

## THE INFLUENCE OF HORIZONTALLY NON-UNIFORM HEATING UPON THE DEVELOPMENT OF STRONG CONVECTIVE MESOSCALE DISTURBANCES

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### ABSTRACT

It is shown by observational data and synoptic analysis that the development of strong convective echo is influenced by the horizontally non-uniform heating, such as the one caused by lake-land distribution. In this paper, a simple linear cell-convection model is established using an appropriate heating field, and the instability of heating convection is theoretically studied. It is found that the heating convection development will be unstable if the heating-caused temperature gradient  $dT_0/dy$  is greater than the critical value  $(dT_0/dy)_c$ , which is approximately  $0.64^\circ\text{C}/10\text{ km}$ , and that the development of convective band has a preferred width of  $12.5\text{ km}$ . It will take 25 min for the initial disturbance to increase intensity by 10 times. All these results are in rather good agreement with the squall line process in the lake-land region of Jiangsu Province on June 8, 1979.

### 1. INTRODUCTION

Recently, the mesoscale cell-convection in the atmosphere has become a topic to be paid close attention to<sup>[1]</sup>. By using the satellite cloud pictures, many research works showed that a lot of cell-convection clouds always lie over the cold-warm ocean currents<sup>[1-3]</sup>. Some useful discussion were also done using the thermodynamic convection theory in the fluid dynamics<sup>[4-6]</sup>. The classical Benard convection discussed more completely in the textbook<sup>[6]</sup> is rather similar to the mesoscale cell-convection in the atmosphere but with some differences, one of the main differences is in the ratio of vertical depth of convective cell to its horizontal scale, i. e.  $H/d$ . The value is the order of 2.22—3.12 for the laboratory experiment and fluid dynamics theory, but  $10^{-1} - 10^{-2}$  in the atmosphere. Two corrections are necessary while Benard's thermal convection theory is extended to the atmosphere. The first one is that the viscosity coefficient of molecule should be substituted by the eddy's, the other is that the temperature gradient should be substituted by potential temperature's.

In the seasons of spring to summer, the strongly convective weather such as thunderstorm complex and squall line etc. usually occurs over the region of Jiangsu and Anhui Provinces if the weather is under the influence of favorable large-scale situation and cold air. Sometimes, the strong convective weather still develops vigorously even though the cold air is very weak and its position is far north from the convective region. Obviously, it is different from

ones such as the squall line etc. triggered by the quickly moving cold front. According to the statistical data of origin area for the mesoscale thunderstorm systems over East China in May and June, 1973—1979, the region near the Hongze Lake is one of the most frequent origins. It means that the formation and development for the strong convective systems are closely related to the horizontally non-uniform heating by the water-land or lake-land distribution in this region. Therefore, it can be approximately treated as a developing cell convection.

In this paper, first we appropriately deal with the heating caused by the water-land distribution, and develop a simple linear model for cell convection. Then, on the basis of the theoretical analysis of thermal convection instability we found that the heating by water-land distribution favors the development of strong convective weather when reaching some degree of intensity. Finally, we have compared the theoretical results with the squall line process developing near the Hongze Lake on June 8, 1979. They are in rather good agreement.

## II. THE THERMAL CONVECTION EQUATIONS EQUIVALENTLY CONTAINING THE HORIZONTALLY NON-UNIFORM HEATING

Shown in Fig. 1 (a) is Benard's cell convection pattern with the vertical difference of temperature, produced by the symmetrically convective cell which converges toward 0—0 axis in the low level and diverges up to high level. However, shown in Fig. 1 (b) is the circulation of lake breeze in ordinary lake-land breeze verified by Li<sup>[6]</sup> using the wind data from surface to 1000 m height at Yueyang Station, Hunan Province. There exist the obvious differences between Fig. 1 (a) and Fig. 1 (b). The former is a thermal convective cell caused by the uniform heating at the bottom and is symmetric to the 0—0 axis. But the latter is the thermal solenoid convection due to the horizontally non-uniform heating for the lake-land distribution and is asymmetric. Generally speaking, the temperature increases quickly, and a warm center forms over the land near the lakeside in the daytime by the solar radiation heating. Therefore, the cell convection occurs as shown in Fig. 1 (a), and the convective cell on the plane perpendicular to the lake bank is also strengthened by the temperature difference between lake and land. Thus the resultant convection is equal to the superimposition of two convective cells of Fig. 1 (a) and Fig. 1 (b). Moreover, the quick increasing of temperature near land surface in the daytime can enlarge the horizontal temperature

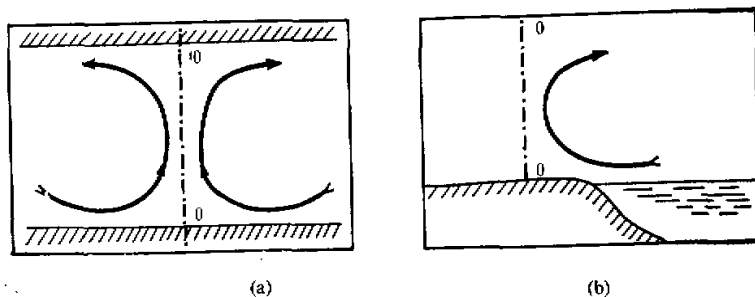


Fig. 1. Sketch maps of Benard convective cell (a) and the lake breeze (b).

gradient between the land and lake, as well as the vertical gradient. Therefore, the horizontally non-uniform heating can be considered as a heating at bottom in discussing the instability of thermal convection. Then we can constitute a set of thermal convection equations which are equivalent to those including the horizontally non-uniform heating such as the situation of Fig. 1 (a) and (b).

Let us assume the following formula

$$\bar{\theta}(y, z) = \theta_{00} + \theta_0(y) + \alpha z, \quad (1)$$

where  $\bar{\theta}$  is the mean potential temperature for environment, and

$$\left. \begin{aligned} \bar{\theta} &= T \left( \frac{p_0}{\bar{p}} \right)^{R/c_p} \\ \frac{d\bar{p}}{dz} &= -\bar{p}g \\ \bar{p} &= \text{const} \times \frac{\bar{p}^{1/\kappa}}{\bar{\theta}} \quad (\kappa = c_p/c_v) \end{aligned} \right\}, \quad (2)$$

where  $\bar{p}$  and  $\bar{\rho}$  are the mean or environmental pressure and density respectively,  $p_0 = 1000$  hPa is the pressure at sea level,  $\theta_{00} = T_{00}$  is the constant temperature at the surface, and  $\theta_0(y) = T_0(y)$  is the horizontal variation of the surface temperature. We take  $dT_0/dy = d\theta_0/dy = \text{const.}$ , and the  $y$  axis perpendicular to lake bank towards the land.  $\alpha$  is the rate of variation of potential temperature with height i. e. the parameter of static stability in the atmosphere and has the form

$$\alpha = \frac{\partial \bar{\theta}}{\partial z} = \frac{\bar{\theta}}{T} (\gamma_d - \gamma) \approx \gamma_d - \gamma, \quad (3)$$

where  $\gamma_d \approx g/c_p$  is adiabatic lapse rate,  $\gamma = -\partial T/\partial z$  is mean temperature lapse rate.

The thermodynamics equation including the eddy diffusion term of heat is

$$\frac{d\theta}{dt} = K_1 \nabla^2 \theta + \frac{\theta}{T} \frac{J}{\rho c_p}, \quad (4)$$

where  $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$ ,  $J$  is the heating rate of exterior source, and  $K_1 = K/\bar{\rho}c_p$  is thermal eddy coefficient. If  $\theta'$  is the potential temperature corresponding to thermal convection, and

$$\theta = \bar{\theta} + \theta', \quad (5)$$

substituting (5) into (4), and taking the adiabatic assumption (i. e.  $J=0$ ) as well as the linearization, we obtain

$$\left( \frac{\partial}{\partial t} - K_1 \nabla^2 \right) \theta' = -\alpha w - \frac{\partial \bar{\theta}}{\partial y} v, \quad (6)$$

where  $(u, v, w)$  is only the velocity field for thermal convection, since the environmental atmosphere is assumed to be stationary.

Not only remain the thermodynamics equation the character of the stratified effect (i. e.  $\alpha$ ) in the process of thermal convection, but also it considers the actions of horizontally uniform heating  $\partial \bar{\theta}/\partial y$ . In Eq. (6), the latter is corresponding to the term of horizontal advection for temperature. Actually, it contains the horizontal advection of surface temperature alone in consideration of Eq. (1). Now, we treat it as a mean value in

the depth  $\delta$  of the thermal boundary layer. Then

$$-v \frac{\partial \theta}{\partial y} = \left( -\frac{1}{\delta} \int_0^\delta v \frac{dT_0}{dy} dz \right) \eta_1(z), \quad (7)$$

where parameter  $\eta_1(z) = v/\bar{v}\delta$ , and  $\bar{v}$  is the mean value for quantity  $v$  in the depth  $\delta$ .

By using Eqs. (6)–(7) the thermal convection equations can be introduced in the form

$$\left. \begin{aligned} \left( \frac{\partial}{\partial t} - \nu \nabla^2 \right) \nabla^2 w &= \beta \nabla_1^2 \theta' \\ \left( \frac{\partial}{\partial t} - K_1 \nabla^2 \right) \theta' &= -aw - \left( \frac{1}{\delta} \int_0^\delta v \frac{dT_0}{dy} dz \right) \eta_1(z) \end{aligned} \right\}, \quad (8)$$

where  $\beta = g/\bar{\theta}$ ,  $\nabla_1^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ , and  $\nu$  is the eddy coefficient. Since Eq. (8) is linearized, the thermal convection due to horizontally non-uniform heating can be separated from the ordinary thermal convection. Then

$$\begin{pmatrix} u \\ v \\ w \\ \theta' \end{pmatrix} = \begin{pmatrix} u_1 \\ v_1 \\ w_1 \\ \theta'_1 \end{pmatrix} + \begin{pmatrix} 0 \\ v_{11} \\ w_{11} \\ \theta'_{11} \end{pmatrix}, \quad (9)$$

where the quantities with subscript II represent those with the thermal convection due to the horizontally non-uniform heating, and  $u_{11} = 0$  while a convective belt is parallel to the lake bank. The ones with subscript I represent those with the ordinary thermal convection. Correspondingly, the governing equations are

$$\left. \begin{aligned} \left( \frac{\partial}{\partial t} - \nu \nabla^2 \right) \nabla^2 w_1 &= \beta \nabla_1^2 \theta'_1 \\ \left( \frac{\partial}{\partial t} - K_1 \nabla^2 \right) \theta'_1 &= -aw_1 \end{aligned} \right\} \quad (10)$$

and

$$\left. \begin{aligned} \left( \frac{\partial}{\partial t} - \nu \nabla^2 \right) \nabla^2 w_{11} &= \beta \nabla_1^2 \theta'_{11} \\ \left( \frac{\partial}{\partial t} - K_1 \nabla^2 \right) \theta'_{11} &= -\left( \frac{1}{\delta} \int_0^\delta v_{11} \frac{dT_0}{dy} dz \right) \eta_1(z) \end{aligned} \right\}. \quad (11)$$

In the same manner as Charney's<sup>[1]</sup> treating of the momentum flux in the boundary layer, differentiating Eq. (1) with respect to  $y$ , and taking the flat surface as the integral boundary condition, i. e.  $w=0$  at  $z=0$ , we get

$$\frac{\partial}{\partial y} \left( -v \frac{\partial \theta}{\partial y} \right) = \left( \frac{1}{\delta} \int_0^\delta \frac{\partial w}{\partial z} \frac{dT_0}{dy} dz \right) \eta_1 = \frac{1}{\delta} \frac{dT_0}{dy} w_\delta \eta_1, \quad (12)$$

where  $w_\delta$  is vertical velocity at the top of thermal boundary layer. It shows that the horizontal convergence of temperature gradient flux on the left-hand side of Eq. (12) is equal to its outflow through the top on the right-hand side, if the temperature advection towards  $y$  axis is considered as the temperature gradient flux. Integrating Eq. (12) with respect to  $y$  in the convective belt of  $l_y$ , we have

$$-v \frac{\partial \bar{\theta}}{\partial y} = \frac{l_y}{\delta} \frac{dT_0}{dy} \eta_1 \bar{w}_\delta, \quad (13)$$

where  $\bar{w}_\delta = \frac{1}{l_y} \int_0^{l_y} w_\delta dy$  is the mean value in the convective belt, and the updraft region of the convective belt is in the area where  $v=0$  at  $y=0$ , i. e.  $0-0$  axis in Fig. 1 (a). The temperature gradient  $dT_0/dy$  transported upward through the top of boundary layer must influence the convective temperature  $\theta'$  on each layer. By introducing parameter  $\eta$ , Eq. (13) becomes:

$$-v \frac{\partial \bar{\theta}}{\partial y} = \frac{l_y}{\delta} \frac{dT_0}{dy} \eta w$$

or

$$-\left(\frac{1}{\delta} \int_0^\delta v_{11} \frac{dT_0}{dy} dz\right) \eta_1(z) = \frac{l_y}{\delta} \frac{dT_0}{dy} \eta w_{11}, \quad (14)$$

where  $\eta = \eta_1 \cdot \eta_2$  and  $\eta_2 = \bar{w}_\delta/w$ , with the result that

$$\eta = \frac{v}{w} \frac{\bar{v}^\delta}{\bar{w}_\delta} \sim 0.1. \quad (15)$$

In this formula, the characters of the cell-convection have been considered as follows: prevailing convergent current  $\bar{v}^\delta/\bar{w}_\delta \sim 10^1$  near the bottom layer and prevailing updraft  $v/w \sim 10^0-10^1$  on the boundary layer. Therefore the influence of horizontal advection of surface temperature upon the thermal convection temperature  $\theta'$  is equivalent to the vertical transport of temperature among several layers on the basis of Eq. (14).

Substituting Eq. (14) into (11), we finally obtain

$$\begin{aligned} \left(\frac{\partial}{\partial t} - v \nabla^2\right) \nabla^2 w_{11} &= \beta \nabla^2 \theta'_{11} \\ \left(\frac{\partial}{\partial t} - K_1 \nabla^2\right) \theta'_{11} &= \frac{l_y}{\delta} \frac{dT_0}{dy} \eta w_{11} \end{aligned} \quad (16)$$

That is the thermal convection equation containing the effect of the horizontally non-uniform heating  $dT_0/dy$ .

### III. THE DEVELOPMENT CONDITION FOR THE CONVECTIVE BELT AND THE THEORETICAL RESULT OF THE GROWING TIME

The ordinary instability of thermal convection has been discussed through Eq. (10) [5,6]. Here, Eq. (16) is solved by the same method and used to explain the influence of  $dT_0/dy$  on the instability of thermal convection. Now, the non-dimensional variables of space and time are introduced

$$\tau = \frac{K_1}{d^2} t, \quad (X, Y) = \frac{1}{d} (x, y), \quad Z = \frac{1}{H} z, \quad (17)$$

where variables  $d$  and  $H$  are the characteristic horizontal and vertical scale respectively. Note that from now on  $(x, y, z)$  is taken to be the non-dimensional variables for convenience. Because Eq. (16) is the partial differential equation of the parabolic type, its solution

contains a growing factor with time as follow:

$$\begin{bmatrix} w_{11}(x, y, z, \tau) \\ \theta'_{11}(x, y, z, \tau) \end{bmatrix} = \begin{bmatrix} \frac{K}{d} W_{11}(z) \\ aH\Theta(z) \end{bmatrix} e^{\sigma\tau} F_{11}(x, y). \quad (18)$$

By substituting Eq. (18) into (16), the horizontal and vertical constitution equations can be obtained respectively, i. e.

$$\frac{\partial^2 F_{11}}{\partial x^2} + \frac{\partial^2 F_{11}}{\partial y^2} + a^2 F_{11} = 0 \quad (19)$$

and

$$\left. \begin{aligned} \left[ \sigma - \left( \frac{d^2}{H^2} D^2 - a^2 \right) \right] \Theta_{11} &= \frac{d}{H} \left( \frac{l_y}{\delta} \frac{dT_0}{dy} \eta \right) W_{11} \\ \left[ \frac{\sigma}{P_r} - \left( \frac{d^2}{H^2} D^2 - a^2 \right) \right] \left( \frac{d^2}{H^2} D^2 - a^2 \right) W_{11} &= \frac{H}{d} R_a a^2 \Theta_{11} \end{aligned} \right\}, \quad (20)$$

where  $\sigma$  is the growing rate for non-dimensional time,  $D = d/dz$ ,  $R_a = -\alpha\beta d^4/\nu K$ , Rayleigh number,  $P_r = \nu/K$ , Prandtl number,  $a$  is a constant inversely proportional to the linear dimension of cell in horizontal plane, and  $l_y/\delta$ ,  $dT_0/dy$ , are all of the non-dimensional quantities. With the coordinate origin at midway in the atmospheric layer and by means of the usual assumption, the boundary conditions are

$$W_{11} = 0 = DW_{11}, \quad \Theta_{11} = 0 \quad \left( \text{at } z = \pm \frac{1}{2} \right),$$

where the first expression shows the non-slip condition to satisfy the equation of continuity, and the second one is the preservation of temperatures at the top and bottom of the atmosphere.

So far as the instability condition of the thermal convection is concerned, it is well-known from the Refs. [5] and [6] that the characteristic solutions for Eqs. (19) and (20) are

$$F_{11}(x, y) = \cos nx \cos my \quad (a^2 = n^2 + m^2) \quad (21)$$

and

$$\begin{bmatrix} W_{11}(z) \\ \Theta_{11}(z) \end{bmatrix} = \begin{bmatrix} \hat{W} \\ \hat{\Theta} \end{bmatrix} \cos qz. \quad (22)$$

Here  $n$ ,  $m$  and  $q$  are non-dimensional horizontal and vertical wave numbers respectively, the value  $q$  given later will make Eq. (22) satisfying the boundary condition of  $W_{11}$ . Considering the belt-type of convection corresponding to the horizontally non-uniform heating, the rectangular cells as represented by Eq. (21) may have

$$n \ll m \text{ or } a^2 \approx m^2. \quad (23)$$

By substituting Eqs. (21) and (22) into Eq. (20), the homogeneous equations with respect to  $\hat{W}$  and  $\hat{\Theta}$  can be obtained, i. e.

$$\left. \begin{aligned} \frac{dT_0}{dy} \hat{W} - [\sigma + Q^2] \hat{\Theta} &= 0 \\ - \left[ \frac{\sigma}{P_r} + Q^2 \right] Q^2 \hat{W} + \frac{H}{d} R_a m^2 \hat{\Theta} &= 0 \end{aligned} \right\} \quad (24)$$

Obviously, the necessary and sufficient conditions for non-trivial solutions are

$$\begin{vmatrix} \frac{dT_0}{dy} & -[\sigma + Q^2] \\ -\left[\frac{\sigma}{P_r} + Q^2\right]Q^2 & \frac{H}{d}R_0m^2 \end{vmatrix} = 0, \quad (25)$$

where

$$\frac{dT_0}{dy} = \frac{d}{H} \left( \frac{l_y}{\delta} \frac{dT_0}{dy} \eta \right), \quad Q^2 = \frac{d^2}{H^2} q^2 + m^2, \quad l_y = 1/m. \quad (26)$$

Setting  $P_r = 1$  in the atmosphere, we get from Eq. (25)

$$[\sigma + Q^2]^2 = \frac{H}{d} \frac{R_0 m^2}{Q^2} \frac{dT_0}{dy} \quad (27)$$

or

$$\sigma = \left( \frac{H}{d} \frac{R_0 m^2}{Q^2} \frac{dT_0}{dy} \right)^{1/2} - Q^2. \quad (28)$$

That is the relationship between the growing rate  $\sigma$  and wave number ( $m, q$ ). If the development of convection is unstable,  $\sigma$  must take positive value. Therefore the first term on the right-hand side of Eq. (28) sets positive symbol, and its value must be larger than the second term. For the condition critically satisfying the convection development i. e.  $\sigma = 0$ ,

$$\left( \frac{dT_0}{dy} \right)_c = Q^2 / \left( \frac{H}{d} R_0 m^2 \right). \quad (29)$$

It is clear that there is a critical value  $(dT_0/dy)_c$  of the horizontally non-uniform heating with respect to the certain atmospheric stratification (i. e.  $R_0$  number). When the actual value of horizontally uniform heating exceeds this critical one, i. e.

$$\frac{dT_0}{dy} > \left( \frac{dT_0}{dy} \right)_c, \quad (30)$$

the convection will develop unstably.

When the atmospheric stratification is unstable, i. e.  $\gamma > \gamma_d$ , then

$$R_0 = -\alpha\beta d^4 / \nu K_1 > 0,$$

and the more unstable the atmospheric stratification, the larger the  $R_0$  number should be. It can be seen from Eq. (29) that in this case the  $(dT_0/dy)_c$  should be much less. As a result, the horizontally non-uniform heating in appropriate amount is able to cause the convection development from Eq. (30). Conversely, if the atmospheric stratification is not very unstable (i. e. the smaller  $R_0$  number), then the value of  $(dT_0/dy)_c$  is larger and the larger value of horizontally non-uniform heating is needed to stimulate the convection development. In addition, while  $\alpha \equiv \gamma_d - \gamma \geq 0$ ,  $R_0$  number should be non-positive and the first term on the right-hand side of Eq. (28) should not be non-zero, then the unstable thermal convection  $\sigma > 0$  should not appear.

Generally speaking, we take a half wavelength of the atmospheric convection cells in the depth concerned, i. e. setting  $q = \pi$ . Under the condition of the atmospheric stratification or  $R_0$  number given,  $(dT_0/dy)_c$  is only a function of horizontal wave number  $m$ . Then from Eqs. (29) and (26) we can obtain

$$\left(\frac{dT_0}{dy}\right)_c = \frac{d}{HR_a} \frac{\left(\frac{d^2}{H^2}\pi^2 + m^2\right)}{m^2} \quad (31)$$

Taking  $d^2/H^2 = 10^2$  and  $R_a = 10^4$  according to Ref. [6] we have

$$\left(\frac{dT_0}{dy}\right)_c = \frac{1}{10^3} \frac{(10^2\pi^2 + m^2)}{m^2} \quad (31')$$

Differentiating this equation with respect to  $m$ , its minimum value can be found as

$$\left(\frac{dT_0}{dy}\right)_{c, \min} = \frac{270}{4} \pi^4 \approx 6575. \quad (32)$$

Thus, when  $(dT_0/dy)$  is larger than its minimum critical value, there is at least a wavelength being unstable disturbance.

According to the general case, we take the following parameters

$$\begin{aligned} d &\sim 10^3 \text{ m}, H \sim 10^4 \text{ m}, \delta \sim 0.5 \times 10^2 \text{ m}, \text{ or } \hat{\delta} \sim 0.5 \times 10^{-2} \\ \alpha &\sim 10^{-5} \text{ deg m}^{-1}, \frac{dT_0}{dy} \sim 0.8 \times 10^{-4} \text{ deg m}^{-1}, \text{ or } \frac{dT_0}{dy} \sim 0.8 \times 10^2 \\ \theta &\sim 10^2 \text{ }^\circ\text{C}, \eta \sim 0.1, g \sim 10^1 \text{ m sec}^{-2}, \nu, K_1 \sim 10^5 \text{ m sec}^{-2} \\ q^2 &\sim \pi^2, L_y \sim \frac{1}{3}d \text{ or } m^2 = l_y^{-2} \sim (6\pi)^2 \end{aligned} \quad (33)$$

where  $\alpha$  is taken as  $10^{-5} \text{ deg m}^{-1}$  corresponding to the lapse rate  $\gamma$  slightly larger than  $\gamma_a$ ,  $\delta$  is taken as  $0.5 \times 10^2 \text{ m}$  due to the horizontally non-uniform heating being restricted only in the layer near ground.  $L_y$  is the wavelength along the  $y$  direction and  $l_y = (1/3)d$  for the convection of belt-type. Because of the non-dimensional wave number  $2\pi d/l_y$ ,  $\frac{1}{2} l_y$  may be considered as a half-width of convective belt. By using these parameters, we can obtain

$$\left. \begin{aligned} \text{characteristic time } d^2/K_1 &= 10^5 \text{ sec} \\ \text{characteristic vertical velocity } K_1/d &= 10^0 \text{ m sec}^{-1} \\ \text{characteristic potential temperature disturbance } \alpha H &= 10^{-1} \text{ deg} \end{aligned} \right\} \quad (34)$$

and

$$\begin{aligned} Q^2 &= 1.342 \times 10^3 \\ R_a &= 10^4 \\ \left(\frac{dT_0}{dy}\right)_c &= 8.50 \times 10^3 \end{aligned} \quad (35)$$

Tritton<sup>[10]</sup> showed that  $R_a$  number is  $10^4$  for Benard convection in laboratories and  $R_a > 10^6$  is corresponding to the thermal convection of high  $R_a$  number. Substituting these parameters and results calculated into Eqs. (28) and (29), we obtain

$$\left. \begin{aligned} \sigma &= 0.157 \times 10^3 \\ \left(\frac{dT_0}{dy}\right)_c &= 0.679 \times 10^4 \\ \left(\text{approximately } \frac{dT_0}{dy} \approx 0.64 \times 10^{-4} \text{ }^\circ\text{C m}^{-1}\right) \\ t^* &= 2.3 \times \frac{d^2}{K_1} \frac{1}{\sigma} = 1.46 \times 10^3 \text{ sec} \end{aligned} \right\} \quad (36)$$



where  $t^*$  is the time for the intensity increased by 10 times from the initial disturbance.

It is known from Eq. (28) that the development of thermal convection may be not only grown by the horizontally non-uniform heating ( $dT_0/dy \neq 0$ ), but also controlled by the thermal diffusion ( $Q^2 \neq 0$ ). The unstable growing occurs only when the horizontally non-uniform heating  $dT_0/dy$  is larger than the critical  $(dT_0/dy)_c = 0.64^\circ\text{C}/10\text{ km}$  as shown in Eq. (29) or (30). And it is also pointed out from Eq. (36) that the time required for the intensity increased by 10 times from initial disturbance is about  $1.46 \times 10^3$  sec (i. e. 25 min roughly). Fig. 2 shows the theoretical curve of  $\sigma$ ,  $t^*$  and  $dT_0/dy$  calculated from the data in Exp. (33). When  $dT_0/dy$  exceeds the critical value the unstable growing rate  $\sigma$  increases but  $t^*$  decreases, with increasing  $dT_0/dy$ . In addition, the growing rate also relates to the width of convective belt  $L_y/2$ . Fig. 3 shows the theoretical curves between the growing rate and the width of convective belt (or  $m$ ) for three cases with  $dT_0/dy = 0.8, 1.0$  and  $1.2$  ( $^\circ\text{C}/10\text{ km}$ ). It is seen that  $L_y/2 = 12.5\text{ km}$  i. e.  $m = 8\pi$  is the preferred width for the convective development.

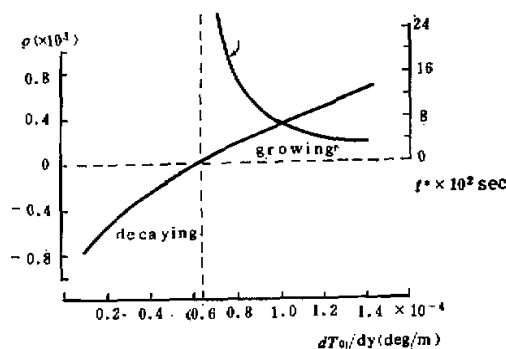


Fig. 2. The theoretical curves among the growing rate ( $\sigma$ ), growing time ( $t^*$ , right) and horizontal gradient of temperature ( $L_y - d/3$ ,  $\alpha = 10^{-5} \text{ deg m}^{-1}$ ).

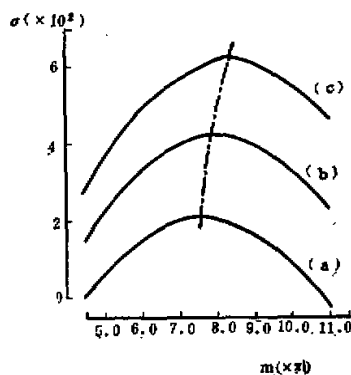


Fig. 3. The growing rate ( $\sigma$ ) as a function of the convection width of belt-type. ( $dT_0/dy \times 10^{-4}$  taking (a) 0.849, (b) 1.061 and (c) 1.273 respectively.

#### IV. EXAMPLE AND DISCUSSION

The strong convective process in six mesoscale systems in the region of Jiangsu and Anhui Provinces on 8 June 1979 has been studied in detail<sup>[11,12]</sup>. Fig. 4 shows one of convective echo belts or squall lines<sup>1)</sup> called B-echo belt. It occurred near north bank of the Hongze Lake at 14:36–40. Shown in Fig. 5 is the distribution of surface temperature and wind in the region of Jiangsu and Anhui Provinces at 14:00 June 8, and the convective

1) It was named as the B-echo belt in the six mesoscale systems according to Ref. [11].

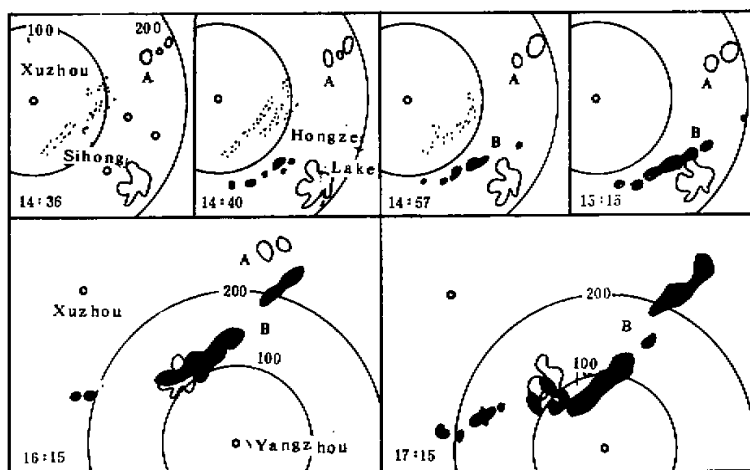


Fig. 4. The convective echo belt (dark area) occurring along north bank of the Hongze Lake, 8 June 1979 afternoon. The convective echo corresponding to the cold front is shown by the fine points.

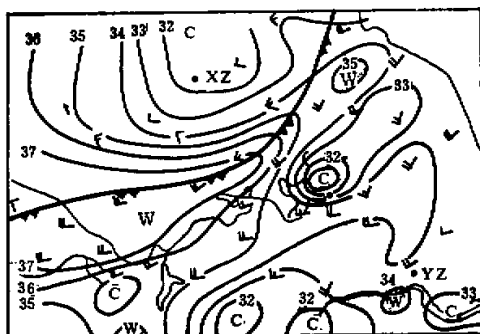


Fig. 5. The distribution of surface temperature (solid lines at 1 °C intervals) and wind at 14:00 June 8, 1979. XZ—Xuzhou; YZ—Yangzhou.

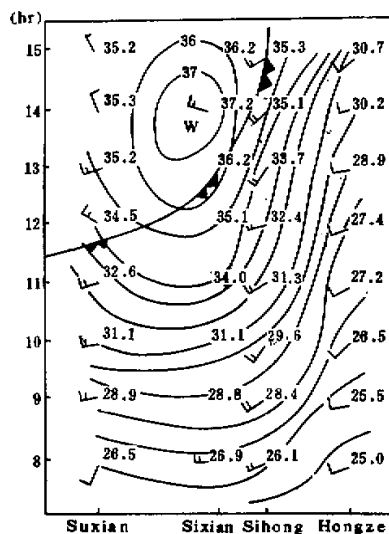


Fig. 6. The time cross-section of surface temperature of the four stations along the line perpendicular to north bank of the Hongze Lake. Hongze county and Suxian county represent the stations of water and land region respectively.

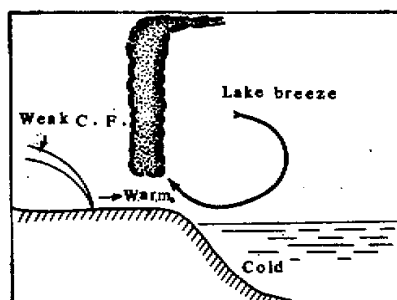


Fig. 7. The schematical representation of convection development caused by the lake breeze ahead of weak cold front.

echo belt is just in the concentrated isotherm region near north bank of the Hongze Lake. Fig. 6 is the time cross-section of four stations along the line perpendicular to north bank of the Hongze Lake. The isotherms of north bank were so concentrated because of the daily variation of temperature or the horizontally non-uniform heating. As pointed in Ref. [11], it also related to the spread toward northeast of warm tongue in the middle reaches of the Huanghe-Huaihe River. Moreover the horizontally non-uniform heating reached maximum strength in the afternoon, then  $dT_0/dy$  along north bank of the Hongze Lake rose nearly to  $1^\circ\text{C}/10\text{ km}$  exceeding the theoretical critical value. It, of course, was easy to trigger off the instability of thermal convection.

In addition, it can be found from Ref. [12] that when the squall line occurred, a weak cold front to its north was dissipating, as shown by fine points in Fig. 4. There was slight difference of temperature but obvious wind shear on both sides of cold front (see Fig. 5). Consequently, the formation process of this squall line, echo B in Fig. 4, was quite different from the ones triggered by the fast moving cold front. However, it is difficult to exclude the relationship between the formation process of squall line and the low level convergence-upward current caused by the wind shear near the cold front. The convection development and the squall line occurrence near the Hongze Lake on 8 June may be schematically shown in Fig. 7.

The actual value of  $dT_0/dy = 1^\circ\text{C}/10\text{ km}$  during the squall echo belt occurred, and the actual growing time and width of convection belt as shown in Fig. 4 were in good agreement with the theoretical results. Analysis of data for several years show that the region near the Hongze Lake is always a preferred source of thunderstorm. The case analysis, statistical facts and convection instability theory support each other and verify that the horizontally non-uniform heating similar to the temperature difference between the lake and land, plays an important role in the occurrence of strong convection or squall line. Therefore, we must pay more attention to it in analysing and forecasting. Finally, it must be pointed out that some treatments and assumption in this paper may be different from the real atmosphere. However, if the issue is only restricted to the initial condition for the instability of thermal convection, the results are well agreeable to the actual example.

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