equiv \pi_U = \frac{p_m}{\sigma_m}. \end{aligned} \quad (1)$$

Both p_m and σ_m are pre-assigned constants. Then we have

- (1) $p=0$, at $\sigma=0$,
- (2) $p=p_m$, at $\sigma=\sigma_m$,
- (3) $p=p_*$, at $\sigma=1$,

where p_* is ground pressure.

If we let $p_m=0$ and $\sigma_m=0$,
then we have $\sigma = p/p_*$,

as commonly used.

According to (1), the pressure is

$$\begin{aligned} p &= \pi\sigma & (p < p_m) \\ p &= \pi(\sigma - \sigma_m) + p_m & (p > p_m) \end{aligned} \tag{2}$$

and vertical velocity is

$$\begin{aligned} \omega &\equiv \frac{dp}{dt} = \pi_L \dot{\sigma} + (\sigma - \sigma_m) \left(\frac{\partial \pi}{\partial t} + V \cdot \nabla \pi \right) & (p > p_m), \\ \omega &= \pi_U \dot{\sigma} & (p < p_m), \end{aligned} \tag{3}$$

where $\dot{\sigma} = d\sigma/dt$. Because the inner boundary at $\sigma = \sigma_m$ is a material surface, it is reasonable that continuity of ω at the surface is desired, i.e.

$$\text{if } \sigma = \sigma_m, \quad \lim_{\sigma \rightarrow \sigma_m^+} \pi_L \dot{\sigma} = \lim_{\sigma \rightarrow \sigma_m^-} (\pi_U \dot{\sigma}) = \omega_{\sigma_m} . \tag{4}$$

At the earth's surface and the top of the atmosphere, the vertical velocity vanishes, i.e.

and

$$\begin{aligned} \pi_U \dot{\sigma} &= 0, & \text{at } \sigma = 0, \\ \pi_L \dot{\sigma} &= 0, & \text{at } \sigma = 1. \end{aligned} \tag{5}$$

2. Governing Equations

Let

$$U = \frac{\pi u}{m}, \quad V = \frac{\pi v}{m}, \quad \dot{\Sigma} = \frac{\pi \dot{\sigma}}{m}$$

and

$$\tilde{L}(a) = m \left[\frac{\partial}{\partial x} \left(\frac{\pi u}{m} a \right) + \frac{\partial}{\partial y} \left(\frac{\pi v}{m} a \right) \right] + \frac{\partial}{\partial \sigma} \left(\frac{\pi \dot{\sigma}}{m} a \right),$$

where m is map factor.

The equation of motions, the thermodynamic equation, the equation of water vapour, the continuity equation, and the hydrostatic equation are as follows:

$$\frac{\partial}{\partial t} \left(\frac{\pi u}{m} \right) = -\tilde{L}(u) + f \frac{\pi v}{m} - \frac{\pi}{m} f v_g + \frac{\pi}{m} F_u, \tag{6}$$

$$\frac{\partial}{\partial t} \left(\frac{\pi v}{m} \right) = -\tilde{L}(v) - f \frac{\pi u}{m} + \frac{\pi}{m} f u_g + \frac{\pi}{m} F_v, \tag{7}$$

$$\begin{aligned} \frac{\partial}{\partial t} \left(\frac{C_p \pi T}{m} \right) &= -\tilde{L}(C_p T + \phi) - \frac{1}{m} \frac{\partial}{\partial \sigma} [(\sigma - \sigma_m) \phi] \frac{\partial \pi}{\partial t} \\ &+ \left(\frac{\pi u}{m} f v_g - \frac{\pi v}{m} f u_g \right) + \frac{\pi}{m} C_p F_T + \frac{\pi}{m} (\dot{H}_R + \dot{H}_S + \dot{H}_L), \end{aligned} \tag{8}$$

$$\frac{\partial}{\partial t} \left(\frac{\pi q}{m} \right) = -\bar{L}(q) + \frac{\pi}{m}(E-C) + \frac{\pi}{m}F_q, \tag{9}$$

$$\frac{\partial \pi}{\partial t} = -m^2 \left[\frac{\partial}{\partial x} \left(\frac{\pi u}{m} \right) + \frac{\partial}{\partial y} \left(\frac{\pi v}{m} \right) \right] - m \frac{\partial}{\partial \sigma} \left(\frac{\pi \sigma}{m} \right), \tag{10}$$

$$\frac{\partial \phi}{\partial \ln p} = -RT, \tag{11}$$

$$\frac{\partial}{\partial \sigma} [(\sigma - \sigma_m) \phi] = \phi - \frac{RT}{\beta}, \quad (\text{If } p > p_m), \tag{11'}$$

where

$$fu_g = -m \left(\frac{\partial \phi}{\partial y} + RT \frac{\partial \ln p}{\partial y} \right),$$

$$fv_g = m \left(\frac{\partial \phi}{\partial x} + RT \frac{\partial \ln p}{\partial x} \right)$$

and

$$\beta = 1 + \frac{p_m}{(\sigma - \sigma_m) \pi_L}$$

If $p < p_m$, we have

$$\frac{\partial \pi}{\partial t} = \frac{\partial \ln p}{\partial x} = \frac{\partial \ln p}{\partial y} = 0.$$

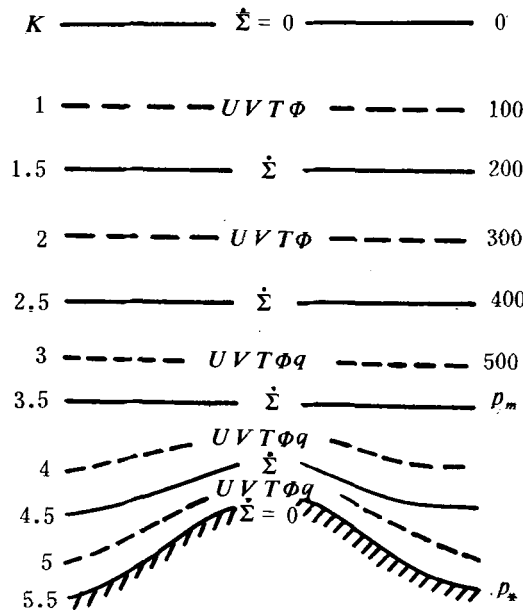


Fig. 1. The model atmosphere.

3. Structure of the Model

The model atmosphere is divided into five layers. The vertical distribution of physical variables is shown in Fig. 1. Note that the physical variables of U, V, T, ϕ and q are carried at a level $\frac{1}{2}\Delta\sigma$ above the Σ levels. On the horizontal grids, all variables are carried at each grid point. The horizontal domain consists of the 51×51 grid points with a 381 km space step (at 60 N).

At the horizontal boundary which lies between the outer two rows of grid points, the normal component of velocity is assumed to vanish.

4. Finite Difference Formulation

Let

$$L(a) \equiv m[(\bar{U}^x \bar{a}^x)_x + (\bar{V}^y \bar{a}^y)_y] + (\bar{\Sigma} \bar{a}^\sigma)_\sigma$$

$$u_g = -\frac{m}{f}(\bar{\phi}^y + R\bar{T}^y \delta_y \ln p^y),$$

$$v_g = \frac{m}{f}(\bar{\phi}^x + R\bar{T}^x \delta_x \ln p^x),$$

and

$$D = \bar{U}_x^x + \bar{V}_y^y$$

Then the difference equations are as follows

$$\frac{\partial U}{\partial t} = -L(u) + fV - \frac{\pi}{m} f v_g + \frac{\pi}{m} F_u, \tag{12}$$

$$\frac{\partial V}{\partial t} = -L(v) - fU + \frac{\pi}{m} f u_g + \frac{\pi}{m} F_v, \tag{13}$$

$$\frac{\partial}{\partial t} \left(\frac{\pi C_p T}{m} \right) = -L(C_p T + \phi) - \frac{1}{m} [(\sigma - \sigma_m) \bar{\phi}^\sigma]_\sigma \frac{\partial \pi}{\partial t} + (U f v_g - V f u_g)$$

$$+ \frac{\pi}{m} C_p F_T + \frac{\pi}{m} (\dot{H}_R + \dot{H}_S + H_L), \tag{14}$$

$$\frac{\partial}{\partial t} \left(\frac{\pi q}{m} \right) = -L(q) + \frac{\pi}{m} (E - C) + \frac{\pi}{m} F_q, \tag{15}$$

$$\frac{\partial \pi_L}{\partial t} = -\frac{m^2}{1 - \sigma_m} \sum_{k=1}^5 D_k \Delta \sigma, \tag{16}$$

$$\dot{\Sigma}_{k+\frac{1}{2}} - \dot{\Sigma}_{k-\frac{1}{2}} = \begin{cases} -\Delta \sigma D_k m & (k=1,2,3), \\ -\Delta \sigma \left(m D_k + \frac{1}{m} \frac{\partial \pi_L}{\partial t} \right) & (k=4), \end{cases} \tag{16'}$$

It is worthwhile to explain the differencing hydrostatic equation used. Applying (11') to the fourth and fifth level and (11) to the first, second and third level. Then by differencing them, the difference equations of hydrostatic equation are obtained as

$$\phi_k = \phi_{k+1} + \frac{R}{2} \ln \left(\frac{p_{k+1}}{p_k} \right) (T_k + T_{k+1}), (k=1,2,3,4), \tag{17}$$

$$\phi_5 = \phi_* + \frac{R}{2} \left[\frac{1}{\beta_4} - \frac{1}{2} \ln \left(\frac{p_5}{p_4} \right) \right] T_4 + \frac{R}{2} \left[\frac{1}{\beta_5} - \frac{1}{2} \ln \left(\frac{p_5}{p_4} \right) \right] T_5,$$

Differentiating Eq.(17) with respect to time, we have the following equations

$$\frac{\partial \phi_5}{\partial t} = \frac{R}{2} \left\{ \left[\frac{1}{\beta_4} - \frac{1}{2} \ln \left(\frac{p_5}{p_4} \right) \right] \frac{\partial T_4}{\partial t} + \left[\frac{1}{\beta_5} - \frac{1}{2} \ln \left(\frac{p_5}{p_4} \right) \right] \frac{\partial T_5}{\partial t} + \left(\frac{2p_m + p_4}{p_4} T_4 + T_5 \right) \frac{1}{2p_4} \frac{\partial p_4}{\partial t} + \left(\frac{2p_m - p_5}{p_5} T_5 - T_4 \right) \frac{1}{2p_5} \frac{\partial p_5}{\partial t} \right\}, \quad (18)$$

$$\frac{\partial \phi_k}{\partial t} = \frac{\partial \phi_{k+1}}{\partial t} + \frac{R}{2} \left\{ \left(\frac{\partial T_k}{\partial t} + \frac{\partial T_{k+1}}{\partial t} \right) \ln \left(\frac{p_{k+1}}{p_k} \right) + (T_k + T_{k+1}) \left(\frac{1}{p_{k+1}} \frac{\partial p_{k+1}}{\partial t} - \frac{1}{p_k} \frac{\partial p_k}{\partial t} \right) \right\} \quad (k=1,2,3,4), \quad (19)$$

where

$$\frac{\partial T}{\partial t} = \frac{m}{C_p \pi} \frac{\partial}{\partial t} \left(\frac{\pi C_p T}{m} \right) - \frac{T}{\pi} \frac{\partial \pi}{\partial t}$$

and

$$\frac{\partial p_k}{\partial t} = \begin{cases} (\sigma_k - \sigma_m) \frac{\partial \pi_L}{\partial t} & (k=4,5), \\ 0 & (k=1,2,3), \end{cases}$$

The total mass, the squared quantities of advective term and the total energy of the difference equations described above (for the time-continuous space difference) are conserved without imposing any constraints on the difference of the hydrostatic equation unless source/sink terms and/or boundary flux present. We can prove it as follows:

For simplicity, assuming uniformity being in the lateral direction (y), then we have

$$\sum_{i,k} \frac{1}{m^2} \frac{\partial}{\partial t} \left(\frac{\pi u^2}{m} \right) = \sum_{i,k} \left[-u(U^x \bar{u}^x)_x + \frac{u^2}{2} U^x_x \right] + \sum_{i,k} \frac{1}{m} \left[-u(\Sigma \bar{u}^o)_o + \frac{u^2}{2} \Sigma \right] + \sum_{i,k} \frac{fV u}{m} - \sum_{i,k} \frac{\pi u}{m} f v_g.$$

If the lateral boundary is assumed to be a rigid wall, we obtain

$$\sum_{i,k} \frac{1}{m^2} \frac{\partial}{\partial t} \left(\frac{\pi u^2}{2} \right) = \sum_{i,k} \frac{f u V}{m} - \sum_{i,k} \frac{\pi u}{m^2} f v_g,$$

similarly, we have

$$\sum_{i,k} \frac{1}{m^2} \frac{\partial}{\partial t} \left(\frac{\pi v^2}{2} \right) = - \sum_{i,k} \frac{f U v}{m}.$$

Therefore, the total kinetic energy is

$$\sum_{i,k} \frac{1}{m^2} \frac{\partial}{\partial t} \left(\pi \frac{u^2 + v^2}{2} \right) = - \sum_{i,k} \frac{\pi u}{m^2} f v_g.$$

The total potential energy is given by

$$\sum_{i,k} \frac{1}{m^2} \frac{\partial}{\partial t} (C_p \pi T) = - \sum_i \frac{1}{m^2} \phi_* (1 - \sigma_m) \frac{\partial \pi_L}{\partial t} + \sum_{i,k} \frac{1}{m} U f v_g.$$

Combining the total potential energy with total kinetic energy yields

$$\sum_i \frac{1}{m^2} \frac{\partial}{\partial t} \left[\sum_k \pi \left(C_p T + \frac{u^2 + v^2}{2} \right) + p_* \phi_* \right] = 0.$$

Thus the total energy of the difference system is conserved.

5. Time Integration

A combined scheme of the leap-frog method and the time smooth method with a 7.5-min time-step is applied to the time integration, and the time smooth operator is

$$\bar{F}(t) = F(t) + \alpha [F(t - \Delta t) + F(t + \Delta t) - 2F(t)],$$

where

$$\alpha = 0.125.$$

III. INITIALIZATION AND PHYSICAL PROCESSES

The objective analysis from BMC provides the heights at five levels (i.e. 850, 700, 500, 300 and 100 mb), sea surface pressure and dew-point depression $T - T_d$ at three levels (i.e. 850, 700, and 500 mb). In the lower mountain model, the 1st, 2nd and 3rd level of the model atmosphere are just the mandatory level, i.e., 100, 300 and 500 mb, respectively. The geopotential heights of the 4th and 5th level, and the pressure at the ground surface p_* are obtained from the geopotential heights of the three p -surface (i.e. 500, 700, 850 mb), the sea surface pressure and ground surface height by the Lagrangian interpolation formula.

The field of temperature T on the σ -surface is obtained from the geopotential height of the σ -surface by using the hydrostatic relations (11) and (11'). The dew-point temperature T_d on the p -surface is found from dew-point depression $T - T_d$ in which the temperature is obtained by linear interpolation formula with respect to $\ln p$ from T on the σ -surface, then the specific humidity on σ -surface is derived by exponential interpolation formula with respect to pressure from the specific humidity on p -surface.

For the time being, the initial wind is obtained from the solution of a non-linear balance equation on σ -surface.

For the limited space, we only give a brief review of the physical processes:

(1) Radiation

The solar radiative heating and long-wave radiative cooling are calculated directly from the differential expressions of the formulae of flux divergence, which is designed by the authors in a 3-layer model^[3]. The empirical formulae for calculating short-wave and long-wave radiation of Wang^[4] and Katayama^[5] are used. In addition, the following points are considered for the operational model:

(a) For simplicity, the feedback influence of cloud is neglected;

(b) Only the effect of water vapour is considered in the calculation of radiation, the specific humidity on the upper two layers is derived through the exponential relation, then the optical depth of every layer can be found.

(2) Sensible heating exchange and evaporation

The planetary boundary layer formulation consists of the use of a simple bulk parameterization with the addition to the following assumptions:

(a) The fluxes of sensible heating and water vapor are assumed to be linear with height and equal to zero at $\sigma = \sigma_m$;

(b) The term of long-wave radiation of ground surface is linearized by $T_*^4 = 4\bar{T}_*^3 T_* - 3\bar{T}_*^4$, where \bar{T}_* is the known mean ground temperature;

(c) At the top of the boundary layer, the upward flux is equal to the sensible heat transport flux from the ground surface.

Then the ground temperature can be found from the analytical relation without iterations, the procedures in detail can be found in [6].

(3) Condensational heating

We neglect the details of condensation in physical mechanism, and assume that the large-scale condensation may begin as soon as the water vapour in the air becomes saturated. In order to avoid the computational instability induced by the latent heat, we distribute this heating to the surrounding grid points.

For the small-scale cumulus heating, Kuo-scheme^[7] have been used by assuming the following conditions for convection.

- 1) $\frac{\partial}{\partial \sigma} (\phi + C_p T + Lq) < 0$,
- 2) $I \equiv \int_0^1 \left(\frac{\partial uq}{\partial x} + \frac{\partial vq}{\partial y} \right) d\sigma - \frac{\omega_5}{p_5} q_5 > 0$,
- 3) $q_5 \geq a(q_5)_s$,
- 4) $\omega < 0$,

IV. TOPOGRAPHY, FRICTION AND DIFFUSION

In the modified σ -coordinates, the pressure gradient below σ_m is also expressed by two terms having the same order of magnitude but different signs. As a result the truncation error is large. We have examined some finite difference schemes of the pressure gradient. For an atmosphere, if its temperature structure is $T(p) = B_1 \ln p + B_2$, we can prove that

$$\bar{\phi}_x^x + R\bar{T}^x \delta_x \ln p^x \equiv 0$$

like Corby's scheme^[8], but p is expressed in terms of (2) in this modified σ -coordinates. Smoothed mountains with a maximum height 2700 m are used in the model.

The surface stress in x and y direction is given by a simple bulk parameterization by assuming the surface wind u_* and v_* to be equal to that on the 5th level of the model atmosphere, and the stress at σ_m -level and above it equals zero. All of the horizontal diffusion terms in momentum, thermodynamic and water vapour equations are written in linear operators.

V. PRELIMINARY RESULTS

The prediction model has been under development since the beginning of 1980 and became operational in september 1981. The 72 hr prognosis is computed on a routine basis. In this paper, we only summarize the results for a selected series of 40 examples before October 1981.

Of primary interests are the correlations between the predicted and observed values, the associated root-mean-square errors (RMSE) and the time-mean errors of 500 mb. Table 1 illustrates the correlation coefficients r , RMSE, standard deviations and S_1 scores for various regions. The 24 hr forecasts show that the better performance is found over Asia, especially over China. This is thought to be attributed to the dense data available for initialization.

Table 1. Correlation Coefficients (r), RMSE, Standard Deviations (σ) and S_1 Scores for Selected Regions Based on the 24hr Forecast Statistics in October 1981

Region			RMSE*		σ^*		S_1	
	500 mb	Sea Surface	500 mb	Sea Surface	500 mb	Sea Surface	500 mb	Sea Surface
China	0.77	0.63	24.8	3.7	20.8	3.5	32.2	63.2
Asia	0.74	0.67	37.2	4.8	34.7	4.6	33.6	64.5
The Northern Hemisphere	0.63	0.65	41.7	4.7	41.3	4.6	39.3	63.7

* RMSE and σ are in m for 500 mb and mb for sea surface.

In order to show the ability of the model to forecast changes in the circulation patterns, results for the observed and predicted square deviations are presented in Fig. 2. They agreed with each other in the gross features.

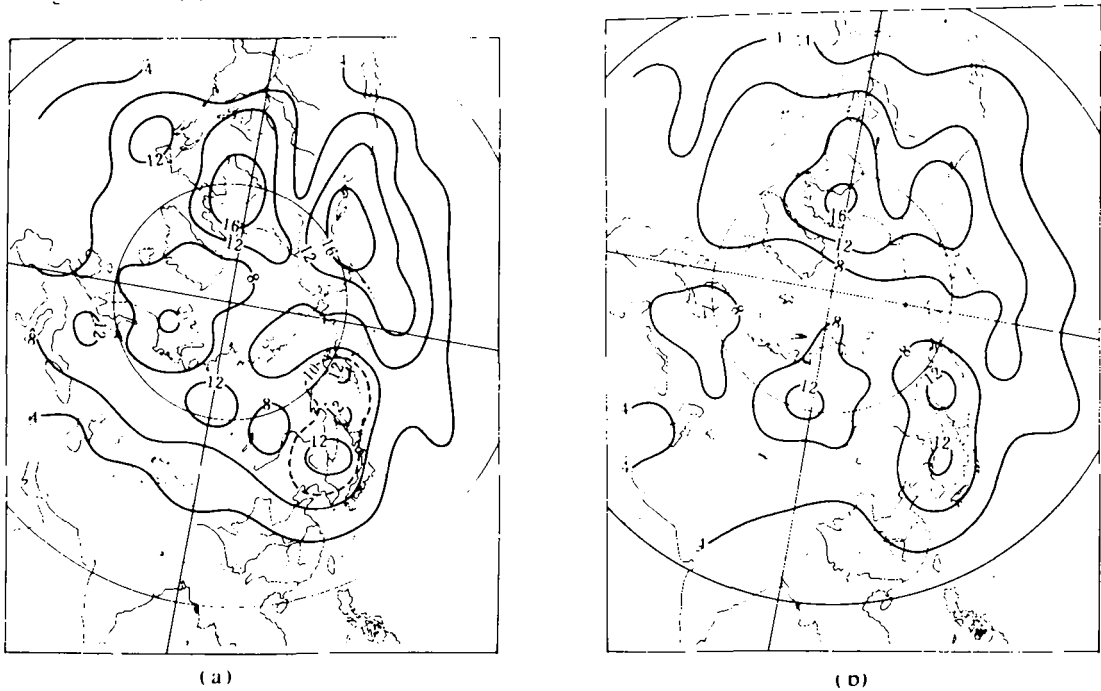


Fig. 2. (a) Observed and (b) 48 hr predicted 500 mb square deviation for Sep. - Oct., 1981. (unit: 10 m)

As a further study of the large-scale forecast-errors, the geographical distribution of the seasonal mean is shown in Fig. 3, which smooths out the small-scale errors, delineating that the large-scale, mean errors tend to be geographically dependent, and the average forecast errors reach up to ± 80 m in 48 hr. The following points may be of interest on the mean-error-chart:

- (a) Positive errors frequently appear on the East-Asian and North-American semi-permanent trough, thus these reflect the model's systematic shallowing of the troughs over these areas.
- (b) Negative errors are found over the Asian and Rocky mountains. This seems to imply that the weaker quasi-stationary anticyclone ridges are forced by the weak topography effect resulting from the

model's lower terrain.

(c) The positive errors occur in the Polar region, most likely due to that the forecasts also tend to shallow the polar vortices.

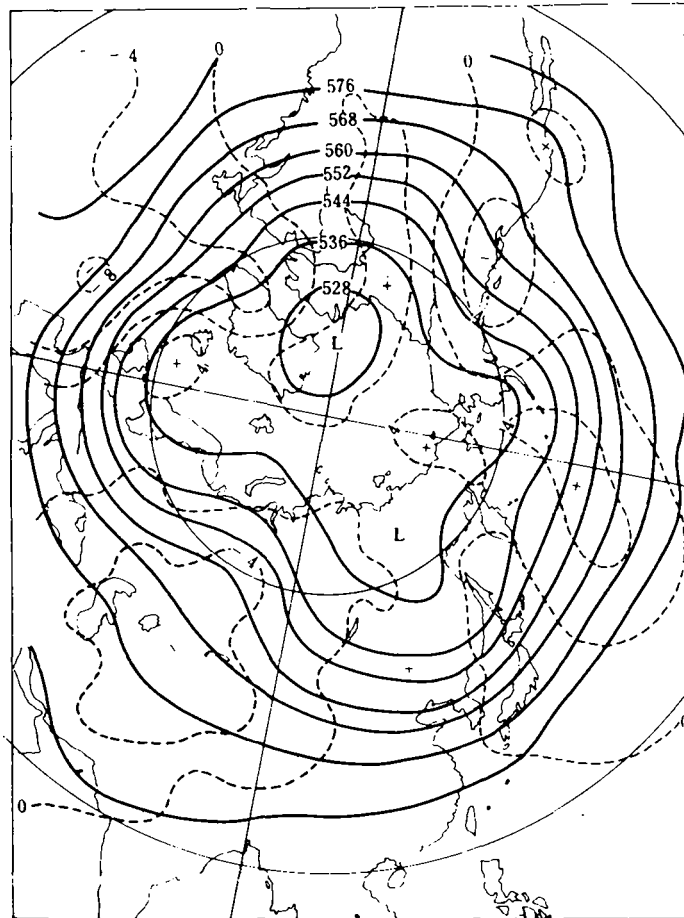


Fig. 3. 500 mb 48 hr mean forecast errors for September-October 1981 (unit: 10 m). Solid lines indicate the observed 500 mb mean heights.

Furthermore, the statistical results of the displacement forecasts for the 48 hr period of 120 samples of cyclones and 76 samples of anticyclones in Euroasia ($10^{\circ} - 160^{\circ}\text{E}$, $30^{\circ} - 60^{\circ}\text{N}$) are summarized in Fig. 4. As regards the predictions of movement, the average phase error is about -4.2 of longitude for troughs and -2.9 for ridges; 70% of the disturbances have forecast-errors $\pm 5^{\circ}$ with respect to the observations during the 48 hr period.

For the forecast-errors of the intensity of weather systems, in general, the results indicate that, the intensity of centres are usually weaker than the observed ones. It is found that 80.5% of cyclones are forecasted to increase less than 80 m, and 31% are ± 20 m with respect to the observations during the 48 hr period. It is also found that 94% of forecasts of anticyclones exhibit a tendency to diminish the intensity less than 80 m, but more than half anticyclones are less than 40 m.

In summary, the verification results are encouraging. The model shows its ability to forecast the

gross circulation features and long-waves in the westerlies, with less ability to simulate the short-waves and the low-latitude systems.

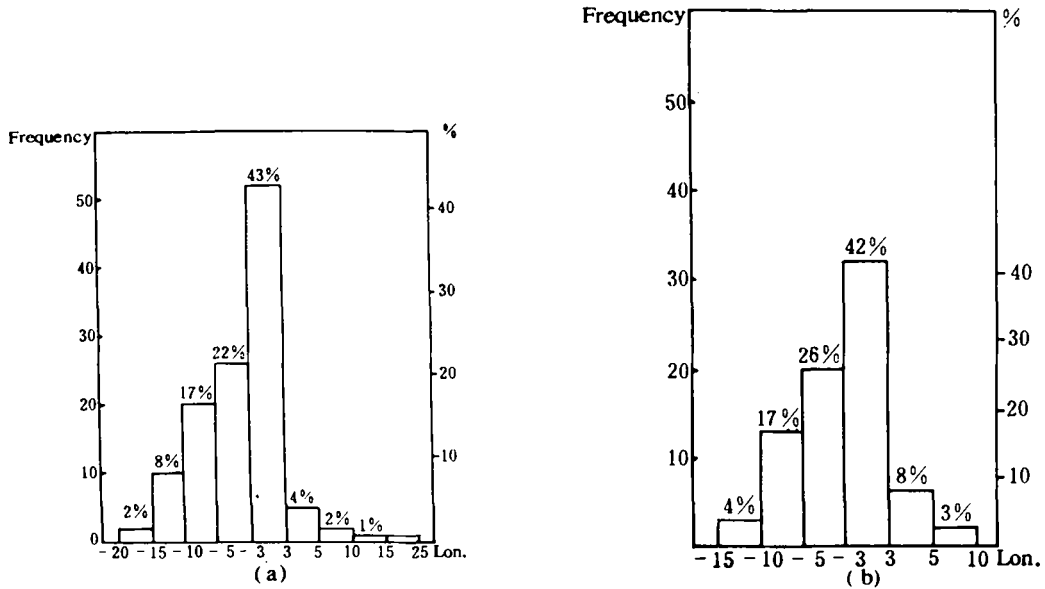


Fig. 4. Frequency distribution of phase errors for (a) troughs and (b) ridges.

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