ANALYSES AND CALCULATION FACTS OF THE BAROCLINIC TERM IN SYNOPTIC-SCALE MOTION OVER TROPICS

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ABSTRACT

Analyses are made of all terms in the vorticity equation for the atmosphere at low latitudes by using the scale analysis theory, with the result that for synotic-scale motion the baroclinic term, i. e. the twisting term and the vorticity vertical-transport term, approximates in order to the relative-vorticity advection, divergence and β term. With intensified atmospheric disturbance ratios of the β term to others become smaller while the others stay in more or less fixed proportions between them. This statement has been confirmed by the results of 22 typhoons calculated covering a large area in low latitudes. Besides, the baroclinic term for the genesis and development of 6 typhoons over 1979—1980 is calculated and the results obtained show that it has significant effect. Finally, the baroclinicity is shown not to be ignored in dealing with synoptics and dynamics of synoptic-scale systems such as typhoons and easterly waves.

I. INTRODUCTION

It is believed that the low-latitude atmosphere has baroclinicity little enough to be completely neglected and is viewed as being barotropic[1]. Yet R. W. Reeves et al. (1979) indicates that results acquired through study of local precipitation over the tropic Atlantic are somewhat incompatible to the traditional hypothesis[2]. Based on the scale analysis of vertical motion and temperature fluctuation of the atmosphere, Cao and Wu (1980) show that only when the tropic circulation has considerably large zonal scale can its motion approach a barotropicized state and quasi-barotropy can hardly exist when the meridian scale is close to the zonal one[3] . This paper is devoted to a further study of the baroclinicity of synoptic-scale motion and demonstrates its importance by a great number of calculation facts.

II. THE NONDIMENSIONAL EQUATION

For a friction-ignorable adiabatic system the vorticity equation can assume the form

$$\frac{\partial}{\partial t}(f+\zeta) = -\mathbf{V}\cdot\nabla(f+\zeta) - (f+\zeta)\nabla\ \mathbf{V}$$
$$-\omega\frac{\partial}{\partial p}(f+\zeta) - \left(\frac{\partial\omega}{\partial x}\frac{\partial v}{\partial p} - \frac{\partial\omega}{\partial y}\frac{\partial u}{\partial p}\right). \qquad (1)$$

For a low-latitude region

$$(f+\zeta)\nabla\cdot\mathbf{V} \doteq \zeta\nabla\cdot\mathbf{V},$$

from which Eq. (1) can be rewritten as

$$\frac{\partial\zeta}{\partial t}=-\left(u\frac{\partial\zeta}{\partial x}+v\frac{\partial\zeta}{\partial y}\right)-v\beta-\zeta\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)$$

$$-\omega\frac{\partial\zeta}{\partial p}-\left(\frac{\partial\omega}{\partial x}\frac{\partial v}{\partial p}-\frac{\partial\omega}{\partial y}\frac{\partial u}{\partial p}\right). \qquad (2)$$

Introducing characteristic scales and supposing that the system has the same distance and speed scales in longitudinal and latitudinal directions, we have

$$\begin{cases} x=L\tilde{x}, & y=L\tilde{y}, & z=H\tilde{z}, & p=P\tilde{p}, \\ u=U\tilde{u}, & v=U\tilde{v}, & \omega=\Omega\tilde{\omega}, & \zeta=\xi\tilde{\zeta}, \\ t=T\tilde{t}, & \beta=B\tilde{\beta}, & f=F\tilde{f}, & D=\mathscr{D}\tilde{D}, \end{cases} \qquad (3)$$

where tilde-over letters denote dimensionless quantities with the order of 10^0, and $\mathscr{D}$, ξ, Ω, L, U, H, P, T, B and F represet the respective order of divergence, vorticity, vertical speed, length, speed, altitude, pressure, time, β and f. For simplicity we assume that in this system

$$\begin{aligned} &\Delta_x\Omega\doteq\Delta_y\Omega\doteq\Delta_p\Omega\sim\Omega, \\ &\Delta_x\xi\doteq\Delta_y\xi\doteq\Delta_p\xi\sim\xi. \end{aligned} \qquad (4)$$

Putting (3) and (4) into (2), we have

$$\frac{\xi}{T}\frac{\partial\tilde{\zeta}}{\partial\tilde{t}}\sim-\frac{U\xi}{L}\left(\tilde{u}\frac{\partial\tilde{\zeta}}{\partial\tilde{x}}+\tilde{v}\frac{\partial\tilde{\zeta}}{\partial\tilde{y}}\right)-UB\cdot\tilde{v}\tilde{\beta}$$

$$-\xi\mathscr{D}\cdot\left(\tilde{\zeta}\frac{\partial\tilde{u}}{\partial\tilde{x}}+\tilde{\zeta}\frac{\partial\tilde{v}}{\partial\tilde{y}}\right)-\frac{\Omega\xi}{P}\cdot\tilde{\omega}\frac{\partial\tilde{\zeta}}{\partial\tilde{p}}$$

$$-\frac{\Omega U}{LP}\cdot\left(\frac{\partial\tilde{\omega}}{\partial\tilde{x}}\frac{\partial\tilde{v}}{\partial\tilde{p}}-\frac{\partial\tilde{\omega}}{\partial\tilde{y}}\frac{\partial\tilde{u}}{\partial\tilde{p}}\right). \qquad (5)$$

A_d and D_i denote orders of the relative-vorticity advection and divergenece term, respectively, V_e of the vorticity vertical-transport term and T_w and B_e of the twisting and β term, separately. And it is apparent from (5) that expressions of these orders are as follows:

$$\begin{cases} A_d\equiv\dfrac{U\xi}{L}, \quad T_w\equiv\dfrac{\Omega U}{LP}, \quad D_i\equiv\xi\mathscr{D}, \\ V_e\equiv\dfrac{\Omega\xi}{P}, \quad B_e\equiv UB. \end{cases} \qquad (6)$$

III. COMPARISON OF THE BAROCLINIC TERM TO OTHERS

1. *Comparisons of the Twisting Term to the Relative Vorticity Advection, Divergence and β Term*

From (6) we get

$$\frac{T_w}{A_d}=\frac{\Omega U}{LP}\Big/\frac{U\xi}{L}=\frac{\Omega}{P\xi}. \qquad (7)$$

The non-dimensional continuous equation is in the form

$$\frac{\Omega}{P}\cdot\frac{\partial\tilde{\omega}}{\partial\tilde{p}}+\mathscr{D}\cdot\left(\frac{\partial\tilde{u}}{\partial\tilde{x}}+\frac{\partial\tilde{v}}{\partial\tilde{y}}\right)=0,$$

where the expression of the characteristic units is

$$\frac{\Omega}{P}\sim\mathscr{D}\ . \tag{8}$$

Putting (8) into (7) gives

$$\frac{T_w}{A_d}=\frac{\mathscr{D}}{\xi},$$

and for low-latitude regions we assume

$$\mathscr{D}\sim\xi,$$

which, when put into (8), gives

$$\frac{T_w}{A_d}\sim10^\circ,\qquad \text{i.e. } T_w\sim A_d, \tag{10}$$

which indicates that the twisting term is close to the relative-vorticity advection term in magnitude.

From (6) we can obtain

$$\frac{T_w}{D_i}=\frac{\Omega U}{LP}\Big/(\xi\mathscr{D}), \tag{11}$$

and from (8) it becomes

$$\frac{T_w}{D_i}=\frac{U}{\xi L}. \tag{12}$$

For a system of synoptic scale L and U can be assumed to be

$$L\sim10^6\,\text{m};\qquad U\sim10^1\ \text{m s}^{-1}. \tag{13}$$

For the atmosphere at low latitudes ξ can be taken as

$$\xi\sim10^{-5}\ \text{s}^{-1}. \tag{14}$$

Substituting (13) and (14) into (12) gives

$$\frac{T_w}{D_i}\sim10^\circ\qquad \text{i.e.}\qquad T_w\sim D_i, \tag{15}$$

which implies that the twisting term is close in order to the relative vorticity divergence term.

Also from (6) the following expression holds

$$\frac{T_w}{B_a}=\frac{\Omega}{LBP}. \tag{16}$$

Inserting (8), (9), (13) and (14) into (16) and assuming[3]

$$B\sim10^{-11}\text{s}^{-1}\text{m}^{-1}, \tag{17}$$

we get

$$\frac{T_w}{B_a}\sim10^\circ,\qquad \text{i.e.}\qquad T_w\sim B_a, \tag{18}$$

which signifies that the twisting term approximates roughly to the β term in order.

2. *Comparisons of the Vorticity Vertical-Transport Term with the Relative Vorticity Advection, Divergence and β Term*

From (6) it can be obtained that

$$\frac{V_e}{A_d}=\frac{\Omega\xi}{P}\Big/\frac{U\xi}{L}=\frac{\Omega L}{PU}, \tag{19}$$

and putting (8), (9), (13) and (14) into (19) gives

$$\frac{V_e}{A_d}\sim 10^0, \qquad \text{i.e.} \qquad V_e\sim A_d; \tag{20}$$

and that

$$\frac{V_e}{D_i}\sim\frac{\Omega\xi}{P}\Big/(\xi\mathscr{D})=\frac{\Omega}{\mathscr{D}P}, \tag{21}$$

and substituting (8) into (21) gives

$$\frac{V_e}{D_i}\sim 10^0, \qquad \text{i.e.} \qquad V_e\sim D_i; \tag{22}$$

and that

$$\frac{V_e}{B_a}\sim\frac{\Omega\xi}{P}\Big/(BU), \tag{23}$$

and inserting (8), (9), (13), (14) and (17) into (23) gives

$$\frac{V_e}{B_a}\sim 10^0, \qquad \text{i.e.} \qquad V_e\sim B_a. \tag{24}$$

These mean that vorticity vertical-transport term approximates in order to the relative-vorticity advection, divergence and β term.

IV. THE BAROCLINIC AND β TERM WITHIN THE REGION COVERED BY A SEVERE TYPHOON

The region covered by a severe typhoon is marked by strong winds. Sometimes the extraordinary typhoon has a maximal speed of about 100 m s^{-1} near its center. Hence we assume the order of wind speed in the typhoon area to be

$$U\sim 10^k \text{ m s}^{-1},$$

where $k=2$ is for the typhoon with the maximal speed of over 50 m s^{-1} near the center and $k=1$ for ordinary typhoons.

Using (8), we have

$$\frac{T_w}{B_a}=\frac{\Omega}{PLB}=\frac{\mathscr{D}}{BL}. \tag{25}$$

In accordance with the definitons of divergence and vorticity the expressions of their orders are

$$\mathscr{D}\sim\frac{U}{L}, \qquad \xi\sim\frac{U}{L}. \tag{26}$$

Putting (26) into (25), and from (13) and (17), we get

$$\frac{T_w}{B_a}\sim 10^{k-1}. \tag{27}$$

Therefore, over an extraordinary-typhoon area with wind speed of over 50 m s^{-1}, $U\sim 10^2$ where $k=2$. Then we have

$$\frac{T_w}{B_a}\sim 10^1, \quad \text{i.e.} \quad T_w>B_a. \tag{28}$$

For ordinary typhoons $k=1$,

$$\frac{T_w}{B_a}\sim 10^0, \quad \text{i.e.} \quad T_w \sim B_a. \tag{29}$$

Employing (8) and (26), we obtain from (23) the following

$$\frac{V_e}{B_a}=\frac{\mathscr{D}\xi}{BU}=\frac{U}{L^2B}. \tag{30}$$

From (13) and (17) we have

$$\frac{V_e}{B_a}\sim 10^{k-1}. \tag{31}$$

For a strong-wind region of an extraordinary typhoon, where $k=2$,

$$\frac{V_e}{B_a}\sim 10^1, \quad \text{i. e.} \quad V_e > B_a; \tag{32}$$

and for a usual typhoon, where $k=1$,

$$\frac{V_e}{B_a}\sim 10^0, \quad \text{i. e.} \quad V_e \sim B_a. \tag{33}$$

It is shown that within the strong-wind region of an extraordinary typhoon the baroclinic term may surpass the β term in order.

It is found in the same way that the ratio of the baroclinic term both to the relative-vorticity advection term and to the divergence term remains more or less constant, without varying with the wind speed of the system. This means that their orders and effects are roughly close to one another. Thus, it is clear from the above analyses that the β term shows decreasing effect with intensified disturbance. It must be noted here that only in synoptic-scale, longwave and extra-longwave motions is such behavior of the β term one of the principal features of low-latitude atmospheric motion[3,4].

V. CALCULATION RESULTS OF ALL TERMS IN THE VORTICITY EQUATION IN THE LOW LATITUDE ATMOSPHERE

The data are used for calculation of 22 observational times from an area covering 90°—155°E and 5°S—35°N and calculation results involve 100°—145°E and 5°—25°N. In the calculated region there occurred 8 severe typhoons whose maximal wind speed>32 m s^{-1} near the center, 8 ordinary typhoons (18—32 m s^{-1}) and 6 typhoon-free occasions when tropical depression activities took place locally. These three categories refer to intense disturbance, disturbance and stable state of the atmosphere, respectively. The data are grid wind data and ω is computed by the continuous equation and then the results are corrected. All terms for the 500 hPa level are evaluated by using data from the surface, 700- and 500-hPa maps. For the sake of analysis and comparison the arithmetic averaging is performed, according to observing times and terms, of absolute values of all the terms obtained through calculation for each grid point to give averages, as shown in Ref. [4].

The table indicates that:

(1) For the low-latitude atmosphere the vertical transport term and twisting term are both close to the divergence term in magnitude, about half the value of the advection term and roughly one third the value of the β term, hence it follows that the baroclinic term cannot be neglected.

Table 1. Averages Obtained through Calculation for Each Observing Time and Total Means for the Three States of the Atmosphere at Low Latitudes

Atmospheric State	$\lvert\overline{A_d}\rvert$	$\lvert\overline{D_t}\rvert$	$\lvert\overline{T_w}\rvert$	$\lvert\overline{V_e}\rvert$	$\lvert\overline{B_a}\rvert$	$\frac{\lvert\overline{D_t}\rvert}{\lvert\overline{A_d}\rvert}$	$\frac{\lvert\overline{T_w}\rvert}{\lvert\overline{A_d}\rvert}$	$\frac{\lvert\overline{V_e}\rvert}{\lvert\overline{A_d}\rvert}$	$\frac{\lvert\overline{T_w}\rvert+\lvert\overline{V_e}\rvert}{\lvert\overline{A_d}\rvert+\lvert\overline{D_t}\rvert}$	$\frac{\lvert\overline{B_a}\rvert}{\lvert\overline{A_d}\rvert}$	$\frac{\lvert\overline{B_a}\rvert}{\lvert\overline{D_t}\rvert}$	$\frac{\lvert\overline{B_a}\rvert}{\lvert\overline{T_w}\rvert}$	$\frac{\lvert\overline{B_a}\rvert}{\lvert\overline{V_e}\rvert}$
Intense Disturbance	0.86	0.59	0.46	0.42	1.06	0.69	0.53	0.49	0.61	1.22	1.78	2.28	2.50
Disturbance	0.47	0.22	0.25	0.20	0.78	0.47	0.53	0.43	0.65	1.66	3.55	3.12	3.90
Stable	0.37	0.15	0.18	0.15	0.69	0.41	0.49	0.41	0.63	1.86	4.60	3.83	4.60
Average*	0.57	0.32	0.30	0.26	0.84	0.56	0.53	0.46	0.63	1.47	2.63	2.80	3.23

*Ratios for the average regime are obtained by calculating averaged values of each term, whose unit is $10^{-10}\ s^{-2}$.

(2) The baroclinic term becomes larger when a typhoon occurs within the calculated area. In general, the more intense the tempest, the more considerable the baroclinicity is. For all these atmospheric states, however, no great difference arises in ratios of the baroclinic term to the advection or the divergence term; the ratio of $\lvert\overline{T_w}\rvert+\lvert\overline{V_e}\rvert$ to $\lvert\overline{A_d}\rvert+\lvert\overline{D_t}\rvert$ stays more stable, over the range of 0.61 to 0.65.

(3) The more stable the atmosphere, the smaller the value of divergence is. For the stable atmosphere and that of normal disturbance, divergence vaule is about half that of the advection and roughly equal to the vertical transport value but slightly less than that of the twisting term.

(4) The β term is the greatest of all, manifesting itself one of the main characteristics of the low-latitude atmospheric motion. The less stable the atmosphere, the greater the β value becomes with increasing. On the other hand, as the instability grows, other terms increase more quickly because of the faster motions, vertical and horizontal. This leads to the fact that the β term grows relatively slowly in terms of its ratio to others. The table summarizes results over a large area. As we expect, if the area is limited only for typhoons, the value of β will not be necessarily larger than those of other terms.

VI. THE SIGNIFICANCE OF THE BAROCLINIC TERM DURING TYPHOON'S GENESIS AND DEVELOPMENT

The 1979—1980 data of 10 observing times from 6 typhoons are used for computation with the longitude-latitude grid distance being 4×4. Others associated with the calculations are the same as part Ⅴ. The results obtained are for the center. Strictly speaking, the center has small (almost zero) wind speed, and therefore only relative-vorticity divergence, twisting and vorticity vertical-transport terms are calculated. For simplicity, averages of absolute values of all terms are given and the correlation coefficients are found of values of all terms with changes in vorticity in the typhoon's center for the next 24 hr. Such correlations (denoted as R) can be obtained by

$$R=\frac{\sum_{i=1}^{m}(Z_i-\bar{Z})(Q_i-\bar{Q})^2}{\sqrt{\sum_{i=1}^{m}(Z_i-\bar{Z})^2\cdot\sum_{i=1}^{m}(Q_i-\bar{Q})^2}},$$

where Q_i is the change in vorticity; Z_i represents D_t, T_w and V_e of all samples; $m=10$.

Table 2. The Mean Absolute Values of D_t, T_w and V_e and the Correlation Coefficient R

	D_t	T_w	V_e
Mean Absolute Values*	0.44 (10^{-2} s^{-2})	0.29 (10^{-2} s^{-2})	0.36 (10^{-2} s^{-2})
R	0.87	0.69	0.82

Table 2 indicates main calculation results, which have the following implications:

(1) In the case of magnitude the twisting term is the smallest and the divergence the largest of these three, which are, however, of the same order.

(2) These terms have high correlation with the change in relative vorticity of the typhoon for a short period of time in advance but that for the twisting is slightly low. Nevertheless the correlations among the three terms are essentially close to each other.

(3) The effect of vorticity vertical-transport and twisting terms are significant and therefore cannot be neglected.

It must be noted that the above results may lack high representativeness because of the data only of 10 observing times from 6 typhoons. It is the limited number of data that keeps us from more calculations.

VII. SUMMARY

(1) For the synoptic-scale system at low latitudes the baroclinic term is close in order to the relative-vorticity advection, divergence and β term, their effects being almost identical except that the baroclinicity has smaller value.

(2) With intensified disturbance of such systems the β term is decreased in its ratio to others and in an extraordinary typhoon the value of β is probably smaller than that of the baroclinic term and others.

(3) During typhoon's genesis and development, the baroclinic term is close in value to the others. They are in more or less positive correlation with variations of intensity of a typhoon.

(4) For the systems such as typhoons and easterly waves, the baroclinic term shows effect too considerable to be ignored.

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