A NUMERICAL INVESTIGATION ON THE INTERACTION OF TURBULENT AND LONG-WAVE RADIATIVE FLUXES IN THE SURFACE LAYER

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ABSTRACT

The interaction of turbulent and radiative transfer applied to a number of plausible atmospheric situations in the surface layer under the stably stratified condition is discussed.

The calculated results show that the long-wave radiative flux has a great influence upon the thermal structure of the surface layer, and that it usually acts in such a way as to weaken the thickness of the constant turbulent heat flux layer. In the case of low wind velocities and strongly stable stratifications, the thickness of the turbulent heat flux layer will become very thin and/or inexistent.

1. INTRODUCTION

It is well-known that, under conditions of stationarity and horizontal homogeneity, the fluxes of turbulent momentum and heat are constant over the first few tens of metres in the atmosphere. For this reason this layer is sometimes defined as constant turbulent flux layer (CTFL). The hypothesis of the CTFL is very widely applied in meteorology. However, Robinson (1950)[1], Brooks (1950)[2], Funk (1960)[3-4], and Garratt et al. (1981)[5] have shown that there exist appreciable variations of the long-wave radiative flux in the lower layer of the atmosphere, not only at night, but also during daytime.

Having analysed the experimental data, Webb (1970)[6], Businger et al. (1971)[7], Panofsky (1973)[8], Ye (1982)[9] and many other authors have shown that, under stable conditions, the profile forms of wind, potential temperature and specific humidity are all those of 'log+linear' laws in the surface layer. But the values of the universal parameters β, which are obtained by many authors are quite different, of which the scattering range of the temperature profile parameter β_T is the largest (from 4.2 to 17). Yaglom (1976)[10] indicates that the existence and the exploration of CTFL itself are worth studying besides the errors from the measuring accuracy of the instruments. Munn (1960)[11] also shows that it is necessary to study the relative importance between the long-wave radiative flux and the turbulent heat flux.

Coantic and Seguin (1970)[12] have studied the case of an underlying water surface under neutral conditions. In order to obtain an analytic solution of the long-wave radiative

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transfer equations, they assume a first-order approximation, and suppose that in the surface layer the distribution of potential temperature with height obeys 'logarithmic law', which leads to very obvious restriction on the solution.

In the present work, by applying the numerical solution of the long-wave radiative transfer equations and the 'log+linear' laws of potential temperature and specific humidity, we have discussed in detail the effects of the long-wave radiative flux and turbulent heat flux upon the 'constant flux layer', when there exist various surface roughness and typical atmospheric situation under stable conditions, so as to give the essential conditions and the relevant thickness of the constant turbulent heat flux layer when it exists.

II. THE APPLICATION OF THE SIMILARITY THEORY IN THE SURFACE LAYER

Under conditions of stationarity and spatial homogeneity and neglecting molecular viscosity, the equations of the fluxes of momentum, water vapour and heat in the surface layer can be given by

$$I=-\rho\overline{u'w'}=I_0, \tag{1}$$

$$Q=\rho\overline{q'w'}=Q_0, \tag{2}$$

$$S+R=\rho C_p\overline{\theta'w'}+R=S_0+R_0, \tag{3}$$

where the variables with subscript 'o' are defined as those on the surface, ρ the air density, C_p the specific heat at constant pressure, u' the turbulent horizontal velocity, w' the turbulent vertical velocity, q' the turbulent specific humidity, θ' the turbulent potential temperature, R the net long-wave radiative fluxes.

Eqs. (1)—(3) indicate that I and Q are constant with height in the surface layer, so is the sum of turbulent heat flux and radiative flux. Therefore, we can not simply consider that there exists a constant turbulent heat flux layer, unless

$$R=R_0. \tag{4}$$

In this case we have

$$S=\rho C_p\overline{\theta'w'}=S_0. \tag{5}$$

In fact, S is given by

$$S=S_0\left(1-\frac{R-R_0}{S_0}\right)=S_0\left(1-\frac{\Delta R}{S_0}\right). \tag{6}$$

Now the problem remained is how we should calculate the thickness of CTFL if the variations of R and S with height are given. We, thereby, may define a small positive number $\delta(\delta<1)$. If

$$\left|\frac{R(Z)-R_0}{S_0}\right|=\left|\frac{\Delta R}{S_0}\right|<\delta, \tag{7}$$

i. e.

$$\left|\frac{S(Z)-S_0}{S_0}\right|=\left|\frac{\Delta S}{S_0}\right|<\delta, \tag{8}$$

then we could consider that the turbulent heat flux is constant with height, and define the relevant h as the depth of the constant turbulent heat flux. It is obvious that the value of h depends on the atmospheric stratification, meteorological conditions and surface roughness.

According to the flux-profile relationships, the turbulent exchange coefficients K_m, K_q,

K_h are given from

$$I=-\rho\overline{u'w'}=\rho K_m\frac{\partial u}{\partial Z},\tag{9}$$

$$Q=\rho\overline{q'w'}=-\rho K_q\frac{\partial q}{\partial Z},\tag{10}$$

$$S=\rho C_p\overline{\theta' w'}=-\rho C_p K_h\frac{\partial\theta}{\partial Z}.\tag{11}$$

And relevant turbulent parameters are defined as

$$u_*=(\tau_0/\rho)^{\frac{1}{2}},\tag{12}$$
$$\theta_*=-S_0/\rho C_p u_*,\tag{13}$$
$$q_*=-Q_0/\rho u_*,\tag{14}$$

and the Monin-Obukhov (M—O) length is given by

$$L=u_*^2\theta_0/kg\theta_*,\tag{15}$$

where k is the Karman constant (0.4 in the present paper), g the gravity acceleration, θ_0 the surface potential temperature.

The dimensionless gradient equations of wind, potential temperature and specific humidity are given by

$$(kZ/u_*)\frac{\partial u}{\partial Z}=\varphi_m(Z/L)\tag{16}$$

$$(kZ/\theta_*)\frac{\partial\theta}{\partial Z}=\varphi_h(Z/L),\tag{17}$$

$$(kZ/q_*)\frac{\partial q}{\partial Z}=\varphi_q(Z/L).\tag{18}$$

From Eqs. (9)—(11) and (16)—(18), the formulae of the turbulent exchange coefficients can be written as

$$K_m=ku_*Z/\varphi_m(Z/L),\tag{19}$$
$$K_h=ku_*Z/\varphi_h(Z/L),\tag{20}$$
$$K_q=ku_*Z/\varphi_q(Z/L),\tag{21}$$

where φ_m, φ_h and φ_q are three universal functions for which the following relationship

$$\varphi=\varphi_m=\varphi_h=\varphi_q\tag{22}$$

is assumed to be held in order to seek solution.

The experimental results have shown that under stable conditions, φ is defined as

$$\varphi=1+\beta\frac{Z}{L}.\tag{23}$$

For making the discussion more universally, let $\beta=5$. Substituting Eq. (23) into (16—18) and respective integrating can obtain the following 'log+linear' profiles

$$u(Z)=\frac{u_*}{k}\left(\ln\frac{Z}{Z_0}+\beta\frac{Z}{L}\right),\tag{24}$$

$$\theta(Z)=\theta_0+\frac{\theta_*}{k}\left(\ln\frac{Z}{Z_0}+\beta\frac{Z}{L}\right),\tag{25}$$

$$q(Z)=q_0+\frac{q_*}{k}\left(\ln\frac{Z}{Z_0}+\beta\frac{Z}{L}\right). \tag{26}$$

Thus if the values of wind, temperature and water vapor at any two levels are given, the turbulent parameters u_*, θ_*, q_* and the M–O length L can be computed from Eqs. (24)—(26) and (15).

III. THE COMPUTATION OF LONG-WAVE RADIATIVE FLUX

In order to compute long-wave radiative flux in the atmosphere, we must know the distributions of temperature and water vapor with height. In section II, the profiles of wind, temperature and humidity in the surface layer are discussed. Above the surface layer, it is assumed that the temperature varies as adiabatic lapse rate, and specific humidity decreases linearly with height.

Many methods of determining the flux divergence with the help of the integral equations governing infrared radiative transfer has been derived by Brooks[2], Funk[4] Rodgers (1967)[13], Paltridge et al (1976)[14], Garratt et al[5]. In this paper the Rodgers' computation method for long-wave radiative flux is adopted, but slight improvement is made.

The net long-wave radiative flux can be given by

$$R=F\uparrow-F\downarrow. \tag{27}$$

Using the emissivity or grey-body approximation ($\epsilon<1$), the radiative flux transfer equations for $F\downarrow(Z)$ and $F\uparrow(Z)$ at the level of reference Z are then obtained

$$F\downarrow(Z)=\int_Z^{\infty}B[T(Z')]\frac{\partial\epsilon\downarrow}{\partial Z'}(Z',Z)dZ', \tag{28}$$

$$F\uparrow(Z)=\int_Z^{0}B[T(Z')]\frac{\partial\epsilon\uparrow}{\partial Z'}(Z,Z')dZ' + [\epsilon_0 B(T_0)-(1-\epsilon_0)F_0\downarrow][1-\epsilon\uparrow(Z_0,0)], \tag{29}$$

where Z' is a dummy variable of integration, $\epsilon(Z, Z')$ the emissivity corresponding to geometric vertical path from height Z to Z', B the Planck black-body radiative function

$$B(T)=\sigma T^4, \tag{30}$$

where σ is the Stefan–Boltzmann constant ($\sigma=5.672\times10^{-8}$ W/(m^2 K^4 s)).

Here, water vapour only needs to be taken into account in computations, and pressure corrections have to be made, so that the corrected mass of absorber $\tilde{W}$ can be given by

$$\tilde{W}=\int_0^Z \rho q\left(\frac{P}{P_0}\right)^n dZ, \tag{31}$$

where the surface air pressure $P_0=1000$ hPa, and corrected parameter $n=1$.

With regard to the form of emissivity function (Z), we use Rodgers' parameterization formulae:

$$\epsilon\uparrow(\tilde{W})=\begin{cases}\sum_{i=0}^{4}a_i(\ln\tilde{W})^i; & \tilde{W}>0.001\ \text{g cm}^{-2}\\ \sum_{i=1}^{4}a'_i\tilde{W}^{i/2}; & \tilde{W}\leq0.001\ \text{g cm}^{-2}\end{cases} \tag{32}$$

$$\in\downarrow(\tilde{W})=\begin{cases}\sum_{i=0}^{4}b_i(\ln\tilde{W}_i)^i; & \tilde{W}>0.001\ \mathrm{g\ cm^{-2}}\\ \sum_{i=1}^{4}b_i'\tilde{W}^{i/2}; & \tilde{W}\leqslant 0.001\ \mathrm{g\ cm^{-2}}\end{cases} \tag{33}$$

where the parameters a_i, a_i', b_i and b_i' are listed in Table 1.

Table 1 Parameters in Rodgers' Parameterization Emissivity Formulae

i	a_i	a_i'	b_i	b_i'
0	0.59830		0.65580	
1	0.15068	9.329	0.12175	8.857
2	0.03404	−446.4	0.01498	−332.8
3	0.00655	824.0	0.00150	14607.0
4	0.00049	259700.0	0.00005	−261900.0

According to the precision of Rodgers' parameterization formulae, we assume $\in\uparrow(\tilde{W})=\in\downarrow(\tilde{W})=0$ when $\tilde{W}<10^{-7}$ g/cm².

Since the present paper mainly discuss the interaction of turbulent heat and long-wave radiative fluxes in the surface layer, we can consider that the variations of the temperature and humidity above 2 km only have little influence upon the long-wave radiative flux in the surface layer. Therefore, the long-wave radiative flux $F\uparrow$ and $F\downarrow$ can be divided into three parts in computations. Now let's define $Z_T=1985$ m, $Z_h=10$ m, it is obvious that $Z_T>Z_h>Z_0$ is held (Z_0 the surface roughness length).

(1) For $Z>Z_T$:

$$\tilde{W}_T=\tilde{W}(Z_r)=\int_{Z_T}^{\infty}\rho q\frac{P}{P_0}dZ \tag{34}$$

$$F_T^{\downarrow}=F\downarrow(Z_T)=\int_{Z_T}^{\infty}B[T(Z')]\frac{\partial\in\downarrow}{\partial Z'}(Z',Z)dZ' \tag{35}$$

When the atmospheric situations are stationary, the distributions of the air density, temperature and water vapor in the free atmosphere do not have considerable variations. For this reason $\tilde{W}_T$ and $F_T^{\downarrow}$ can be treated as constant values, which will be specified according to the statistical values of radiative cooling rate in the free atmosphere. Here, $\tilde{W}_T=0.2$ g/cm² and $F_T^{\downarrow}=148$ W/m².

(2) For $Z_h<Z\leqslant Z_T$

If $T(Z_h)=T_h=\theta(Z_h)$,

so

$$T(Z)=T_h-\Gamma_d(Z-Z_h), \tag{36}$$

where Γ_d is the dry adiabatic lapse rate. Similarly we assume

$$q(Z)=q_h-\frac{\Delta q}{\Delta Z}(Z-Z_h). \tag{37}$$

Thus, the values of the radiative flux $F\uparrow$ and $F\downarrow$ can be calculated from Eqs. (28)—(37).

(3) For $Z_0\leqslant Z\leqslant Z_h$,

Finally using Eqs. (28)—(33), we can calculate $F\downarrow$ (Z) and $F\uparrow$ (Z) in which the distributions of θ(Z) and q(Z) can be determined by Eqs. (25)—(26).

In the computation of the radiation on the surface,

$$\in'_0 B(T_0) = \in_0 B(T_0) - (1-\in_0) F\downarrow \tag{38}$$

is taken, where $\in'_0 = 0.95$.

IV. THE PROCEDURES AND RESULTS OF THE COMPUTATION

In Table 2, the distributions of typical meteorological elements, wind speed, temperature and specific humidity quantities for every computation procedure, are explained. They are divided into three parts with the order of the surface roughness length, and in each part twelve cases are classified in accordance with the conditions of the distribution of wind, temperature and water vapor.

Table 2 The Classification of the Computation Parameters

	A											
Z_0(m)	0.1											
T(10m)$-T(Z_0)$(℃)	0.5						2.0					
T(10m)$-q(Z_0)$(g/kg)	−3.4									−0.5		
$U(Z=10\text{m})$(m/s)	1			10								
$T(Z_0)$(℃)	30	20	5	30	20	5	30	20	5	30	20	5
$q(Z_0)$(g/kg)	27	8	4	27	8	4	27	8	4	27	8	4
No.	1	2	3	4	5	6	7	8	9	10	11	12

	B											
Z_0(m)	0.01											
T(10m)$-T(Z_0)$(℃)	0.5						0.2					
T(10m)$-q(Z_0)$(g/kg)	−3.4									−0.5		
$U(Z=10\text{m})$(m/s)	1			10								
$T(Z_0)$(℃)	30	20	5	30	20	5	30	20	5	30	20	5
$q(Z_0)$(g/kg)	27	8	4	27	8	4	27	8	4	27	8	4
No.	1	2	3	4	5	6	7	8	9	10	11	12

	C											
Z_0(m)	0.001											
T(10m)$-T(Z_0)$(℃)	0.5						2.0					
T(10m)$-q(Z_0)$(g/kg)	−3.4									−0.5		
$U(Z=10\text{m})$(m/s)	1			10								
$T(Z_0)$(℃)	30	20	5	30	20	5	30	20	5	30	20	5
$q(Z_0)$(g/kg)	27	8	4	27	8	4	27	8	4	27	8	4
No.	1	2	3	4	5	6	7	8	9	10	11	12

On the basis of the values in Table 2, the turbulent parameters U_*, θ_*, and q_* along with stability parameter L can be obtained from Eqs. (15), (24)—(26).

In the vertical direction, the numerical difference grids are divided into two sections 99 levels altogether, computing the long-wave radiative flux in the surface layer for the lowest 20 levels from Z_0 to Z_h, and two kinds of numerical difference scheme systems are used. The first one, equidistant grids, is given by

$$Z = 0.5(I-1) + Z_0. \quad (I = 1, 2, 3, \cdots, 21) \tag{39}$$

The second one, nonequidistant grids, is given by

$$Z = 0.01(I-1)^n + Z_0, \quad (I = 1, 2, 3, \cdots, 21) \tag{40}$$

where, $n = (\ln 999)/\ln 20$. The nonequidistance difference scheme is used in order to carefully inspect the distribution of the radiative flux divergence $\partial R/\partial Z$ within 1 m height.

The height from Z_h to Z_T is divided into 79 equidistant grids, in which 25 meter interval is taken. The difference scheme

$$Z = Z_h + 25(I-21), \quad (I = 22, 23, \cdots, 100) \tag{41}$$

is used for numerical computation.

The values of turbulent parameters U_*, θ_*, q_* and M-O length L obtained by computation under various conditions are listed in Table 3.

Table 3 The Calculated Turbulent Parameter Values under Various Conditions

		1	2	3	4	5	6	7	8	9
A	U_*(m/s)	0.017	0.014	0.010	0.862	0.861	0.861	0.841	0.840	0.838
	θ_*(℃)	0.008	0.007	0.005	0.043	0.043	0.043	0.168	0.168	0.168
	q_*(g/kg)	0.057	0.049	0.035	0.293	0.293	0.293	0.286	0.285	0.286
	L(m)	2.6	2.1	1.5	1330	1280	1200	325	313	297
B	U_*(m/s)	0.011	0.010	0.007	0.574	0.574	0.574	0.560	0.560	0.559
	θ_*(℃)	0.006	0.005	0.003	0.029	0.029	0.029	0.112	0.112	0.112
	q_*(g/kg)	0.038	0.032	0.023	0.195	0.195	0.195	0.191	0.190	0.190
	L(m)	1.7	1.4	0.98	890	860	850	2.7	209	198
C	U_*(m/s)	0.008	0.007	0.005	0.431	0.431	0.430	0.420	0.420	0.419
	θ_*(℃)	0.004	0.004	0.003	0.022	0.022	0.022	0.084	0.084	0.084
	q_*(g/kg)	0.028	0.024	0.0180	0.146	0.146	0.146	0.143	0.143	0.142
	L(m)	1.3	1.1	0.7	670	640	610	163	157	149

(1) Let's define $\Delta_{10}=(S_{10}-S_0)/S_0$ and $\Delta=(S(Z)-S_0)/S_0$, the results show that the variations of the surface roughness Z_0 and turbulent friction velocity U_* have little effect on the long-wave radiative flux divergence $\partial R/\partial Z$, but $\partial R/\partial Z$ is quite sensitive to the thermal turbulent factors. In fact, any variation of the turbulent thermal field shall lead to a corresponding variation of the turbulent kinetic field. For example, the variation of Z_0 and U_* shall probably influence on the variation of θ_* and q_*. Therefore the radiative flux divergence is able to be indirectly affected by kinetic field. From Fig. 1, we can see that in the stable conditions at night, the air about 2—3 cm over the ground is firstly heated and then cooled above that height by long-wave radiative flux divergence. It is worth indicating that the maximum of the radiative cooling rate is situated about 6 cm above the ground, the absolute values of $\partial R/\partial Z$ decrease with height Z. Thus the absolute values of S always increase with height Z from Eq. (6).

(2) The relationships between Δ_{10} and M-O length L are given in Table 4. We can see that for a constant roughness Z_0, the variational rate of the turbulent heat flux with height is inversely proportional to the value of M-O length L. In other words, the stronger the stability, the larger the value of Δ_{10}. It shows that the increase of stability will restrain the development of turbulence ($S_0\rightarrow0$), but the variation of $\partial R/\partial Z$ usually is not large. Therefore, the heat flux in the surface layer is mainly determined by the long-wave radiative flux at this time.

(3) For various stratification conditions, the relationships between Δ_{10} and roughness length Z_0 are shown in Fig. 2. It can be seen that Δ_{10} are inversely proportional to Z_0. This phenomenon indicates that the mechanical turbulence is relatively feeble for the small roughness, and the heat flux is the major part. For the larger roughness, the relative variance of the heat flux between upper and low levels decreases gradually with roughness in the surface layer, but the mechanical turbulence increases. We also find that the larger the

specific humidity, the bigger the values of Δ_{10}. In addition, it is worth indicating that the values of Δ_{10} increase rapidly with stable stratification (or with decrease of L). Thus, with the increasing of stability, the turbulent heat flux is restrained, and the long-wave radiative flux plays an important role at this time.

Table 4 The Relationships between Δ_{10} and M-O Length L for Various Roughness Length Z_0

Z_0	0.1						0.01						0.001					
q	27		8		4		27		8		4		27		8		4	
L	1333	2.6	1288	2.1	1211	1.5	888	1.0	858	1.6	814	1.7	644	1.3	644	1.1	610	0.7
$\frac{\Delta S}{S_0}$	0.15	40.5	0.09	31.6	0.06	39.2	0.23	92.3	0.20	57.7	0.13	88.4	0.39	162.6	0.34	134.1	0.25	163.1

(4) For various atmospheric conditions, the relative vertical variation of the turbulent heat flux is shown in Fig. 3 (here only several typical curves are given). If $\delta=0.1$, the computational results show that the thickness of CTFL is more than ten meters in cases of A_5-A_9, B_7-B_9, C_8-C_9, about 1—4 meters in cases of A_4, B_4-B_6, C_4-C_9 and even hardly reach to 1 m in cases of A_1-A_3, B_1-B_3, C_1-C_3. For this reason we can see that the thickness of CTFL is determined by Z_0, L, $q(z)$, $T(z)$, and $U(z)$ in common. However, the stratification factor L plays an important role in affecting the thickness of CTFL.

In addition, from Fig. 3 we can find that in the same conditions, Δ is also inversely proportional to Z_0. In the same roughness length Z, the thickness of CTFL increases gradually from A_1 to A_{12} (in the same way from B_1 to B_{12} and from C_1 to C_{12}).

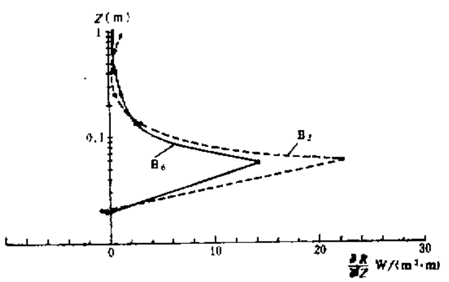

Fig. 1. The variation of $\partial R/\partial Z$ with height over the first meter from the ground for various meteorological conditions.

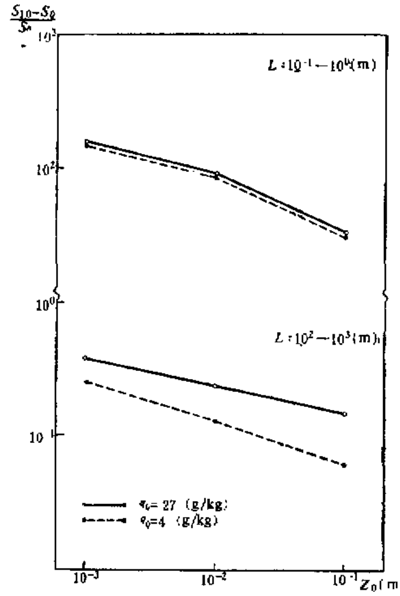

Fig. 2. The relationships between Δ_{10} and the roughness length Z_0.

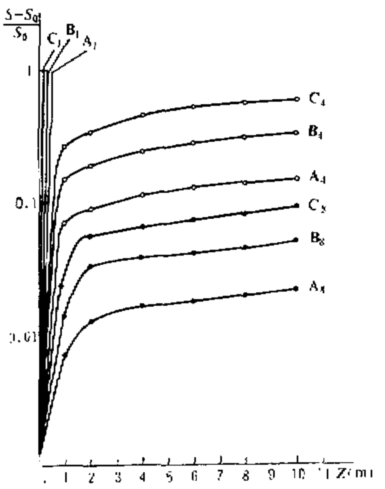

Fig. 3. Relative vertical variation of the turbulence heat flux Δ with height for various parameters.

V. CONCLUSION

In the stably stratified surface layer, the long-wave radiative flux has great influence upon thermal structure, paticularly for small roughness length. As the stability increasing, the heat flux in the surface layer is mainly determined by the radiative flux. For this reason the long-wave radiative flux often reduces the thickness of CTFL. In the cases of low wind speeds and strongly stable stratifications, the thickness of CTFL becomes very thin, even exists impossibly.

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