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EVOLUTION OF LARGE SCALE DISTURBANCES AND THEIR INTERACTION WITH MEAN FLOW IN A ROTATING BAROTROPIC ATMOSPHERE——PART II

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ABSTRACT

In part I of this paper, we have discussed two problems: the general properties of two-dimensional barotropic motion and the evolution and structure of both Rossby wave packet and inertio-gravity wave packet. In this part, we shall continue our discussion. Third, normal modes and continuous spectra of both quasi-geostrophic and non-geostrophic models, their different behaviour, and the comparison of normal mode approach to the wave packet approach. Fourth, weakly nonlinear theory of interaction based on the analysis of eddy transports. A nonzonal basic flow as well as non-geostrophic model is also included in the consideration. The last, the fully nonlinear theory, making emphasis on the conditions for the maintenance of nonzonal disturbances and the conditions for their continuous and complete absorption by the zonal flow. A comparison of Rossby wave absorption to energy cascade in the two-dimensional turbulence is also given.

IV. NORMAL MODE APPROACH

1. *Normal Modes and Continuous Spectrum In the Quasi-Geostrophic Model*

As mentioned above, the representation of disturbances in the form of wave packet and the correspondent WKB method have the advantages of simple and clear geometric picture and excellent mathematic formulas; but the disadvantage is that they can be applied only to those disturbancse whose temporal and spatial scales are small compared with the basic flow. It is better to apply the normal mode method to the general initial value problem.

The normal mode method has been successfully developed in the study on the problems of hydrodynamic instability (see, Lin[1], 1955, for example), and applied to the investigation of barotropic instability in a rotating atmosphere first by Kuo[2] (1949). However, in an inviscid fluid normal modes do not construct a complete system and there must exist continuous spectrum.

Let

$$\psi'(\lambda,\theta,t)=\Psi(\theta)e^{im(\lambda-ct)}. \tag{79}$$

substituting (79) into (16), we have

$$(\bar{\lambda}-c)\left\{\frac{d}{dy}(1-y^2)\frac{d\Psi}{dy}-\left[\frac{m^2}{a^2(1-y^2)}+K\frac{f_c^2}{\bar{\phi}}\right]\Psi\right\}-\frac{m}{a}\frac{\partial\bar{q}}{\partial y}\Psi=0, \tag{80}$$

where $y\equiv a\cos\theta$ and the basic flow is assumed to be zonal and steady with a continuous $\partial\bar{q}/\partial y$; c is called as an eigenvalue or discrete spectrum if the correspondent solution to (80), Ψ, and its derivative of second order are finite and continuous everywhere in $-1\leqslant y\leqslant +1$, and such Ψ is called as an eigenfunction or a (discrete) mode. Usually, ψ'

determined by (79) *from an eigenvalue* c *and eigenfunction* Ψ is also called as a normal mode. It is clear that a normal mode is a special solution to (1). Each eigenvalue c must be either a complex number (unstable case) or a real number located in $c<\bar{\lambda}_{min}$ or $c>\bar{\lambda}_{max}$, where $\bar{\lambda}_{min}\leqslant\bar{\lambda}(y)\leqslant\bar{\lambda}_{max}$. More strict bounds of discrete spectra are given by the so-called semicircle theorem (see, Howard[3], 1961; Pedlosky[4], 1964; Kuo[5], 1973 and Wang[6], 1983).

Every constant c satisfying $\bar{\lambda}_{min}\leqslant c\leqslant\bar{\lambda}_{max}$ makes (80) have a singular point y_c, where $\bar{\lambda}(y_c)=c$, hence, the correspondent Ψ has logarithmic singularity at y_c and is not an ordinary solution to (2). The region $\bar{\lambda}_{min}\leqslant c\leqslant\bar{\lambda}_{max}$ constructs the continuous spectrum.

Denote the discrete spectra and modes as C_{mn} and Ψ_{mn} (y) respectively for a given m, $n=1,2,\cdots$; and the continuous modes as $\Psi_m(y, c)$. Every ordinary solution of (1) can be represented as follows

$$\Psi'(\lambda,y,t)=\text{Re}\left\{\sum_m\left[\sum_n A_{mn}\Psi_{mn}(y)e^{im(\lambda-c_{mn}t)}+\int_{\bar{\lambda}_{min}}^{\bar{\lambda}_{max}}A_m(c)\Psi_m(y,c)e^{im(\lambda-ct)}dc\right]\right\}$$

$$\equiv\text{Re}\left\{\sum_m\left[\psi_{md}(\lambda,y,t)+\psi_{mc}(\lambda,y,t)\right]\right\}, \tag{81}$$

where A_{mn} and A_m (c) are the coefficients of expansion, (Zeng et al[7]., 1981, Lu et al[8]., 1983).

2. *The Behaviour of Quasi-Geostrophic Disturbances*

Let

$$\Psi_{mn}(y)\equiv\Psi_{mn}^{(r)}(y)+i\Psi_{mn}^{(i)}(y)\equiv|\Psi_{mn}(y)|e^{i\sigma_{mn}(y)}, \tag{82}$$
$$(\Psi_{mn}^{(r)}(y)\not\equiv0),$$

we have that $\Psi_{mn}^{(i)}$ is a const. (no phase shear) for the real C_{mn}, while $d\Psi_{mn}^{(i)}$ (y)/dy$\not\equiv$0 for a complex c with nonzero imaginary part (Wang[6], 1983). These are just required by the fact that a single neutral mode (C_{mn} is real) does not interact with the basic flow, but an unstable mode, either developing or decaying, must have energy exchange with basic flow, hence $u'v'$ and d$\Psi_{mn}^{(i)}$/dy are not identically equal to zero.

Of course, every decaying normal mode or their linear combination in the unstable case will finally be absorbed completely by the basic zonal flow. Besides, unlike a single neutral mode, a combination of several neutral normal modes usually interacts with the zonal flow due to their non-orthogonality even if the zonal flow satisfies the sufficient condition for stability. However, this interaction is transient, and the disturbance always remains.

On the contrary to the discrete modes in the stable case, the continuous spectrum, i. e. Ψ'_{mc}, always interacts with the basic zonal flow. Zeng et al[7]. (1983) have proved that both ψ'_{mc} and $|\partial\psi'_{mc}/\partial y|$ approach zero as $t\to\infty$. This means that there is a complete absorption of a disturbance consisting only of continuous spectrum and its energy no matter whether the disturbance grows or decays in the initial stage. However, its enstrophy must remain in the stable case due to the conservation law (18), hence, as $t\to\infty$ the limit is an ideal function.

From the above analyses we conclude that, after a long time, a disturbance leaves the part consisting of a combination of neutral modes and unstable growing modes, if any, then that part consisting of continuous spectrum or a combination of unstable decaying modes becomes very weak. In the stable case the necessary as well as sufficient condition for complete absorption of a disturbance is that the disturbance consists only of continuous spectrum;

while in the unstable case growing modes do not exist with the disturbance.

Numerical calculation by using finite difference method but with a beta-plane approximation (Lu et al[6]., 1983) shows that there are only a few discrete spectra in the stable case, and the modes have rather large meridional scale. It also shows that a continuous spectrum does exist (in the region $\bar{\lambda}_{min}\leqslant c\leqslant\bar{\lambda}_{max}$ the eigenvalues of the matrix approximating the original differential operator are denser and denser as the grid size becomes smaller and smaller), and the function $\Psi_m(y, c)$ usually has small scale, like a local disturbance (Fig. 7). This means that by representing a local disturbance or a wave packet in terms of (81), the main component just corresponds to the continuous spectrum.

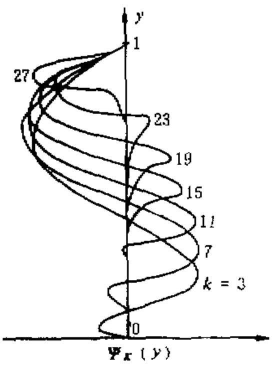


Fig. 7. Examples of spectral functions $\Psi_m(y, c)$ corresponding to the continuous spectrum but computed by using finite difference method. $\Psi_m(y, c_k)$ is simply denoted as $\psi_k(y)$. $\bar{u}=\bar{u}_0+\bar{u}_1 y$, $0\leqslant y\leqslant 1$ on a β-plane, $\bar{u}_0$, $\bar{u}_1$ and β_y are all const., and $m=1$.

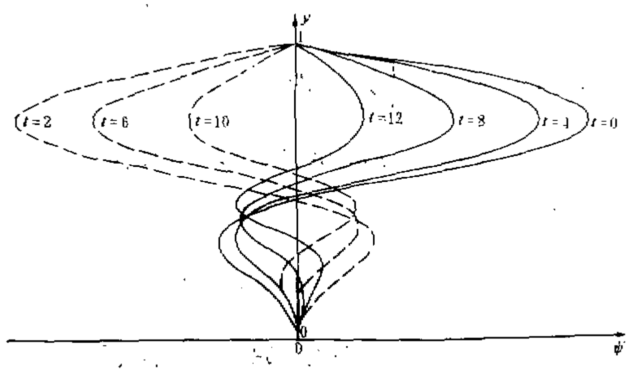


Fig. 8. The evolution of a disturbance at $x=0$. The disturbance consists of continuous spectrum (some examples are given in Fig. 7), and its trough-ridge lines initially are directed along the meridian.

It is very interesting that a disturbance consisting of linear combination of such modified $\Psi_m(y, c)$ and having no initial meridional shear of the trough-ridge lines does continuously decay for a long time, and its meridional wavelength becomes smaller and smaller (Fig. 8). This is a good demonstration of the behaviour of a wave packet and a confirmation of the conclusions obtained by the WKB method. In fact, by dividing $\bar{\lambda}_{min}\leqslant c\leqslant\bar{\lambda}_{max}$ into several subsets with distance δc_k small enough, ψ'_{mc} in (81) can be represented in a form of linear combination of wave packets,

$$\psi'_{mc}(\lambda, y, t)=\sum_k \psi'_{mck}(\lambda, y, t),$$

$$\begin{aligned}\psi'_{mck}(\lambda, y, t)&\equiv\int_{c_k-\delta c_k/2}^{c_k+\delta c_k/2} A_m(c)\Psi_m(y, c)e^{im(\lambda-ct)}dc\\&=\left\{\int_{c_k-\delta c_k/2}^{c_k+\delta c_k/2} A_m(c)\left|\Psi_m(y, c)\right| e^{i[\alpha_m(y,c)-\alpha_m(y,c_k)-(c-c_k)t]}dc\right\}\\&\quad\times e^{i[\alpha_m(y,c_k)+m\lambda-mc_k t]}\\&\equiv A_k(\lambda, y, t)e^{i[\alpha_m(y,c_k)+m\lambda-mc_k t]},\end{aligned}\tag{83}$$

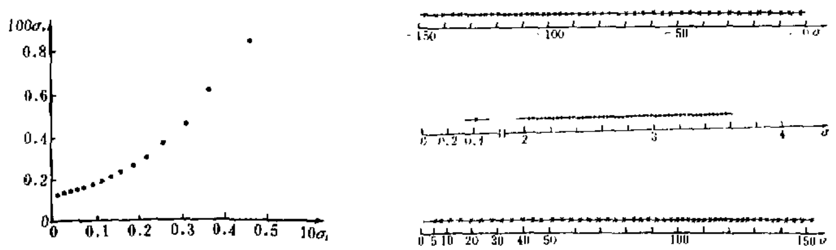
Every one has a critical line at $y=y_k$, where $\bar{\lambda}(y_k)=C_k$, but the scale of amplitude, A, is not necessarily larger than that of $a_m(y, c_k)$. Of course, the result obtained by normal mode approach is more strict.

Yamagata[9] (1976) has found an analytic solution to the linearized vorticity equation in an infinite β- plane with a constant shear of $\bar{u}$ or $\bar{v}$, and has also shown the complete absorption of disturbances. Recently, Tung[10] (1983) has made a more strict analysis, and has shown the asymptotic behaviour of this absorption in detail.

3. *Non-Geostrophic Model*

Using the finite difference method and beta-plane approximation Li and Zeng[11] (1983) have recently carried out a numerical analysis of the modes and continuous spectrum in a barotropic atmosphere with a steady zonal flow as the basic flow.

There are three branches of spectra corresponding to the Rossby or vortical wave, downstream propagating and upstream propagating inertio-gravity waves respectively. The Rossby or vortical wave branch essentially does not differ from that obtained by the quasi-geostrophic model. Namely, there are discrete spectra and continuous spectrum. The number of discrete spectra is finite in the stable case, but, probably, infinite in the unstable case, although the imaginary part of the spectra is very small except a few of them (Fig. 9a). However, two inertio-gravity waves branches both have only discrete spectra (Fig. 9b) again. This fact indicates that the inertio-gravity waves can not be completely absorbed by a steady zonal flow in an inviscid barotropic atmosphere unless the waves propagate in an infinite plane.



(a) (b)

Fig. 9. (a) Discrete spectra with complex σ for an unstable zonal flow $\bar{u}=8ye^{(0.5-y^2/0.08)}$ in a primitive equation model, $f=1$, $m=2$ and $0\leqslant y\leqslant 1$, and $\sigma\equiv mc=\sigma_r+i\sigma_i$.

(b) Three branches of spectra for a stable zonal flow $\bar{u}=1+0.8y$, $f=1$, $m=2$ and $0\leqslant y\leqslant 1$. One branch located in $-1<\sigma<3.6$ corresponds to the vortical waves. There is only one discrete mode in this branch, and the region $2<\sigma<3.6$ corresponds to the continuous spectrum degenerated as very dense "discrete" spectra by using the finite difference method. The two branches located in $\sigma<-2$ and $\sigma>4$ correspond to upstream and downstream propagating inertio-gravity waves respectively.

V. THE INFLUENCE OF DISTURBANCES ON THE BASIC FLOW——WEAKLY NONLINEAR THEORY

1. *Interaction of Rossby Wave with the Zonal Flow*

There is not only energy transformation between the disturbances and zonal flow but also redistribution of the zonal momentum as the evolution of disturbances goes on.

The redistribution of zonal flow can be directly determined by equation (17) as the meridional flux of potential vorticity $\overline{v'_\theta q'}\sin\theta$ caused by the disturbances is known. However, in order to facilitate the physical interpretation it is more convenient to take the (dimensional) angular momentum equation

$$\frac{\partial a\bar{v}_\lambda \sin\theta}{\partial t} = -\frac{\partial}{a\sin\theta\partial\theta}(\overline{v'_\lambda v'_\theta}a\sin^2\theta) - 2\omega\cos\theta\cdot\bar{v}_\theta a\sin\theta, \tag{84}$$

where the first term on the right-hand side is the convergency of the meridional flux of angular momentum, $\overline{v'_\lambda v'_\theta}a\sin^2\theta$, and the second term, $-2\omega\cos\theta\cdot\bar{v}_\theta a\sin\theta$, is the effect of wave-induced mean meridional velocity. By use of the quasi-geostrophic approximation, i. e. $2\omega\cos\theta$ in (84) is replaced by $f_0\equiv 2\omega\cos\theta_0$ and the continuity equation is approximated by

$$f_0 K\frac{\partial\bar{\Phi}}{\partial t} + \tilde{\Phi}\frac{\partial\bar{v}_\theta\sin\theta}{a\sin\theta\partial\theta} = 0, \tag{85}$$

the wave-induced mean meridional velocity is determined by the following equation

$$\sin\theta\frac{\partial}{\partial\theta}\left(\frac{\partial\bar{v}_\theta\sin\theta}{\sin\theta\partial\theta}\right) - K\frac{f_0^2 a^2}{\tilde{\Phi}}\bar{v}_\theta\sin\theta = K\frac{f_0^2}{\tilde{\Phi}}f_0^{-1}\frac{\partial}{\sin\theta\partial\theta}(\overline{v'_\lambda v'_\theta}a\sin^2\theta). \tag{86}$$

It is clear that $\bar{v}_\theta\equiv 0$, and the evolution of zonal flow is simply determined by the convergency of angular momentum flux if $K=0$, i. e., in a non-divergent barotropic atmosphere. However, as the effect of two-dimensional divergency of the flow is taken into account there is always a wave-induced $\bar{v}_\theta$ and a more detailed analysis of the waves' interaction is needed, although the unique factor of influence is the angular momentum flux, $\overline{v'_\lambda v'_\theta}\, a\sin^2\theta$ which also governs the field $\bar{v}_\theta$ in accordance with (86).

By using the observed wind data or observed geostrophic wind fields the statistics of angular momentum flux have been carried out by many investigators in the general circulation of the atmosphere. On the other hand, the transport properties of normal modes have been thoroughly analyzed by Kuo[11] (1951). Namely, a neutral mode does not generate nonzero flux of angular momentum due to no phase shear of the trough-ridge lines (see, (82)), but an unstable mode, either growing or decaying, does. It is not difficult to think that a single unstable mode has its definite profile of $\overline{v'_\theta v'_\lambda}\, a\sin^2\theta$, hence the zonal flow is accelerated in a certain definite latitudinal zone and decelerated in another definite zone, although the total zonal energy decreases in the case of growing mode and increases in the case of decaying mode. In fact, there is a very simple relationship between the fluxes of angular momentum and potential vorticity,

$$\overline{v'_\theta q'}a^2\sin^2\theta = -\frac{\partial}{\partial\theta}(\overline{v'_\lambda v'_\theta}\, a\sin^2\theta), \tag{87}$$

hence, we have

$$\frac{\partial}{a\partial\theta}(\overline{v'_\lambda v'_\theta}\, a\, \sin^2\theta)=\frac{a^2\sin^2\theta}{\partial\bar{q}/\partial\theta}\,\frac{\partial}{\partial t}\left(\frac{\overline{q'^2}}{2}\right) \tag{88}$$

and

$$\frac{\partial}{\partial t}(\bar{v}_\lambda a\sin\theta)+\frac{a^2\sin\theta}{\partial\bar{q}/\partial\theta}\,\frac{\partial}{\partial t}\left(\frac{\overline{q'^2}}{2}\right)=-2\omega\cos\theta\cdot\bar{v}_\theta a\sin\theta. \tag{89}$$

Integrating (88) over the whole region with the boundary conditions $\overline{v'_\theta\sin\theta}=0$, at $\theta=0,\ \pi$ yields (18), so that the conservation of total weighted enstropy $q'^2\sin\theta/2(\partial\bar{q}/a\partial\theta)$ is required by the vanishing of angular momentum flux at the two poles or two boundaries. According to (89), in a nondivergent atmosphere the zonal flow is accelerated wherever $(\partial q'^2/\partial t)/(\partial\bar{q}/\partial\theta)<0$, but decelerated wherever $(\partial q'^2/\partial t)/(\partial\bar{q}/\partial\theta)>0$, so that the zonal flow is accelerated in the region of $\partial\bar{q}/\partial\theta>0$ and decelerated in the region of $\partial\bar{q}/\partial\theta<0$ if a decaying normal mode is superimposed on it, and the opposite is true if a growing mode is superimposed. When the effect of divergency is taken into account one can obtain a tendency of the zonal flow by solving (17) i. e.

$$\frac{\partial\bar{v}_\lambda}{\partial t}=\int_0^\pi\frac{\partial G(\theta,\theta')}{\partial\theta}\left[\frac{\partial}{a\sin\theta'\partial\theta'}\left(\frac{\partial\overline{v'_\lambda v'_\theta}\, a\sin^2\theta'}{a\sin\theta'\partial\theta'}\right)\right]\sin\theta' d\theta'\ , \tag{90}$$

where

$$G(\theta,\theta')=\frac{1}{2}\sum_{n=0}^{\infty}\frac{2n+1}{n(n+1)+f_0^2a^2/\bar{\Phi}}P_n(\cos\theta)P_n(\cos\theta')$$

$$=\ln\left(\sin\frac{|\theta-\theta'|}{2}\right)+\text{regular terms}. \tag{91}$$

Substituting (88) into (90), a qualitative analysis can still be made, but it is rather complicated. If there is a neutral mode, $\overline{v'_\lambda v'_\theta}$ is zero everywhere in accordance with (88) because $\partial\overline{q'^2}/\partial t\equiv 0$ for a single neutral mode, hence $\bar{v}_\theta\equiv 0$ and $\partial\bar{v}_\lambda/\partial t\equiv 0$ everywhere no matter whether $K=0$ or 1. A more strict analysis of the mean zonal acceleration can be found in Andrews and McIntyre's paper[13] (1976) and Pedlosky's[14] book (1979), although for baroclinic atmosphere.

However, due to the non-orthogonality of the modes a linear combination of normal modes or a disturbance consisting of continuous spectrum greatly differs from a single mode, they always cause some transport of angular momentum and interact with the zonal flow (Zeng[26], 1982). When the disturbance is represented by a single wave packet, we have

$$\overline{v'_\theta v'_\lambda}a\sin^2\theta=U^{*2}L^{*-2}a\,\overline{\frac{\partial\psi'}{\partial x}\,\frac{\partial\psi'}{\partial y}}, \tag{92}$$

$$-\overline{\frac{\partial\psi'}{\partial x}\,\frac{\partial\psi'}{\partial y}}=-\frac{1}{2\Delta X}\int_0^X mn|\Psi_0|^2dX+0(\varepsilon)\equiv-\frac{\overline{mn}}{2}\,|\Psi_0|^2+0(\varepsilon), \tag{93}$$

in accordance with the results obtained in §III-1, where $\Delta X=2\pi\varepsilon a$ and ψ' (x,y,t) are nondimensional. (92) and (93) indicate that a wave packet always causes divergency-convergency of angular momentum flux due to the variation of its amplitude $|\Psi_0|$, and even the variation of its orientation mn. Supposing there is a single jet in the westerlies with the maximum of $\bar{\lambda}$ (i. e. $\bar{u}$, defined in §.III-1 at Y_0 (θ_0)), $\partial\bar{\lambda}/\partial\theta<0$ as $\theta>\theta_0$ (i. e. $\partial\bar{u}/\partial Y>0$ as $Y<Y_0$), and $\partial\bar{\lambda}/\partial\theta>0$ as $\theta<\theta_0$ (i. e. $\partial\bar{u}/\partial Y<0$ as $Y>Y_0$); and there is a typical decaying wave packet with $mn>0$ at $Y>Y_0$ and $mn<0$ at $Y<Y_0$; $\overline{mn|\Psi_0|^2}$ reaches its extreme value at Y_1, Y_2, where $Y_2>Y_1$; $\partial\,\overline{mn|\Psi_0|^2}/\partial Y=\max.$ at Y'_0, and zero at Y'_1, Y'_2, where $Y'_1<Y'_0<Y'_2$, and $Y'_1<Y_0$, $Y'_2>Y_0$; and $(\sin\theta)^{-2}\partial\,\overline{mn|\Psi_0|^2}/\partial Y=\max.$ at Y''_0, zero at Y''_1, Y''_2. We have the convergency of angular momentum flux in $Y'_1<Y<Y'_2$; and convergency of potential vorticity flux in $Y''_1<Y<Y''_2$, but its divergency in $Y''_0<Y<Y''_2$. For simplicity, assume that Y'_0 and Y''_0 are closed to Y_0 (this is the case if the wave packet is symmetric about the jet), then we have the acceleration of westerlies on both sides of the jet in accordance with (90) and (91) except those latitudinal zones far away from the jet. Differentiating (90) with respect to t, we come to the conclusion that the absolute shear of the zonal flow is also enlarged.

Performing the similar analysis we come to the following conclusion: a growing wave packet with its centre located near the jet in the westerlies decelerates the zonal west flow as well as weakens the absolute shear of zonal flow on both sides of the jet (Fig. 10).

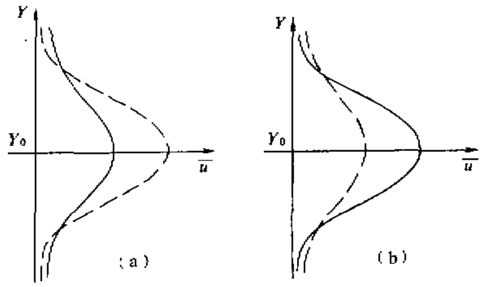


Fig. 10. The evolution of zonal flow due to the interaction with a Rossby wave disturbance located symmetrically about the jet. Solid line is the initial profile of $\bar{u}$, the dashed is the redistributed. (a) with a decaying disturbance; (b) with a growing one.

2. *Nonzonal Basic Flow*

Taking the quasi-geostrophic model (16) and a basic flow consisting of ultra-long waves, we have

$$\left(\frac{\partial}{\partial t}+\bar{v}_\lambda\frac{\partial}{a\sin\theta\partial\lambda}+\bar{v}_\theta\frac{\partial}{a\partial\theta}\right)\bar{q}=-\nabla\cdot\mathscr{F}+\bar{S}, \tag{94}$$

where the bar denotes some spatially and temporally averaging operator rather than a zonal mean, $\bar{S}$ is the source (because in the real atmosphere the basic flow is always forced), and $\mathscr{F}$ denotes the flux of potential vorticity generated by the perturbations,

$$\mathscr{F}\equiv\boldsymbol{\theta}^{\circ}\,\overline{v'_\theta q'}\sin\theta+\boldsymbol{\lambda}^{\circ}\,\overline{v'_\lambda q'}. \tag{95}$$

If we again adopt a linearized model to describe the evolution of the perturbation,

$$\left(\frac{\partial}{\partial t}+\bar{v}_\lambda\frac{\partial}{a\sin\theta\partial\lambda}+\bar{v}_\theta\frac{\partial}{a\partial\theta}\right)q'+v'_\lambda\frac{\partial\bar{q}}{a\sin\theta\partial\lambda}+v'_\theta\frac{\partial\bar{q}}{a\partial\theta}=0, \tag{96}$$

(40) is the dimensionless form of (96), then (96) and (94) together construct a weakly interacting system.

Taking a time-filter with an appropriate time-scale, some investigators have performed systematical calculations of $\mathscr{F}$ by the observed atmospheric data. The results show that the divergency $|\nabla\cdot\mathscr{F}|$ is large indeed. This means that the spatially and temporally averaged flow is forced not only by the source but also by the influence of perturbations. At present much effort of investigators is devoted to the evolutionary process of the basic flow owing to its practical importance in the middle-long range predictions of weather and climate. However, for a better understanding it is very important to study the interaction mechanism.

Some questions arise. First, Eq. (94) is obtained by taking an averaging operator (filter) on (1) and neglecting the terms such as $\overline{\bar{v}_\theta \partial q'/a\partial\theta}$ and $\overline{v'_\theta \partial \bar{q}/a\partial\theta}$, i. e.

$$\overline{Fq'}\approx 0, \tag{97}$$

where F and q denote two functions. Eq. (97) takes place only if the scale of the filter (the "bar") is much larger than that of perturbation (the "prime"). This is a practical constraint for the filter. Second, the behaviour of ultra-long waves is not well described by (16) or (94) and, instead of (94), a more accurate equation is needed although it is not difficult to be found. Third, the averaged forcing $\bar{S}$ is not very well known, although we have a good observational data set of $\bar{\Phi}$, so that for making a prediction it is better to divide $\bar{\Phi}$ into two parts, $\bar{\Phi}_t$(transient) and $\bar{\Phi}_s$ (quasi-stationary), by adopting a new filter (denoted by symbol "$\sim$") with a long enough period,

$$\bar{\Phi}_s\equiv(\tilde{\bar{\psi}}), \quad \bar{\Phi}_t\equiv\bar{\Phi}-\bar{\Phi}_s. \tag{98}$$

$\bar{\Phi}_s$ is known from the observations although $\bar{S}_s$ is not known very well. Now, the equation for predicting $\bar{\Phi}_t$ can be written as follows

$$\left(\frac{\partial}{\partial t}+\bar{v}_{\lambda s}\frac{\partial}{a\sin\theta\partial\lambda}+\bar{v}_{\theta s}\frac{\partial}{a\partial\theta}\right)\bar{q}_t+\left(\bar{v}_{\lambda t}\frac{\partial}{a\sin\theta\partial\lambda}+\bar{v}_{\theta t}\frac{\partial}{a\partial\theta}\right)\bar{q}_s$$
$$=-\nabla\cdot(\mathscr{F}-\mathscr{F}_s)+\bar{S}_t-\bar{\mathscr{R}}_t, \tag{99}$$

where

$$\bar{\mathscr{R}}_t\equiv-\frac{1}{a\sin\theta}\left\{\frac{\partial}{\partial\lambda}[\bar{v}_{\lambda t}\bar{q}_t-\widetilde{\bar{v}_{\lambda t}\bar{q}_t}]+\frac{\partial}{\partial\theta}[\sin\theta(\bar{v}_{\theta t}\bar{q}_t-\widetilde{\bar{v}_{\theta t}\bar{q}_t})]\right\}. \tag{100}$$

$\bar{\mathscr{R}}_t$ might probably be omitted and subscript s denotes the quasi-stationary state. In (99) we have simply take (94) as the governing equation as an example. Fourth, when the perturbation ψ' is very strong one should even take a nonlinear equation instead of (96), i. e. adding the nonlinear terms similar to (100). By doing so, it seems that the calculation is not simple and even more complicated than directly integrating (16), but we benefit by the separation of the scale (especially the time-scale) and the elimination of some unknown factors.

3. *Non-Geostrophic Model*

In the range of weakly nonlinear theory the perturbation equations are written as (69), and the dimensional equations for the evolution of basic flow should be taken as follows (see, Zeng[6], 1979)

$$\begin{cases} \left(\dfrac{\partial}{\partial t}+\bar{u}\dfrac{\partial}{\partial x}+\bar{v}\dfrac{\partial}{\partial y}\right)\bar{u}-f\bar{v}+\dfrac{\partial\bar{\phi}}{\partial x}=-\dfrac{1}{\bar{\phi}}\left[\dfrac{\partial}{\partial x}\left(\bar{\phi}\,\overline{u'u'}-\dfrac{\overline{\phi'^2}}{2}\right)+\dfrac{\partial}{\partial y}(\bar{\phi}\overline{u'v'})\right]+\bar{F}_x, \\ \left(\dfrac{\partial}{\partial t}+\bar{u}\dfrac{\partial}{\partial x}+\bar{v}\dfrac{\partial}{\partial y}\right)\bar{v}+f\bar{u}+\dfrac{\partial\bar{\phi}}{\partial y}=-\dfrac{1}{\bar{\phi}}\left[\dfrac{\partial}{\partial x}(\bar{\phi}\overline{u'v'})+\dfrac{\partial}{\partial y}\left(\bar{\phi}\,\overline{v'v'}+\dfrac{\overline{\phi'^2}}{2}\right)\right]+\bar{F}_y, \\ \dfrac{\partial\bar{\phi}}{\partial t}+\dfrac{\partial\bar{\phi}\bar{u}}{\partial x}+\dfrac{\partial\bar{\phi}\bar{v}}{\partial y}=0, \end{cases} \tag{101}$$

where a local standard coordinate system has been used for simplicity, otherwise the terms $\theta^0 a^{-1}\ \overline{v_\lambda'^2}\,\mathrm{ctg}\theta+\lambda^0(-a)^{-1}\,\overline{v_\lambda' v_\theta'}\,\mathrm{ctg}\theta$ and $\theta^0 2a^{-1}\bar{v}_\lambda v_\lambda'\,\mathrm{ctg}\theta+\lambda^0(-a)^{-1}(\bar{v}_\lambda v_\theta'+v_\lambda'\bar{v}_\theta)\times\mathrm{ctg}\theta$ would have to be added to the left sides of the equations of motion for the basic flow and the perturbation respectively.

The unusual terms $(\bar{\phi})^{-1}(-\nabla\cdot\overline{\phi'^2}/2)$ arise due to the fact that there is a transport of available potential energy, while the terms $\partial\overline{\phi'u'}/\partial x+\partial\overline{\phi'v'}/\partial y$ disappear because the zero or small mass transport either by the vortical wave or inertio-gravity waves.

This weakly interactive system possesses the conservation of total energy

$$\frac{\partial}{\partial t}(\bar{E}+E')=0, \tag{102}$$

where

$$\begin{cases} \bar{E}=\dfrac{1}{2}\displaystyle\iint_S\bar{\phi}(\bar{u}^2+\bar{v}^2+\bar{\phi})ds, \\ E'=\dfrac{1}{2}\displaystyle\iint_S[\bar{\phi}(u'^2+v'^2)+\phi'^2]ds, \end{cases} \tag{103}$$

if $\bar{F}_x=\bar{F}_y=0$.

Suppose that there is a zonal basic flow with a jet, and that there is a downstream propagating inertio-gravity wave propagating toward the jet from the source located on one side of the jet. According to the results obtained in § III-4, this wave decays before it reaches the jet axis, and the zonal flow is fed by the wave energy. After the wave propagates into the other side of the jet axis, the wave becomes a growing one, hence it draws the zonal energy and decelerates the zonal flow. Therefore, the jet axis will continuously move to the source region if the source acts continuously. This is one possible mechanism of interaction of the inertio-gravity wave on the synoptic or planetary scale atmospheric motions, although it is difficult to be separated from the very strong interaction of Rossby waves in the baro-tropic components of the real atmosphere.

It might be important to study the interaction between the basic flow and the tide or the synoptic system and the mesoscale or small scale inertio-gravity waves. In the former the wave is generated continuously, and in the latter the wave has a rather large amplitude. These problems need more investigations.

VI. NONLINEAR CONSIDERATIONS

1. *Special Solutions Consisting of Nonzonal Disturbances*

So far most the results obtained are based on the linear theory or the theory of weakly nonlinear interaction. They should be corrected by considerations of nonlinearity.

As the nonlinearity is taken into account only some classes of the neutral modes obtained by the linearized quasi-geostrophic model are solutions to the nonlinear quasi-geostrophic equation and are the propagating waves without change in their shapes. Usually, such solutions are also simply called as Rossby wave (in a β-Plane) or Rossby-Haurwitz wave (in a spherical atmosphere). As well known, they have been found as

$$\psi(\theta,\lambda,t) = -\bar{\lambda}_0 a^2 \sin\theta + A_0 P_n(\cos\theta) + \sum_{m=1}^{n} A_m P_n^m(\cos\theta)\cos(m\lambda - mct), \quad (104)$$

in the spherical atmosphere, where $\bar{\lambda}_0$, A_0, A_m ($m=1,2,\ldots,n$) are all arbitrary const., and the phase velocity c is given by

$$c = \frac{\bar{\lambda}_0[n(n+1)-2]-2\omega}{n(n+1)+Kf_0^2a^2/\bar{\Phi}}. \quad (105)$$

However, according to Hoskins[15] (1970), these waves are unstable with respect to the small perturbation either when m is large enough or one of the amplitudes A_m is large enough.

Taking a Rossby wave or Rossby-Haurwitz wave as a zero-order approximation, and expanding the solution into series in power of Rossby Number, Zeng[15] (1979) suggested a method to find slowly propagating non-interactive waves. This has been extended and tested in numerical experiments by Zhang and Zeng[16,17] (1983) and Zhang[18] (1983). Another class of solutions but for an infinite plane has been found by Lius[19] (1983).

The above-mentioned non-interactive waves are useful in the numerical weather prediction for testing the numerical schemes and the schemes of initialization. However, they are not very much interesting in the dynamic studies.

Other class of special solution to the barotropic quasi-geostrophic model has been found by Lorenz[20] (1960), Longuet-Higgins and Gill[21] (1967), Wu[22] (1979), Li[23] (1982) and many others. This class of solutions consists of triad resonantly interactive waves, demonstrating the Fjørtoft's type of energy cascade. Let the triad consists of $\Psi_j(\theta,\lambda,t) \equiv \Phi_j(\theta,\lambda,t) + \Phi_j^*(\theta,\lambda,t)$, where $\Phi_j = A_j(t)P_{n_j}^{m_j}(\cos\theta)\exp\{m_j\lambda - \sigma_j t\}i$, Φ_j^* is the complex conjugate of Φ_j, $\sigma_j = m_j c_j$ and c_j is given by (105) but with $\bar{\lambda}_0 = 0$, $j=1,2,3$, the conditions for constructing a closed resonant triad are as follows

$$\begin{cases} \sum_{j=1}^{3} m_j(-1)^j = 0, \\ \sum_{j=1}^{3} \sigma_j(-1)^j = 0. \end{cases} \quad (106)$$

Otherwise, the set might be a non-interactive one or a unclosed one which interacts with whole spectrum. If one of the components in the triad is the zonal flow, it will undergo a periodic vacillation and result in an index cycle.

2. *The Conditions for the Maintenance of Nonzonal Disturbances*

The above special solutions are some too particular cases. In general it is desirable to seek the conditions for the maintenance of nonzonal disturbances. As predicted by the linear theory, the necessary condition for the maintenance of nonzonal disturbances is either that the zonal flow is unstable with $(\bar{\lambda}_0 - \bar{\lambda})/\beta_y < 0$ in some area, or that the nonzonal disturbance has no critical line or consists of some of discrete modes. These conditions should be corrected by the nonlinearity on the one hand and, on the other hand, in the cases of critical line or continuous spectrum the nonlinear results will much differ from those obtained by the linear theory. According to the linear theory, the conditions for complete absorption of disturbances do not depend on the intensity, but they do in accordance with nonlinear consideration. In fact, we have the conservation of potential vorticity and the conservation of the inertial axis of the atmospheric motion in the nonlinear theory (see, § II-3). Therefore we come to the conclusion:

Nonzonal disturbances will maintain and will not be completely absorbed by a jet-like zonal flow in a barotropic atmosphere without forcing and dissipation, if one of the following conditions is satisfied: (1) there are three or more centres of potential vorticity at the whole sphere, and (2) the atmospheric inertial axis does not coincide with the axis of the earth rotation.

The zonal flow has two potential vorticity centres, each at one pole in the two hemispheres. Note that an individual vortex (cyclone or anticyclone) intensive enough is usually associated with a centre of potential vorticity, hence there is an additional centre at the sphere along with the above-mentioned main two, and this nonzonal disturbance can maintain to some extent. The part of energy maintained with the disturbance also depends on how large its

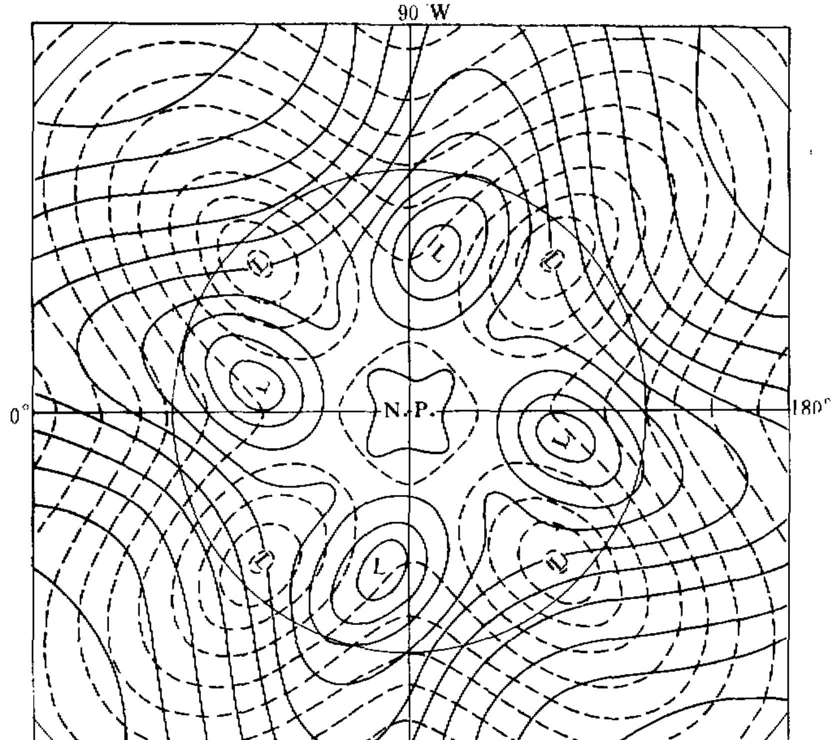


Fig. 11. Numerical experiment with hemispheric primitive equations of barotropic atmosphere. The dashed lines are the isopleths of initial geopotential, consisting of 4 strong vortical centres superimposed on a zonal flow with 4 Rossby waves. The solid lines are the isopleths on the 10th day.

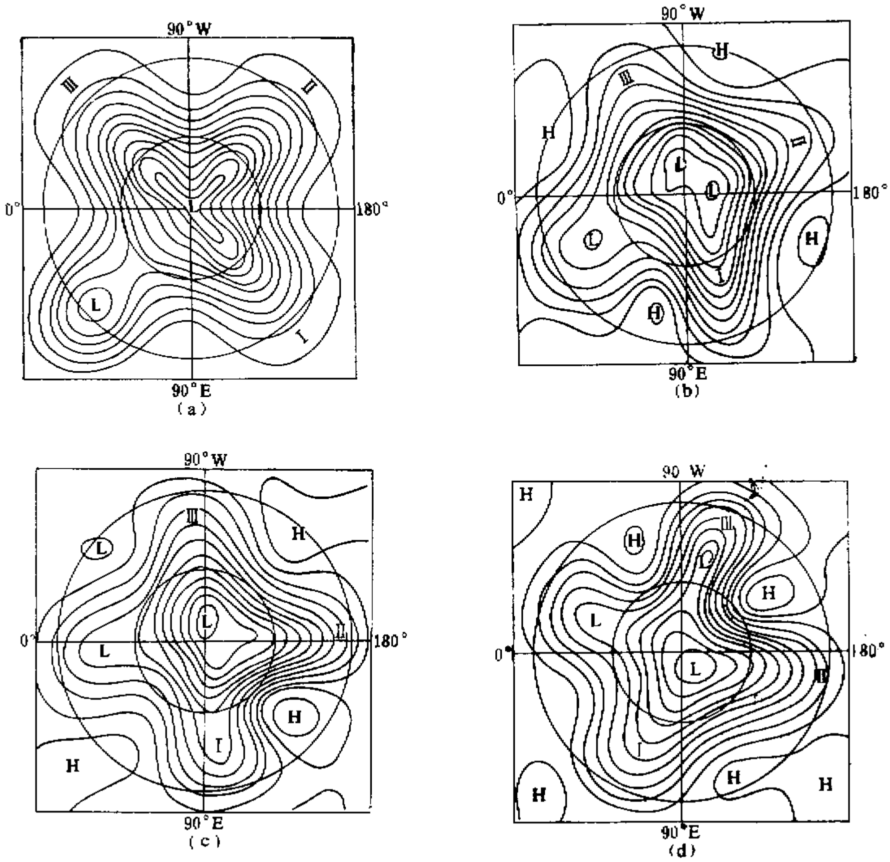


Fig. 12. As in Fig. 11, except that the initial field consists of one additional vortical centre superimposed on almost the same zonal flow and the Rossby waves. (a) initial, (b) on the 4th day, (c) on the 8th day, and (d) on the 13th day.

meridional scale is and how weak the zonal shear of the flow is.

Second, the atmospheric inertial axis does not coincide with the axis of the earth's rotation if there is a strongly asymmetric flow pattern, for example, an intensive ultra-long wave with longitudinal wavenumber 1.

It is also interesting to point out that there are always several potential vorticity centres if the zonal flow is unstable and the disturbances are located in the area with $\bar{\beta}_y < 0$. Basides, a strong flow across the equator also creates more potential vorticity centres in the equatorial region. These are all the favorable conditions for the maintenance of nonzonal disturbances.

Along with the above-mentioned conditions, the orographic and thermal forcing as well as the baroclinity of the atmosphere, of course, are the other factors for the generation and maintenance of disturbances.

Some numerical experiments with hemispheric barotropic model have been carried out by Zeng et al.[14] (1980) and Yuan et al.[15] (1982). An initial zonal flow like the climatological one at 500 hPa in the Northern Hemisphere is taken. When the disturbances with additional potential vorticity centres are superimposed, the non-disturbances are all maintained indeed in the calculations. Fig. 11 shows the maintenance of four intensive vortices superimposed on four planetary waves with a jet-like zonal flow. Fig. 12 shows the influence of an intensive vortex resulting in a very typical downstream effect and alternatively deepenning of the downstream troughs.

3. *The Rotational Adaptation*

On the contrary, the nonzonal disturbances might be completely absorbed by the zonal flow (rotational adaptation) under some conditions. As predicted by the linear theory the necessary and sufficient condition for rotational adaptation is that the nonzonal disturbance consists of continuous spectrum. However, taking account of the nonlinearity, we can indicate the only necessary conditions, i. e., there are two and only two centres of potential vorticity on the whole sphere, and the inertial axis of the atmospheric motion coincides with the axis of earth rotation (Zeng[29], 1979).

The absorption of nonzonal energy depends on the shear of the zonal jet as well as the scale of the disturbance. In fact, according to (58) we have

$$\frac{\partial E'}{E'\partial T}=\frac{2}{E'}\iint_{W}\left(\frac{mn}{\nu^2}\right)\frac{\partial \bar{u}}{\partial Y}\left(\frac{1}{2}\nu^2|\Psi_c|^2\right)dXdY=2\left(\frac{mn}{m^2+n^2+\rho^2}\,\frac{\partial \bar{u}}{\partial Y}\right)^*, \quad (107)$$

where the asterisk denotes the average over the whole wave packet with energy density $\nu^2|\Psi_0|^2/2$ as a weight. When the shear $|\partial\bar{u}/\partial Y|$ of the zonal flow is large but the scale of the disturbance is small (either m or n is large), the absorption is rapid.

Zeng et al.[14] (1980) have carried out some hemispheric numerical experiments with an initial zonal flow similar to the climatological one on 500 hPa and disturbances satisfying the above necessary conditions. The disturbances do gradually be absorbed by the zonal flow. Fig. 13 is an example in which the initial field is the modificaton of that in Fig. 11 by removing the additional centres of potential vorticity. Changing m from 2 to 8 (but fixing the meridional structure of the disturbances), the time interval required for absorption of half initial nonzonal energy varies from 17 to 4 days. Zeng and Held (1981, unpublished)

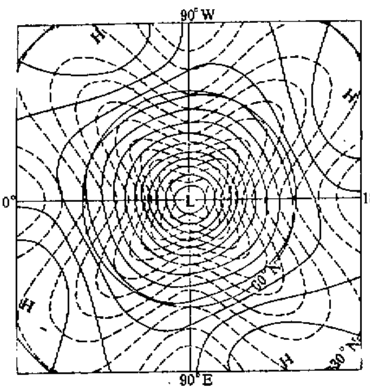


Fig. 13. As in Fig. 11, except that 4 initial vortical centres are removed. The solid lines are the isopleths on the 8th day.

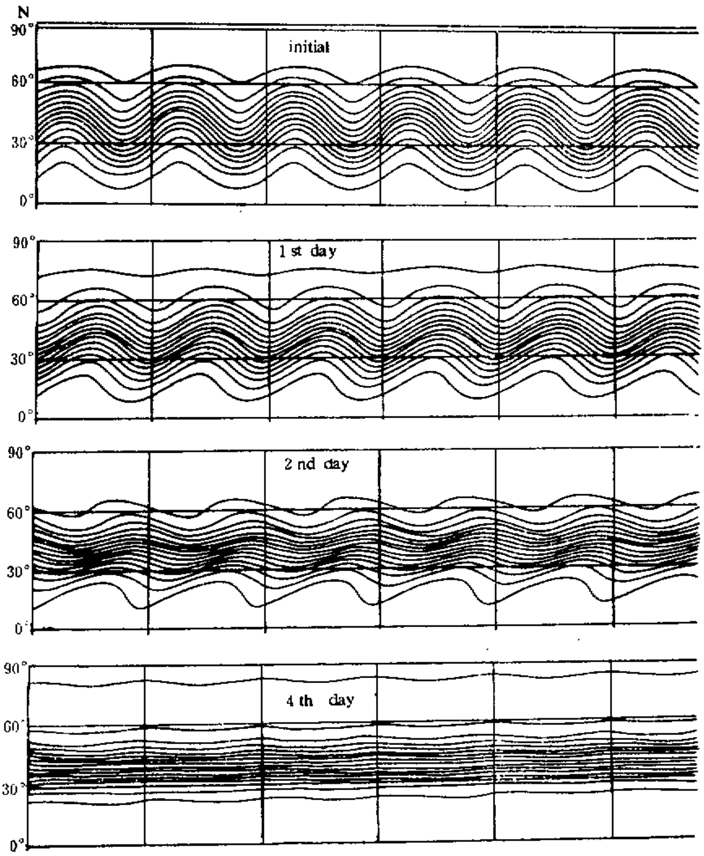


Fig. 14. As in Fig. 13, except that the initial zonal flow is rather strong and given by $v_\lambda = a\alpha\omega\,[0.02\sin\theta + 0.1\sin^2\theta\times\sin^2 2\theta]$, and the disturbance consists of 6 Rossby waves, $\psi' = A\sin(\pi\cos\theta)\times\sin 6\lambda$. The successive patterns are displayed on a Mercator's projection. Only the Northern Hemisphere is shown.

have performed another numerical experiments with an initial zonal flow similar to the mean zonal flow on 200 hPa. Owing to a larger shear of the zonal flow, the absorption is very rapid, and a half of initial disturbance with $m=6$ is absorbed within only 2 days. Fig. 14 shows the evolution of geopotential field for the first 4 days.

The question whether a disturbance can be completely absorbed still remains unsolved in the nonlinear theory. However, the linear theory applied to the weak disturbance as well as the results obtained by numerical experiments shows that the absorption of disturbances superimposed on a jet-like zonal flow and satisfying the above necessary conditions are practically complete.

It should be pointed out that whenever there is a large initial shear of the zonal flow and the disturbances are represented in a form of wave packet, a considerable amount of disturbances is always absorbed even if the disturbances are intensive enough to violate the above-mentioned necessary conditions, but some portion of energy remains with the disturbances.

4. *A Comparison of Rossby Wave Absorption with Energy Cascade in the Two-Dimensional Turbulence*

Based on the analysis of two-dimensional incompressible turbulence equation in a β-plane

$$\frac{\partial}{\partial t}\nabla\psi+J(\psi,\Delta\psi)+\beta\frac{\partial\psi}{\partial x}=\nu\Delta^2\psi+F, \tag{108}$$

we have realized that if the forcing F is given at a fixed wavenumber, the energy flux will be directed from this wavenumber to smaller wavenumber but the enstrophy flux will be to the higher wavenumbers in accordance with Fjϕrtoft's[30] (1953) and Kraichnan's[29] (1967) nonlinear theory, and then the energy goes into the zonal flow due to the separation of cyclones from anticyclone by the β-effect in accordance with Kuo's theory[11] (1951).

Rhines[31] (1975) has indicated that if the turbulence is purely generated by a random isotropic forcing with characteristic scale L^* and velocity V^* such that $L^*\ll l^*$, where $l^*=(v^*/\beta)^{1/2}$ the (nonlinear) two-dimensional turbulence energy cascade is dominant, and after the energy goes into the waves with zonal scale l^*, or if $L^*\gg l^*$, or the forcing with $L^*\ll l^*$ is located periodically in a distance larger than l^*, the Rossby waves are dominant due to the smallness of the nonlinear terms $J(\psi,\Delta\psi)$ as compared with the β—effect, $\beta\partial\psi/\partial x$ and the energy cascade is slowed down (see also Williams[27], 1978). Rhines' theoretic considerations were tested by his and Williams' numerical experiments, in which $L^*\ll l^*$ and a turbulent flow with scales smaller than l^* is generated in the beginning. This flow gradually has larger and larger zonal component, and finally a zonal flow alternative with a characteristic meridional width l^* is statistically established. The energy feedback seems to go to the zonal flow directly, and Rossby waves seem not to occur.

It must be pointed out that in Rhines' and Williams' experiments there is no zonal flow initially, and their theoretical analysis did not pay attention to the influence of zonal flow. The establishment of zonal flow is so slow that it takes about 70 days even for the Jovian atmosphere. Second, the dissipation term $\nu\Delta^2\psi$ is needed for balancing the forcing F as well as for smoothing out the small-scale structures of the flow, otherwise the small scale vorticity centres might always be pronounced.

Now, a question arises: what happens after the zonal flow is established or the influence of the zonal flow becomes larger? To investigate this problem it is better to use the following equations

$$\frac{\partial}{\partial t}\Delta\psi'+\bar{u}\frac{\partial}{\partial x}\Delta\psi'+\bar{\beta}_y\frac{\partial\psi'}{\partial x}+J(\psi',\Delta\psi')-\overline{J(\psi',\Delta\psi')}=\nu\Delta^2\psi'+F', \tag{109}$$

$$\frac{\partial}{\partial t}\Delta\bar{\Psi}+\overline{J(\psi',\Delta\psi')}=\nu\Delta^2\bar{\Psi}+\bar{F}, \tag{110}$$

where the symbols bar and prime denote the zonal part and the departure respectively. Eq. (109) is essentially the same as (40) but with adding the nonlinear terms $J(\psi',\Delta\psi')-\overline{J(\psi',\Delta\psi')}$, which can usually be neglected. Now, we have Rossby wave, and the advective term $\bar{u}\frac{\partial\Delta\psi'}{\partial x}$ caused by the zonal flow is as large as the β-effect term, $\bar{\beta}_y\partial\psi'/\partial x$. Therefore, the behaviour of Rossby waves is controlled by $(\partial/\partial t+\bar{u}\partial/\partial x)\Delta\psi'+\bar{\beta}_y\partial\psi'/\partial x$ rather than the pure β-effect, and there occurs strong interaction between the zonal flow and the Rossby waves. Trough–ridge lines are always overturned by the meridional shear of zonal flow, and the separation of cyclones from anticyclones is also enhanced by the enlarged β–effect for a jet–like zonal flow since β is replaced by $\bar{\beta}_y$. According to our results, the energy cascade does not slow down. In fact, our numerical experiments described in § Ⅳ–3 show that even in the earth's atmosphere the characteristic time for the absorption of Rossby waves is at least one order of magnitude shorter than that for generation of zonal flow by the pure small scale turbulences. Besides, the short–wave absorption is even more rapid in accordance with (107) if the eddies are not too intensive.

As suggested by Zeng et al.[24] (1980), Yuan et al.[25] (1982) have carried out further numerical experiments, changing parameters such as the angular velocity ω of the planet, the thickness H of the atmosphere and so on. Analysing all the numerical results by using the nondimensional parameters: $\varepsilon\equiv(T^*f^*)^{-1}$, $\mu\equiv L_0/L^*$, $M_0^{-1}\equiv\sqrt{gH}/U^*$, and $\delta\equiv L^*/a$, and plotting them in a $\ln\varepsilon^{-1}$-μ diagram, we have Fig. 15, where T^* is the time for $E'(T^*)=E'(0)/2$, f^* is the Coriolis parameter at middle latitudes, L^* and U^* are the characteristic length and velocity of disturbances respectively, L_0 is the deformation radius of the atmosphere under consideration, and a and g are the radius and gravity of the planet respectively. In all these numerical experiments, the meridional structure of initial flow patterns are rather similar, and M_0^{-1} (or $R_0\equiv M_0\mu$) and δ do not change very much, so that ε is nearly a function of μ. Comparing this diagram with Williams' numerical experiments, we may have some ideas about the efficiency of Rossby wave absorption and energy cascade by pure two–dimensional turbulence.

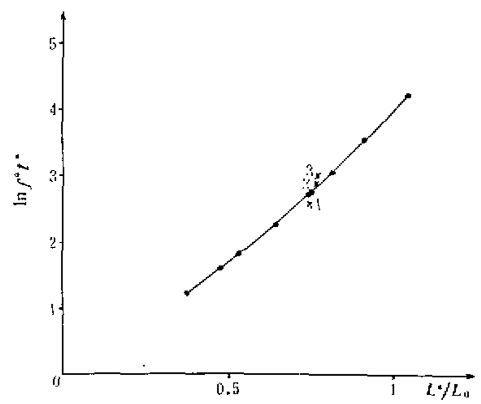


Fig. 15. A diagram showing the dependence of nondimensional characteristic time scale f^*t^* on the nondimensional characteristic length L^*/L of the disturbances, where f^* is the Coriolis parameter, and L_0 is the Rossby deformation radius. The dark dots are referred to $R_0=0.13$ under the conditions of earth's atmosphere, and crosses 1—3 respectively to $R_0=0.115$, 0.14 and 0.23.

In the Jupiter the ω and g are larger than in the earth, and the Jovian atmosphere is much deeper than the earth's atmosphere. For the same δ, i. e., the same zonal wave number, μ^{-1} is much less, hence the characteristic time T^* is much less than that for our atmosphere. For example, T^* becomes less than 12 hours for the disturbances with 8 planetary waves ($m=8$). Note that in the Jovian atmosphere there are many jets, whose shear, $|\partial\bar{u}/\partial Y|$, and meridional wave number are both much larger than those of our atmosphere. These factors make T^* even greatly shorter than 12 hours. Therefore, planetary waves, if any, should be soon absorbed by the zonal circulation. This may explain why the zonal structure is dominant, why there is not any large-amplitude Rossby wave observed in the Jovian atmosphere, and why there is no feedback of turbulence energy to the planetary waves but to the zonal flow. However, intensive disturbances exist in Jovian atmosphere, for example, the Great Red Spot, which undoubtedly is a very intensive vortical centre, hence is always maintained in accordance with our analysis in § Ⅳ-2.

It seems that in such a medium as the rather homogeneous ocean and, probably also, the Jovian atmosphere, where the relatively small-scale forcing is dominant, the zonal flow or a weak but very large scale flow pattern is generated by the energy cascade due to the two-dimensional turbulence; while as a zonal flow is established or in a medium such as in the earth's atmosphere, where large-scale forcing is dominant, the zonal flow is generated or, especially maintained by its interaction with Rossby waves.

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