DETERMINATION OF THE DISTANCE BETWEEN TWO ADJACENT STATIONS, THE OBSERVATIONAL VERTICAL INCREMENT AND THE OBSERVATIONAL TIME INTERVAL IN OPTIMUM SENSE

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ABSTRACT

Considering the observational error. the truncation error and the requirements of numerical weather prediction, three formulas for determining the distance between two adjacent stations d_1, the observational vertical increment Δp_1 and the observational time interval Δt_1 in optimum sense, have been derived. Since they depend on the shortest wavelength concerned and the ratio of maximum observational error to wave amplitude, the results are quite different for different scale systems.

For the filtered model the values of d_1, Δp_1, and Δt_1 in general come near those required in the MANUAL on the GOS published in 1980 by WMO. But for the primitive equation model the estimated value of Δt_1 is much less than those required in the filtered model case.

Therefore, it is improper to study the fast moving and developing processes of the atmospheric motion only on the basis of the conventional observations. It seems to be necessary to establish an optimum composite observational system including the surface-based system and the space-based system.

I. INTRODUCTION

It is necessary to have enough observational data firstly for making weather predictions. The accuracy of the predictions, to some extent, depends on the accuracy of the data, the observational time interval and data distribution. Apparently, high-quality data are favourable to predictions, especially to numerical weather predictions (hereinafter the abbreviation NWP is used),but extremely uneven data distribution and sparse data, are much unfavorable. However, these problems have seldom been studied as formulations of mathematical physics, since the early days of making predictions by using modern weather maps.

In 1970, based on statistics and climatology, Alaka[1] and Drozdov and Spelevskij[2] gave some formulas to compute the admissible distance between two adjacent observational stations. However, since no observational data requirements for NWP are taken into account in their derivation, those formulas are applicable only to designing the climatological network.

In 1983, the author et al. presented two formulas for determining the optimum distance between two adjacent stations of a horizontal plane and the optimum observational time interval, but no consideration in the vertical was taken. It seems that how to design a meteorological station network to meet increasing demands of NWP remains an unsettled problem. On the other hand, in recent years, in order to predict severe weather caused by meso- and small-scale systems straightforwardly, many meteorologists tend to take the grid as fine as possible to reduce truncation error. How fine should it be? Does there exist any limitation to it? These problems are still to be solved.

In the following sections we shall discuss the problems mentioned above in turn.

II. BASIC EXPRESSIONS

If all the stations coincide with all the grid points, the distance between two adjacent stations may be regarded as identical with the grid length. For this reason it is enough to discuss the latter only.

The central difference quotient $\nabla_x f$ of a function $f(x, y, p, t)$, within second order accuracy, may approximately be written as

$$\nabla_x f = \frac{\partial f}{\partial x} + \frac{1}{6}\frac{\partial^3 f}{\partial x^3}\Delta x^2, \tag{1}$$

where $\nabla_x f = \Delta_x f/2\Delta x, \Delta_x f = f_{i+1} - f_{i-1}, \Delta x = x/i, i = 0, 1, \cdots, L.$

If f represents a certain meteorological element with observational error ε and true value f_1, then

$$f = f_1 + \varepsilon, \tag{2}$$

substituting (2) for f in (1), we have

$$\nabla f - \frac{\partial f_1}{\partial x} = \frac{1}{6}\frac{\partial^3 f_1}{\partial x^3}\Delta x^2 + \nabla_x \varepsilon. \tag{3}$$

In similar manner replacing x in the above expression by y, p and t respectively, similar expressions can be obtained immediately.

In the expression (3), the first term on the right hand side represents the truncation error caused by replacing the derivative with the central difference quotient, the second term represents the error caused by observation. It can readily be seen that the truncation error increases with d, while the error caused by observation decreases with d. Therefore, it seems to exist an optimum d to make minimum the total sum of $(\nabla_x f - \partial f_1/\partial x)^2$ at all the stations in the entire computational domain during time period T.

If the above treatment is applied to similar derivatives and similar difference quotients with respect to y, p and t, then a function

$$F = \sum_{\tau=0}^{T}\sum_{i=1}^{L}\sum_{j=1}^{M}\sum_{k=1}^{N}\left\{\left(\nabla_x f - \frac{\partial f_1}{\partial x}\right)^2 + a_1\left(\nabla_y f - \frac{\partial f_1}{\partial y}\right)^2 + a_2\left(\nabla_p f - \frac{\partial f_1}{\partial p}\right)^2 + a_3\left(\nabla_t f - \frac{\partial f_1}{\partial t}\right)^2\right\} \tag{4}$$

can be constructed, where $\nabla_y(\ ) = \Delta_y(\ )/2\Delta y$, $\Delta_y(\ ) = (\ )^{(\tau)}_{i,j+1,k} - (\ )^{(\tau)}_{i,j-1,k}$, $j = y/\Delta y, j = 0, 1, \cdots, M$; $\nabla_p(\ ) = \Delta_p(\ )/2\Delta p$, $\Delta_p(\ ) = (\ )^{(\tau)}_{i,j,k+1} - (\ )^{(\tau)}_{i,j,k-1}$, $k = p/\Delta p$, $k = 0, 1, \cdots, N$; $\nabla_t(\ ) = \Delta_t(\ )/2\Delta t$, $\tau = 0, 1, \cdots, T$; a_1, a_2 and a_3 are weighting coefficients.

Requiring F in (4) to be minimum under the conditions

$$\frac{\partial F}{\partial \Delta x} = \frac{\partial F}{\partial \Delta y} = 0, \quad \frac{\partial F}{\partial \Delta p} = 0 \quad \text{and} \quad \frac{\partial F}{\partial \Delta t} = 0, \tag{5}$$

we find

$$\sum_{\tau=0}^{T}\sum_{i=1}^{L}\sum_{j=1}^{M}\sum_{k=1}^{N}\left\{\frac{1}{18}\left(\frac{\partial^3 f_1}{\partial x^3}\right)^2\Delta x^3+\frac{1}{12}\frac{\partial^3 f_1}{\partial x^3}\Delta_x\varepsilon-\frac{1}{4}(\Delta_x\varepsilon)^2\frac{1}{\Delta x^3}\right\}_{i,j,k}^{(\tau)}=0. \tag{6}$$

In the above expression, if x is replaced by y, p, or t, Δx by Δy, Δp or Δt, and $\Delta_x\varepsilon$ by $\Delta_y\varepsilon$, $\Delta_p\varepsilon$ or $\Delta_t\varepsilon$, we can obtain three other similar expressions. Eq. (6) and the other similar expressions are the basic expressions in this paper. Given function forms of f_1 and ε, we can find the optimum value of Δx, Δy, Δp and Δt from these expressions.

III. THE FORMULAS FOR DETERMINING d AND Δp

1. *Function Forms of f_1 and ε*

Considering the distribution of meteorological elements and their time variation usually to be in sinusoidal form, we assume

$$f_1=A\sin(lx+my+np-\Omega t), \tag{7}$$

where A is the wave amplitude, l, m and n the wave numbers in x, y and p directions respectively, $l=2\pi/L_x$, $m=2\pi/L_y$, $n=2\pi/L_p$, L_x, L_y and L_p the wavelengths in x, y and p directions respectively, Ω the frequency, $\Omega=2\pi/T_1$, T_1 the period.

Usually the function forms of ε is very complicated. For convenience of discussion, two special cases will be considered below.

(1) *Case 1*

$$|\Delta_x\varepsilon|=|\Delta_y\varepsilon|=|\Delta_p\varepsilon|=|\Delta_t\varepsilon|=C, \tag{8}$$

where C is a constant. Suppose $\varepsilon_{\sin}=\max\limits_{i,j,k,\tau}|\varepsilon_{i,j,k}^{(\tau)}|$. Then $0\leqslant C\leqslant 2\varepsilon_{\max}$.

(2) *Case 2*

$$\varepsilon=E\sin(l_1x+m_1y+n_1p-\Omega_1t), \tag{9}$$

where E is the wave amplitude, l_1, m_1 and n_1 the wave numbers in x, y and p directions respectively; Ω_1 the frequency.

2. *Determination of d_1 and Δp_1 in Optimum Sense*

(1) *Case 1*

For simplicity, suppose the length of the computational domain in x-direction to be n_1L_x where n_1 is a positive integer. Substituting (7) and (8) into (6), taking $C=2\,\varepsilon_{\max}$ and using the trigonometric formulas

$$\sum_{i=1}^{L}\cos(lx_i+my_j+np_k-\Omega t_\tau)=0 \tag{10}$$

and

$$\sum_{i=1}^{L}\cos^2(lx_i+my_j+np_k-\Omega t_\tau)=\frac{L}{2}, \tag{11}$$

we can find

$$d=\frac{L_1}{\pi}\sqrt[3]{\frac{3\varepsilon_{max}}{4A}}\,. \tag{12}$$

Replacing x by p and making use of (6) lead to

$$\Delta p=\frac{L_p}{\pi}\sqrt[3]{\frac{3\varepsilon_{max}}{4A}}\,. \tag{13}$$

Here the assumptions $\Delta x=\Delta y=d$ and $L_x=L_y=L_1$ have been made.

If the shortest wavelength in x direction considered in NWP and the corresponding optimum distance between two adjacent stations are denoted by L_{min} and d_1 respectively, then d_1 can be obtained by replacing d by d_1 and L by L_{min} from (12). Similarly, if the shortest wavelength in p direction and the corresponding optimum vertical observational increment are denoted by $(L_p)_{min}$ and Δp_1 respectively, then replacing L_p by $(L_p)_{min}$ and Δp by Δp_1 can obtain Δp_1 from (13).

From (12) put $\varepsilon_{max}/A=1/10$ and $L_{min}=10^3$km, then the curve of d_1 varying with L_{min} or ε_{max}/A can be illustrated in Figs. 1 and 2. Comparing these two curves we can see that d_1 varies much faster with L_{min} than with ε_{max}/A. Therefore, we may conclude preliminarily that d_1 is mainly determined by L_{min}. This is the essential difference between the formula (12) and those obtained by Alaka and Drozdov et al. In the latter formulas the distance between two adjacent stations depends on the root mean square of observational standard deviation and is independent of the horizontal wavelength.

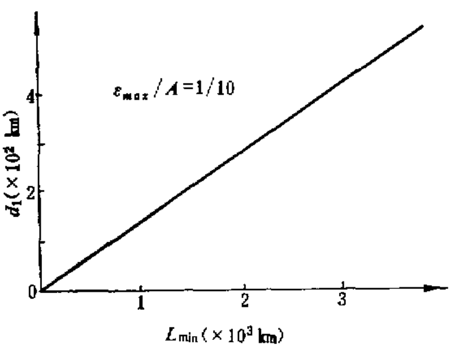

Fig. 1. Relation between d_1 and L_{min}.

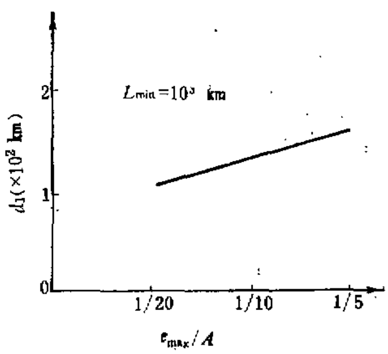

Fig. 2. Relaton between d_1 and ε_{max}/A.

The formulas (12) and (13) are quite similar. Put $(L_p)_{min}=10^3$ hPa. Then the variation of Δp_1 with ε_{max}/A is the same as that shown by the curve in Fig. 2 provided that d_1 is replaced by Δp_1 and the length unit km by the pressure unit hPa. In this case, if the value of ε_{max}/A is taken to be 1/10, then for the wave with $(L_p)_{min}$ of 1000 hPa, $\Delta p_1\approx130$ hPa. This value is equivalent to dividing the atmosphere into 8 layers. For the wave with $(L_p)_{min}$ of 500 hPa, the atmosphere can be divided into 15—16 layers.

(2) *Case* 2

Substituting (7) and (9) for f_1 and ε in (6) respectively and considering the case in which $l_1=l$, $m_1\neq m$, $n_1\neq n$ and $\Omega_1\neq\Omega$, we have

$$d_1=\frac{L_1}{\pi}\sqrt[3]{\frac{3}{4}\mu\frac{E}{A}}\ , \tag{14}$$

where $\mu=\sqrt{2}\,|\sin l_1 d|/2$.

Comparing (14) and (12) we can see that if ε_{max} is replaced by E, the terms under the cubic roots on the right hand sides in both expressions are different by a factor μ. When the wavelength of ε in x direction is taken to be $2d$, $\mu=0.7071$. This is the possible maximum value we can obtain. However, even in this case, μ has no much effect on d_1. For example, when $L_{min}=10^3$ km and $\varepsilon_{max}/A=1/10$, from (12) we have $d_1\approx 130$ km, and from (14) we have $d_1\approx 118$ km. The latter value diminishes by 1/10 only.

3. *The Problem of Reduction of Truncation Error by Decreasing the Grid Length*

The above formula for determining d_1 may also be used to determine the grid length. It will be seen from (6) that d_1 has a definite value for given L_{min} and function forms of f_1 and ε. Thus, improving the accuracy of computation only by decreasing the grid length is not always true. It might be, in some cases, ineffective. That is to say the advantage of decreasing the grid length is limited. Therefore, improving the accuracy of computation needs not only fine grid length, but also data with less observational errors.

4. *The Shortest Wavelength Truncated in Spectral Models*

In most spectral models, the choice of the shortest wavelength truncated, to some extent, is arbitrary, or depends on experiences. Actually, this problem can be settled by (12), for if d_1 is given, we can find L_{min} from (12), namely

$$L_{min}=\pi d_1\sqrt[3]{\frac{4A}{3\varepsilon_{max}}}\ . \tag{15}$$

Then putting $d_1=100$ km and using the given value of ε_{max}/A mentioned above, we get $L_{min}\approx 766$ km. This value comes near those determined from experiences.

IV. FORMULAS FOR DETERMINING THE OPTIMUM OBSERVATIONAL TIME INTERVAL

Now we shall derive some formulas for determining the optimum observational time interval.

Replacing x by t and substituting (7) and (8) for d_1 and $\Delta_t\varepsilon$ in (6), we have

$$\Delta t_1=\frac{1}{\Omega}\sqrt[3]{6\frac{\varepsilon_{max}}{A}}\ . \tag{16}$$

Here C is taken to be $2\varepsilon_{max}$. However, owing to the connection of Ω with l, m and n, we can not determine Δt without given relation between them.

The following two cases are taken into account,

1. *The Filtered Model*

Adopting the linearized vorticity equation

$$\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}\right)\frac{\partial^2\psi}{\partial x^2}+\beta\frac{\partial\psi}{\partial x}=f_0\frac{\partial\omega}{\partial p} \tag{17}$$

and the linearized thermodynamic equation

$$\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}\right)\frac{\partial\psi}{\partial p}=-\frac{\sigma}{f_0}\omega \tag{18}$$

and eliminating ω from them, we can obtain

$$\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}\right)\left(\frac{\partial^2}{\partial x^2}+\frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\psi+\beta\frac{\partial\psi}{\partial x}=0, \tag{19}$$

where ψ is the stream funtion, $\psi=\phi/f_0$; ϕ the geopotential height of isobaric surfaces; f_0 the Coriolis parameter at a given latitude; $\omega=dp/dt$; $\sigma=-\frac{1}{\rho}\cdot\frac{\partial\ln\theta}{\partial p}$; ρ the density of air; θ the potential temperature. Here σ and the speed of basic zonal wind u are assumed to be constants respectively.

Replacing f_1 by ψ and assuming that the above equation has the solution like (7) in form, we obtain

$$\Omega=lu-\frac{\beta l}{l^2+\frac{f_0^2}{\sigma}n^2} \tag{20}$$

The Ω in the above expression represents the frequency of Rossby waves. Since, in general, $l^2\gg f_0^2n^2/\sigma$ in middle and high latitude regions, the Ω determined by the above expression approximates that in the barotropic filtered model case. Given $u=15$ m/s, $f_0=10^{-4}\,s^{-1}$ $\beta=1.6\times10^{-11}\text{m}^{-1}\text{s}^{-1}$ and $\sigma=3\times10^{-2}\text{m}^2\text{s}^{-2}\text{hPa}^{-2}$, we can obtain the optimum observational time interval Δt_1 for different horizontal wavelengths and vertical wavelengths from (16) and (20). The results are shown in Table 1. It can be seen from the table that the values of Δt_1 computed from (16) have no much differences from those required by the MANUAL on the GOS published by WMO in 1980.

Table 1 The Optimum Observational Time Interval (hr) in the Filtered Model Atmosphere

$(L_p)_{min}$ (hPa) \ L_{min}(km)	10^3	2×10^3	3×10^3
500	2.51	5.07	7.60
1000	2.54	5.21	7.94

2. *The Primitive Equation Model*

For the barotropic atmosphere, according to Morel[4], we have

$$\Omega_e = lu + \mu_e \sqrt{gHl^2 + f_0^2}, \tag{21}$$

where $e=1, 2, 3$; μ_1, μ_2 and μ_3 are equal to 0, 1 and -1 respectively; H the mean height of the free surface of fluid.

When $\mu_e \neq 0$, the above expression represents the frequency of gravity waves. Since gHl^2 is generally over one order of magnitude greater than f_0^2, for the case of $\mu_2=1$, we can replace (21) accurately by

$$\Omega \approx (u+c)l, \tag{22}$$

where $c=\sqrt{gH}$. Putting $g=10$ m s^{-2}, $H=9\times10^3$ m, and using (16) and the values of Ω computed by (22), we can obtain the values of Δt_1. The results are shown in Table 2.

Table 2 The Optimum Observational Time Interval (hr) in the Case of the Barotropic Primitive Equation Model

L_{min} (km)	10^3	2×10^3	3×10^3
Δt_1 (hr)	0.12	0.24	0.35

For the baroclinic atmosphere, adoption of the linearized set of equations

$$\frac{\partial u}{\partial t} - f_0 v = -\frac{\partial \phi}{\partial x}, \tag{23}$$

$$\frac{\partial v}{\partial t} + f_0 u = -\frac{\partial \phi}{\partial y}, \tag{24}$$

$$\frac{\partial^2 \phi}{\partial t \partial p} + \sigma\omega = 0, \tag{25}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} = 0, \tag{26}$$

and elimination of u, v and ϕ from them lead to

$$\left(\frac{\partial^2}{\partial t^2} + f_0^2\right)\frac{\partial^2 \omega}{\partial p^2} + \sigma\nabla^2\omega = 0. \tag{27}$$

If the solution of (27) has the form like (7), the frequency equation

$$\Omega^2 = f_0^2 + \frac{\sigma}{n^2}(l^2+m^2) \tag{28}$$

can be derived. The Ω in the above expression represents the frequency of internal inertia-gravity waves. If the values of f and σ are the same as those mentioned above, and $\varepsilon_{max}/A = 1/10$, $l=m$, then the values of optimum observational time intervals for different horizontal wavelengths and vertical wavelengths can be computed from (16). The values are shown in Table 3.

Comparing Tables 1, 2 and 3, we can see that the values of the optimum observational time interval in the case of the filtered model are generally greater than those in the case of the primitive equation model, especially in the barotropic case. Evidently, it results from the difference in the propagation velocity between Rossby waves and gravity waves. This is

Table 3 The Optimum Observational Time Interval (hr) in the Case of the Baroclinic Primitive Equation Model

L_{min}(km) / $(L_p)_{min}$(hPa)	10^3	2×10^3	3×10^3
500	1.48	1.99	2.17
1000	0.88	1.48	1.89

similar to the results caused by the difference of the time steps which is obtained from the theory of computational stability in the two models with a common definite grid length.

If we compare the values in the above three tables with those required by international convention, then only the values in Table 1 approximates the latter. Therefore, it seems reasonable to adopt Δt_1 equal to 3 hours for Rossby waves in the case of the shortest wavelength being 10^3 km (such as in the case of large-scale short-range prediction). However, in the case of the shortest wavelength being 2×10^3 km (such as in the case of large-scale medium-range prediction or global prediction), adopting Δt_1 to be 6 hours might be desirable.

V. SURFACE-BASED OBSERVATIONAL SYSTEM

Based upon the results of section III and Table 1 the requirements of observational data for the global, the regional and the national surface-based network can be listed in Table 4. *It will be seen that the required networks are finer than those in the Guide on* the GOS. Although it is just to meet the requirements in the case of Rossby waves, it is probably difficult to establish such networks completely. We need wide, close and long-term international cooperation.

Table 4 Requirements for the Surface-Based Observational System

Observational System	Global	Regional	National
d_1 (km)	300	300	150
No. of Layers	Stratosphere 6 Troposphere 9	Same as the Global	Same as the Global
Δt_1 (hr)	6	6	3

If we want the surface-based observational system to meet the requirements of NWP in the case of the primitive equation model, it is necessary to overcome many tremendous difficulties. Owing to scientific, technical and economic causes, it seems impossible to establish such surface-based system in the near future. However, such situation would have no effect on the formulation of NWP with one-instant initial values, but would have effect on that with multi-instant initial values or time derivatives at initial instant, or on some related research studies. On the other hand, if we still want to eliminate such effect, it is necessary to establish a composite observational system which includes both surface-based system and space-based system (the satellite observational system being its main part) and to keep improving the accuracy, space resolution and observational frequency of the system. Of course, to establish a reasonable, economical, optimum and efficient system, many comprehensive investigations, experiments, analyses and comparing works have to be done.

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