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AN ANALYSIS OF THE CAUSES OF MEANDERING TRACKS OF TYPHOONS

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ABSTRACT

Criteria of the free meandering tracks of typhoons are derived from the general solutions of typhoon motion equations. It is suggested that the meandering motion of a typhoon is caused by the combination of the internal force, the initial speed of the typhoon and the average pressure gradient force of the typhoon volume affected by the environmental pressure field. It is also revealed that under specified circumstances, the meandering may be caused only by the typhoon's internal force. Finally two examples of fitting calculation are given for illustration.

I. INTRODUCTION

The meandering typhoon tracks can be classified, in general, into two categories. One is the forced meandering. When a typhoon moves in the variable environmental flows, its track sways due to the change of the steering direction of the basic current at a different location and a meandering track is thus formed. The meandering process of this category will not be discussed here as it is forced apparently by the change of the flow pattern, and lacks regularity in the periodicity and equality in amplitude. The other category is free meandering. When a typhoon moves in less variable environmental flows, a meandering track may occur under certain circumstances and is called free meandering as, in this category, the forcing effect of the environmental flows on the typhoon motion is not evident. The periodic regularity of this kind of meandering is very evident and its amplitude is large. As the free meandering typhoon track is one of erratic typhoon tracks and often causes troubles in operational forecast, its causes are thus analysed and discussed here.

Many authors have discussed the causes of the free meandering typhoon track, and most of them have attributed it to the internal force of typhoons[1-4]. Some believe that the free meandering typhoon track should occur in a uniform pressure field or opposite steering field. In this paper we have used the general solutions of the typhoon motion equations to derive the criteria of the free meandering in the stable pressure field and have found that the conclusions drawn in the past[1-4] only cover two special cases. Generally speaking, the occurrence of the free meandering typhoon track is mainly related to the mean pressure gradient force of the whole volume of a typhoon influenced by the environmental pressure field and to the initial speed of the typhoon, besides to the internal force. The internal force of the typhoon, however, may increase or decrease the amplitude of the free meandering of a typhoon according to its respective condition. Finally, satisfactory results have been obtained from the fitting calculations for the meandering tracks of Typhoons 5904 (Joan) and 8211 (Cecil).

II. ANALYSIS OF THE GENERAL SOLUTIONS OF TYPHOON MOTION EQUATIONS

The differential equation of the motion of a typhoon center can be written as

$$\begin{cases} \dfrac{d^2x}{dt^2}-f\dfrac{dy}{dt}=P_x+I_x\,, \\[2ex] \dfrac{d^2y}{dt^2}+f\dfrac{dx}{dt}=P_y+I_y\,, \end{cases} \tag{1}$$

where $P_x=-g\dfrac{\partial \bar{z}}{\partial x}$, $P_y=-g\dfrac{\partial \bar{z}}{\partial y}$ are two components of the mean pressure gradient force of the typhoon volume influenced by the environmental pressure field. The components of the internal force of the typhoon are expressed as

$$I_x=-2\Omega\cdot\cos\varphi\cdot\left(\bar{\omega}-\frac{r_0}{3R}\,\bar{v}_r\right),$$

$$I_y=\frac{2r_0}{3R}\cdot\Omega\cdot\cos\varphi\cdot\bar{v}_\lambda,$$

where $\bar{\omega}$, $\bar{v}_r$ and $\bar{v}_\lambda$ are the mean vertical velocity, radial velocity and tangential velocity in the typhoon area, r_0 is the radius of the typhoon circulation, R is the radius of the earth. For $\bar{\omega}\ll\bar{v}_\lambda$, and $\bar{v}_r\ll\bar{v}_\lambda$ then $I_x=0$. Assume that P_x, P_y, I_y and f are taken as constants, and set the initial condition $t=0$, $x=x_0$, $y=y_0$; $u=u_0$, and $v=v_0$, the general solutions of Eq. (1) are

$$\begin{cases} u=\dfrac{dx}{dt}=A\cdot\sin(f\cdot t+\theta)+\dfrac{P_y+I_y}{f}\,, \\[2ex] v=\dfrac{dy}{dt}=A\cdot\cos(f\cdot t+\theta)-\dfrac{P_x}{f}\,, \end{cases} \tag{2}$$

$$\begin{cases} x=-\dfrac{A}{f}\cdot\cos(f\cdot t+\theta)+\dfrac{P_y+I_y}{f}\cdot t+c\,, \\[2ex] y=\dfrac{A}{f}\cdot\sin(f\cdot t+\theta)-\dfrac{P_x}{f}\cdot t+d\,. \end{cases} \tag{3}$$

Eq. (2) is for the moving speed of the typhoon center and (3) is for the displacement of the typhoon center. The first on the right is the swaying term of the typhoon track and the second is the advective term. The integrating constants are taken as

$$A=\sqrt{\left(u_0-\frac{P_y+I_y}{f}\right)^2+\left(v_0+\frac{P_x}{f}\right)^2}\,,$$

$$\theta=\mathrm{tg}^{-1}\left[\left(u_0-\frac{P_y+I_y}{f}\right)\Big/\left(v_0+\frac{P_x}{f}\right)\right],$$

$$c=x_0+\frac{1}{f}\left(v_0+\frac{P_x}{f}\right),$$

$$d=y_0-\frac{1}{f}\left(u_0-\frac{P_y+I_y}{f}\right).$$

If the geostrophic wind to which P_x and P_y correspond (i.e. the geostrophic steering current of the basic current field) is $u_g=\frac{P_y}{f}$ and $v_g=-\frac{P_x}{f}$, the integrating constant A can also be written as

$$A=\sqrt{\left(u_0-u_g-\frac{I_y}{f}\right)^2+(v_0-v_g)^2}\ .$$

For details, three behaviors are to be discussed as follows.

1. *Uniform Straight Motion*

When the initial speed $u_0=u_g+I_y/f$ and $u_0=v_g$, thus $A=0$; i.e. the swaying term in (3) is zero and therefore the equation can simplified as a uniform-speed straight motion equation

$$\begin{cases} x=u_0\cdot t+c, \\ y=v_0\cdot t+d. \end{cases} \tag{4}$$

This indicates that when the initial speed of a typhoon satisfies this condition, the typhoon will keep its moving behavior, say, remaining the uniform speed straight motion at the initial speed.

2. *Free Meandering Track*

If $0<A\leqslant \max\left(\left|\frac{P_y+I_y}{f}\right|,\ \left|\frac{P_x}{f}\right|\right)$, the typhoon will have a free meandering track while it moves in the direction of the basic current, as is shown in Fig. 1 [where max (a, b) means that a larger value between a and b will be chosen]. To illustrate this, the meandering track to the northwest quadrant, for example, is described. Since the trend of a typhoon's meandering track is consistent with the geostrophic steering current of the environmental pressure field, the typhoon when moving towards the northwest quadrant (i.e. moving in the direction between N—W) should have the following expression:

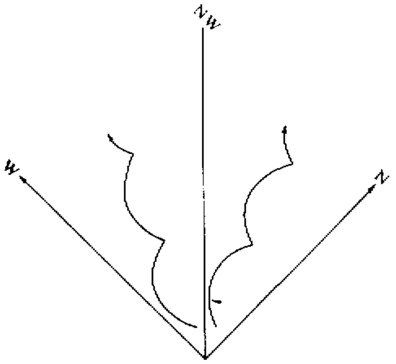

Fig. 1. The meandering track towards the northwest quadrant.

$$u_g \leqslant 0, \quad v_g \geqslant 0, \quad \text{i.e.} \frac{P_x}{f} \leqslant 0, \quad \frac{P_y+I_y}{f} \leqslant 0.$$

(1) If the basic moving direction of the meandering typhoon track is within W—NW, we have $|u_g| \geqslant v_g$; and $u<0$, while v here can be positive or negative. It is known from (2)

$$u = A \cdot \sin(f \cdot t + \theta) + \frac{1}{f}(P_y + I_y)$$

that only when

$$0 < A \leqslant \left| \frac{1}{f}(P_y + I_y) \right|,$$

i.e.

$$0 < \sqrt{\left(u_0 - u_g - \frac{I_y}{f}\right)^2 + (v_0 - v_g)^2} \leqslant \left| u_g + \frac{I_y}{f} \right|, \tag{5}$$

can $u<0$ be ensured. Eq. (5) is thus the criterion of the meandering motion of the typhoon at the W—NW direction.

(2) If the basic direction of the free meandering typhoon track is NW—N, the criterion can be obtained from the same reason

$$0 < A \leqslant \left| \frac{P_x}{f} \right|,$$

i.e.

$$0 < \sqrt{\left(u - u_g - \frac{I_y}{f}\right)^2 + (v_0 - v_g)^2} \leqslant |v_g|. \tag{6}$$

Eqs. (5) and (6) represent that the free meandering typhoon track results from the combination of the internal force of the typhoon, the initial speed and the mean pressure gradient force of the typhoon volume influenced by the environmental pressure field. The amplitude of the meandering is

$$S = \frac{A}{f} = \frac{1}{f}\sqrt{\left(u_0 - u_g - \frac{I_y}{f}\right)^2 + (v_0 - v_g)^2}. \tag{7}$$

Here S is inversely proportional to $\sin\varphi$, i.e., the lower the latitude, the larger the amplitude; and it is proportional to the value of $|u_0 - u_g|$ and $|v_0 - v_g|$, i.e., the larger the difference between the typhoon initial speed and the speed of the geostrophic steering current of the basic flows exerting on the typhoon, the larger the amplitude. The value of the difference can not be extremely large as it is restricted by (5) or (6).

The effects of the typhoon internal force on the free meandering of the typhoon are different in different situations. Generally speaking, the internal force is small while the value of $|u_0 - u_g|$ is relatively large, hence $|u_0 - u_g| > \frac{I_y}{f}$. Therefore when $u_0 - u_g > 0$, the larger the I_y, the smaller the $\left[(u_0 - u_g) - \frac{I_y}{f}\right]^2$ and the amplitude $\frac{A}{f}$. This means that when a typhoon enters a relatively strong and stable pressure field with a small initial speed, the internal force of the typhoon will hinder the typhoon from free meandering. On the contrary, when $u_0 - u_g \leqslant 0$, then the larger the I_y is, the larger the amplitude of the meandering swaying will be. That is to say, when a typhoon enters a relatively weak pressure field with a larger initial speed, the internal force will be helpful to the free meandering of the typhoon.

To further demonstrate the contributions of the three parameters, three special cases are discussed as follows.

(A) When $I_x=I_y=0$, i.e., the typhoon's internal force is negligible, A, in Eq. (3), equals $\sqrt{(u_0-u_g)^2+(v_0-v_g)^2}$. When $0<A\leqslant\max(|u_g|, |v_g|)$, the typhoon may also have a free meandering track. Such an example is Typhoon 5904. In the calculation, $u_0\approx -7.2\text{m s}^{-1}$, $v_0\approx 4.6\text{m s}^{-1}$; $u_g\approx -7.2\text{m s}^{-1}$, $v_g\approx 2.1\text{m s}^{-1}$, and when the effects of the typhoon internal force are completely neglected, then $A=2.5\text{m s}^{-1}$ to satisfy the criterion of Eq. (5). The amplitude $S=A/f\approx 0.37$ latitude. This indicates that (u_0-u_g) and (v_0-v_g) are indeed one of the causes of the free meandering of the typhoon. Of course, the typhoon's internal force always exists. The above assumption is made only to demonstrate the effects of the difference between the typhoon initial speed and the geostrophic steering current's speed of the basic flow field on the typhoon.

(B) If $P_x=P_y=0$, i.e., when the typhoon is located in the uniform pressure field, $0<A\leqslant\max\left(\left|\frac{P_y+I_y}{f}\right|, \left|\frac{P_x}{f}\right|\right)$ can be satisfied only in the special cases when $u_0=0$, $v_0=0$. Therefore the quantities in (3) will be $A=\frac{I_y}{f}$, $\theta=-\frac{\pi}{2}$, $c=x_0$, $d=y_0+\frac{I_y}{f^2}$. We can thus obtain the same conclusion as Refs. [1,2]

$$\begin{cases} x=-\dfrac{I_y}{f^2}\cdot\sin ft+\dfrac{I_y}{f}\cdot t+x_0\,, \\ y=-\dfrac{I_y}{f^2}\cdot\cos ft+\dfrac{I_y}{f^2}+y_0\,. \end{cases} \tag{8}$$

Here the swaying is caused only by the typhoon internal force. But Eq. (8) is the formula for the free meandering track when a typhoon moves due east. *It is shown from observations,* however, that most of the typhoons have their free meandering sways in the northwest quadrant and very few have their sways in the northeast quadrant. The eastward meandering track assumed above has not been found. Therefore, generally speaking, the free typhoon meandering track should occur in the stable non-uniform pressure field instead of the uniform field.

(C) If $u_0=u_g$, and $v_0=v_g$, i.e., when the initial speed of the typhoon is same as the speed of the geostrophic steering current of the basic flow field, we have $A=I_y/f$. In such a case. the criterion $0<A\leqslant\max\left(\left|\frac{P_y+I_y}{f}\right|, \left|\frac{P_x}{f}\right|\right)$ can generally be satisfied. At the same time, by simplifying (3), we can draw the same conclusions as Refs. [3] and [4].

$$\begin{cases} x=-\dfrac{I_y}{f^2}\cdot\sin ft+\left(u_g+\dfrac{I_y}{f}\right)\cdot t+x_0\,, \\ y=-\dfrac{I_y}{f^2}\cdot\cos ft+v_g t+\dfrac{I_y}{f^2}+y_0\,. \end{cases} \tag{9}$$

The swaying here is also caused by the typhoon's internal force only. However it is shown from the examples that the amplitude of the swaying due to the typhoon's internal force is small. In the fitting calculation for Typhoon 5904, assuming $u_0=u_g$, $v_0=v_g$ and the typhoon's internal force $I_y=9.6\times10^{-5}$ m s^{-2}, the amplitude of the meandering caused only by the internal force $S=I_y/f^2=0.23°$ latitude. The actual amplitude of the meandering track is 0.5° latitude. Therefore, the free meandering track of the typhoon in the stable pressure field, in general, results from the combination of the typhoon's internal force, initial speed and the steering current of the environmental flow field. Only the last two special situations mentioned above are caused by the internal force alone.

3. *Looping Track*

When $A<\max\left(\left|\frac{P_y+I_y}{f}\right|, \left|\frac{P_x}{f}\right|\right)$, a typhoon will have a clockwise looping track while moving along the direction of the steering current. The looping track to the northwest quadrant is an example. From Eq. (2), it can be seen that when the above condition is met, i.e., when $u>0$ or $v<0$ in a certain time interval, the typhoon will loop in the opposite direction of the steering current for a short time. This will not be discussed here in detail.

III. EXAMPLE FITTING CALCULATION

We have selected the WNW meandering track of Typhoon 5904 and the northerly meandering track of Typhoon 8211 for fitting calculation. Data are determined in the calculation according to the methods in Ref. [4]. (The radius of the typhoon r_0 is estimated from the intensity of the typhoon and the size of the circulation on surface, 700 hPa and 500 hPa charts, The mean tangential velocity is obtained from the average of the maximum wind speeds near the typhoon center at the surface and 700 hPa from aircraft reports and the tangential wind velocities at r_0 read on the surface, 700 hPa and 500 hPa synoptic charts. The average pressure gradient force P_x, P_y exerting on the typhoon volume is derived from the mean value estimated at the 700 hPa and 500 hPa.)

The track of Typhoon 5904 is shown in Fig. 2. This is an example of meandreing of rapid moving. From 0800 August 28 to 0200 August 30 (Beijing Time and hereafter), the typhoon had two sways of which the amplitude was about 0.5° latitude. The sways occurred in the pressure field with strong gradient (Fig. 3). To the due north of the typhoon was the east-west oriented subtropical high with its east tip retreating somewhat to the south. To the west and south of the typhoon was the depression area. The mean pressure gradient force of the typhoon area was large and therefore the typhoon moved fast. It is shown from calculation that

$$\begin{cases} P_y\approx-44.70\times10^{-5}\ m\ s^{-2}, \\ P_x\approx-13.24\times10^{-5}\ \mathrm{m\ s^{-2}}. \end{cases}$$

During the period from 0800 August 28 to 2000 August 29, $|\Delta P_x|\leqslant2.76\times10^{-5}$m s^{-2}, $|\Delta P_y|\leqslant4.96\times10^{-5}$m s^{-2}. The variation is very small and can be taken as a constant. Based on the average moving speed in the 6 hours before the typhoon's swaying, the initial speed can be defined as $u_0=7.2$ m s^{-1}, $v_0\approx4.6$ m s^{-1}. The Coriolis parameter is taken at 25°N, i.e., $f=6.167\times10^{-5}$ s^{-1}. The radius of the typhoon $r_0=5°$ latitude and the mean tangential velocity $\bar{v}_\lambda\approx25$ m s^{-1} within the range of r_0. Thus the typhoon internal force can be estimated as

$$I_y=\frac{2r_0}{3R}\cdot\Omega\cdot\cos\varphi_0\cdot\bar{v}_\lambda\approx9.6\cdot10^{-5}\ \mathrm{m\ s^{-2}}\,,$$

$$\left|\frac{P_y+I_y}{f}\right|\approx5.6\ \mathrm{m\ s^{-1}},$$

$$A=\sqrt{\left(u_0-\frac{P_y+I_y}{f}\right)^2+\left(v_0+\frac{P_x}{f}\right)}\approx2.96\ \mathrm{m\ s^{-1}}\,.$$

The criterion of the meandering motion in WNW direction is satisfied. The estimated amplitude of the meandering $S=\frac{A}{f}=48000\ \mathrm{m}=0.5°$ latitude, the initial phase $\theta=\sin^{-1}\left[\left(u_0-\frac{P_y+I_y}{f}\right)\Big/A\right]\approx-32.72°\approx-0.571$ rad. The swaying period $T=\frac{2\pi}{f}\approx28$ hr . Substituting the above parameters to (3), we have the fitting equation for the meandering track of Typhoon 5904

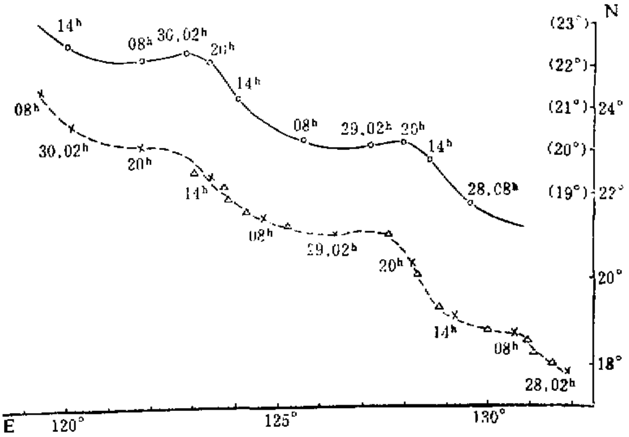

Fig. 2. The track for Typhoon 5904. The solid line is the fitted track. The dashed line is the actual track.
× represents the real-time positions by the Central Meteorological Office (Beijing). Δ gives the positions from aircraft reconnaissance. The figures in the parentheses are the latitudes corresponding to the fitted track, and those before hours denote date.

$$\begin{cases} x=-0.5\cdot\cos\left(\frac{2\pi}{28}\cdot t-0.571\right)-0.5\cdot\frac{t}{28}+c, \\ y=0.5\cdot\sin\left(\frac{2\pi}{28}\cdot t-0.571\right)+2.0\cdot\frac{t}{28}+d, \end{cases} \tag{10}$$

where the time t is in hr. and x, y in deg. lat.

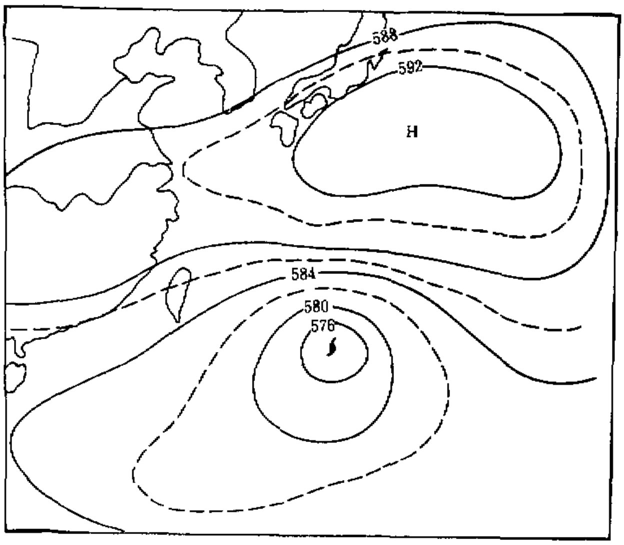

Fig. 3. The 500 hPa synoptic chart at 0800 August 28, 1959.

The solid line in Fig. 2 is the fitted track plotted from the above equation. The trend of the meandering swaying agrees well with the observed.

Typhoon 8211 is an example of slow-moving, northward meandering track, as is shown in Fig. 4. The solid line in the figure is plotted from the hourly observed positions of the typhoon eye by the Dongtou Radar Station. During the period from 2000 August 9 to 0800 August 12, the typhoon had two sways while moving to the north slowly. The amplitude of the sways was 0.3—0.5° latitude and the period was about one day. The meandering sways for this typhoon occurred in the pressure field of weak gradient. From 2000 August 9, as the typhoon entered an area enclosed by four highs (Fig. 5), it decelerated suddenly. As the Korea high to the north of the typhoon was slightly stronger than the tropical high to the south and the Pacific high to the east was stronger than the continental high, the typhoon was still present in the non-uniform pressure field. It is shown from the calculation that $P_x = -14.7 \times 10^{-5}$ m s^{-2} (towards west), and $P_y = -7.3 \times 10^{-5}$ m s^{-2} (towards south). From 2000 August 9 to 0800 August 12, because the westerly trough bottom in the Lake Baikal continued to move to the north and resulted in the continual northward advance of the Korea high and the ridge of the subtropical high, which kept pace with the northward motion of the typhoon, the typhoon center hence was always about 10° latitude apart from the ridge of the subtropical hgih (figure omitted). Meanwhile, the tropical high to the south of the typhoon moved slightly to the north. Therefore, although the circulation pattern in the East Asia during this time changed a lot, the flow pattern near the typhoon circulation had little change and the mean pressure gradient force exerting on the typhoon area P_x, P_y can be treated as a constant. The size of Typhoon 8211 is small, $r_0 = 3°$ latitude. Within r_0, the mean tangential velocity $\bar{v}_\lambda = 30$ m s^{-1}. It is estimated that $I_y \approx 6.6 \times 10^{-5}$ m s^{-1}, just the same as P_y in magnitude but in opposite direction, i.e., $P_y + I_y \approx 0$. Therefore the typhoon was actually affected only by the westerly pressure gradient force and tended to move northward. From the mean moving speed observed by the radar 3 hours before swaying, the initial speed is defined as $u_0 = -2$ m s^{-1}, $v_0 = 2$ m s^{-1}, and f is taken at 30° N to be 7.29×10^{-5} s^{-1}. It is estimated that $A = 2$ m s^{-1}, $\left|\frac{P_x}{f}\right| = 2.2$ m s^{-1}, satisfying criterion (6). From the calculation, the amplitude $S = \frac{A}{f} = 0.3°$ latitude, initial phase $\theta \approx -\frac{\pi}{2}$, and period $T \approx 24$ hr. Substituting the above parameters into (3), we have got

$$\begin{cases} x = -0.3 \sin \frac{2\pi}{24} t + x_0, \\ y = -0.3 \cos \frac{2\pi}{24} t + 1.6 \cdot \frac{t}{24} + y_0 + 0.3. \end{cases} \tag{11}$$

Here t is in hr. and x, y in deg. lat.

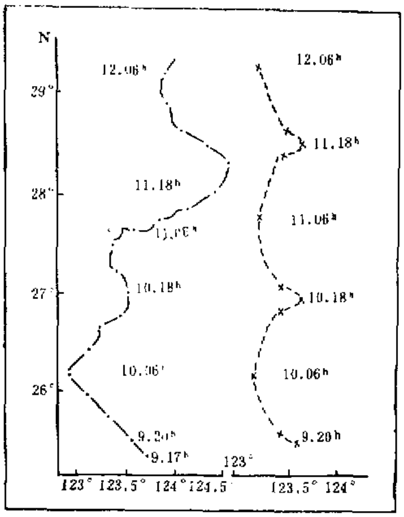

Fig. 4. The track of Typhoon 8211. The solid line is the actual track and the dashed line is the fitted track. The figures before decimal point along curves represent date.

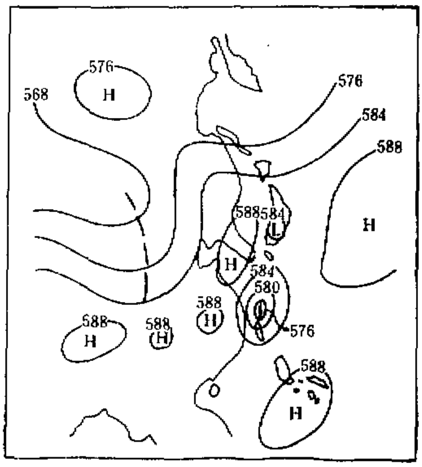

Fig. 5. 500 hPa chart at 2000 August 9, 1982.

The dashed line in Fig. 4 is the fitted track dotted from the above equation and agrees with the actual track. Generally speaking, Typhoons 5904 and 8211 are characterized by the free meandering in the northwest quadrant. The results of fitting indicate that the above explanation of the causes of a typhoon's free meandering is reasonable. It should be noted, however, that the above fitting calculation only roughly delineates the basic features of a typhoon's free meandering and there must be some discrepancies between the fitted and observed tracks.

This is mainly because 1) the pressure gradient force of the environmental flows on the typhoon is calculated by only the readings from the synoptic charts, leading to some errors; 2) the average pressure gradient force exerting on the volume of the typhoon should be integrated from the surface to the top of the cylinder-like typhoon and then be averaged. In the calculation here, however, the two-level average of 700 and 500 hPa in the middle troposphere is used instead of the volume average due to the limitation of data available; and 3) the internal force is only approximated as the formulae to estimate the internal force of the typhoon are imperfect and the sounding data are limited.

IV. CONCLUSIONS

(1) When $u_0=u_g+\frac{I_y}{f}$, and $v_0=v_g$, a typhoon in the stable pressure field will have a uniform straight track at the initial speed. When $0<A\leqslant\max\left(\left|\frac{P_y+I_y}{f}\right|, \left|\frac{P_x}{f}\right|\right)$ is satisfied, a typhoon will produce a free meandering track. When $A>\max\left(\left|\frac{P_y+I_y}{f}\right|, \left|\frac{P_x}{f}\right|\right)$ is met, a typhoon will move along a looping track.

(2) The free typhoon meandering results from the combination of typhoon's internal force, initial speed and the steering current for the basic current exerting on the typhoon. Only when $P_x=0$, $P_y=0$, $u_0=0$, $v_0=0$, or $u_0=\frac{P_y}{f}, v_0=-\frac{P_x}{f}$, may the free meandering result just from the typhoon internal force.

(3) The amplitude of a typhoon's meandering sways is inversely proportional to the sine of the latitude; it is directly proportional to the difference between the initial moving speed of the typhoon and the steering current velocity of the basic flow field, i.e., it is proportional to $|u_0-u_g|$, $|v_0-v_g|$; the relation to the internal force is defined from the value of u_0-u_g. For the meandering track in the northwest quadrant ($u_0<0$, $u_g<0$), when $u_0-u_g\leqslant0$, in other words, when a typhoon moves into a weak pressure field at a high initial speed, the amplitude will be proportional to the typhoon internal force; and when a typhoon enters the strong pressure field at a small initial speed, i.e., $u_0-u_g>0$, the amplitude will be inversely proportional to the typhoon internal force.

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