THE EFFECTS OF EARTH PARTIAL SPECULAR REFLECTION ON THE QUANTITATIVE RAINFALL-RATE MEASUREMENTS BY RADAR

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ABSTRACT

The equations for calculating the echo power from meteorological targets and the energy distribution within radar beam were derived, by taking the earth curvature, atmospheric refractivity gradient, reflection factor and the roughness of the earth's surface into consideration. The estimation based on these equations shows that the rain echo power may deviate from its normal value by a factor of −3 to +6 db depending on the radar height, antenna elevation, wave length, beam width, surface reflectivity and roughness.

I. INTRODUCTION

In rainfall-rate measurements by radar, the effects of ground objects are shown in three respects: 1. The clutter produced by the ground scattering imposes upon the rain echo signal. 2. A portion of radar beam is obscured by the ground object. 3. The wave reflected from the earth electromagnetically interfers with the direct wave, as a result, the energy distribution in the radar beam changes. These effects make the measurements of rain echo strength different from those obtained without the earth effects. In this paper, the theoretical consideration refers to the effect of ground specular reflection on the energy distribution within the radar beam and the quantitative rainfall-rate measurements is presented, and some data useful to practical work are listed in table.

II. THE PATH LENGTH DIFFERENCE OF THE DIRECT WAVE AND REFLECTED WAVE

In the case of low elevation angle scanning of radar, some rays within the transmitted beam would strike the ground at some distance. The incident point is Q_2 (see Fig. 1). The incident ray would be reflected from the earth at a reflective elevation angle α_r. some other portion of the beam that has a relative high initial elevation angle would also have a local elevation angle α_r when it reaches some height over point Q_2. This portion of beam, not striking the earth, may be called the direct wave. Because some portion of the reflected wave and the direct wave are parallel, the interference between them occurs. The strength of composite wave depends not only on the strength of these two waves, but also on their path length difference, which determines the relative phase of their carrier frequencies. For convenience we discuss the length difference at first.

In a spherically stratified atmosphere, the exact differential equation for a ray is[1]

$$\frac{d^2h}{dx^2}-\left(\frac{2}{R+h}+\frac{1}{n}\frac{dn}{dh}\right)\left(\frac{dh}{dx}\right)^2-\left(\frac{R+h}{R}\right)^2\left(\frac{1}{R+h}+\frac{1}{n}\frac{dn}{dh}\right)=0, \qquad (1)$$

where h is the height of ray above the earth, x is the horizontal distance from the radar, R is the radius of the earth, n is the atmospheric refractive index, as a function of height.

When the ray height under consideration is not high, the following equations that show the variation of the ray elevation angle and ray height with horizontal distance can be derived from Eq. (1):

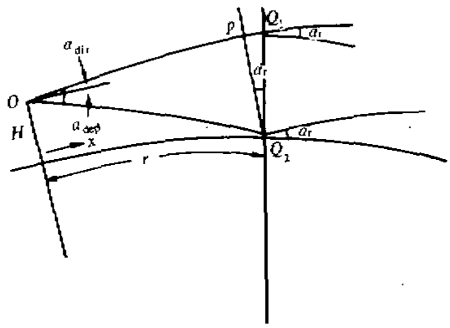

Fig. 1. Diagram showing the path length difference between the direct wave and the earth-reflected wave.

$$\alpha=\left(\frac{1}{R}+K\right)x+\alpha_{ini}, \qquad (2)$$

$$h=\frac{1}{2}\left(\frac{1}{R}+K\right)x^2+\alpha_{ini}x+H, \qquad (3)$$

where α and α_{ini} are the ray elevation angles on the path and on the radar station respectively, H the antenna height above the earth, K the vertical gradient of atmospheric refractive index, and $K=dn/dh$. In this paper, we take the value of K corresponding to that for the standard atmosphere, i. e., $K=-4\times10^{-8}$ m^{-1}, and 6370 km for the radius of the earth.

If the initial elevation angle has a negative value, the ray may strike the earth, the horizontal distance from radar to the striking point may be derived from Eq. (3), i. e.

$$r=\frac{-\alpha_{dep}-(\alpha_{dep}^2-2EH)^{1/2}}{E}, \qquad (4)$$

where α_{dep} represents the depression angle (given in negative value) of the ray when it starts from the radar, and $E=\left(\frac{1}{R}+K\right)$.

The elevation angle (a negative value) of the ray when it touches the ground is

$$\alpha_i=-(\alpha_{dep}^2-2EH)^{1/2}. \qquad (5)$$

Then the elevation angle of the reflected ray is

$$\alpha_r=(\alpha_{dep}^2-2EH)^{1/2}. \qquad (6)$$

The direct ray, which has the elevation angle of α_r on the distance r, may have a initial elevation that can be deduced from Eq. (2), i. e,

$$\alpha_{dir}=\alpha_r-Er. \qquad (7)$$

Then the relation between the individual initial elevation angle α_{dir} and α_{dep} of the direct ray and the reflected ray parallel to each other beyond the distance r, may be deduced from the Eqs (4), (6) and (7), that is

$$\alpha_{dep}=-\frac{1}{3}\left[\alpha_{dir}+2(\alpha_{dir}^2+6EH)^{1/2}\right]. \tag{8}$$

For calculating the difference in path lengths for the direct ray and the reflected ray to determine the phase relation between them, one must calculate the arc lengths $\widehat{OP}$ for the direct ray and $\widehat{OQ_2}$for the earth-striking ray, where P is a point in the direct ray but being on the same wavefront in the composite wave as Q_2 's (see Fig. 1). Because weather radars always work on the microwave band, the phase relation between the two rays is very sensitive to the path length difference. In calculating the ray path lengths, the generally-used flat earth assumption may not be used, one has to treat the problem directly in the global polar coordinate system with the center at the midst of the earth. If we symbolize the arc length of section OQ_1 in the direct ray (Q_1 is a point in the direct ray path just above point Q_2) as S_1' and the arc length of section OQ_2 as S_2, then starting from Eq. 3, after some deduction, one may get the expression for the length difference between S_2 and S_1', that is

$$\begin{aligned}S_2-S_1'=&\frac{1}{4ER}(\alpha_{dep}-\alpha_{dir})[2(1+ER)Er^2+(\alpha_{dep}+\alpha_{dir})(ER\\&+2(1+ER))r]\\&+\frac{1}{2E\sqrt{ER(1+ER)}}[\alpha_{dep}^2\ln Y_2-\alpha_{dir}^2\ln Y_1]\\&+\frac{R}{2\sqrt{ER(1+ER)}}\ln\frac{Y_1}{Y_2},\end{aligned} \tag{9}$$

where

$$Y_1=\frac{\alpha_{dir}+\sqrt{\dfrac{ER(1+\alpha_{dir}^2)}{1+ER}}}{Er+\alpha_{dir}+\sqrt{\dfrac{ER(1+\alpha_{dir}^2)}{1+ER}+2E\alpha_{dir}r+E^2r^2}},$$

$$Y_2=\frac{\alpha_{dep}+\sqrt{\dfrac{ER(1+\alpha_{dep}^2)}{1+ER}}}{Er+\alpha_{dep}+\sqrt{\dfrac{ER(1+\alpha_{dep}^2)}{1+ER}+2E\alpha_{dep}r+E^2r^2}}.$$

If the arc length of section in the direct ray OP is expressed *as* S_1, the path length difference is

$$\Delta S=S_2-S_1=\widehat{OQ}_2-\widehat{OQ}_1+\widehat{PQ}_1=S_2-S_1'+h_r\sin\alpha_r \tag{10}$$

where h_r is the height of the direct ray over the point Q_2, and may be calculated from Eq. (3). Hence, for any direction of the direct ray, one may calculate the length differences ΔS of the direct ray and the reflected ray by starting from α_{dir} and through α_{dep}, α_r, r ,h_r.

III. THE ENERGY FLUX DISTRIBUTION IN THE BEAM COMPOSED OF THE DIRECT WAVE AND THE EARTH-REFLECTED WAVE

Assume that the beam axis elevation angle at radar station is α_0, the angle between the direct ray and the beam axis is expressed as ϕ, the angle between the depression ray and the beam axis as ϕ_1, both ϕ and ϕ_1 are in vertical direction and taken to be positive if the ray is above the beam axis. Hence we have

$$\phi=\alpha_{dir}-\alpha_0, \qquad \phi_1=\alpha_{dep}-\alpha_0. \tag{11}$$

In the original beam (transmitted by radar and unaffected by the ground), the pattern function of field strength is assumed to be $f(\phi)$, then the field strength of the direct wave on S_1 slant range is[2]

$$E_1=\frac{\sqrt{60PG_0}}{S_1}f(\phi)e^{-jks_1}=|E_1|e^{-jks_1}, \tag{12}$$

where P is the radar transmitted power, G_0 the antenna gain along the beam axis, k the wave number, $k=2\pi/\lambda$, λ the wave length and $|E_1|=\frac{\sqrt{60PG_0}}{S_1}f(\phi)$

The microwave complex reflection coefficient of the earth may be expressed as

$$\Gamma=\gamma e^{-j\beta}, \tag{13}$$

where γ is the reflection coefficient for rough surfaces, $\gamma=\Gamma_0 R_s$, Γ_0 is the reflection coefficient for smooth surface, R_s is the specular reflective factor for surface, β is the phase shift caused by the reflection from the earth. For extremely flat ground surfaces, one has $R_s=1$. Each of Γ_0, R_s and γ has a value between 0 and 1.

If the slant range transversed by the depression ray before it touches the ground is S_2, the field strength of the reflected wave is

$$E_2=\Gamma\frac{\sqrt{60PG_0}}{S_2}f(\phi_1)e^{-jks_2}\approx|E_1|\gamma\frac{f(\alpha_{dep}-\alpha_0)}{f(\alpha_{dir}-\alpha_0)}e^{-j(\beta+ks_2)}, \tag{14}$$

and the field strength of the composite wave is

$$E=E_1+E_2=|E_1|e^{-jks_1}\left[1+\gamma\frac{f(\alpha_{dep}-\alpha_0)}{f(\alpha_{dir}-\alpha_0)}e^{-j(\beta+k\Delta s)}\right] \tag{15}$$

where $\Delta S=S_2-S_1$ determined by Eq. (10).

Because the grazing angle is very small in operational applications of radar rainfallrate measurements, we have $\beta=\pi$. Let $D=\gamma\frac{f(\alpha_{dep}-\alpha_0)}{f(\alpha_{dir}-\alpha_0)}$, the ratio of the field strength of the composite wave to that of the direct wave can be obtained from Eqs. (15) and (13) as follows

$$F=\frac{|E|}{|E_1|}=[1+D^2-2D\cos(k\Delta S)]^{1/2}. \tag{16}$$

It is accurate enough for radar quantitative rainfall-rate measurements if the antenna gain pattern is regarded as Gaussian distribution. Therefore we may give the antenna gain pattern

$$G=G_0e^{-4\ln 2\left[\left(\frac{\theta}{\theta_0}\right)^2+\left(\frac{\phi}{\phi_0}\right)^2\right]}, \tag{17}$$

where θ is the horizontal angle of the ray with respect to the beam axis, θ_0 and ϕ_0 are the beam width at half-power points. When the effect of ground reflection exists, the gain pattern of the composite beam becomes

$$G'=GF^2=G_0[1+D^2-2D\cos(k\Delta S)]e^{-4\ln 2\left[\left(\frac{\theta}{\theta_0}\right)^2+\left(\frac{\phi}{\phi_0}\right)^2\right]}, \quad (18)$$

the composite beam has a lower bound which is the critical depression angle making the ray tangential to the earth surface. Thus one can deduce the critical depression angle from Eq. (5) in the from

$$\alpha_{dep,cri}=-(2EH)^{1/2}. \quad (19)$$

Eq. (18) shows, as a result of the interference of the direct wave and the reflected wave, beam splits into several lobes, each of them contains a maximum. Fig. 2 shows the

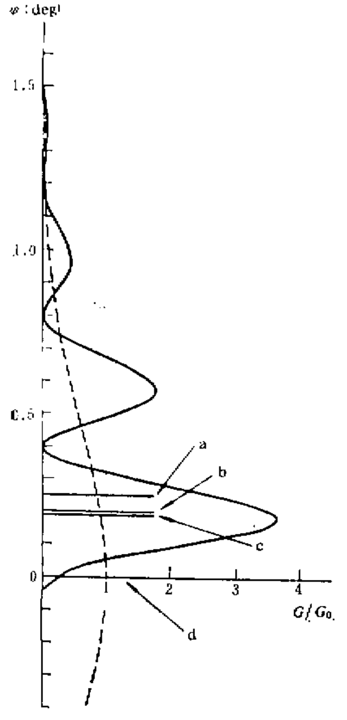

Fig. 2. The relative power flux as a function of the angle with respect t the original beam axis. The diagram is for the model–713 radar transmitting horizontally at a height of 4 m above the ground. Earth reflection coefficient is assumed to be 1. The angles denoted by letters represent respectively: a—the medium angle of transmitted energy distribution in the composite beam, 0.25°; b—the medium angle of echo energy distribution, 0.20°; c—the angle of maximum radiaion, 0.19°; d—the original beam axis elevation angle, 0°.

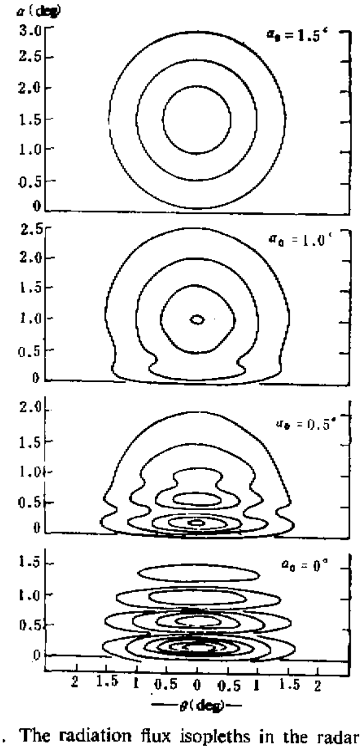

Fig. 3. The radiation flux isopleths in the radar transmitted composite beam at different antenna elevation angles. The example is for the model–713 radar at a height of 4 m above the ground. The earth reflection coefficient is 1. The relative power flux isopleths are in turn 0.01, 0.1, 0.5, 1, 2, 3 inwardly.

relative power flux in the composite beam as a function of the elevation angle of the ray with respect to the original beam axis. Fig. 3 shows the distribution of relative power flux on beam cross section when a model-713 weather radar (made in China) works at some beam axis elevations.

The figure shows, that with increasing elevation, the power flux maximum and the split degree decrease, the beam gradually becomes unaffected by the earth. When the antenna elevation is larger than twice the half angle width of the beam, the power pattern will recover its original form.

The results for the model -711 radar ($\lambda \sim 3.2$ cm, beam width $\sim 1.5°$) and the model-714 radar ($\lambda \sim 10.7$ cm, beam width $\sim 2°$) show similar characteristics. The shorter the wave length, or the higher above the ground the antenna, the more the interference lobes. Besides, the earth reflection coefficient also plays an important role in affecting the gain pattern, see Figs. 4 and 5.

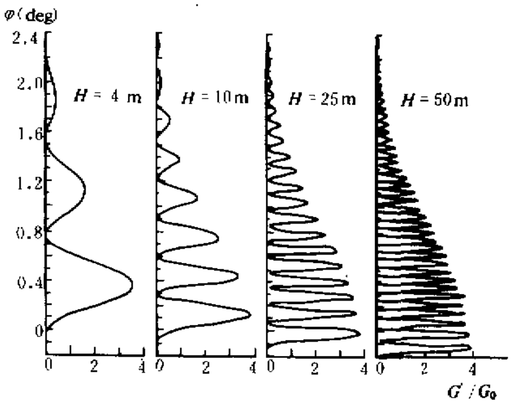

Fig. 4. The relative power flux as a function of the angle with respect to original beam axis for different heights of a model-714 radar transmitting horizontally. The earth reflection coefficient is assumed to be 1.

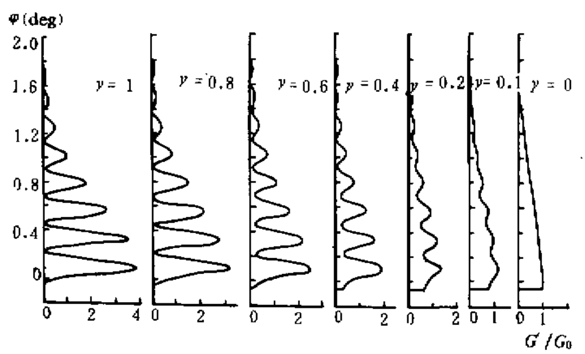

Fig. 5. The ralative power flux as a function of the angle with respect to original beam axis for a model-711 radar transmitting horizontally for different earth reflection coefficients. The antenna height is taken to be 4 m.

IV. THE EFFECTS OF EARTH REFLECTION CHARACTERISTICS ON THE RAIN ECHO SIGNAL STRENGTH

If the gain pattern function of antenna is expressed as $G(\theta, \phi)$, the echo power from rain may be expressed as

$$P_r = \frac{P_t \lambda^2 L\eta}{2(4\pi)^3 r^2} \iint G^2(\theta,\phi) d\theta d\phi, \tag{20}$$

where P_t is the transmitted power, L the pulse length, and η the precipitation reflectivity. If $G(\theta,\phi)$ is in the form of Gaussian distribution, Eq. (20) turns to

$$P_r = \frac{P_t G_0^2 \lambda^2 \theta_0 \phi_0 L}{1024(\ln 2)\pi^2 r^2}\eta, \tag{21}$$

which is the well-known Probert-Jones meteorological radar equation[3]. When the earth influence is taken into consideration, the antenna gain pattern will be described as Eq. (18) and the echo power becomes

$$P_r' = \frac{P_t G_0^2 \lambda^2 L\eta}{2(4\pi)^3 r^2} \int e^{-8\ln 2\left(\frac{\theta}{\theta_0}\right)^2} d\theta \int B^2(\phi) d\phi, \tag{22}$$

where

$$B(\phi) = [1 + D^2 - 2D\cos(k\Delta S)] e^{-4\ln 2\left(\frac{\phi}{\phi_0}\right)^2}. \tag{23}$$

Comparing (22) with (21), we may have

$$\frac{P_r'}{P_r} = \frac{\int B^2(\phi) d\phi}{\dfrac{\sqrt{\pi}\phi_0}{2\sqrt{2\ln 2}}}, \tag{24}$$

Table 1 The Values of the Influence Factor under Conditions of Different Surface Reflection for a Model-713 Radar

Antenna Height / Antenna Elevation Angle	γ=1		γ=0.8		γ=0.6		γ=0.4		γ=0.2		γ=0	
	4m	50m	4m	50m	4m	50m	4m	50m	4m	50m	4m	50m
−0.1°	3.03	3.99	1.94	2.61	1.20	1.66	0.74	1.06	0.50	0.72	0.44	0.61
0.0°	3.11	3.83	2.06	2.58	1.34	1.71	0.88	1.14	0.63	0.82	0.56	0.72
0.1°	3.00	3.45	2.06	2.41	1.41	1.67	0.98	1.18	0.74	0.90	0.68	0.81
0.2°	2.73	2.95	1.96	2.14	1.41	1.57	1.04	1.18	0.83	0.96	0.78	0.88
0.3°	2.36	2.41	1.78	1.84	1.35	1.43	1.07	1.15	0.90	0.99	0.86	0.93
0.4°	1.98	1.94	1.57	1.57	1.27	1.29	1.07	1.11	0.95	1.00	0.91	0.96
0.5°	1.65	1.57	1.38	1.35	1.19	1.18	1.06	1.07	0.98	1.00	0.95	0.98
0.6°	1.39	1.32	1.23	1.20	1.12	1.10	1.04	1.04	0.99	1.00	0.98	0.99
0.7°	1.21	1.16	1.13	1.10	1.07	1.05	1.02	1.02	1.00	1.00	0.99	1.00
0.8°	1.11	1.08	1.07	1.05	1.03	1.03	1.01	1.01	1.00	1.00	0.99	1.00
0.9°	1.05	1.03	1.03	1.02	1.02	1.01	1.01	1.00	1.00	1.00	1.00	1.00
1.0°	1.02	1.01	1.01	1.01	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.1°	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.2°	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

which is the ratio of echo power with to without the influence of the earth. This ratio may be called the influence factor, its reciprocal is the correction factor to echo power.

The values of influence factor when a model–713 radar antenna is at the heights of 4m and 50 m are shown in Table 1. When the antenna is at between these two heights, the influence factor will have values between these two sets of correspondent values. The table shows that the effect of earth reflection is evident when the radar scans near horizontally. The rainfall rate measurement will not be affected by the earth only if the antenna elevation angle is higher than a value equal to the beam width.

The value of echo power influence factor varies with the earth reflection coefficient. Hence one should assess the value γ according to the condition of topography, vegetation, ground constituent and its wetness around the radar station.

E. CONCLUSIONS

1. The results under different conditions show that the high value of antenna gian always appears at the lower part of composite beam when antenna elevation angle is rather low; at $\gamma=1$, the maximum relative gain has a value near 4.

2. The upper limited value of the influence factor is also 4 if the earth is very smooth and reflective. The influence factor decreases with decrease of the earth reflection coefficient. When the earth is extremely rough, one has $\gamma=0$. In this case, the echo power from rain for a radar transmitting horizontally will be only one half of original value.

3. Under the influence of earth reflection, the energy within the beam distributes asymmetrically in some relative small interference lobes. The direction of peak transmission at any lobe can not be used adequately to represent the main radiation direction of the whole beam. Under such a condition, for assessing the height of scatterrs in rain detection, a new representative elevation angle value must be chosen. For example, one may use the medium angle of echo power distribution or the medium angle of composite beam energy distribution. When the antenna elevation angle is higher than a value equal to the beam width, the medium angle of echo power, the medium angle of transmitted energy, and the angle of maximum transmitted energy flux, will coincide with the antenna beam axis elevation.

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