APPLICATION OF SODAR SOUNDING TO ATMOSPHERIC DISPERSION—MIXING DEPTH AND CONCENTRATION AT THE GROUND

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ABSTRACT

It is the intent of this paper to illustrate how to apply acoustic radar data on the variation of mixing depth in the study of atmospheric dispersion. The box model, as an example of the routine usage of acoustic sounding, has been modified. A case of the development of the structure of mixed layer, resulting from some synoptic process is discussed and the results show that the ordinary model calculations regarding atmospheric dispersion will mislead the assessment of air quality if no remote techniques, such as the acoustic radar, are associated with.

I. INTRODUCTION

Acoustic sounder can be used for measuring, with real-time advantage, such quantities in the atmospheric boundary layer as velocities, mixing height, capping inversion thickness and strength, which are necessary for investigating the transport and diffusion of atmospheric pollutants. In particular, it can provide fairly reliable results in that case where an ordinary model calculation would not work well because certain synoptic processes bring about a singular variation of the structure of boundary layer.

In this work, the theoretic formula for box model and its operation method are modified. The local parameters in the formula are obtained by using sodar records. The modified box model is useful for the quality assessment of atmospheric environment. Besides, a special example of mixed layer development and its effect on the surface layer concentration are studied. These reveal the important role of the acoustic radar in the investigation of atmospheric dispersion.

II. BOX MODEL

The concentration of a specific pollutant that diffuses in a one-dimensional flow field is given by

$$\frac{\partial \bar{c}}{\partial t}+\bar{u}\,\frac{\partial \bar{c}}{\partial x}+\frac{\partial}{\partial x}\overline{u'c'}=s, \tag{1}$$

where s standing for the source or sink of the species, is not a function of $\bar{c}$. Considering the boxes shown in Fig. 1, we concentrate on the space-time averaged concentration, variation of the averaged concentration with time and asymptotical behavior of this variation, while ignoring the details about the pollutant distribution in a box. Besides, we assume

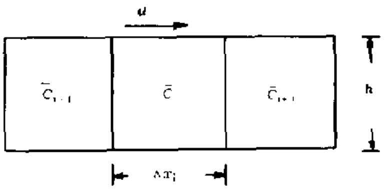

Fig. 1. Conceptual representation of the boxes.

that, for a given box, the arrangement of the source strength on the x–y plane is of probability preserving. Under these conditions, we obtain from (1) the equation of pollutant budget for box i

$$\frac{\partial \bar{c}_i}{\partial t}+\frac{\bar{u}}{\Delta x_i}(\bar{c}_i-\bar{c}_{i-1})+\frac{\partial K u_c}{\Delta x_i}\left(\bar{c}_i-\frac{\bar{c}_{i+1}+\bar{c}_{i-1}}{2}\right)=\frac{Q_i}{h\Delta x_i\Delta y_i}, \tag{2}$$

where h is the mixing height, K the eddy diffusivity, Δx the length of box along the wind, Q_i the total area effluent strength in box i and u_t the turbulent characteristic velocity. The third term on the left hand side in Eq. (2) has been obtained by assuming an exchange of air parcels with identical volume in the adjacent two boxes. If we ignore the diffusion term and assume that j, $\bar{u}$ and $\bar{c}_{i-1}$ are constants ($\bar{c}_{i-1}$ can be considered approximately as constant for the case, for example, when box i-1 is located at a rural site upwind of the city concerned), the solution of Eq. (2) is

$$\bar{c}_i=\frac{Q_i}{h\bar{u}\Delta y_i}+\bar{c}_{i-1}+\left[\bar{c}_i(0)-\left(\frac{Q_i}{h\bar{u}\Delta y_i}+\bar{c}_{i-1}\right)\right]\exp-\frac{\bar{u}}{\Delta x_i}t, \tag{3}$$

where $\bar{c}_i(0)=\bar{c}_i(t)|_{t=0}$. Let $\frac{\bar{u}}{\Delta x_i}t\to\infty$ and $Q_i\to 0$, we have

$$\bar{c}_i=\bar{c}_{i-1}, \quad \text{or} \quad \bar{c}_i\propto\bar{u}^0. \tag{4}$$

In this circumstance the wind speed has no significant effect on the concentration distribution, as often happens in the areas with few local pollutant sources. Eq. (4) is supported by the data quoted in Ref. [1]. If $\frac{\bar{u}}{\Delta x_i}t\to\infty$ and $\bar{c}_{i-1}\to 0$ the well-known formula for a box model is deduced to

$$\bar{c}_i=\frac{Q_i}{h\bar{u}\Delta y_i}. \tag{5}$$

It must be pointed out that if Δx_i is as large as 20 km and $\bar{u}$ is as small as 2 $\mathrm{m\,s^{-1}}$, the decaying of $c_i(0)$ by one order of magnitude will take as long as 5 hr, as indicated by Eq. (3). During this period the structure of atmospheric boundary layer, such as h and the drainage term $\frac{Q_i}{\Delta x_i\Delta y_i}$, will vary significantly, so that Eq. (5) will no longer be valid. For the smaller values of $\frac{\bar{u}}{\Delta x_i}t$, we have

$$\exp\left(-\frac{\bar{u}}{\Delta x_i}t\right)\approx 1-\frac{\bar{u}}{\Delta x_i}t,$$

substituting into Eq. (3) yields

$$\bar{c}_i=\left[\bar{c}_i(0)-\left(\frac{Q_i}{h\bar{u}\Delta y_i}+\bar{c}_{i-1}\right)\right]\left(1-\frac{\bar{u}}{\Delta x_i}t\right)+\frac{Q_i}{h\bar{u}\Delta y_i}+\bar{c}_{i-1}. \tag{6}$$

Taking emissions from both industrial and domestic activities into account, if for a specific period $Q_i=0$ and $\bar{c}_{i-1}=0$, Eq. (6) becomes

$$\bar{c}_i(0)-\bar{c}_i(t)=\frac{\bar{c}_i(0)t}{\Delta x_i}\bar{u},\quad \text{or}\quad \bar{c}_i(0)-\bar{c}_i(t)\propto\bar{u}, \tag{7}$$

which indicates that the pollutant is diluted in direct propertion to $\bar{u}$.

In case when $\bar{c}_{i-1}$ can not be considered as a constant while box i is being studied, we may start with the box in the suburb to the upwind of a city. Denote this box as i-2. Because Q_{i-2} is very small, for the first approximation, $\bar{c}_{i-2}$ may be considered as a constant, we obtain from Eq. (2)

$$\bar{c}_i=\frac{A\Delta x_i\Delta x_{i-1}}{\bar{u}(\Delta x_{i-1}-\Delta x_i)}\exp\left(-\frac{\bar{u}}{\Delta x_{i-1}}t\right)+\left[\bar{c}_i(0)-\frac{A\Delta x_i\Delta x_{i-1}}{\bar{u}(\Delta x_{i-1}-\Delta x_i)}-\frac{1}{h\bar{u}}\left(\frac{Q_i}{\Delta y_i}+\frac{Q_{i-1}}{\Delta y_{i-1}}\right)\right]\exp\left(-\frac{\bar{u}}{\Delta x_i}t\right)+\frac{1}{h\bar{u}}\left(\frac{Q_i}{\Delta y_i}+\frac{Q_{i-1}}{\Delta y_{i-1}}\right),$$
$$\text{for}\quad \Delta x_i\neq\Delta x_{i-1}, \tag{8}$$

$$\bar{c}_i=\left[\bar{c}_i(0)+At-\frac{1}{h\bar{u}}\left(\frac{Q_i}{\Delta y_i}+\frac{Q_{i-1}}{\Delta y_{i-1}}\right)\right]\exp\left(-\frac{\bar{u}}{\Delta x_i}t\right)+\frac{1}{h\bar{u}}\left(\frac{Q_i}{\Delta y_i}+\frac{Q_{i-1}}{\Delta y_{i-1}}\right),\quad \text{for}\quad \Delta x_i=\Delta x_{i-1}, \tag{9}$$

where
$$A=\frac{\bar{u}}{\Delta x_i}\left(\bar{c}_{i-1}(0)-\frac{Q_{i-1}}{h\bar{u}\Delta y_{i-1}}\right).$$

In deriving the above equations we have omitted the turbulent term in Eq. (2) and put $\bar{c}_{i-2}=0$ for simplicity. The discussion similar to that for Eq. (3) can be conducted for Eqs. (8) and (9), the only difference is that the procedure for $\bar{c}_i$ to reach its stationary values is dependent on the arrangement of Δx_i and Δx_{i-1}.

Likewise, a similar discussion can be given to Eq. (2) for the case when $\bar{u}=0$, with $u_e=1-2\text{m s}^{-1}$ typically. Of course, there are some other factors, such as the fluctuations of wind direction, that influence the distribution of pollutants, All of them make us believe that, generally speaking, the concentration of a species in the box model is not proportional to $\bar{u}^{-1}$. From Eqs. (4), (5) and (7) we may obtain a semiempirical formula for the box model, that is

$$\bar{c}=a\bar{u}^b, \tag{10}$$

where a and b are constants. The same expression was first proposed by Benarie[1].

It should be noted that a and b can be considered as constants only for specific time periods. Thus if the daily average is taken as a unit, it would deviate from the physical conditions of the model. Therefore we use the box model following the procedure given below.

(1) Divide the time of a day into four periods, $\Delta\tau_j$ ($j=1,2,3,4$) according to the sodar records. Each of these periods corresponds to a specified step in daily evolution of the atmospheric boundary layer. For $\Delta\tau_1$ (Beijing time 05:00 to 08:00 for autumn), the radiation inversion formed at night is being dissipated, and the concentrations of pollutants at the ground usually have their maximum values of the day since the pollutant storaged in the stable layer would be transported down to the ground. During the period $\Delta\tau_2$ (08:00

to 11:00), the radiation inversion has been dissipated to disappear, the concentrations of pollutants begin to decrease but this decrease is limited by the mixing height and the strength of capping inversion. As to the period $\Delta\tau_3$ (11:00—17:00), the capping inversion will either disappear or become very weak so that the ascending air parcels generated by convective activities could penetrate across it, and the mixed layer reaches its maximum height of the day. The concentrations of pollutants at the ground will therefore significantly decrease. In the period $\Delta\tau_4$ (17:00—5:00), the elevated sources have no effects on the concentrations of pollutants in the weakly mixed layer formed by disturbances of the urban dynamic roughness and heat island. h_j, the mixing height with respect to $\Delta\tau_j$ can be determined conveniently according to the sodar records.

(2) Calculate the space-time average of wind speeds obtained by both acoustic Doppler radar and metorological tower for each of $\Delta\tau_j$.

(3) Carry out regressive analysis to Eq. (10) by using the concentrations of pollutants obtained from continuous observations to determine the values of a_j and b_j.

Figs. 2—4 show the comparisons of SO_2 concentration between the results calculated by

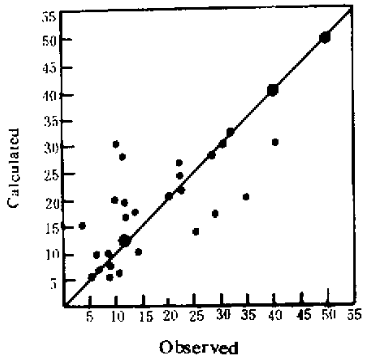

Fig. 2. A comparison of calculated and observed values for SO_2 concentrations in the urban district, Beijing, November 24—December 3, 1981. (10^{-2} mg m^{-3})

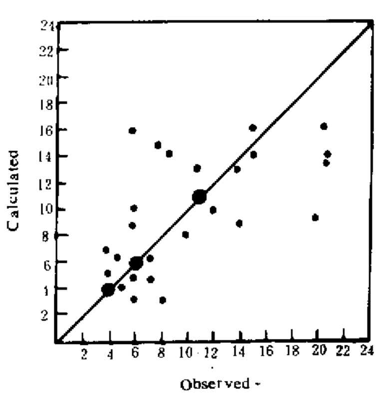

Fig. 3. As in Fig. 2, except for the northwest suburb.

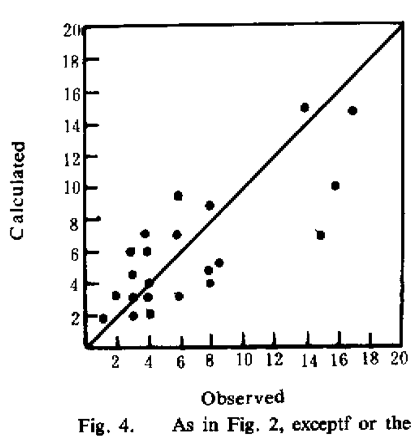

Fig. 4. As in Fig. 2, exceptf or the southeast suburb.

Eq. (10) and those measured by the instrument KZL–SO_2. The data used for calculation are from November to December, 1981 whereas the values of a_j and b_j are specified by using the data of October 1981. Of course, an appropriate adjustment has been given to the source strength term. In drawing these results, the profiles of wind from 320 m meteorological tower and an assumption that the shape of the profiles is not affected by ground roughness are utilized. Besides if no inversion can be found from the sodar records, the mixing height is considered as 1 km, and if the mixing layer develops to higher than 320 m, the mean wind speed within it is considered to be equal approximately to that within 320 m.

The values of a_j and b_j are presented in Table 1. Inspecting these values we can find out that, on an average, the correlation of concentrations at the ground in the suburbs to wind speeds is higher than that in the urban section, and that the concentrations in the suburb areas for nighttime have higher correlation to wind than for daytime. $|b_j|$ can be greater than 1, and this may be resulted from the strong interactions between turbulence and advection during nighttime.

Table 1 The Values of a_j and b_j

Locations	Periods	a_j	b_j
The Northwest Suburb	$\Delta\tau_1$	0.20	−0.39
	$\Delta\tau_2$	0.13	−0.42
	$\Delta\tau_3$	0.16	−0.70
	$\Delta\tau_4$	0.30	−1.0
The Urban	$\Delta\tau_1$	0.50	−0.31
	$\Delta\tau_2$	0.20	−0.50
	$\Delta\tau_3$	0.30	−0.84
	$\Delta\tau_4$	0.30	−0.30
The Southeast Suburb	$\Delta\tau_1$	0.13	−0.06
	$\Delta\tau_2$	0.18	−0.72
	$\Delta\tau_3$	0.12	−0.73
	$\Delta\tau_4$	0.50	−1.44

The dots in Figs. 2—4 are somewhat sporadic, and this is partly ascribed to scattered limitied data we have had. Furthermore, the limited data implicate a possibility of non-constant a_j and b_j. In spite of these deficiencies, the results shown in Fig. 2—4 still support the modified box model. The rms errors, by taking Fig. 3 as an example, are 0.06, 0.028, 0.05 and 0.049 respectively for periods $\Delta\tau_1$ to $\Delta\tau_4$.

III. A CASE STUDY OF MIXED LAYER DEVELOPMENT AND ITS EFFECT ON THE CONCENTRATIONS OF POLLUTANTS AT THE GROUND

In the field of atmospheric dispersion studies, we are concerned not only with the air quality under normal meteorological conditions but also with that in special synoptic processes. In this section we will give an example to discuss the ability of the acoustic radar in studies for special pollutant events.

Fig. 5 shows the facsimile photograph of the variation of mixing height measured by sodar on November 19, 1981 in Beijing. At local time 16:00 the mixing height is 520 m, and the strength of the capping inversion is rather weak, After that, the mixing height continuously increases and the capping inversion tends to be broken down. But, by contraries, the sodar facsimile photograph shows a decreasing mixing height and a rising capping inversion strength,

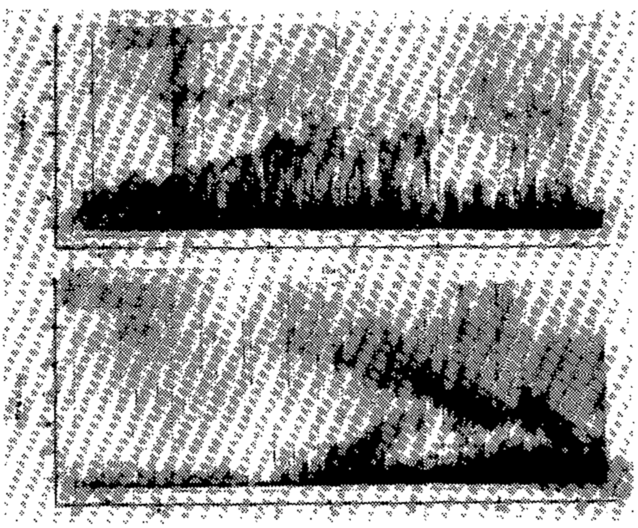

Fig. 5. A special development of mixing height, November 19, 1981, Beijing.

and the descending capping inversion goes to superpose upon the radiation inversion. Apparently, under the influence of this process, the concentrations of pollutants at the ground will increase and the usual model calculations will not be able to predict it. For being specific, let us consider the Gaussian plume model for $x=y=0$[2],

$$\bar{c} = \frac{Q}{2\pi u\sigma_y\sigma_z}\left\{\sum_{-\infty}^{+\infty}\exp\left[-\frac{(2Nh-h_Q)^2}{2\sigma_z^2}\right]+\sum_{N=-\infty}^{+\infty}\exp\left[-\frac{(2Nh+h_Q)^2}{2\sigma_z^2}\right]\right\}, \tag{11}$$

where h is the mixing height, h_Q the source's effective height and N an integral. If $\sigma_z \ll h$,

$$\frac{\partial \bar{c}}{\partial h} = 0. \tag{12}$$

It indicates that the capping inversion does not affect the concentration distribution in the mixed layer. If $\sigma_z \sim h$, $h \gg h_Q$ and $N=1$ (the condition $N=1$ is usually employed for $\sigma_z \sim h$), it follows from Eq. (11) that

$$\frac{\partial \ln \bar{c}}{\partial h} = -\frac{\Delta h}{\sigma_z^2} \approx -\frac{4}{h}. \tag{13}$$

If $\sigma_z \geqslant h$, the plume will uniformly distribute between the ground and the base of the capping inversion, i.e. the mean concentrations in the air column can be recognized as the surface concentrations. Having considered the concentration distribution for the plume in the infinite atmosphere and puting the total pollutant mass into the layer with a thickness h, we obtain the mean concentrations between the ground and the base of the inversion,

$$\langle \bar{c} \rangle = \frac{1}{h}\int_{-\infty}^{+\infty} \bar{c}_0\, dz = \frac{Q}{2\pi u\sigma_y\sigma_z h}\int_{-\infty}^{+\infty}\exp\left[\frac{-(z-h_Q)^2}{2\sigma_z^2}\right]dz = \frac{Q}{\sqrt{2\pi}\,\sigma_y u h}, \tag{14}$$

or

$$\frac{\partial\langle \bar{c}\rangle}{\partial h} \propto -\frac{1}{h^2}, \quad \text{for} \quad \sigma_z \geq h. \tag{15}$$

From Eqs. (12), (13) and (15) it is readily seen:

(1) For a specified pollutant source, the effect of the mixing height on the concentration distribution is significant only for downstream regions of wind at some distance from the source. This effective distance would decrease with the growth in strength of turbulent activities, since σ_z is controlled by turbulence.

(2) The smaller the h, the greater the effect on the concentration, in particular, when $\sigma_z > h$. In general, the mixed layer begins to grow within 1—2 hr after sunrise. At this time, the structure of atmospheric boundary layer is dynamically non-stationary, which makes it very difficult to determine theoretically the initial value of h. Acoustic radar then can be used to provide visual photograph to cope with the difficulty.

Table 2 shows the effect of the circumstance described in Fig. 2 on the concentration at the ground after local time 16:00. In the scheme we assume that wind velocity $u=1$ m s^{-1}, the source strength $Q=30$ g sec^{-1}, the effective source height $h_Q=80$ m and the downstream distance $x=1$, 5 km.

Table 2 The Effects of the Special Development of the Mixed Layer on the Concentrations (in mg m^{-3})

Local Time	Concentrations			
	$x=1$ km		$x=5$ km	
	B	A	B	A
16:00	1.86×10^{-1}	1.86×10^{-1}	2.50×10^{-2}	2.50×10^{-2}
17:00	1.87×10^{-1}	1.86×10^{-1}	3.00×10^{-2}	1.20×10^{-2}
18:00	1.87×10^{-1}	1.86×10^{-1}	3.20×10^{-2}	1.20×10^{-2}
19:00	2.34×10^{-1}	1.86×10^{-1}	4.20×10^{-2}	1.20×10^{-2}

The results in column A are obtained by usual model computation, and that in column B by model computation adjusted according to the sodar records. It is clear that at $x=5$ km, large errors would have occurred if there had been no real time observation.

IV THE INITIAL DEVELOPMENT OF ATMOSPHERIC BOUNDARY LAYER AND THE POLLUTANT CONCENTRATION

Near sunrise the mixing depth changes markedly, as observed by our acoustic radar. At this time, the dynamical structure of the surface layer is rather complicated, which will lead to complex behavior of pollutant dispersal. The time scale for the variance of boundary condition at the ground level is

$$T_1=(\overline{w\theta})_0 \Big/ \frac{d(\overline{w\theta})_0}{dt},$$

where $C_p\rho(\overline{w\theta})_0$ is the turbulent heat flux. On the other hand, a bulk time scale for the mixed layer is

$$T_2=h/u_f,$$

where h is the mixing depth and

$$u_f=\left[\frac{g}{\Theta}(\overline{w\,\theta})_0 h\right]^{1/3}$$

is the characteristic velocity for free convection. Within 1—2 hr after sunrise, the value of T_1 is increasing from $T_1=0$ to a value not too large. At the same time, u_f is also increasing. Taking notice of the fact that h is increasing at a rate greater than that for u_f (there are some factors which give rise to a quickly increasing in h, such as the surface roughness and the momentum transport, etc.), we can find out that, near surnise, the condition

$$T_1 \leqslant T_2$$

would occur. This indicates that the dynamic process is non-stationary. In this case we should be cautious in utilizing the models which are derived under the assumption of spatial and temporal uniformity in the structure of boundary layer. What is important is how to consider the pollutant transport in the vertical direction. For simplicity, we use the following equation,

$$\frac{\partial \bar{c}}{\partial t}=-\frac{\partial}{\partial z}\overline{w'c'}. \tag{16}$$

In the initial development of the mixed layer, the turbulent energy is mainly used for the entraining processes so that the fluctuations with low frequencies have significant effects on the dispersion process. Therefore, we can write

$$-\frac{\partial}{\partial z}\overline{w'c'}\propto u_f\frac{\Delta\bar{c}}{h}=\left[\frac{g}{\Theta}(\overline{w\,\theta})_0 h\right]^{1/3}\frac{\Delta\bar{c}}{h}, \tag{17}$$

where $\Delta\bar{c}$ expresses the concentration difference between the surface level and a layer of the inversion, which is going to be dissipated. According to the analysis of mixed layer, we have

$$\frac{h^2}{t}\propto\frac{(\overline{w\,\theta})_0}{\Gamma}, \tag{18}$$

where Γ is the potential temperature gradient of the background field. Substitution of Eq. (18) into (17) gives

$$\frac{\partial \bar{c}}{\partial t}\propto\frac{N^{2/3}\Delta\bar{c}}{t^{1/3}}, \tag{19}$$

where

$$N=\left[\frac{g}{\Theta}\Gamma\right]^{1/2}.$$

Near sunrise, t is small while $\Delta\bar{c}$ has a large magnitude. The Eq. (19) shows that during the initial development of mixed layer the pollutant concentration at the ground level would quickly increase. In support of this conclusion, the observations show that the pollutant concentrations at the ground reach their maximum within 1—2 hr after sunrise. From Table 1, we can find out that, for a given box, the value of b for $\Delta\tau_1$ is the smallest. One important cause is the vertical transport induced by the process discussed above. The acoustic radar can be used to find the time when the radiative inversion has been dissipated and the probable time when the pollutant concentration reaches its maximum as well.

V. CONCLUDING REMARKS

For the achievement of more quantitative and more reliable estimates of atmospheric pollutant transport and diffusion it is necessary to place emphasis on developing application of remote techniques. Acoustic Doppler techniques appear to be vital candidates for this area. By using acoustic radar measurements of the total wind vector and the temperature structure, as well as the mixing heights and capping inversion thicknesses, etc., the model computations can be adjusted to give more reasonable and reliable results.

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