A TWO-DIMENSIONAL ENERGY BALANCE CLIMATE MODEL INCLUDING RADIATION AND ICE CAPS-ALBEDO FEEDBACK

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ABSTRACT

A simplified two-dimensional energy balance climate model including the solar and infrared radiation transports, the turbulent exchanges of heat in vertical and horizontal directions and the ice caps-albedo feedback is developed. The solutions show that if the atmosphere is considered as a grey body and the grey coefficient depends upon the distributions of absorption medium and cloudiness, both horizontal and vertical distributions of temperature are identical to the observation.

On the other hand, comparing the models that the atmosphere is considered as a grey body with ones that the infrared radiation is parameterized as a linear function of temperature, as was considered by Budyko, Sellers(1969), then the results show that even though both of them can obtain the earth's surface temperature in agreement with the observation, the sensitivity of the climate to the changes of solar constant is very different. In the former case, the requirement for the ice edge to move southward from the normal 72°N to 50°N(i.e. where the glacial climate would take place) is that the solar constant should decrease by 13% to 16%. However, in the latter case, the climate is highly sensitive to the changes of solar radiation. In this case, the requirement of solar radiation occurring in the glacial climate should decrease by 2% to 6%. According to the investigations mentioned above we must be careful when the parameterizations of the radiation and other processes are conducted in a climate model, otherwise the reliability of the results is suspicious.

I. INTRODUCTION

For a climate model, the distribution of atmospheric temperature in agreement with the observation should be simulated at least. Considering that the physical processes controlling climate are rather complicated, the detailed general circulation model (GCM) is generally used. However, apart from saving a lot of computer time, compared with detailed GCM'S, the simplified dynamical model or thermal model has the advantage to gain an insight into different physical mechanisms for the formation of climate.

The simplest climate model is one-dimensional energy balance model (EBM) which was developed by Budyko[1] and Sellers[2]. They obtained the surface temperature as a function of latitude being consistent with the observation and discussed the sensitivity of climate to the changes of solar radiation. However, one-dimensional EBM can not give the distribution of temperature with height, so a two-dimensional EBM which is not vertically integrated has to be developed. But the calculations of the radiation absorption and emission in vertical direction are also very complicated. Fortunately, Kuo[3] presented a simplified method to calculate the flux of infrared radiative transfer and obtained the daily changes of temperature in agreement with the facts. Using a simple two-dimensional EBM including the solar and infrared radiation transport and ice caps-albedo feedback developed by means of Kuo's method in this paper, we want to simulate the present climate and discuss the effects of some factors and parameterizations on the results.

II. MODEL

When the radiative energy is in equilibrium with the turbulent transport of heat, the basic equations may be expressed as follows:

$$\frac{K\partial}{a^2\partial\chi}(1-\chi^2)\frac{\partial T}{\partial\chi}+\frac{\partial}{\partial z}\left(k_t\frac{\partial T}{\partial z}\right)+\sum_i\alpha_i'\rho_c(A_i+B_i-2E_i)+\alpha''\rho_c Q=0, \tag{1}$$

$$\frac{\partial A_i}{\partial z}=\alpha_i'\rho_c(A_i-E_i), \tag{2}$$

$$\frac{\partial B_i}{\partial z}=\alpha_i'\rho_c(E_i-B_i), \tag{3}$$

$$\frac{\partial Q}{\partial z}=\alpha''\rho_c Q, \tag{4}$$

where T is atmospheric temperature, α_i' and α'' are the absorption coefficients of infrared radiation of the wavelength λ_i and of the solar radiation respectively, A_i and B_i are the upward and downward fluxes of the infrared radiation in wavelength interval $\Delta\lambda_i$ respectively, Q is the flux of the solar radiation, E is the radiative energy in $\Delta\lambda_i$, ρ_c is the density of absorption medium, K, k_t are coefficients of the turbulent exchange of heat in horizontal and vertical directions respectively, and a is the radius of the earth. $X=\sin\theta$, θ is the latitude, the other symbols are used as usual.

If Eq. (1) is integrated over the total absorption spectra, there must be a troublesome calculation, and what is more, it is inconvenient to deal with analytically. Kuo[3] has presented a simplified method to calculate the flux of long-wave radiative transfer. According to

$$\alpha_i\rho_c \gtrless \frac{d}{dz}\,,$$

he divided the absorption spectra into the intense (s) and weak (w) absorption regions. The mean absorption coefficient in the two regions is defined as follows:

$$\alpha_{s,w}=\frac{\int_{s,w}\alpha_\lambda E_\lambda d\lambda}{E_{s,w}}=\frac{\int_{s,w}\alpha_\lambda G_\lambda d\lambda}{G_{s,w}}, \tag{5}$$

where

$$E_{s,w}=\int_{s,w}E_\lambda d\lambda,\quad G_{s,w}=\int_{s,w}G_\lambda d\lambda,\quad G_{s,w}=A_{s,w}+B_{s,w}\,,$$

and the relations

$$E_s=rE,\ E_w=(1-r)E,\ E=\sigma T^4F \tag{6}$$

are used. Here r expresses the proportion (in %) of radiative energy in intense absorption region to the total radiative energy. σ is the Stefan-Boltzmann constant. F is the coefficient of the grey body.

From Eqs. (2) and (3), we get

$$\left(\frac{\partial^2}{\partial z^2}-\alpha_{s,w}^2\rho_c^2\right)G_{s,w}=-2\alpha_{s,w}^2\rho_c^2E_{s,w}. \tag{7}$$

Rewriting Eq. (1) as

$$\alpha_s\rho_c(G_s-2E_s)+\alpha_w\rho_c(G_w-2E_w)$$

$$=-\left[\frac{K}{a^2}\frac{\partial}{\partial\chi}(1-x^2)\frac{\partial T}{\partial\chi}+\frac{\partial}{\partial z}(k_t\frac{\partial T}{\partial z})+\alpha''\rho_c Q\right], \qquad (8)$$

and taking operator $(\frac{\partial^2}{\partial z^2}-\alpha_s^2\rho_c^2)(\frac{\partial^2}{\partial z^2}-\alpha_w^2\rho_c^2)$ upon this equation, we get

$$(\frac{\partial^2}{\partial z^2}-\alpha_s^2\rho_c^2)(\frac{\partial^2}{\partial z^2}-\alpha_w^2\rho_c^2)\left[\frac{K}{\alpha^2}\frac{\partial}{\partial x}(1-x^2)\frac{\partial T}{\partial x}+\frac{\partial}{\partial z}(k_s\frac{\partial T}{\partial z})+\alpha''\rho_c Q\right]$$

$$=2\left[r\alpha_s\rho_c(\frac{\partial^2}{\partial z^2}-\alpha_w^2\rho_c^2)+(1-r)\alpha_w\rho_c(\frac{\partial^2}{\partial z^2}-\alpha_s^2\rho_c^2)\right]\frac{\partial^2 E}{\partial z^2}.$$

Since $\alpha_s^2\rho_c^2\gg\frac{d^2}{dz^2}$ and $\alpha_w^2\rho_c^2\ll\frac{d^2}{dz^2}$, the above formula may be reduced to

$$\frac{\partial^2}{\partial z^2}\left[\frac{K}{\alpha^2}\frac{\partial}{\partial\chi}(1-x^2)\frac{\partial T}{\partial x}+\frac{\partial}{\partial z}(k_t\frac{\partial T}{\partial z})+\alpha''\rho_c Q\right]$$

$$=2\left[(1-r)\alpha_w\rho_c-\frac{r}{\alpha_s\rho_c}\frac{\partial^2}{\partial z^2}\right]\frac{\partial^2 E}{\partial z^2}. \qquad (9)$$

Integrating Eq. (9) twice with z, we have

$$\frac{K}{\alpha^2}\frac{\partial}{\partial x}(1-x^2)\frac{\partial T}{\partial x}+\frac{\partial}{\partial z}(k_t\frac{\partial T}{\partial z})+\alpha''\rho_c Q$$

$$=2\left[(2-r)\alpha_w\rho_c E-\frac{r}{\alpha_s\rho_c}\frac{\partial^2 E}{\partial z^2}\right]+C_0+C_1 z. \qquad (10)$$

When $z\to\infty$, each physical quantity is finite, hence $C_1=0$.

Taking account of $\frac{\partial T}{\partial z}\approx\frac{1}{F4\sigma\overline{T}^3}\frac{\partial E}{\partial z}$, and introducing the optical thickness,

$$\xi=\frac{\alpha''}{\alpha_s\xi_0}\int_z^\infty\alpha_s\rho_c dz, \qquad \xi_0=\frac{\alpha''}{\alpha_s}\int_0^\infty\alpha_s\rho_c dz, \qquad (11)$$

we may rewrite Eq. (10) as

$$D\frac{\partial}{\partial x}(1-x^2)\frac{\partial E}{\partial x}+\frac{\partial}{\partial\xi}(k_t+k_r)\frac{\partial E}{\partial\xi}-N^2E=-\mathfrak{S}\xi_0 Q+C, \qquad (12)$$

and Eq. (4) as

$$\frac{\partial Q}{\partial\xi}=-\xi_0 Q, \qquad (13)$$

in which

$$D=\frac{\xi_0^2}{(\alpha''\rho_c)^2\alpha^2}K, \qquad k_r=\frac{8r\sigma\overline{T}^3F}{\alpha^s\rho_c}, \ \overline{S}=\frac{4\xi_0\sigma\overline{T}^3F}{\alpha''\rho_c},$$

$$N^2=\frac{8(1-r)\alpha_w\rho_c\xi_0^2\sigma\overline{T}^3F}{(\alpha''\rho_c)^2}. \tag{14}$$

In comparison with k_t, k_r can be named as the coefficient of radiative exchange and N^2 is the Newtonian radiative cooling coefficient. In addition, from Eq. (13) we have

$$Q=Q_0(x)e^{-\xi_0\xi},$$
$$Q_o(x)=(1-\Gamma_a)S_o/4, \tag{15}$$

where S_o is the net flux of solar radiation at the top of the atmosphere. Γ_a is the planet albedo of earth atmosphere.

Now the boundary conditions are as follows:

(1) At the atmospheric top $\xi=0$, we suppose that

(i) there is not downward flux of long wave radiation, i.e.

$$A_s=A_w=0. \tag{16}$$

(ii) There are no turbulent processes, i.e. pure radiative balance is satisfied. Then from Eqs. (1) and (16), we get

$$\alpha_s\rho_c(B_{s0}-2E_{s0})+\alpha_w\rho_c(B_{w0}-2E_{w0})+\alpha''\rho_cQ_0=0. \tag{17}$$

From Eq. (7), according to the simplified promise and taking Eq. (16) into consideration, we have

$$B_{s0}=2E_{s0}=2rE_0. \tag{18}$$

From Eq. (17), we immediately obtain

$$B_{wo}=2(1-r)E_0-\frac{\alpha''}{\alpha_w}Q_0. \tag{19}$$

(iii) The net solar radiation reaching the earth as a whole must be in equilibrium with the outgoing long wave emission to space, i.e.,

$$\int_0^1(B_{so}+B_{wo})dx=\int_0^1 Q_0(x)dx. \tag{20}$$

Let $\int_0^1 Q_0(x)dx=\overline{Q}_0$ and suppose that $Q_0(x)=\overline{Q}_0s(x)$, then we get

$$\int_0^1 S(x)dx=1.$$

After taking account of Eqs. (18) and (19), Eq. (20) becomes

$$\xi=0, \qquad \int_0^1 E_0dz=\frac{1}{2}\left(1+\frac{\alpha''}{\alpha_w}\right)\overline{Q}_0. \tag{21}$$

(iv) The upper atmosphere is an isothermal layer, i.e.

$$\xi=0, \ \frac{\partial E}{\partial \xi}=0. \tag{22}$$

(2) On the surface, $\xi=1$. When the heat storage in ocean or land is neglected, we have the condition of heat balance

$$A+(1-\Gamma)Q=B-k_t\frac{\partial T}{\partial z}, \tag{23}$$

where Γ is the albedo of the surface. Then Eq. (23) can be written as

$$A-B=\frac{k_t\alpha''\rho_c}{F4\xi_0\sigma T^3}\left(\frac{\partial E}{\partial \xi}\right)-(1-\Gamma)\overline{Q}_0e^{-\xi_0}S(x), \tag{24}$$

In the intense absorption region, from Eqs. (2) and (3), we obtain

$$A_s-B_s=-\frac{\alpha''}{\alpha_s\xi_0}\cdot\frac{\partial G_s}{\partial \xi}. \tag{25}$$

According to the simplified promise and Eq.(7), we obtain the approximate expression $G_s=2E_s$ and Eq. (25) becomes

$$A_s-B_s=-\frac{2r\alpha''}{\alpha_s\xi_0}\frac{\partial E}{\partial \xi}. \tag{26}$$

On the other hand, in the weak absorption region, according to Eq.(7), we approximately get

$$\frac{\partial^2 G_w}{\partial \xi^2}=-2(1-r)\frac{\xi_0^2\alpha_w^2}{\alpha''^2}E. \tag{27}$$

From Eqs. (2) and (3), wo have

$$\frac{\partial^2 G_w}{\partial \xi^2}=\alpha_w\rho_c(A_w-B_w). \tag{28}$$

Differentiating Eq. (28) and using ξ as the vertical coordinate, we get

$$\frac{\partial^2 G_w}{\partial \xi^2}=-\frac{\alpha_w\xi_0}{\alpha''}\frac{\partial(A_w-B_w)}{\partial \xi}. \tag{29}$$

Taking account of Eq. (27), we get

$$\frac{\partial(A_w-B_w)}{\partial \xi}=2(1-r)\frac{\xi_0\alpha_w}{\alpha''}E.$$

Integrating this formula, we get

$$(A_w-B_w)|_{\xi=1}=2(1-r)\frac{\xi_0\alpha_w}{\alpha''}\int_0^1 Ed\xi-B_{w0}.$$

Eliminating B_{w0} by using Eq. (19), we obtain

$$\xi=1,\ A_w-B_w=2(1-r)\frac{\xi_0\alpha_w}{\alpha''}\int_0^1 Ed\xi-2(1-r)E_0+\frac{\alpha''}{\alpha_w}Q_0. \tag{30}$$

Adding Eq. (30) to Eq. (26), we finally obtain

$$\xi=1,\ A-B=-\frac{2r\alpha''}{\xi_0\alpha_s}\frac{\partial E}{\partial \xi}+2(1-r)\frac{\xi_0\alpha_w}{\alpha''}\int_0^1 Ed\xi$$
$$-2(1-r)E_0+\frac{\alpha''}{\alpha_w}Q_0. \tag{31}$$

Substituting Eq. (31) into Eq. (24), we obtain another boundary condition, i. e.

$$\xi=1,\ (k_t+k_r)\frac{\partial E}{\partial \xi}-N^2\int_0^1 Ed\xi=-\frac{\alpha'' N^2}{\alpha_w \xi_0}E_0 + \bar{S}\left[\frac{\alpha''}{\alpha_w}+(1-\Gamma)e^{-\xi_0}\right]Q_0. \tag{32}$$

On the other hand, lateral boundary conditions may be given by

$$x=0,1,\qquad (1-x^2)^{\frac{1}{2}}\frac{\partial E}{\partial x}=0. \tag{33}$$

Now let us calculate integral constant C in Eq. (12). Integrating over the total earth and using Eqs. (22), (32) and (33), we obtain

$$C=\bar{S}Q_0\left[r\left(1+\frac{\alpha''}{\alpha_w}\right)-e^{-\xi_0}\int_0^1 \Gamma S(x)dx\right] \tag{34}$$

Finally, our model is summarized as

$$D\frac{\partial}{\partial x}(1-x^2)\frac{\partial E}{\partial x}+ +\frac{\partial}{\partial \xi}(k_t+k_r)\frac{\partial E}{\partial \xi}-N^2E$$

$$=-\bar{S}\bar{Q}_0\xi_0 e^{-\xi_0\xi}S(x)+\bar{S}\bar{Q}_0\left[r\left(1+\frac{\alpha''}{\alpha_w}\right)-e^{-\xi_0}\int_0^1 \Gamma S(x)dx\right], \tag{35}$$

$$\xi=0,\ \int_0^1 E_0(x)dx=\frac{1}{2}\left(1+\frac{\alpha''}{\alpha_w}\right)\bar{Q}_0\ , \tag{36}$$

$$\xi=1,\ (k_t+k_r)\frac{\partial E}{\partial \xi}-N^2\int_0^1 Ed\xi=-\frac{\alpha'' N^2}{\alpha_w \xi_0}E_0 + \bar{S}\bar{Q}_0\left[\frac{\alpha''}{\alpha_w}+(1-\Gamma)e^{-\xi_0}\right]S(x), \tag{37}$$

and the lateral condition (33). So far, these equations are closed.

III. SOLUTIONS

If all the coefficients in the equations are constants, the analytical solution can be easily obtained. According to Budyko's theory [1], the albedo is taken as

$$\Gamma(x,x_s)=\begin{cases}\Gamma_1=0.62, & \text{when } x>x_s\ ,\\ \Gamma_0=0.32, & \text{when } x<x_s\ ,\end{cases} \tag{38}$$

where $x_s=\sin\theta_s,\theta_s$ is the latitude of ice edge, which is defined by $T=-10°C$. Only the symmetric solution with respect to equator is taken here.

When the values of the parameters are taken to be $\bar{T}=283$K, $r=0.5$, $\alpha=0.25$ cm^2/g, $\alpha_w=1.25$ cm^2/g, $\alpha_s=100$ cm^2/g, $F=1$, $K=3\times 10$ cal/cm·sec·K, $\kappa_t=50$ cal/cm·sec·K, $\rho_c=4\times 10^{-6}$ g/cm^3, we get $\xi_0=0.4$. If the present solar constant and the ice edge are taken as follows:

$S_o=1.92$ cal/cm^2·min,

$x_s=0.95$, (i.e. $\theta_s=72°$N),

we obtain the distribution of temperature of surface (thin solid line in Fig. 1.). It shows that it is in agreement with the observations[4] (represented by circle). However, the temperature of atmosphere as a function of height is not very similar to the observations.

In order to get the distributions of temperature with height in agreement with the facts, the effects of the absorption medium in atmosphere and cloudiness on the radiation must be taken into account. But distinguishing in detail the absorption and emission of various mediums to different wave lengths we will meet a lot of problems, therefore, we have to find a simple parameterization by means of grey coefficient F which expresses the effects of medium and clouds on the radiation. Because the content of water vapor near surface is larger, the infrared radiation is stronger and it produces the larger grey coefficient F[5]. Since the clouds will play an important role in the middle troposphere and the dense clouds can be approximately considered as black body[5] to long-wave radiation, larger grey coefficient F can be taken. Because all the coefficients in Eq. (35) are not equal to constants, we will numerically get the solution by means of finite-difference equation. The boundary conditions are taken as

$$x=0,1,\quad \frac{\partial E}{\partial x}=0 \tag{39}$$

and

$$\xi=0,\quad E=\frac{1}{2}\left(1+\frac{\alpha''}{\alpha_w}\right)\bar{Q}_o\,. \tag{40}$$

The absorption gases in atmosphere are primarily water vapor (H_2O) and CO_2. Taking $\rho_{c_{H_2O}}=1.5\times10^{-6}e^{-0.4z}$ g/cm^3, $\rho_{c_{CO_2}}=10^{-6}e^{-0.1z}$g/cm^3, we get $\xi_0=0.4$. The height z as function of the optical thickness ξ is expressed in Table 1. The vertical ranges corresponding to the step length of optical thickness are taken to be 0.1 and 0.01 in lower and higher atmosphere respectively. The step length of x is taken to be 0.05. Using iteration method, when the difference of E between two adjacent intervals is less than 10^{-6} cal/cm·min, it is considered that the correct solution has been obtained.

Table 1. The Height as a Function of the Optical Thickness

z(km)	0	1.0	2.0	3.0	4.0	6.0	8.0	10.0	12.0	14.0	∞
ξ	1	0.69	0.47	0.33	0.23	0.12	0.07	0.04	0.03	0.02	0

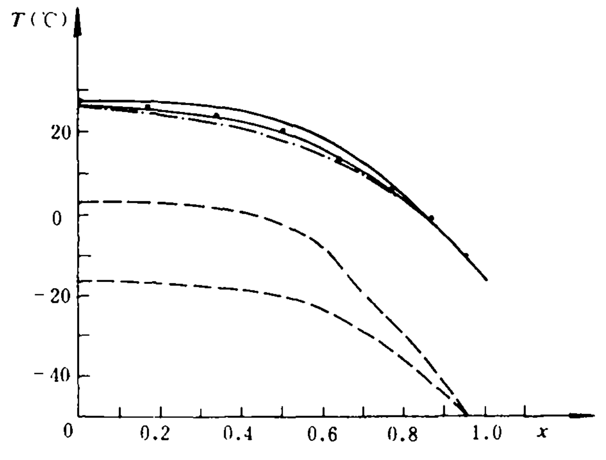

Fig. 1. Temperature of the earth's surface as a function of latitude. The black and thin solid curves are the numerical and analytical results of the two-dimensional model respectively. The dash-dotted curve is the result of the two-dimensional model. The dash curves are the two climate states corresponding to $Q/Q_o=0.75$.

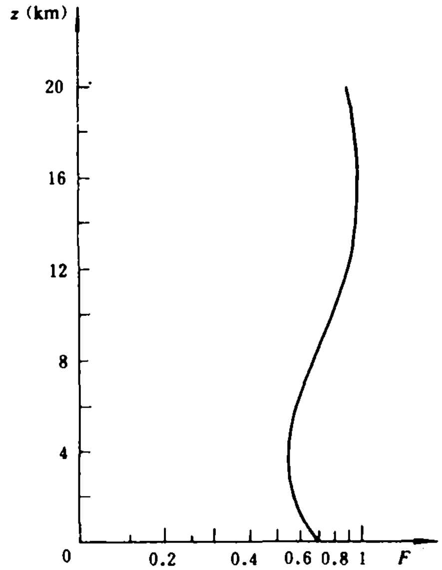

Fig. 2. The grey body coefficient F as a function of height.

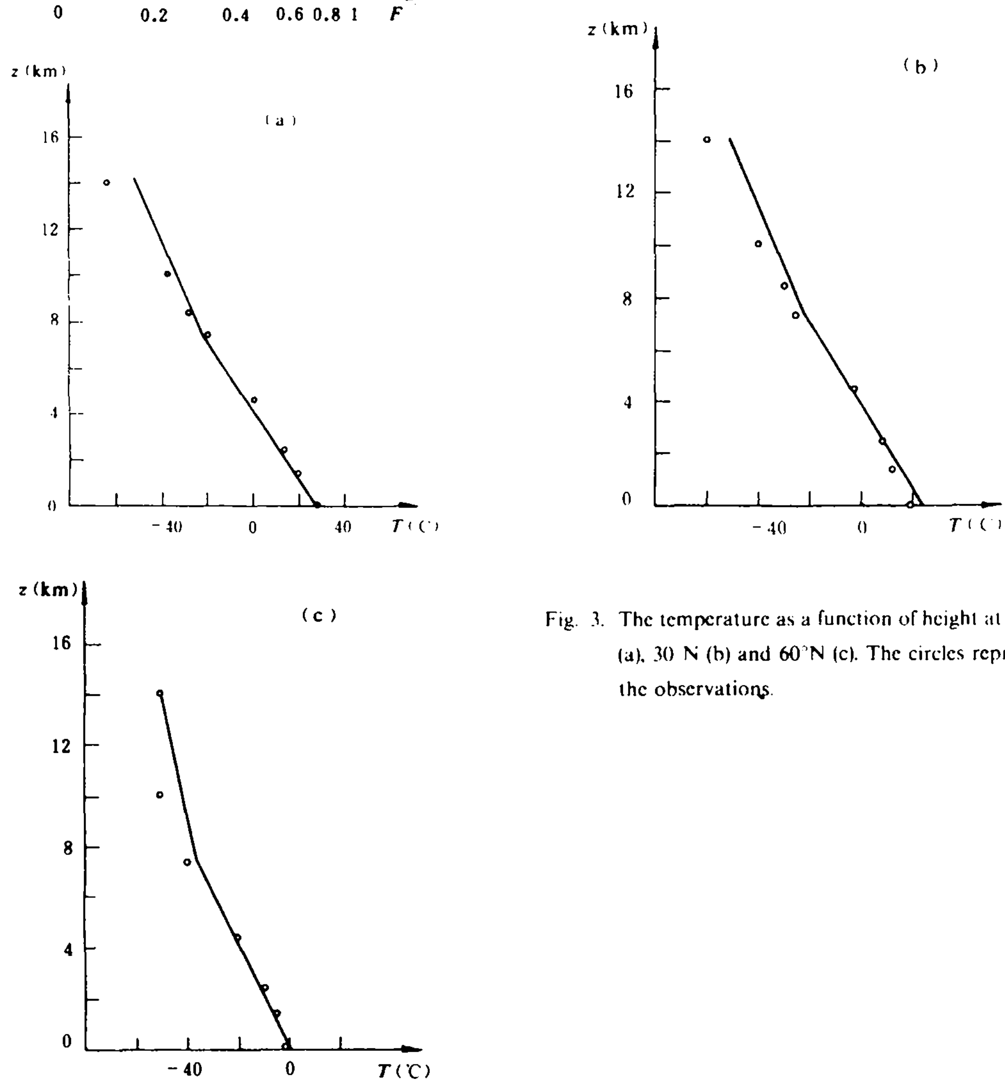

Fig. 3. The temperature as a function of height at 10°N (a), 30°N (b) and 60°N (c). The circles represent the observations.

Taking $k_t = 29$ cal/cm·sec·K, $\Gamma_\alpha = 0.35$, the grey coefficient F as a function of height as expressed in Fig. 2, and the same other parameters as the above mentioned, we get the temperature of the earth's surface as a function of latitude expressed by black solid line in Fig. 1. Fig. 3 is the temperature as the functions of height at 10°N, 30°N and 60°N. It is found that the calculative values are similar to the observations. Like the analytical solution, the temperature has double value corresponding to the same solar constant. The two climate states are illustrated by dash lines in Fig. 1 corresponding to $Q/Q_0 = 0.75$. One of them has ice edge $x_s = 0.62$, another corresponds to the climate of ice-covered earth. The former can be achieved if the initial iterated value $E_o \geqslant 0.3$ is taken and the latter corresponds to $E_o = 0.2$. If $Q/Q_o = 1.0$, no matter what value of E_o is given, the climate is unique.

IV. THE INFLUENCE OF THE CHANGES OF SOLAR RADIATION ON THE ICE EDGE

The ice edge as a function of different solar constants obtained from analytical solution is illustrated by thin solid line in Fig. 4. It is shown that if ice edge would move from the present 72°N of ice ages, the solar constant should be 15% less than the present value. The bifurcation point is presented when Q/Q_o is nearly equal to 0.65. The corresponding results of numerical solution is illustrated by black solid line in Fig. 4. The requirement of solar radiation occurring in ice ages should decrease by

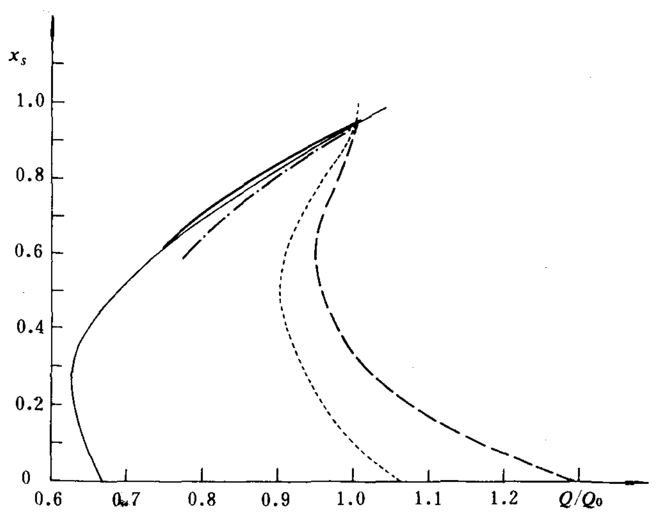

Fig. 4. Latitude of ice edge as a function of solar constant. The black and thin solid curves are the results of two-dimensional model, the dash-dotted one is the result of one-dimensional model. The dotted and dashed curves are the results that the infrared radiation is parameterized by a linear function of temperature in one-dimensional model.

16% which is similar to the result in analytical solution. It is seen from the black solid line in Fig. 4 that when $Q/Q_o \leqslant 0.75$, the climate is ice-covered earth and that when $Q/Q_o = 0.75$, two climate states can be obtained.

V. COMPARISON WITH ONE-DIMENSIONAL MODEL

Integrating Eqs. (35)—(37) and (33) from $\xi = 0$ to 1, they may be transformed into the one-dimensional model, i.e.

$$D^*\frac{d}{dx}(1-x^2)\frac{d\overline{E}}{dx}=E_0-\left[(1-\Gamma)e^{-\xi_0}+\frac{\alpha''}{\alpha_w}\right]Q_0+\bar{Q}_0(e^{-\xi_0}-1)S(x)$$
$$+\bar{Q}_0\left[r\left(1+\frac{\alpha''}{\alpha_w}\right)-\int_0^1 e^{-\xi_0}\Gamma S(x)dx\right], \tag{41}$$

where $\overline{E}=\int_0^1 Ed\xi$ is mean radiative energy over the total column of air, and

$$D^*=\frac{K\xi_0}{a^2\alpha''\rho_c}\cdot\frac{1}{4\sigma\overline{T}^3F}. \tag{42}$$

The solution can be numerically gotten when $F=0.68$, $\alpha''=0.08$, $\alpha_w=2$, the step length of x is 0.01 and other parameters are the same as those in section III. The distribution of surface temperature is illustrated by dash-dotted line in Fig. 1. The ice edge as a function of changes of solar radiation is illustrated by the same line in Fig. 4. In this case the decreased requirement of solar constant occurring in glacial climate is 13%.

Using spectrum analysis method to solve Eq. (41), we obtain

$$\overline{E}=\bar{Q}_0\sum_n\frac{(2n+1)\int_0^1 M(x)P_n(x)dx}{D^*n(n+1)+1}P_n(x), \tag{43}$$

where

$$M(x)=\left(\Gamma e^{-\xi_0}-1-\frac{\alpha''}{\alpha_w}\right)S(x)+\frac{1}{2}\left(1+\frac{\alpha''}{\alpha_w}\right)-\int_0^1 e^{-\xi_0}\Gamma S(x)dx. \tag{44}$$

Taking $D^*=0.4$ and $\alpha''/\alpha_w=0.1$, using the same other parameters as those mentioned above and letting

$$\overline{E}=A+BT, \tag{45}$$

where $A=201.4$ W/m^2, $B=1.45$W/m$^2\cdot$°C as used by North [6] the surface temperature as a function of latitude in agreement with the facts can also be achieved. The ice edge as a function of the changes of solar radiation is illustrated by dotted line in Fig. 4. The requirement of solar constant occurring in ice ages should decrease by 5% from thr present value.

Eq. (41) may be further simplified. Suppose that atmospheric thickness is so thin that optical thicknes tend to zero, i.e. $\xi_0\rightarrow 0$, then the solar radiation absorbed by this layer may be neglected. Thus it is equivalent to $\alpha''\rightarrow 0$, and $E_0\rightarrow\overline{E}$. Considering

$$\int_0^1 \Gamma S(x)dx\approx\Gamma\int_0^1 S(x)dx=\Gamma\approx 0.5. \tag{46}$$

Eq. (41) can be simplified as follows:

$$D^*\frac{d}{dx}(1-x^2)\frac{d\overline{E}}{dx}=\overline{E}-(1-\Gamma)Q_0. \tag{47}$$

Obviously, it is the Sellers-type model used by North [6]. We can get the solution as follows:

$$\overline{E}=\bar{Q}_0\sum_n\frac{(2n+1)\int_0^1(1-\Gamma)S(x)P_n(x)dx}{D^*n(n+1)+1}P_n(x). \tag{48}$$

The parameterization of E is the same as Eq. (45). The distribution of surface temperature in agreement with observation can also be obtained. In this case, if the ice edge shifts from 72°N to 50°N, solar constant would only decrease by 2% (see the dash curve in Fig. 4).

VI. THE INFLUENCE OF PARAMETERIZATION OF INFRARED RADIATION ON THE RESULTS

Some authors[7-8] have pointed out that the sensitivity of climate to the changes of solar constant greatly upon the parameterization of infrared rediation in one-dimensional model. When Eq. (45) is taken and $B=1.45$ W/m$^2\cdot$°C, the requirement of solar constant occurring in ice ages should decrease by 2% from the present value. However, if $B=2.23$ W/m$^2\cdot$°C, the above value is 12%. If the atmosphere is considered as a grey body, the above value is 13% (see previous section).

The long-wave radiation is linearized in two-dimensional model as follows:

$$E=4F\sigma\overline{T}^3\cdot T+A. \tag{49}$$

Although the distribution of temperature in agreement with the observations can be obtained, the decreased requirement of solar radiation occurring in ice ages is about 6% rather than 16% as described in section III.

we can come to the conclusion that the difference of sensitivity of climate is caused by parameterization of radiation. In particular, when the relationship between radiation and temperature is linear, the sensitivity of climate to the changes of solar constant will be much more larger.

VII. CONCLUSIONS

A simple two-dimensional energy balance climate model including the solar and infrared radiation transports in vertical direction, the turbulent exchanges of heat in vertical and horizontal directions and the ice caps-albedo feedback is developed in this paper. The analytical and numerical solutions show that if the atmosphere is considered as a grey body and the grey body coefficient depends upon the distributions of absorption medium and cloudiness, both horizontal and vertical distributions of temperature which are identical with the facts are very similar to the observation. It is found that the atmospheric components (including clouds) influencing radiation will play an important role in the formation of earth climate.

Comparing the results that the atmosphere is considered as a grey body with ones that the infrared radiation is parameterized by a linear function of temperature, the results show that even though both of them can obtain the climate in agreement with the facts, the sensitivity of the climate to the changes of the solar constant is very different. In the former case, no matter whether the model is two-or one-dimensional, the ice edge is extremely independent of the changes of solar radiation, for instance, the requirement for the ice edge to move southward from the the normal 72°N to 50°N (i.e. where the glacial climate would take place) is that the solar constant should decrease by 13% to 16%. However, in the latter case, the climate is highly sensitive to the changes of solar radiation. In this case, the requirement of solar radiation occurring in the glacial climate should decrease by 2% to 6%. It is shown that we must be careful when the parameterizations of the radiation and other processes are taken, otherwise, the reliability of the results is suspicious. Thorefore, it is seen that the research of radiation processes in the atmosphere is very important, particularly, the research of the influence of cloud on the radiation should be emphasized in future.

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