

## A GLOBAL ANNUALLY-AVERAGED CLIMATE MODEL WITH CLOUD, WATER VAPOR AND CO<sub>2</sub> FEEDBACKS

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### ABSTRACT

In consideration of the radiation transfer, latent and sensible heat exchange between oceans and atmosphere, a three-dimensional autonomous nonlinear ordinary differential equation is established by statistical parameterization method. The variables of the model are the mean ocean surface temperature  $T_s$ , mean atmospheric temperature  $T_a$  and atmospheric relative humidity  $f$ , and the feedbacks of clouds, water vapor and CO<sub>2</sub> are involved. The steady state corresponding to the present-day climate can be obtained from this model. The analysis of parameter sensibility in the steady state indicates that clouds have considerable negative feedback effects and water vapor may affect the sign of CO<sub>2</sub> feedback. The stability analysis of the steady state to small disturbance indicates that with increase of the positive feedback effect of clouds, the steady state goes through such a structural variance series as a stable node→a stable focal point→an unstable focal point→an unstable node, and when the steady state becomes unstable it undergoes a subcritical Hopf bifurcation. When the steady state is at a focal point, the periodic oscillation solutions of damping or amplifying can be obtained with the period being about two years.

### 1. INTRODUCTION

The atmosphere is a very complex system that involves many feedback processes. It is one of the main contents for the present-day climate theory to study various feedback mechanisms so as to determine their contributions, positive or negative, strong or weak, and their influences on the climatic change. In 1969, Budyko<sup>[1]</sup> and Sellers<sup>[2]</sup> established a radiation energy balance climate model involving ice-cap-albedo feedback independently. While considering the climate sensibility to solar constant variation, they found the multi-equilibrium states of climate. It was further studied several years after<sup>[3-6]</sup>. Many investigators came to a conclusion that the climate is very sensible to the change of solar constant, that is, a small change of  $Q_0$  (about 2%) would result in a crisis of climate. However, Chao et al.<sup>[6]</sup> held a contrary opinion that only when  $Q_0$  decreases about 15%, could the climate have a crisis. In the study of ice-cap-albedo feedback, many feedback processes, such as water vapor and clouds, were introduced into various climate models<sup>[7-10]</sup>. Through studying the sensibility of climate model to some external conditions, such as  $Q_0$ , CO<sub>2</sub> content and aerosol variation, the effect of feedback on climate can be determined. These issues are well summarized by Cess<sup>[11]</sup> and Schneider<sup>[12,13]</sup>. Now most authors are considering the importance of negative feedback effect of clouds<sup>[11,14]</sup>, but some other calculations indicate that there is a balance between positive and negative feedback of clouds,

or the positive feedback is slightly superior to the negative<sup>[10]</sup>. Thus, Cess et al.<sup>[12]</sup> think that it is too early to get a definite answer.

In recent years, the action of feedback has been further studied in the climatic self-oscillation of various time scales. Considering the effect of ice-cap-albedo feedback on the climate, Saltzman (1978)<sup>[13]</sup> has established a two-dimensional autonomous ordinary differential equation, of which the mean ocean temperature and the ice border latitude are taken as variables. In this simple system, Saltzman has found out a climatic oscillation of several thousand years. Then he and Moritz (1980)<sup>[17,18]</sup> have established another climate model (S-M model) involving more feedback processes, the CO<sub>2</sub> feedback is also included. It is found that as CO<sub>2</sub> feedback increases, the solution to the model would bifurcate from a stable state to a limit-cycle, indicating the possibility that the climate takes a periodic self-oscillation in the form of limit-cycle (under the ideal condition without disturbances). Making use of a zonally-averaged two-dimensional ocean-atmosphere coupled model, Ji (1982)<sup>[19]</sup> has studied the phenomena of the multi-year oscillation. In his model there are three feedback processes and when any of the three is removed, the system would not generate periodic oscillation.

In this paper, we try to study the effect of clouds, water vapor and CO<sub>2</sub> feedback on climate by use of a simple ocean-atmosphere coupled climate model. The climate system under consideration is divided into two parts: the mixing layer of ocean surface and the atmosphere. The thermodynamic process of the ocean-atmosphere interaction, i. e., radiation transfer and latent-sensible heat exchange, is mainly considered. The temperature distribution and ice-snow effect are omitted here. Taking the mean ocean surface temperature, mean atmosphere temperature and mean atmospheric relative humidity as variables, we set up the equations describing the evolution of these variables. Although the global-averaged climate model is quite simple, it can more clearly be used to discuss the effect of various feedback processes on the climate.

## II. THE ESTABLISHMENT OF WATER VAPOR EQUATION

The cloud amount has a close relation to the water vapor content in the atmosphere, but it is very complicated. And different authors have established their own relationship between clouds and water vapor. Sligo<sup>[20]</sup> has obtained the relation as follows:

$$\begin{aligned} n_L &= (f-80)^2/400, & f \geq 80 \\ n_M &= (f-65)^2/1225, & f \geq 65 \\ n_H &= (f-80)^2/400, & f \geq 80 \end{aligned} \quad (1)$$

where  $n_L$ ,  $n_M$  and  $n_H$  are low, medium and high cloud cover respectively, and  $f$  is the relative humidity(%). Using the relation  $I = I_0 (1 - C'n) = I_0 [1 - (C'_L n_L + C'_M n_M + C'_H n_H)]$  where  $I$  and  $I_0$  are net long-wave radiation at ocean surface with and without clouds respectively,  $C'$ ,  $C'_L$ ,  $C'_M$  and  $C'_H$  are constants, and putting  $C' = 0.75$ ,  $C'_L = 0.7$ ,  $C'_M = 0.5$ ,  $C'_H = 0.15$  (see Ref. [21]), then one can get the relationship between the total cloud cover  $n$  and  $f$ :

$$n = \frac{1}{C'} (C'_L n_L + C'_M n_M + C'_H n_H). \quad (2)$$

This is a fractional quadratic function and shown in Fig. 1. It is seen from Fig. 1 that when  $f$  approaches 1, then  $n > 1$ , that is unreasonable and is caused by the rude selection of the coefficients in Eq. (2). In order to make up this shortage and deal simply with it, we

use a fractional linear function to approximate Eq. (2), that is

$$n = \alpha_i f + \gamma_i. \quad (3)$$

If  $f = 1$  then  $n = 1$ . The coefficients  $\alpha_i$  and  $\gamma_i$  are taken as follows:

$$\alpha_i = \begin{cases} 4.3875 \\ 0.8167 \\ 0 \end{cases}, \quad \gamma_i = \begin{cases} -3.3875 \\ -0.531 \\ 0 \end{cases} \quad \begin{matrix} f > 0.8 \\ 0.65 < f < 0.8 \\ f < 0.65 \end{matrix}$$

Curve 2 in Fig. 1 is just the trace of the function in Eq. (3).

Assuming the total water vapor in the atmosphere is  $W$ , it comes mainly from the evaporation of oceans and then rises, cools and forms clouds, and at last falls down to the earth surface in the form of precipitation. Thus the variation of  $W$  can be expressed as

$$\frac{dW}{dt} = E_0 - P, \quad (4)$$

where  $E_0$  is the evaporation rate and  $P$  is the transformation rate of water vapor into cloud. They are discussed as follows:

(1) Assume that the water vapor density in the atmosphere is  $\rho_v$ , the atmosphere elevation is  $H$ , then

$$W = \int_0^H \rho_v dz \approx \bar{\rho}_v H = \frac{e}{R_v T_a} H, \quad (5)$$

where  $e$  is the average vapor pressure,  $R_v$  is the gas constant of water vapor.

The saturation vapor pressure of the atmosphere is  $e_s = 6.11 \exp \left( 7.451 \ln 10 \frac{T_a - 273}{T_a - 38} \right)$

(hPa). Using  $e = e_s f$ , we obtain

$$\frac{dW}{dt} = \frac{d}{dt} \left( \frac{H f e_s}{R_v T_a} \right) \approx \left[ (v_1 - v_0) \frac{dT_a}{dt} + \frac{df}{dt} \right] \frac{H e_s}{R_v T_a}, \quad (6)$$

where  $v_1 \approx 7.451 \ln \left[ 10 \times \frac{235}{(T_a^* - 38)^2} \right] \approx 0.073$ ,  $v_0 = \frac{1}{T_a} \approx \frac{1}{T_a^*} = 0.004$ , and  $T_a^* = 261$  K.

Thus  $v_0$  can be omitted in Eq. (6).

(2) In general,  $E_0$  can be expressed as<sup>[11]</sup>

$$E_0 = K U_{z_0} (e_s - e_{z_0}), \quad (7)$$

where  $K$  is a constant;  $U_{z_0}$ , the velocity at some elevation  $z_0$ , above sea level;  $e_s$ , the saturation vapor pressure at  $T_s$ ;  $e_{z_0}$ , the real vapor pressure at  $z_0$ . Because of the dynamical process inside the atmosphere such as the diffusion and transfer of water vapor there exists a relationship between  $e_{z_0}$  and  $e$ . We have no attempt to introduce the dynamical process into the interior atmosphere, so it is necessary to get the parameterized form of the relation.

In general, the water vapor density in the standard atmosphere varies with height in terms of  $\rho_v(z) = \rho_{v_0} e^{-\beta z}$ , thus the averaged water vapor density can be obtained from

$$\bar{\rho}_v = \frac{1}{H} \int_0^H \rho_v(z) dz = \frac{\rho_{v_0}}{H\beta} (1 - e^{-\beta H}) \approx \frac{\rho_{v_0}}{H\beta}, \quad (8)$$

where  $\rho_{v_0}$  is the water vapor density at  $z_0$ ,  $\beta$  is a constant. Besides, the variation of the air temperature with height follows  $T_a(z) = T_{a_0} - \gamma(z - z_0)$ , then the mean air temperature is

$$T_a = \frac{1}{H} \int_0^H T_a(z) dz = T_{a_0} - \frac{1}{2} \gamma (H - 2z_0). \quad (9)$$

Using ideal gas state equation and Eqs. (8) and (9), we obtain

$$e_{s_0} = \rho_{v_0} R_v T_{s_0} = \delta e + e' \quad (10)$$

where  $\delta = \rho_{v_0}/\bar{\rho}_0 = H\beta$ ,  $e' \approx \rho_{v_0}^* R_v \frac{1}{2} \gamma (H - 2z_0)$ . Let  $H=10$  km,  $\beta=0.5/\text{km}$ ,  $\gamma=6$

K/km,  $z_0=10$  m,  $\rho_{v_0}^*=6$  g/m<sup>3</sup>, then we have  $\delta \approx 5$ ,  $e'=0.78$  hPa.

(3) Assuming the averaged density and thickness of clouds are  $\bar{\rho}_c$  and  $H_c$  respectively, and according to Eq. (3), we have  $\frac{dn}{dt} = \alpha_i \frac{df}{dt}$ , and then

$$P = \frac{d}{dt} (n \bar{\rho}_c H_c) = \bar{\rho}_c H_c \alpha_i \frac{df}{dt}. \quad (11)$$

Combining Eqs. (4), (6), (7), (10) and (11), we obtain

$$\begin{aligned} & \frac{1}{KU_{s_0}} \left( 1 + \frac{\bar{\rho}_c H_c}{W_s} \alpha_i \right) \frac{H}{R_s T_{s_0}} \frac{df}{dt} \\ &= \frac{e_s}{e_0} - \delta \frac{e}{e_0} - \frac{e'}{e_0} - \frac{1}{KU_{s_0}} \frac{H}{R_s T_{s_0}} \gamma_i f \frac{dT_{s_0}}{dt}, \end{aligned} \quad (12)$$

where  $W_s = H e_s / R_s T_{s_0}$ ,  $\bar{\rho}_c H_c / W_s$  is the ratio of cloud precipitable to atmospheric saturated vapor content and assumed to be 1. We make an approximation  $H/(R_s T_{s_0}) = H/(R_s T_{s_0}^*) = \alpha$  in Eq. (12), and  $W$  and  $e$  are measured in g/cm<sup>2</sup> and hPa respectively, then  $\alpha \approx 1$  [g/(cm<sup>2</sup> hPa)]. Besides,  $e_s/e_0$  and  $1/e_0$  can be expanded into the linear function of  $T_s$  and  $T_0$  in the vicinity of  $T_s^*=288$  K and  $T_0^*=261$  K, i. e.,  $e_s/e_0 = b_5(T_s - T_0) - b_6$ ,  $1/e_0 = b_7 - b_8 T_0$ , putting  $b_5=0.41$ ,  $b_6=8$ ,  $b_7=0.033$ , and  $b_8=9.13$ . In the meanwhile, assuming  $m_s = \alpha \nu_i / (KU_{s_0} C_A)$ ,  $C_f = 2(1 + \alpha_i) \alpha / KU_{s_0}$ , where  $C_A$  is the atmospheric heat capacity, finally we obtain the evolution equation of  $f$  as

$$\frac{1}{2} C_f \frac{df}{dt} = b_5(T_s - T_0) - b_6 - \delta f + e' b_7 T_0 - e' b_8 - f m_s C_A \frac{dT_{s_0}}{dt}. \quad (13)$$

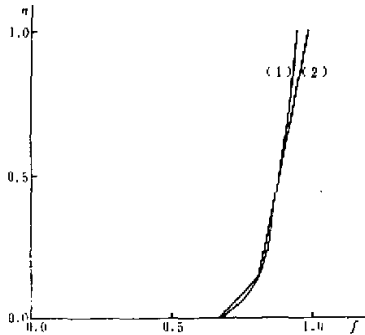


Fig. 1. Relation between  $n$  and  $f$ . Curve (1) is fractional quadratic function, and curve (2) is fractional linear function.

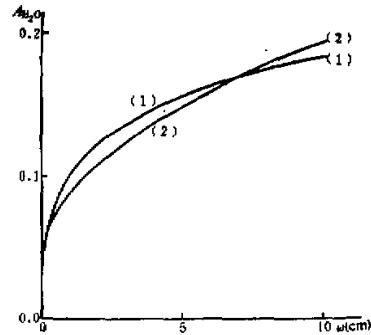


Fig. 2. The comparison between Eqs. (15) and (16). Curve 1 is for Eq. (15), and 2 for Eq. (16).

The physical meaning of every term in Eq. (13) is as follows:  $b_5 T_s$  expresses that the

increase of  $T_s$  will cause the increase of evaporation, and then the increase of relative humidity; because of  $b_s \gg e/b_s$ , then  $-(b_s + e/b_s)T_s - (f m_0 C_A) dT_s/dt \approx -b_s T_s - (f m_0 C_A) dT_s/dt$ , these two terms express that as  $T_s$  increases, the saturation vapor pressure also increases, and it makes  $f$  decrease;  $-\delta f$  corresponds to the reverse force of relative humidity itself.

### III. THE ESTABLISHMENT OF $T_a$ AND $T_s$ EVOLUTION EQUATIONS

In considering the heat exchange between the ocean and atmosphere, it is easy to obtain the parameterized evolution equation of  $T_a$  and  $T_s$ ,

$$\begin{aligned} C_A \frac{dT_a}{dt} &= Q_0 \bar{A} - (\varepsilon_a^* + \varepsilon_s^*) \sigma T_a^4 + \varepsilon_o \sigma T_s^4 + H_s + H_e, \\ C_S \frac{dT_s}{dt} &= Q_0 (1 - \bar{A} - \tau_a) (1 - \tau_s) + \varepsilon_a^* \sigma T_a^4 - \sigma T_s^4 - H_s - H_e, \end{aligned} \quad (14)$$

where  $C_A$  and  $C_S$  are the heat capacities of atmosphere and oceans respectively, and can be expressed as  $C_A = P_s C_a / g$ ,  $C_S = \rho_w C_w h$ , where  $P_s$  is the atmospheric pressure at surface;  $g$  is the gravity acceleration;  $C_a$  and  $C_w$  are specific heat of atmosphere and sea water respectively;  $h$  is the thickness of the mixing layer of ocean surface.

In Eq. (14),  $Q_0 \bar{A}$  is the solar short-wave radiation absorbed by the atmosphere;  $(\varepsilon_a^* + \varepsilon_s^*) \times \sigma T_a^4$ , the radiation of atmosphere;  $\varepsilon_o \sigma T_s^4$ , the surface long-wave radiation absorbed by the atmosphere;  $H_s$ , the sensible heat;  $H_e$ , the latent heat;  $Q_0 (1 - \tau_a - \bar{A}) (1 - \tau_s)$ , the solar short-wave radiation absorbed by the ocean surface,  $\varepsilon_a^* \sigma T_a^4$  the atmospheric long-wave radiation absorbed by the ocean surface;  $\sigma T_s^4$ , the long-wave radiation of ocean surface.

The coefficients in Eqs. (13) and (14) are not constant, but they are related to cloud cover, water vapor and  $\text{CO}_2$ . We discuss them term by term below.

(1)  $\bar{A}$  is the atmospheric short-wave absorptivity related mainly to water vapor, cloud and  $\text{O}_3$ , i.e.,  $\bar{A} = A_{\text{cloud}} + A_{\text{H}_2\text{O}} + A_{\text{O}_3}$ , where  $A_{\text{H}_2\text{O}}$  is the absorptivity due to water vapor which can be written as<sup>[23]</sup>

$$A_{\text{H}_2\text{O}} = 2.9W / [(1 + 141.5W)^{0.635} + 5.925W], \quad (15)$$

where  $W$  is the water vapor content used previously. For simplicity, the following formula

$$A_{\text{H}_2\text{O}} = 0.039 + 0.049\sqrt{W} \quad (16)$$

is used instead of Eq. (15). The comparison between Eqs. (15) and (16) is seen in Fig. 2.

Assuming the linear relation between  $A_{\text{cloud}}$  and cloud cover  $n$ , and without considering the ozonic variation, then we can express  $\bar{A}$  as

$$\bar{A} = \chi_0 + \chi_1 \sqrt{W} + \chi_n n, \quad (17)$$

where  $\chi_0 = 0.09$ ,  $\chi_1 = 0.049$ ,  $\chi_n = 0.04$  (see Saltzman<sup>[18]</sup>).

(2)  $\varepsilon_a^*$  is the downward atmospheric long-wave emissivity, and  $H_s^{(2)*} = \sigma T_a^4 - \varepsilon_a^* \sigma T_s^4$  is the net upward radiation of earth surface. According to the relations  $H_s^{(2)*} = \sigma T_a^4 (1 - \varepsilon) \times (1 - \beta n)$ ,  $\varepsilon = \varepsilon_o + \varepsilon_{\text{CO}_2} + \varepsilon_s - \varepsilon_*$  and  $\varepsilon_{\text{CO}_2} = 0.0235 \ln [\text{CO}_2]^* + 0.0537 + 0.001(T_s - T_s^*)$ , given by Saltzman<sup>[18]</sup>, and according to  $\varepsilon_a^* \sigma T_a^4 = \sigma T_s^4 (a + b\sqrt{e})$ , given by Budyko<sup>[21]</sup>,  $\varepsilon_a^*$  can be written as

$$\varepsilon_a^* = \varepsilon_o^* + \eta_1 \sqrt{e} + \xi_1 n + a_1 \ln [\text{CO}_2]^* + \beta_1 (T_s - T_s^*), \quad (18)$$

where  $\varepsilon_o^* = 0.7715$ ,  $\beta_1 = 0.65^{[21]}$ ,  $a_1 = 0.61$ , and  $b = 0.058^{[21]}$ . Comparing them we obtain  $\eta_1 = bT_s^{*4}/T_s^{*4} = 0.086$ ,  $\xi_1 = T_s^{*4}(1 - \varepsilon)\beta/T_s^{*4} = 0.225$ ,  $a_1 = 0.0225T_s^{*4}/T_s^{*4} = 0.035$ , and  $\beta_1 = 0.001T_s^{*4}/T_s^{*4} = 0.0015$ . Putting  $\varepsilon_o^* = 0.846$ , thus when  $[\text{CO}_2]^* = 300$  ppm,  $e = e^* = 2$  hPa,

$n=0.5$  and  $T_s=T_s^*$ , we get  $\varepsilon_s^{\dagger}=1.25$ , which accords with the data given by Taylor<sup>[24]</sup>.

(3)  $\varepsilon_a$  is the atmospheric long-wave absorptivity, and can be written as  $\varepsilon_a=\varepsilon_{an}+\varepsilon_{as}+\varepsilon_{ac}$ . Assume  $\varepsilon_{an}=\varepsilon'_{an}+\xi_s n$  and the absorptivity of cloud to be 1, in general,  $\varepsilon_a=0.93$ , then  $\varepsilon_a=\varepsilon'(1-n)+n=0.93$ , we get  $\xi_s=1-\varepsilon'=0.14$ . According to Kondratyev<sup>[25]</sup>, the absorptivity of water vapor  $\varepsilon_{as}$  can be expressed as

$$\varepsilon_{as}=1-\frac{1}{4}\sum_{j=1}^4 e^{-K_j W}. \quad (19)$$

Every coefficient  $K_j$  can be found in Ref. [25]. For simplicity, we use the formula

$$\varepsilon_{as}=0.53+0.138\sqrt{W} \quad (20)$$

instead of Eq. (19). The comparison between Eqs. (19) and (20) is given in Fig. 3.

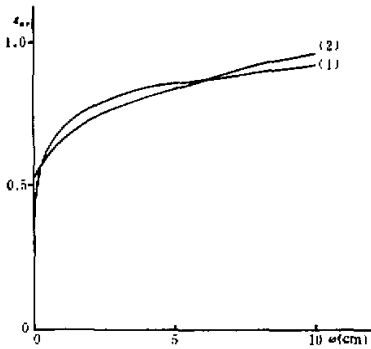


Fig. 3. The comparison between Eqs. (19) and (20). Curve 1 is for Eq. (19), and 2 for Eq. (20).

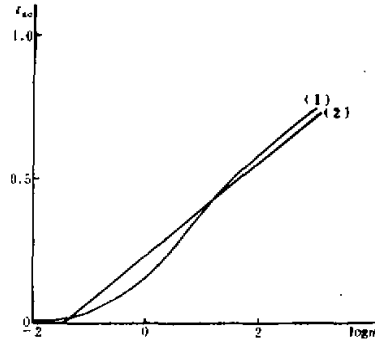


Fig. 4. The comparison between Eq. (21) and the real data. Curve 1 is for the real data, and 2 for Eq. (21).

According to the data given by Kondratyev, the approximate relation between the long-wave absorptivity  $\varepsilon_{as}$  of CO<sub>2</sub> and its content,

$$\varepsilon_{as}=0.1631 \lg u+0.235 \quad (21)$$

can be found, where  $u$  in cm is the thickness of CO<sub>2</sub> under standard temperature and pressure. When the content of CO<sub>2</sub> is measured in ppm, Eq. (21) becomes

$$\varepsilon_{as}=0.071 \ln [\text{CO}_2]+0.196. \quad (22)$$

Fig. 4 gives the comparison between Eq. (21) and real data. Therefore in the same way as Saltzman's assumption<sup>[18]</sup>, we have  $\varepsilon_{as}=a_s \ln [\text{CO}_2]^*+0.196+\beta_s(T_s-T_s^*)$ , where  $\beta_s=a_s \delta [\text{CO}_2]/[\text{CO}_2]^*(T_s-T_s^*)=a_s \beta_{10}$ ,  $\beta_{10}=0.05^{[26]}$ . Eventually,

$$\varepsilon_a=\varepsilon_{ao}+\eta_s \sqrt{W}+\xi_s n+a_s \ln [\text{CO}_2]^*+\beta_s(T_s-T_s^*), \quad (23)$$

where  $\varepsilon_{ao}=0.26$ ,  $\eta_s=0.138$ ,  $\xi_s=0.14$ ,  $a_s=0.071$ , and  $\beta_s=0.0035$ .

(4)  $\varepsilon_s^{\dagger}$  is the atmospheric emissivity outward to space,  $\varepsilon_s^{\dagger}=\varepsilon_s^{\dagger_s}+\varepsilon_s^{\dagger_c}+\varepsilon_s^{\dagger_n}+\varepsilon_s^{\dagger_{\text{other}}}$ . Assume again  $\varepsilon_s^{\dagger_s}=\varepsilon'_{sn}+\xi_s n$ . In order to estimate  $\xi_s$ , we express  $\varepsilon_s^{\dagger}$  as  $\varepsilon_s^{\dagger}=\varepsilon_s^{\dagger_s}(1-n)+\varepsilon_s^{\dagger_n} n$ , where the first term on the right stands for clear atmospheric radiation and the second stands for cloudy atmospheric radiation. Based on Wittman's data listed by Lion<sup>[27]</sup>, we have

$$\begin{aligned} \varepsilon_s^{\dagger_s}(1-n)\sigma T_s^{\dagger} &= 0.33 Q_0, \\ \varepsilon_s^{\dagger_n} n \sigma T_s^{\dagger} &= 0.36 Q_0. \end{aligned}$$

Putting  $T_a = T_a^*$ ,  $n=0.5$ , and  $Q_0=338.25 \text{ W/m}^2$ , then  $\xi_i = e_i^{*+} - e_i^{*+} = 0.92 - 0.84 = 0.08$ .

In order to estimate  $\varepsilon_i^+$ , and  $\varepsilon_i^*$ , first we should formulate an expression. According to the basic theory of the atmospheric radiation transfer, the upward and downward radiations are

$$\begin{cases} U_j(w) = p_j B(w) - (1 - \varepsilon_0) \varepsilon_j e^{-k_j w} - p_j \int_0^w \frac{dB}{d\mu} e^{-k_j(w-\mu)} d\mu, \\ G_j(w) = p_j B(w) - p_j B(w_\infty) e^{-k_j(w_\infty-w)} + p_j \int_0^{w_\infty} \frac{dB}{d\mu} e^{-k_j(\mu-w)} d\mu, \\ \varepsilon_j = p_j B(w_\infty) e^{-k_j w_\infty} - p_j \int_0^w \frac{dB}{d\mu} e^{-k_j \mu} d\mu, \end{cases} \quad (24)$$

where  $U_j$  and  $G_j$  are the upward and downward radiations of the  $j$ th equivalent waveband of component  $w$ ;  $w$  is the content of water vapor or  $\text{CO}_2$  below elevation  $z$ ;  $\varepsilon_0$ , long-wave emissivity of earth surface;  $k_j$ , absorption coefficient;  $p_j$ , the ratio factor of the  $j$ th waveband radiation to the whole radiation;  $B(w)$ , blackbody radiation. From Eq. (24), we obtain the net radiation of earth surface as

$$F_{0,i} = U_i(0) - G_i(0) = -(1 - \varepsilon_0) \varepsilon_i + p_i B(w_\infty) e^{-k_i w_\infty} - p_i \int_0^{w_\infty} \frac{dB}{d\mu} e^{-k_i \mu} d\mu = \varepsilon_0 \varepsilon_i. \quad (25)$$

Thus the net radiation of the  $i$ th component is  $F_{0,i} = \sum_{j=1}^n F_{0,i,j} = \varepsilon_0 \sum_{j=1}^n \varepsilon_{i,j}$ . Here the subscript  $i$  stands for the  $i$ th atmospheric component,  $j$  stands for the  $j$ th equivalent waveband of the  $i$ th component, and one can make summation over  $j$ . The same meaning is used in the following derivation. Therefore, the upward and downward atmospheric radiation are

$$\begin{aligned} U_i(w_\infty) &= p_i B(w_\infty) - (1 - \varepsilon_0) \varepsilon_i e^{-k_i w_\infty} - p_i \int_0^{w_\infty} \frac{dB}{d\mu} e^{-k_i(w_\infty-\mu)} d\mu \\ &= p_i B(w_\infty) + p_i B(w_\infty) e^{-k_i w_\infty} - \varepsilon_i - (1 - \varepsilon_0) \varepsilon_i e^{-k_i w_\infty}, \end{aligned} \quad (26)$$

and

$$G_i(0) = p_i B(0) - p_i B(w_\infty) e^{-k_i w_\infty} + p_i \int_0^{w_\infty} \frac{dB}{d\mu} e^{-k_i \mu} d\mu = p_i B(0) - \varepsilon_i. \quad (27)$$

Because  $\varepsilon_0$  is close to 1, then  $(1 - \varepsilon_0) \varepsilon_i e^{-k_i w_\infty} \ll \varepsilon_i$  and the last term in Eq. (26) can be omitted. Thus,

$$\begin{aligned} \varepsilon_{0,i} \sigma T_a^* &= U_i(w_\infty) = \sum_{j=1}^{n_i} U_i(w_\infty) B(w_\infty) \left( 1 + \sum_{j=1}^{n_i} p_j e^{-k_j w_\infty} \right) - \sum_{j=1}^{n_i} \varepsilon_{i,j} \\ &= \sigma T_{H_i}^* (2 - \varepsilon_{0,i}) - F_{0,i} / \varepsilon_0, \end{aligned} \quad (28)$$

$$\varepsilon_{0,i} \sigma T_a^* = G_i(0) = \sum_{j=1}^{n_i} G_i(0) = B(0) \sum_{j=1}^{n_i} p_j - \sum_{j=1}^{n_i} \varepsilon_{i,j} = \sigma T_i^* - F_{0,i} / \varepsilon_0. \quad (29)$$

Combining Eqs. (28) and (29), we obtain

$$\varepsilon_{0,i}^* = (2 - \varepsilon_{0,i}) T_{H_i}^* / T_a^* + \varepsilon_{0,i}^* - T_i^* / T_a^*, \quad (30)$$

where  $\varepsilon_{0,i}$  is the long-wave radiation absorptivity of the  $i$ th component;  $H_i$  is a specific elevation, above which the content of the  $i$ th component is considered so little that it has no

considerable influence on the total content; and  $T_{H_i}$  is the temperature at the corresponding elevation. Assuming  $\varepsilon_s^+$  has the form

$$\varepsilon_s^+ = \varepsilon_s^+ + \eta_s \sqrt{e} + \xi_s n + a_s \ln[\text{CO}_2]^* + \beta_s (T_s - T_s^*), \quad (31)$$

then from Eq. (30) it is easy to obtain  $\eta_s = \eta_s - \eta_s T_{H_i}^+ / T_s^{*+}$ ,  $a_s = a_s - a_s T_{H_i}^+ / T_s^{*+}$ , and  $\beta_s = \beta_s - \beta_s T_{H_i}^+ / T_s^{*+}$ . As far as the radiation budget is concerned, water vapor can reach the tropopause and CO<sub>2</sub> above 20 km. Therefore we take  $H_s = 10$  km,  $H_o = 20$  km, and corresponding temperatures  $T_{H_s} = 230$  K,  $T_{H_o} = 212$  K, thus acquiring  $\eta_s = 0.003$ ,  $a_s = 0.004$ ,  $\beta_s = 0.0002$  and  $\varepsilon_s^+ = 0.79$ . In fact, it is required that  $\eta_s \geq 0$ ,  $a_s \geq 0$  and  $\beta_s \geq 0$ , and thus  $T_{H_s} \leq \sqrt{\eta_s T_s^{*+} / \eta_s} \approx 232$  K,  $T_{H_o} \leq \sqrt{a_s T_s^{*+} / a_s} \approx 218$  K. Because the atmospheric vertical temperature structure needs  $H_s \geq 10$  km and  $H_o \leq 25$  km (CO<sub>2</sub> has reached above 20 km), which is not in contradiction with the averaged measurements observed so far, it is indicated that the parameters obtained previously are reasonable.

(5)  $r_s$  is the albedo of ocean surface and is assumed to be constant, i. e.,  $r_s = 0.1$ .

(6)  $\bar{r}_o$  is atmospheric albedo,  $\bar{r}_o = r_{\text{cloud}} n + r_{\text{clear}} (1 - n) = r_o + r_s n$  putting  $r_{\text{cloud}} = 0.54^{(11)}$ ,  $r_{\text{clear}} = 0.14^{(11)}$ , then  $r_o = 0.14$ ,  $r_s = 0.4$ .

(7) As for  $H_s$ , Newton's approximation is used, i. e.

$$H_s = b_o (T_s - T_o) - C_o, \quad (32)$$

where  $b_o = 4 \text{ W m}^{-2} \text{ K}^{-1}$ ,  $C_o = 40.8 \text{ W m}^{-2(11)}$ .

(8) From  $E_o$  obtained previously, the latent exchange between the ocean and atmosphere is

$$\begin{aligned} H_E &= L_v E_o = L_v K U_s e_o [b_s (T_s - T_o) - b_s - \delta f + e' b_s T_o - e' b_s] \\ &= v_s (b_s T_o - b_s) [b_s (T_s - T_o) - b_s - \delta f + e' b_s T_o - e' b_s], \end{aligned} \quad (33)$$

where  $v_s = L_v K U_s$ ,  $L_v$  is evaporation latent heat.  $e_o$  has been expanded into the linear function of  $T_o$  near  $T_s^*$ , i. e.,  $(b_s T_o - b_s)$ , with  $b_s = 0.2$  and  $b_s = 48.9$ .

(9)  $\sqrt{W}$  and  $\sqrt{e}$  exist in many coefficients and now they are expressed as functions of  $T_o$  and  $f$ , i. e.,  $W = a e$  and  $\sqrt{e} = \sqrt{e_o} \sqrt{f}$ . Let  $f_p = \sqrt{f}$ , and expand  $\sqrt{e_o}$  into the linear function of  $T_o$  near  $T_s^*$ , i. e.,  $\sqrt{e_o} = b_1 T_o - b_1$ , with  $b_1 = 0.0515$  and  $b_1 = 11.587$ .

#### IV. THE COMPLETE EQUATIONS

Substituting  $\bar{A}$ ,  $\varepsilon_s^+$ ,  $\varepsilon_o^+$ ,  $e_o$ ,  $H_s$ ,  $H_E$ ,  $\bar{r}_o$  and  $r_s$  obtained previously into Eq. (14), and expanding all  $T^+$  into linear function near  $T_o = 273$  K,  $\sigma T^+ = \sigma T_o^+ + 4\sigma T_o^+ (T - T_o) = 4C_1 T - 3C_2$ , with  $C_1 = \sigma T_o^+$  and  $C_2 = \sigma T_o^+$ , then combining and simplifying them as well as considering Eq. (13), we finally obtain a complete mathematic model describing zero-dimensional climate

$$\begin{aligned} C_A \frac{dT_o}{dt} &= A_0 + A_1 T_o + A_2 T_s + A_3 f_p + A_4 T_o T_s + A_5 T_o f_p + A_6 T_s f_p + A_7 T_s^2 \\ &\quad + A_8 f_p^2 + A_9 T_o f_p T_s + A_{10} T_o f_p^2 + A_{11} T_s^2 f_p + A_{12} T_s f_p^2 \\ &\quad + v_s (b_s T_o - b_s) (D_0 + D_1 T_o + D_2 T_s + D_3 f_p^2) \\ &= G_1(T_o, T_s, f_p), \\ C_s \frac{dT_s}{dt} &= B_0 + B_1 T_o + B_2 T_s + B_3 f_p + B_4 T_o T_s + B_5 T_o f_p + B_6 f_p^2 \\ &\quad + B_7 f_p^2 T_o + B_8 T_s^2 f_p - v_s (b_s T_o - b_s) (D_0 + D_1 T_o + D_2 T_s + D_3 f_p^2) \\ &= G_2(T_o, T_s, f_p), \end{aligned} \quad (34)$$



$$C_f \frac{df_p}{dt} = \frac{1}{f_p} \left( D_0 + D_1 T_a + D_2 T_s + D_3 f_p^2 - C_A m_0 f_p^2 \frac{dT_a}{dt} \right) \\ = G_s(T_a, T_s, f_p).$$

The coefficients in Eq. (34) are given in the Appendix.

#### V. THE PARAMETER SENSIBILITY ANALYSIS OF STEADY STATE

Letting the left of Eq. (34) be zero and combining them, we obtain the following equations

$$\begin{aligned} P_{12}(f_p) &= 0, \\ T_a &= F_1(f_p), \\ T_s &= F_2(f_p, T_a), \end{aligned} \quad (35)$$

where  $P_{12}$  is a twelve-order polynomial of  $f_p$ , from which  $f_p$  can be solved with numerical method and then  $T_a$  and  $T_s$  can be solved from the following two formulae. When each parameter  $p_i$  takes the reference value given in Table 1, only one valid solution is obtained, i. e.,  $f_p = 0.921$  ( $f = 0.85$ ),  $T_a = 256.07$  K and  $T_s = 287.02$  K (the real values are  $f = 0.75$ ,  $T_a = 261$  K and  $T_s = 288$  K), whereas other solutions keep  $f_p$  outside  $[0, 1]$ , so they are not taken into account.

Since the steady state depends on parameters in Eq. (34), then when parameter  $P_i$  undergoes a small change, the steady state also has a small variance. This is the sensibility problem of steady states to parameters. Table 1 lists the calculated results. In order to make the comparison conveniently, Table 1 also gives the change  $\Delta T'_i$  in the case of  $P'_i/P_i = 0.1$ . This change can be calculated from the values of other three columns in Table 1 by using  $\Delta T'_i = 0.1 P_i \Delta T_a / \Delta P_i$ .

It is seen from Table 1 that  $T^*$ ,  $T_a^*$ ,  $\alpha_1$ ,  $\gamma_1$ ,  $\chi_0$ ,  $b_0$ ,  $C_0$ ,  $b_1$ ,  $b_2$ ,  $b_3$ ,  $b_4$ ,  $\epsilon_{10}^+$ ,  $\epsilon_{10}^-$  and  $\epsilon_{00}$  have a strong influence on the steady state, which is easily understood.  $T^*$  and  $T_a^*$  are reference values, so their changes will certainly have a great influence on the steady state.  $\chi_0$ ,  $\epsilon_{10}^+$ ,  $\epsilon_{10}^-$  and  $\epsilon_{00}$  are directly related to the energy budget of atmospheric radiation. If all the feedbacks were taken away, these parameters would be the atmospheric equivalent absorptivity and emissivity. Therefore they have a great influence on the steady state.  $\alpha_1$ ,  $\gamma_1$ ,  $b_0$ ,  $C_0$ ,  $b_1$ ,  $b_2$ ,  $b_3$  and  $b_4$  are all parameterized coefficients of a certain process,  $\alpha_1$  and  $\gamma_1$ ,  $b_0$  and  $C_0$ ,  $b_1$  and  $b_2$ , as well as  $b_3$  and  $b_4$  are parameterized coefficients for the relationship of cloud with water vapor, the sensible heat, latent heat, and square root of saturation vapor pressure respectively. They emerge in pairs, and if either of them has a change, there will be a great influence on the way and the energy budget of the corresponding process and, consequently on the steady state. Owing to that their changes are in pairs, the total effect would not cause such a great sensibility as listed in Table 1, and this can be seen from the inverse change of the state caused by their increases.

The sensibility of steady state to solar constant  $Q_0$  is extremely concerned by scientists. The relations of  $T_s - Q_0$  and  $f_p - Q_0$  are shown in Fig. 5. In general, the sensibility of  $T_s$  to  $Q_0$  is defined as  $\beta = Q_0 dT_s/dQ_0$  (taking  $dQ_0/Q_0 = 0.01$ , generally). It is seen from Fig. 5b that, when  $Q/Q_0 > 0.94$ ,  $\beta = 0.57$  K and when  $Q/Q_0 < 0.94$ ,  $\beta = 1.67$  K. This difference is caused by the cloud feedback. From Section II,  $dn/df = \alpha_i$ , so when  $\alpha_i$  is larger,  $n$  varies greatly with  $f$ . Thus, the cloud feedback effect added to the model is also larger, and on the contrary, it is smaller when  $\alpha_i$  is smaller. In Fig. 5a, when  $f_p > 0.89$  ( $f > 0.8$ ,

$Q/Q_0 > 0.94$ ),  $\alpha_1 = 4.4$ ; when  $0.81 < f_p < 0.89$  ( $0.65 < f < 0.8$ ,  $0.87 < Q/Q_0 < 0.94$ ),  $\alpha_1 = 0.82$ ; and when  $f_p < 0.81$  ( $f < 0.65$ ,  $Q/Q_0 < 0.87$ ),  $\alpha_1 = 0$ . The correspondent  $\beta$  in Fig. 5b is 0.57, 1.67 and 1.92 ( $\alpha_1 = 0$  not shown in Fig. 5b). Comparing with the standard value  $\beta = 0.7$  K<sup>(1.1+1.5)</sup> under radiation balance without feedback, it is seen that when  $\alpha_1$  is larger, the negative albedo feedback of clouds is greater than the positive feedback of greenhouse; and when  $\alpha_1$  is smaller or zero, the feedback effect of clouds is small or vanished, then the positive feedback effect (greenhouse effect) of water vapor is in a dominant position, and results in a larger value of  $\beta$ .

Table 1. The Sensibility of the Steady State to Parameters

Parameter $P_i$	Reference Value	$\Delta P_i$	$\Delta T_s(K)$	$\Delta T'_s(K)$
$T_{H_v}$	230 K	5 K	0.06	0.276
$T_{H_o}$	212 K	5 K	-0.02	-0.085
$T_s^*$	288 K	2 K	-0.84	-12.11
$T_a^*$	261 K	5 K	-0.5	-2.61
$\alpha$	1 g cm <sup>-2</sup> hPa <sup>-1</sup>	0.25	1.75	0.7
$\alpha_1$	4.3875	0.1	-1.11	-4.86
$\gamma_1$	-3.3875	-0.1	1.45	4.91
$\chi_1$	0.049	0.005	0.94	0.92
$\chi_a$	0.09	0.03	3.81	1.14
$\chi_n$	0.04	0.02	0.83	0.17
$r_a$	0.14	0.05	-2.0	-0.56
$r_1$	0.4	0.05	-0.58	-0.46
$r_s$	0.1	0.02	-0.56	-0.28
$\zeta_1$	0.225	0.03	0.42	0.32
$\zeta_2$	0.08	0.03	0.51	0.14
$\zeta_3$	0.14	0.03	-1.1	-0.51
$\eta_1$	0.086 hPa <sup>-1/2</sup>	0.005 hPa <sup>-1/2</sup>	-0.05	-0.086
$\eta_s$	0.138 hPa <sup>-1/2</sup>	0.05 hPa <sup>-1/2</sup>	0.084	0.023
$a_1$	0.035	0.005	0.005	0.0035
$a_3$	0.071	0.005	-0.06	-0.085
$b_a$	4 W m <sup>-2</sup> K <sup>-1</sup>	0.2 W m <sup>-2</sup> K <sup>-1</sup>	2.2	4.4
$C_0$	40.8 W m <sup>-2</sup>	2 W m <sup>-2</sup>	-0.69	-1.44
$b_1$	0.515 K <sup>-1</sup> hPa <sup>1/2</sup>	0.005 K <sup>-1</sup> hPa <sup>1/2</sup>	3.41	35.12
$b_2$	11.587 hPa <sup>1/2</sup>	0.1 hPa <sup>1/2</sup>	-2.1	-24.3
$b_3$	0.41 K <sup>-1</sup>	0.01 K <sup>-1</sup>	-2.3	-9.43
$b_6$	8	0.5	4.89	7.82
$e_{a0}^+$	0.764	0.02	-2.15	-8.21
$e_{s0}^+$	0.847	0.02	-1.56	-6.61
$e_{a0}$	0.26	0.02	4.06	5.28

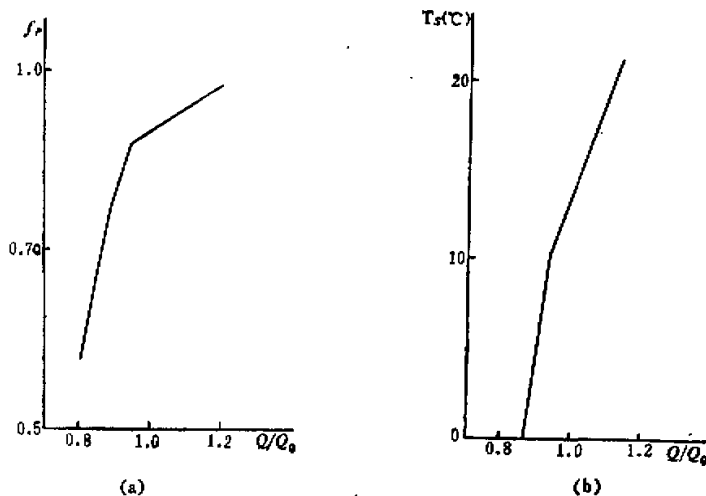
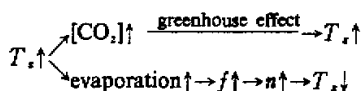


Fig. 5. The sensibility curve of steady state to solar constant  $Q_0$ . (a)  $f_p$ - $Q_0$ ; (b)  $T_s$ - $Q_0$ .

## VI. CO<sub>2</sub> FEEDBACK

In section III,  $\beta_{10}$  describes the relativity between the change of ocean temperature and CO<sub>2</sub> content in the atmosphere. The variation of CO<sub>2</sub> content will influence ocean temperature through greenhouse effect, so it forms a feedback process<sup>[10,26]</sup>. It is known from section III that  $\beta_3 = a_3\beta_{10}$ ,  $\beta_2 = \beta_1 - \beta_3 T_{H_0}^4 / T_s^{*4}$ , and  $\beta_1 = a_1\beta_{10}$ <sup>[10]</sup>. If  $\beta_{10} = 0$ , then  $\beta_1 = \beta_2 = \beta_3 = 0$ , causing this feedback process to vanish. Consequently, the magnitude of  $\beta_{10}$  reflects whether the CO<sub>2</sub> feedback is strong or weak. When  $\beta_{10} = 0$  and the other parameters are kept to be reference values, the steady state changes to  $f_p = 0.922$ ,  $T_s = 256.39$  K and  $T_e = 287.35$  K. Comparing with  $f_p = 0.921$ ,  $T_s = 256.07$  K and  $T_e = 287.03$  K under  $\beta_{10} = 0.05$ , it is seen that as  $\beta_{10}$  increases,  $f_p$  changes a little and temperature decreases, indicating the total feedback effect caused by  $\beta_{10}$  is negative, which is contrary to Saltzman's result<sup>[17]</sup>. It is thought to be caused by the strong negative cloud feedback in the model. In fact, when  $T_s$  increases, it causes the increase of [CO<sub>2</sub>], and so does the greenhouse effect. On the other hand, it also makes the evaporation increase, thus the water vapor and the cloud cover increase too, and the negative feedback strengthens. This process can be drawn in the form



The total effect caused by  $\beta_{10}$  is the competitive result of these two processes. If the cloud feedback effect is larger, the result will be negative; if the [CO<sub>2</sub>] effect is larger, the result will be positive. For instance, if let  $\xi_1 = 0.375$ , which corresponds to increasing the greenhouse effect of cloud (see next section) and decreasing the total negative cloud feedback, the steady states for  $\beta_{10} = 0$  and  $\beta_{10} = 0.05$  will be  $f_p = 0.918$ ,  $T_s = 257.97$  K,  $T_e = 288.74$  K and  $f_p = 0.920$ ,  $T_s = 258.20$  K,  $T_e = 288.99$  K, respectively. It is seen from above that as  $\beta_{10}$  increases, the temperature increases, which is due to the fact that the greenhouse effect of CO<sub>2</sub> has an advantage over the others and causes a positive total feedback effect.

It can be seen from the above analysis that the effect of CO<sub>2</sub> feedback on climate will depend on whether other feedback processes are positive or negative and strong or weak, that is, these feedback processes are interacting with and influencing on each other. In S-M model, there exists a strong ice-cap-albedo positive feedback, therefore it is easy to understand that increasing  $\beta_0$  will result in positive feedback effect.

The CO<sub>2</sub> feedback process considered above is caused by the exchange of CO<sub>2</sub> between atmosphere and oceans. If the CO<sub>2</sub> content in the atmosphere is doubled by human activity, then the calculated temperature increment of the model is about 8 K. It is quite close to the value of 9.6 K calculated by Moller<sup>(12)</sup>.

## VII. THE STABILITY AND BIFURCATION OF SOLUTION

The stability of steady state is determined by the eigenvalues of the Jacobian of the right functions in Eq. (34), i. e.

$$P(\lambda) = \begin{vmatrix} \frac{1}{C_A} \frac{\partial G_1}{\partial T_a} - \lambda & \frac{1}{C_A} \frac{\partial G_1}{\partial T_s} & \frac{1}{C_A} \frac{\partial G_1}{\partial f_p} \\ \frac{1}{C_s} \frac{\partial G_2}{\partial T_a} & \frac{1}{C_s} \frac{\partial G_2}{\partial T_s} - \lambda & \frac{1}{C_s} \frac{\partial G_2}{\partial f_p} \\ \frac{1}{C_f} \frac{\partial G_3}{\partial T_a} & \frac{1}{C_f} \frac{\partial G_3}{\partial T_s} & \frac{1}{C_f} \frac{\partial G_3}{\partial f_p} - \lambda \end{vmatrix} = 0, \quad (36)$$

where every partial derivative takes its value at the steady state. The eigenvalues obtained are  $\lambda_1 = -89.14 \text{ a}^{-1}$ , ( $a$  denotes year in SI units),  $\lambda_2 = -12.68 \text{ a}^{-1}$ , and  $\lambda_3 = +0.88 \text{ a}^{-1}$ , indicating that the steady state is stable to small disturbance. And because the three eigenvalues are of real and negative, the steady state is a stable node.

There are three factors controlling the greenhouse effect of clouds, i.e.  $\xi_1$ ,  $\xi_2$ , and  $\xi_3$ . It is seen from Table 1 that, increasing  $\xi_1$  and  $\xi_2$  or decreasing  $\xi_3$  will increase temperature, and thus increase the positive feedback. This process can be shown as follows:

$$\begin{array}{l} \xrightarrow{\xi_1} \epsilon_0^+ \uparrow \rightarrow \epsilon_0^+ \sigma T_a^4 \uparrow \rightarrow T_a \uparrow \\ \xrightarrow{\xi_2} \epsilon_0^+ \uparrow \rightarrow T_a \downarrow \rightarrow f \uparrow \rightarrow \text{evaporation} \downarrow \rightarrow T_s \uparrow \\ \xrightarrow{\xi_3} \epsilon_0^+ \uparrow \rightarrow T_a \uparrow \rightarrow f \downarrow \rightarrow \text{evaporation} \uparrow \rightarrow T_s \downarrow \end{array}$$

Let  $\xi_2$  and  $\xi_3$  take the reference values in Table 1 and change  $\xi_1$  only. It is seen from the above analysis that with increasing  $\xi_1$ , the positive feedback effect increases, hence the instability of the climate. This may change the structure and stability of the steady state. The calculated results prove that the situation is true. Fig. 6 shows the variation of eigenvalues with  $\xi_1$ . It is seen that when  $\xi_1$  rises to  $\xi_{10} \approx 0.3$ , the three negative real eigenvalues become a negative real number and a pair of conjugate complex eigenvalues with negative real part, that is,  $\lambda_{1,2} = -a \pm ib$ ,  $\lambda_3 = -c$ , where  $a > 0$ ,  $b > 0$ ,  $c > 0$  ( $\lambda_3$  not shown in Fig. 6). Then the stable node becomes a stable focal point. If an initial value near the steady state is given, the solution will tend toward a steady state in the form of oscillation. Fig 7 shows the variation of solution  $T_s$  to Eq. (34) with time  $t$  and the orbit in the phase space, when  $\xi_1 = 0.37$ . The correspondent eigenvalues are  $\lambda_{1,2} = -0.25 \pm i 2.79$

( $\text{a}^{-1}$ ),  $\lambda_3 = -107.7 \text{ a}^{-1}$ , thus the damping oscillation period is  $T_0 = \frac{2\pi}{b} = \frac{2\pi}{2.79} \approx 2.25 \text{ a}$ , and

the damping time constant is  $\tau_0 = \frac{1}{a} = \frac{1}{0.25} = 4$  a.

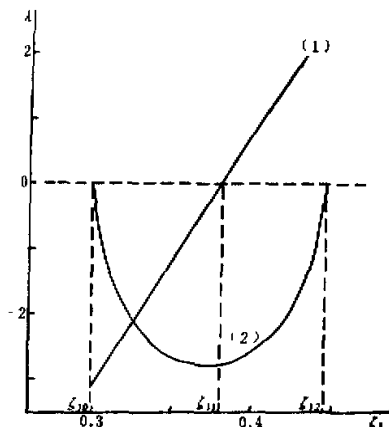


Fig. 6. Variation of eigenvalues with  $\zeta_1$ . Curve (1) is for real part, and (2) for imaginary part.

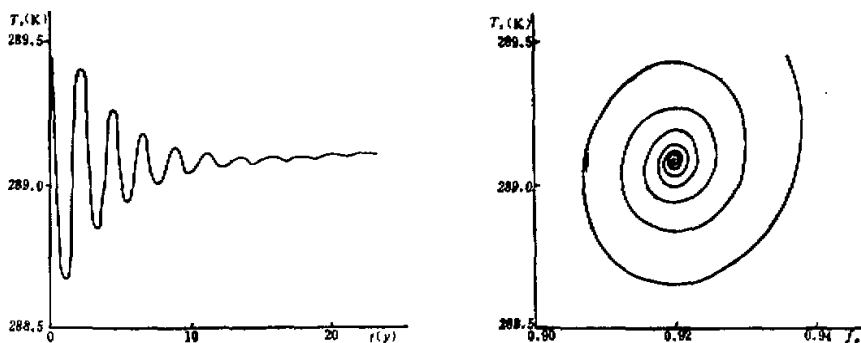


Fig. 7. The property of solution near steady state when  $\zeta_1 = 0.37$ . (a) The variation of  $T_s$  with time  $t$  (left). (b) The projection onto the  $T_s$ - $f_p$  plane of the damping oscillatory solution in phase space (right).

As  $\zeta_1$  increases,  $a$  approaches to zero and  $\tau_s$  becomes longer and longer, then the oscillation property of solution gets strengthened and the instability increases. When  $\zeta_1 \geq \zeta_{11} \approx 0.3764$ , the real part of the pair conjugate complex eigenvalues becomes positive, so the steady state becomes an unstable focal point. At  $\zeta_1 = \zeta_{11}$ , the eigenvalues satisfy Hopf bifurcation conditions:  $a=0$ ,  $da/d\zeta_1 \neq 0$ , and  $b \neq 0^{[12]}$ , which is easy to see from Fig. 6. Therefore, at  $\zeta_{11}$ , the steady state undergoes a Hopf bifurcation. When  $\zeta_1 > \zeta_{11}$ , if any initial value near the steady state is given to solve Eq. (34), then the solution will eventually run to some other invalid state. It means that no stable periodic limit-cycle is found, so the Hopf bifurcation is subcritical. When  $\zeta_1$  rises to  $\zeta_1 \geq \zeta_{11}$ , the imaginary part of the eigenvalues vanishes and the eigenvalues become three real numbers with two positive and one negative. Then the steady state becomes an unstable node.

The other two parameters  $\zeta_2$  and  $\zeta_3$  have the same influence on the structure of the

steady state as  $\xi_1$ , but the decreasing process of  $\xi_2$  corresponds to the increasing process of  $\xi_1$ .

From the above analysis it is seen that when the positive feedback of clouds becomes strong (increasing  $\xi_1$ ,  $\xi_2$  or decreasing  $\xi_3$ ), the structure variation of the steady state can be formulated as follows: a stable node  $\rightarrow$  a stable focal point  $\xrightarrow{\text{Hopf bifurcation}}$  an unstable focal point  $\rightarrow$  an unstable node. Near the bifurcation point, there exist both the attenuation and growth oscillation solution with oscillation period being about two years ('growth' means that the real part of the eigenvalues is greater than zero and there is no stable limit-cycle). Ji (1982)<sup>(12)</sup> has studied this oscillation in a two-dimensional air-sea coupled model. His model involved thermodynamic and dynamic process, as well as three feedbacks (one positive and two negative). He has found that when the intensity of convection  $L_r$ , which describes the degree of positive feedback, rises to above a certain critical value, or either of the two negative feedbacks is taken out of the model, then the oscillation solution will vanish. This is easily seen from the cloud feedback in this paper. It should be noted that the oscillation property of the climate is not completely determined by the imaginary part of eigenvalues. From Fig. 6, it is seen that when  $\xi_1 > \xi_{10}$ , the imaginary part increases from zero to 2.79, and the oscillation period decreases from infinite to 2.25 years. However, owing to the influence of  $\tau_0$ , long-term oscillation can not be displayed. In fact, it is required that  $\tau_0 \geq T_0$ , i. e.  $b \geq 2\pi a$ . In this condition, the solution can then have properties of oscillation (otherwise it will damp out quickly). Only when  $b$  is near 2.77, can it satisfy this condition, i. e. the period is a few years.

Not only  $\xi_1$ ,  $\xi_2$  and  $\xi_3$  have the above influence on the structure variance of the steady state, but also some other parameters have similar effect. The relationship between  $U_{s0}$  and eigenvalues is shown in Fig. 8, which is similar to Fig. 6, with an exception that the increase of  $\xi_1$  corresponds to the decrease of  $U_{s0}$ . In this case the steady state undergoes the same structure variance as that of increasing  $\xi_1$ , and the Hopf bifurcation is also subcritical. It is easy to understand the above behavior of  $U_{s0}$ , because  $U_{s0}$  determines the ocean evaporation in the full extent, hence the cloud feedback. The total effect of decreasing  $U_{s0}$  is to decrease the negative feedback of clouds, resulting in the increase of instability of the steady state.

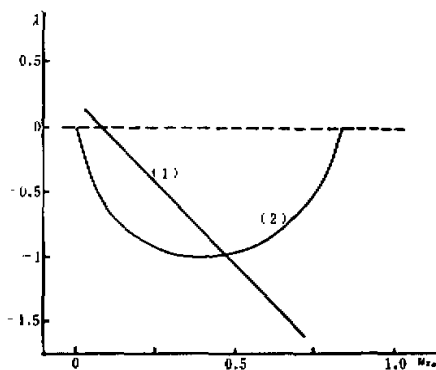


Fig. 8. Relation of  $U_{s0}$  with eigenvalues. Curve (1) is for real part, and (2) for imaginary part.

## VIII. CONCLUSIONS

In this paper, a global annually-averaged zero-dimensional climate model is established. This model describes the evolutions of the mean atmosphere temperature  $T_a$ , mean ocean temperature  $T_o$  and atmospheric relative humidity  $f$ . The model involves clouds, water vapor and  $\text{CO}_2$  feedbacks. From the analysis and calculation, we can draw some conclusions as follows:

(1) The negative feedback of reflection from clouds is greater than the positive feedback of greenhouse, so the total feedback effect of clouds is negative.

(2) The sign of  $\text{CO}_2$  feedback depends on other feedback processes, i.e., it can be positive or negative in different cases.

(3) There exists a damping and amplifying oscillation with about two-year period. This indicates that when the positive and negative feedbacks on some time-scale reach a competitive balance, the climate system will have a self-oscillatory property. Therefore it is very important to give a more realistic description of the positive and negative feedback processes in studying the problem of climate change.

During the establishment of the model, Slingo's parameterized form of cloud is employed, the cloud cover  $n$  is expressed as a fractional linear function of  $f$ . The water vapor evaporation  $E_o$  and the sensible heat  $H_s$  are also parameterized. These parameterized processes are very rude. In fact, because of the effect of the atmospheric turbulence, these dynamic and thermodynamic processes are very complex and give rise to great difficulties in the establishment of relatively precise parameterized equations. This is a problem that has remained unsolved so far. However, the analysis of some feedback processes in the simple climate model is helpful to determine the relative importance in consideration of more complex and more real climate model.

## APPENDIX

Table A1. Coefficient Expressions in Eq. (34)

$$\begin{aligned}
 A_0 &= Q_o X_o + 3C_2 ((e_{o0}^{\dagger} + e_{o0}^{\ddagger} - \epsilon_{o0}) - (\beta_1 + \beta_2 - \beta_3) T_o^* + (\zeta_1 + \zeta_2 - \zeta_3) \gamma_i) + Q_o X_o \gamma_i - b_o \\
 A_1 &= -4C_1 (e_{o0}^{\dagger} + e_{o0}^{\ddagger} - (\beta_1 + \beta_2) T_o^* + (\zeta_1 + \zeta_2) \gamma_i) - C_o \\
 A_2 &= 4C_1 (\epsilon_{o0} - \beta_3 T_o^* + \zeta_3 \gamma_i) + C_o + 3C_2 (\beta_1 + \beta_2 - \beta_3) \\
 A_3 &= -Q_o X_o b_2 - 3C_2 b_2 (\eta_1 + \eta_2 - \eta_3) \\
 A_4 &= -4C_1 (\beta_1 + \beta_2) \\
 A_5 &= Q_o X_o b_2 + 4C_1 b_2 (\eta_1 + \eta_2) + 3C_2 b_1 (\eta_1 + \eta_2 - \eta_3) \\
 A_6 &= -4C_1 b_2 \eta_3 \\
 A_7 &= 4C_1 \beta_3 \\
 A_8 &= (Q_o X_o + 3C_2 (\zeta_1 + \zeta_2 - \zeta_3)) a_1 \\
 A_9 &= 4C_1 b_1 \eta_3 \\
 A_{10} &= -4C_1 (\zeta_1 + \zeta_2) a_1 \\
 A_{11} &= -4C_1 b_1 (\eta_1 + \eta_2) \\
 A_{12} &= 4C_1 \zeta_3 a_1 \\
 B_0 &= Q_o (1 - r_s) (1 - X_o - r_s - (r_1 + X_o) \gamma_i) - 3C_2 (e_{o0}^{\dagger} - \beta_1 T_o^* - 1 + \zeta_1 \gamma_i) + b_o \\
 B_1 &= 4C_1 (e_{o0}^{\dagger} - \beta_1 T_o^* + \zeta_1 \gamma_i) + C_o
 \end{aligned}$$

$$B_2 = -3C_2\beta_1 - C_2 - 4C_1$$

$$B_3 = Q_0\chi_1 b_2(1-r_s) + 3C_2 b_2 \eta_1$$

$$B_4 = 4C_1\beta_1$$

$$B_5 = -Q_0\chi_1 b_1(1-r_s) - 4C_1 b_1 \eta_1 - 3C_2 \eta_1 b_1$$

$$B_6 = -(Q_0(\chi_n + r_s)(1-r_s) + 3C_2 \zeta_1) \alpha_1$$

$$B_7 = 4C_1 \zeta_1 \alpha_1$$

$$B_8 = 4C_1 b_1 \eta_1$$

$$D_0 = -b_0 - e' b_6$$

$$D_1 = -b_5 + e' b_7$$

$$D_2 = b_3$$

$$D_3 = -\delta$$

## REFERENCES

- [1] Budyko, M. I., *Tellus*, **21**(1969), 611—619.
- [2] Sellers, W. D., *J. Appl. Meteor.*, **8**(1969), 392—400.
- [3] North, G. R., *J. Atmos. Sci.*, **32**(1975), 1301—1307, 2033—2043.
- [4] Chylek, P. and Coakley, J. A., *ibid.*, **32**(1975), 675—679.
- [5] Suarez, M.I. and Held, I. M., *J.G.R.*, **84**(1979), C8: 4825—4836.
- [6] 巢纪平, 陈英仪, *中国科学*, 1979, 第12期, 1198—1207.
- [7] Manabe, S. and Wetherald, R. T., *J. Atmos. Sci.*, **24**(1967), 241—259.
- [8] Weare, B. C. and Snell, F. M., *ibid.*, **31**(1974), 1725—1734.
- [9] Temkin, R. L. and Snell, F. M., *ibid.*, **33**(1976), 1671—1685.
- [10] Roads, J. O. and Vallis, G. K., *Tellus*, **36A**(1984), 236—250.
- [11] Cess, R. D., *J. Atmos. Sci.*, **33**(1976), 1831—1843.
- [12] Schneider, S. H., *ibid.*, **32**(1975), 2060—2066.
- [13] ———, *GARP Publi. Ser.*, **11**(1979), 22:728—749.
- [14] Ohring, G. and Clapp, P., *J. Atmos. Sci.*, **37**(1980), 447—454.
- [15] Cess, R. D., Briegleb, B. P., and Lian, M. S., *ibid.*, **39**(1982), 53—59.
- [16] Saltzman, B., *Advances in Geophysics*, **20**(1978), 283—304.
- [17] ———, *Tellus*, **34**(1982), 97—112.
- [18] Saltzman, B. and Moritz, R. E., *ibid.*, **32**(1980), 93—118.
- [19] Ji Jinjun, *Sci. Sin.*, **25**(1982), 630—654.
- [20] Slingo, J. M., *Quart. J.R. Met. Soc.*, **106**(1980), 747—770.
- [21] Budyko, M. I., *Climate and Life*, in *Inter. Geophy. Ser.*, Academic Press, N.Y., 1974, 508pp.
- [22] Defont, A., *Physical Oceanography*, Vol. 1, Pergamon Press, Oxford, London, 1961, 219—242.
- [23] G.W. 帕尔特里奇, C.M.R. 普拉特, *气象学和气候学中的辐射过程*, 科学出版社, 北京, 1981, 207pp.
- [24] Taylor, K., *J. Appl. Meteor.*, **15**(1976), 1129—1138.
- [25] Kondratyev, K. Y., *Radiation in the Atmosphere*, Academic Press, N.Y. and London, 1969, 912pp.
- [26] Eriksson, E., *J.G.R.*, **68**(1963), 3871—3876.
- [27] Liou K-N, *An Introduction to Atmospheric Radiation*, Academic Press, N.Y., 1980, 392pp.
- [28] Moller, F., *J.G.R.*, **68**(1963), 3877—3886.
- [29] Iooss, G. and Joseph, D., *Elementary Stability and Bifurcation Theory*, Springer-Verlag, New York, 1980, 123—156.