

## THE NONLINEAR INTERACTION BETWEEN DIFFERENT WAVE COMPONENTS AND THE PROCESS OF INDEX CYCLE OF GENERAL CIRCULATION

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### ABSTRACT

By using a two-level quasi-geostrophic truncated spectral model taking account of the nonlinear interaction between different wave components (*i. e.* basic current, ultra-long waves and long waves), the index cycle of general circulation is investigated. The calculated results show that the circulation index has a quasi-periodic vacillation with a period of 8 to 16 days, which can be created by the nonlinear interaction and that the nonlinear interaction between different wave components may cause the tilted-trough vacillation, amplitude vacillation of wave pattern and quasi-periodic change of wave number of flow pattern.

### I. INTRODUCTION

The evolution of general circulation over the Northern Hemisphere always presents a translation between high index circulation and low index circulation. The index cycle is a quasi-periodic process. Generally speaking, the period is about 2—4 weeks. To investigate the mechanism of index cycle is very important for understanding the evolution of the medium-range weather process. Zeng et al.<sup>[1]</sup> have stated that the barotropic-rotational adaptation process plays a dominant part during the translation of low index circulation into high index circulation, and that the nonlinear interaction between different wave components is a main mechanism of rotational adaptation. Zhu<sup>[2]</sup> and Lorenz<sup>[3]</sup> have pointed out that the nonlinear interaction between the ultra-long wave and basic current may create the quasi-periodic index cycle of general circulation. But they considered the nonlinear interaction occurs only between some ultra-long wave and basic current. In this paper, we use a two-level quasi-geostrophic model including the nonlinear interaction among the basic current, ultra-long waves and long waves to investigate their contribution to the formation of index cycle.

### II. BASIC EQUATIONS AND SOLUTION

We use a two-level quasi-geostrophic model, and the basic equations<sup>[4]</sup> are as follows:

$$\begin{aligned} \frac{\partial}{\partial t} [\nabla^2 \phi_1 - \lambda_2 (\phi_1 - \phi_3)] &= -\mathbf{V}_1 \cdot \nabla [\nabla^2 \phi_1 - \lambda_2 (\phi_1 - \phi_3)] - \beta \frac{\partial \phi_1}{\partial x} \\ &+ A \nabla^2 [\nabla^2 \phi_1 - \lambda_2 (\phi_1 - \phi_3)] - L_1 \left[ (\phi_1 - \phi_3) - \frac{3}{4} R T_2 \right] - S_1 W, \end{aligned} \quad (1)$$

$$\begin{aligned} \frac{\partial}{\partial t} [\nabla^2 \phi_3 + \lambda_2(\phi_1 - \phi_3)] &= -\mathbf{V}_s \cdot \nabla [\nabla^2 \phi_3 + \lambda_2(\phi_1 - \phi_3)] - \beta \frac{\partial \phi_3}{\partial x} \\ &+ A \nabla^2 [\nabla^2 \phi_3 + \lambda_2(\phi_1 - \phi_3)] + \frac{K}{2} (3 \nabla^2 \phi_3 - \nabla^2 \phi_1) + S_1 W \\ &+ L_1 \left[ (\phi_1 - \phi_3) - \frac{3}{4} R T_2 \right], \end{aligned} \tag{2}$$

where

$$\begin{aligned} L_1 &= \lambda_2 M' \frac{4 \sigma T_2^3}{R} \left( \frac{R}{C_p} \frac{\bar{\alpha} \rho_\omega}{\rho_2} \right), & S_1 &= \lambda_2 b \left( \frac{R}{C_p} \frac{\bar{\alpha} \rho_\omega}{\rho_2} \right) e^{-5.5 \lambda_1 \rho_2}, \\ M' &= M / \sigma T_2^3, & \lambda_2 &= f_0^2 / \alpha_2 T R, \end{aligned}$$

$L_1$  and  $S_1$  are the parameters relative to long-wave and short-wave radiation respectively;  $K$  is the horizontal eddy viscosity;  $\alpha$  is a parameter relative to hydrostatic stability; and  $W$  is the amount of solar radiation at the top of the atmosphere.

The geopotential heights  $\phi_1$  and  $\phi_3$  may be expressed as the double Fourier series:

$$\begin{cases} \phi_1 = \sum_{n=-N}^N \sum_{m=-M}^M [(A_{m,n} \cos kmx + B_{m,n} \sin kmx) \cos lny \\ \quad + (A'_{m,n} \cos kmx + B'_{m,n} \sin kmx) \sin lny], \\ \phi_3 = \sum_{n=-N}^N \sum_{m=-M}^M [(C_{m,n} \cos kmx + D_{m,n} \sin kmx) \cos lny \\ \quad + (C'_{m,n} \cos kmx + D'_{m,n} \sin kmx) \sin lny], \end{cases} \tag{3}$$

where  $A_{m,n}$ ,  $B_{m,n}$ , etc. are the functions of time;  $k = 2\pi/L$ ;  $l = 2\pi/d$ ;  $L$  is the length of  $45^\circ$  latitude circle; and  $d/2$  is the pole-to-equator distance on the earth.

Taking

$$\begin{aligned} A_{-m,0} &= A_{m,0}, & B_{-m,0} &= -B_{m,0}, & A'_{0,n} &= -A'_{0,n}, & A_{0,-n} &= A_{0,n}, \\ A_{-m,n} &= A_{m,-n} = A_{-m,-n} = A_{m,n}, & -B_{-m,n} &= -B_{m,-n} = B_{m,n} = B_{m,n}, \\ A'_{-m,n} &= -A'_{m,-n} = -A'_{-m,-n} = A'_{m,n}, & B'_{-m,-n} &= -B'_{-m,n} = -B'_{m,-n} = B'_{m,n}, \end{aligned}$$

and taking  $C_{m,n}$ , ...,  $D_{m,n}$  in a way analogous to  $A_{m,n}$ , ...,  $B_{m,n}$ , then substituting them into Eq. (3), we get

$$\begin{cases} \phi_1 = A_{00} + 2 \sum_{n=1}^N (A_{0,n} \cos lny + A'_{0,n} \sin lny) + 2 \sum_{m=1}^M (A_{m,0} \cos kmx + B_{m,0} \sin kmx) \\ \quad + 4 \sum_{n=1}^N \sum_{m=1}^M [(A_{m,n} \cos kmx + B_{m,n} \sin kmx) \cos lny + (A'_{m,n} \cos kmx \\ \quad + B'_{m,n} \sin kmx) \sin lny], \\ \phi_3 = C_{00} + 2 \sum_{n=1}^N (C_{0,n} \cos lny + C'_{0,n} \sin lny) + 2 \sum_{m=1}^M (C_{m,0} \cos kmx + D_{m,0} \sin kmx) \\ \quad + 4 \sum_{n=1}^N \sum_{m=1}^M [(C_{m,n} \cos kmx + D_{m,n} \sin kmx) \cos lny + (C'_{m,n} \cos kmx \\ \quad + D'_{m,n} \sin kmx) \sin lny]. \end{cases} \tag{4}$$

The albedo  $\Gamma(x, y)$  is also expressed as the double Fourier series (not shown). Substituting Eq. (4) into Eqs. (1) and (2), multiplied by corresponding trigonometric

functions, then integrating them over the entire domain, we get a set of nonlinear ordinary differential equations:

$$\begin{aligned} \frac{dP_1}{dt} &= -\frac{L_1}{4} (A_{00} - C_{00}) + \frac{S_1}{2} W_0 \left[ \Gamma_{0,1}^{(1)} + \left( \frac{1}{4} + 0.63 \sin \lambda_{\odot} \right) \cdot \Gamma_{0,1}^{(2)} \right] \\ \frac{dP_{i+2,N+M+Mz+m}}{dt} &= \frac{1}{f_0} \left\{ (km)(ln)[(ln)^2 - (km)^2] B_{m,0} \cdot A'_{0,n} \right. \\ &+ \frac{1}{2} \sum_{j=1}^N (km)[(km)^2 + (lj)^2] [I(j+n)(A_{0,j+n} B'_{m,i} - A'_{0,j+n} B_{m,i}) \\ &+ I(j-n)(A_{0,j-n} B'_{m,i} - A'_{0,j-n} B_{m,i})] \\ &+ \frac{1}{2} \sum_{i=1}^M (ln)[(ki)^2 + (ln)^2] [k(i+m)(A_{i+m,0} B'_{i,n} + B_{i+m,0} A'_{i,n}) \\ &+ k(i-m)(A_{i-m,0} B'_{i,n} + B_{i-m,0} A'_{i,n})] \\ &+ \frac{1}{2} \sum_{i=1}^N (km)(lj)^2 (B_{m,i+n} A'_{0,i} - B'_{m,i+n} A_{0,i} + B_{m,i-n} A'_{0,i} - B'_{m,i-n} A_{0,i}) \\ &+ \frac{1}{2} \sum_{i=1}^M (ki)^2 (ln) (B'_{i+m,n} A_{i,0} - A'_{i+m,n} B_{i,0} + B'_{i-m,n} A_{i,0} - A'_{i-m,n} B_{i,0}) \\ &+ \frac{1}{4} \sum_{j=1}^N \sum_{i=1}^M [(ki)^2 + (lj)^2] \\ &\cdot \{ B'_{i,j} [(kiln - kmlj)(A_{i+m,j+n} - A_{i-m,j-n}) - (kiln + kmlj) \\ &\cdot (A_{i+m,j-n} - A_{i-m,j+n})] - A'_{i,j} [(kiln - kmlj)(B_{i+m,j+n} - B_{i-m,j-n}) \\ &- (kiln + kmlj) \cdot (B_{i+m,j-n} - B_{i-m,j+n})] \\ &- B_{i,j} [(kiln - kmlj)(A'_{i+m,j+n} - A'_{i-m,j-n}) - (kiln + kmlj) \\ &\cdot (A'_{i+m,j-n} - A'_{i-m,j+n})] \\ &+ A_{i,j} [(kiln - kmlj)(B'_{i+m,j+n} - B'_{i-m,j-n}) - (kiln + kmlj) \\ &\cdot (B'_{i+m,j-n} - B'_{i-m,j+n})] \} \\ &- \frac{A_2}{f_0} \{ km ln (D_{m,0} A'_{0,n} - B_{m,0} C_{0,n}) \\ &- \frac{1}{2} \sum_{j=1}^N km [I(j+n)(A_{0,j+n} D'_{m,i} - A'_{0,j+n} D_{m,i}) + I(j-n) \\ &\cdot (A_{0,j-n} D'_{m,i} - A'_{0,j-n} D_{m,i})] \\ &- \frac{1}{2} \sum_{i=1}^M ln [k(i+m)(A_{i+m,0} D'_{i,n} + B_{i+m,0} C'_{i,n}) + k(i-m) \\ &\cdot (A_{i-m,0} D'_{i,n} + B_{i-m,0} C'_{i,n})] \\ &- \frac{1}{2} \sum_{j=1}^N kmlj (B_{i,j+n} C'_{0,i} + B_{i,j-n} C'_{0,i} - B'_{i,j+n} C_{0,i} - B'_{i,j-n} C_{0,i}) \end{aligned}$$

$$\begin{aligned}
& -\frac{1}{2} \sum_{i=1}^M kiln(B'_{i+m,n}C_{i,0} + B'_{i-m,n}C_{i,0} - A'_{i+m,n}D_{i,0} - A'_{i-m,n}D_{i,0}) \\
& -\frac{1}{4} \sum_{j=1}^N \sum_{i=1}^M \{D'_{i,j}[(kiln - kmlj)(A_{i+m,j+n} - A_{i-m,j-n}) - (kiln + kmlj) \\
& \quad \cdot (A_{i+m,j-n} - A_{i-m,j+n})] \\
& - C'_{i,j}[(kiln - kmlj)(B_{i+m,j+n} - B_{i-m,j-n}) - (kiln + kmlj) \\
& \quad \cdot (B_{i+m,j-n} - B_{i-m,j+n})] \\
& - D_{i,j}[(kiln - kmlj)(A'_{i+m,j+n} - A'_{i-m,j-n}) - (kiln + kmlj) \\
& \quad \cdot (A'_{i+m,j-n} - A'_{i-m,j+n})] \\
& + C_{i,j}[(kiln - kmlj)(B'_{i+m,j+n} - B'_{i-m,j-n}) - (kiln + kmlj) \\
& \quad \cdot (B'_{i+m,j-n} - B'_{i-m,j+n})]\} \\
& - \beta km B_{m,n} + \frac{S_1}{2} W_0 \left[ \Gamma_{m,j,n+1}^{(0)} + \Gamma_{m,j,n-1}^{(1)} + \left( \frac{1}{4} + 0.63 \sin \lambda_{\odot} \right) \right. \\
& \quad \left. \cdot (\Gamma_{m,j,n+1}^{(2)} - \Gamma_{m,j,n-1}^{(3)}) \right] \\
& + [(km)^2 + (ln)^2][kmlj + (ln)^2] \cdot A_{m,n} + [(km)^2 + (ln)^2] \\
& \quad \cdot \lambda_2 A - L_1 x'_{i+zN+M+Mn+m} \\
& \quad n=1, 2, \dots, N; \quad m=1, 2, \dots, M. \\
& \dots\dots
\end{aligned} \tag{5}$$

where  $P_i$  ( $i=1, 2, \dots, 2+4N+4M+8MN$ ) satisfies a set of linear algebraic equations (Eq. (6), omitted) including Fourier coefficients  $A_{m,n}$ ,  $B_{m,n}$ , ...,  $C'_{m,n}$ ,  $D'_{m,n}$ . Eqs. (5) and (6) can be numerically integrated. First, by giving the initial Fourier coefficient  $A_{0,0}^{(0)}$ ,  $A_{0,0}^{(1)}$ , ...,  $D_{m,n}^{(0)}$ , and calculating  $P_i$  from Eq. (6), then by integrating numerically Eq. (5) using Runge-Kutta method,  $P_i^{(t)}$  ( $t_i = t_0 + \Delta t$ ) can be solved from Eq. (5).  $A_{m,n}^{(t)}$ ,  $B_{m,n}^{(t)}$ , ...,  $D_{m,n}^{(t)}$  can be solved from Eq. (6).  $\phi_1^{(t)}$ ,  $\phi_2^{(t)}$  are calculated from Eq. (4). Repeating the calculation mentioned above, we are able to obtain future potential height field.

### III. RESULTS AND DISCUSSION

In numerical experiments, we neglect the terms of radiation, lateral eddy diffusion and friction in Eq. (5). Take  $M=6$ ,  $N=2$ ,  $L=2\pi R \cos 45^\circ$ ,  $d=2 \times 10^4$  km,  $\lambda_2=1.2 \times 10^{-12}$  m $^{-2}$ ,  $f_0=10^{-4}$  sec $^{-1}$ ,  $\beta=1.6 \times 10^{-11}$  m $^{-1}$ s $^{-1}$  and the following cases as initial fields:

Case 1: Disturbances  $A_{2,2}$ ,  $A_{4,2}$ ,  $B_{2,2}$ ,  $B_{4,2}=196$  m $^2$  s $^{-2}$  and  $C_{2,2}$ ,  $D_{2,2}$ ,  $C_{4,2}$ ,  $D_{4,2}=98$  m $^2$  s $^{-2}$ , where the disturbances are stronger over the regions of high and low latitudes, but they are nothing at 45°N;

Case 2:  $A'_{2,2}$ ,  $A'_{4,2}$ ,  $B'_{2,2}$ ,  $B'_{4,2}=196$  m $^2$  s $^{-2}$  and  $C'_{2,2}$ ,  $C'_{4,2}$ ,  $D'_{2,2}$ ,  $D'_{4,2}=98$  m $^2$  s $^{-2}$ , where the disturbances are stronger over the middle latitude region and are nothing at 90°N and equator;

Case 3:  $A_{4,0}$ ,  $B_{3,0}=137$  m $^2$  s $^{-2}$  and  $C_{4,0}$ ,  $D_{4,0}=68.6$  m $^2$  s $^{-2}$ , where they are analogous to a wave-like basic current; and

Case 4, which is a zonal flow superimposed with weak disturbances from wave number 1 to 6.

(1) *Nonlinear interaction and the quasi-periodic vacillation of circulation index*

The variation of general circulation index has a feature of quasi-periodic vacillation. Monin<sup>[9]</sup> has calculated spectral density of circulation index, revealing that it has obvious maxima at the period of 12, 16 and 24 days. The vacillation of circulation index corresponds to the translation between meridional and zonal circulations. In this paper, we use

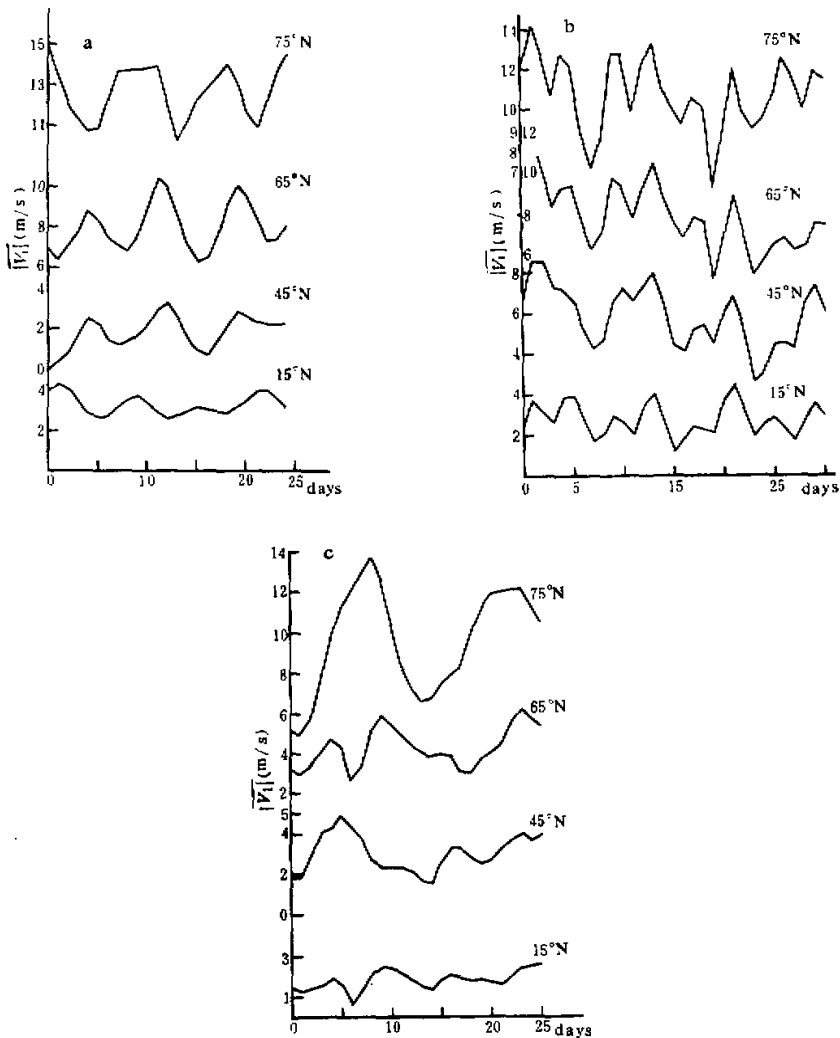


Fig. 1. The predicted  $|V|$  with the effect of nonlinear advection, (a) for Case 1, (b) for Case 2, and (c) for Case 3.

the zonal mean value of geostrophic wind in north-south direction  $|\overline{V}_1|$  to indicate circulation index. The smaller  $|\overline{V}_1|$  corresponds to the higher index. For the purpose of understanding the relationship between the nonlinear interaction and the vacillation of circulation index, we reserve only the nonlinear advection term in Eq. (5), and integrate it to 20–40 days for Cases 1–4. According to the predicted Fourier coefficient of geopotential height field, we calculate  $|\overline{V}_1|$  at 15°, 25°, ..., 75°N. The changes of circulation index for high, middle and low latitudes calculated from Cases 1 and 2 are shown respectively in Fig. 1a and Fig. 1b.

It can be seen from Fig. 1 that the circulation index has the feature of quasi-periodic vacillation, and their periods are 8 days and 12 days respectively. We have also calculated the variations of  $|\overline{V}_1|$  for Cases 3 and 4, their periods being about 12–15 days. These results are in good agreement with Monin's. Thus we may say that the process of index cycle with two-week period is a kind of quasi-periodic vacillation related to the atmosphere itself, and the nonlinear interaction between different wave components is a main mechanism for creating the index cycle.

For analyzing the effect of  $\beta$  term, we reserve the advection term and  $\beta$  term in Eq. (5) and calculate the variation of index cycle for Case 1. In this case the period is still about 8 days, almost unchanged at all, but the initial time of vacillation is postponed.

(2) *The variable tendency of circulation index in high and low latitudes and the meridional structure of height field*

Ye et al.<sup>[6]</sup> have pointed out that the variation of circulation index at high latitudes is opposite to that of low latitudes during the circulation adjustment.

It is seen from Figs. 1a–1c that there are two kinds of tendency of variation in  $|\overline{V}_1|$  along meridional direction. One of them (see Fig. 1a) is that the tendency of  $|\overline{V}_1|$  variation over the middle-high latitudes (45°–65°N) is opposite to the middle-low latitudes (15°–35°N). The tendency of  $|\overline{V}_1|$  variation over the polar region (75°N) is similar to that of low latitudes. However, there are some differences. It can be seen from Fig. 1a that there are three wave belts corresponding to three different trends of variation. It is noticed from Fig. 1b that the tendencies of  $|\overline{V}_1|$  variation in different latitude circles are consistent. The results show that although the nonlinear interaction is a main factor for the formation of index cycle, there exists another more important cause for the variable tendency of circulation index at high latitudes opposite to that at low latitudes. It is seen that the initial fields of Cases 2 and 3 have a feature in common, i. e. they have only a "wave belt", the pressure systems are in phase along north-south direction from high to low latitudes (see Fig. 3a). The height field in Case 1, however, possesses different structure along north-south direction, the pressure systems (troughs and ridges) are in opposite phase (a trough in the north with a ridge in the south), and the troughs are located in middle-high latitudes and the ridges are located in middle-low latitudes along the same meridional circle. The perturbation at 45°N is the weakest. The height field is equivalent to two wave belts. So, the fact that the variable tendency of circulation index in high latitudes is opposite to that in low latitudes is mainly due to the existence of two (or more) wave belts or a couple of high and low pressures in the meridional direction of the hemisphere. Ye et al.<sup>[6]</sup> have posed this view, and it is proved by our calculated results as well. It

is known that the elementary features of real general circulation in the troposphere is that the low pressure belt is located in the northern part of hemisphere with the high pressure belt in the south, and that there always exist two branches of westerly jet streams. However, its meridional structure is quite complex. Therefore, the opposite trend of circulation index for the high and low latitudes is a basic feature in the real atmosphere. It should also be pointed out that in comparison of the variable tendency of circulation index over polar region with that over middle-high latitudes, there are also many differences, as shown in Case 1.

(3) *The vacillation of the tilted-trough with spiral structure*

Not only tilts the trough line of planetary wave in the rotating atmosphere, but also it possesses the spiral structure. Chao et al.<sup>[7]</sup> and Liu et al.<sup>[8]</sup> have pointed out that these features are due to shear rotational current and the change of Coriolis parameter. Actually, not only would the planetary waves have trough and ridge lines with spiral structure, but also the tilted directions of their trough and ridge lines have quasi-periodic vacillation. Qiu<sup>[9]</sup> has analysed the composite map of mean zonal height field and wave 3, observing that the tilted directions of trough and ridge lines are of the feature with quasi-periodic change (see Fig. 2).

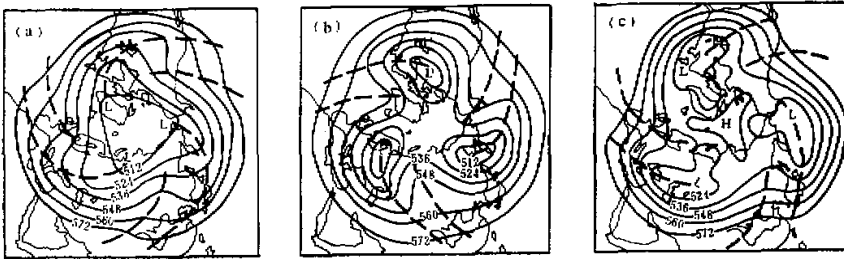


Fig. 2. The composite map of mean zonal height field and wave 3.  
(a) 5 Jan. 1977, (b) 9 Jan. 1977, and (c) 18 Jan. 1977.

The calculated evolution chart of circulation field ( $\phi$ , field) with the consideration of nonlinear advective effect is given in Fig. 3, which is the result calculated from Case 2. In the initial field, the trough line orientates to NS. Five days later, the trough line to the north of jet stream lies to NE-SW direction. Ten days later, it changes into NW-SE direction, but the trough line to the south of jet stream presents a NE-SW direction, and shows the feature of spiral structure. Twenty days later, the trough line transforms its direction into the opposite. The vacillation period of tilted trough line is about 10–12 days. For the circulation field calculated from Cases 1 and 3, the variation of trough line direction also possesses the feature of quasi-periodic vacillation. It is basically in agreement with the result shown in Fig. 2. Thus the vacillation of the tilted trough can be thought to be due to the nonlinear advective effect.

The vacillation of tilted direction of trough line can also be obtained by Fultz's experiments<sup>[10]</sup> of rotational tank. When the tilt of trough line changes, the sign of meridional eddy angular momentum flux also changes. It is well known that, when the wave in the north of westerly jet stream is a leading one (the trough line in directed to NW-SE) and

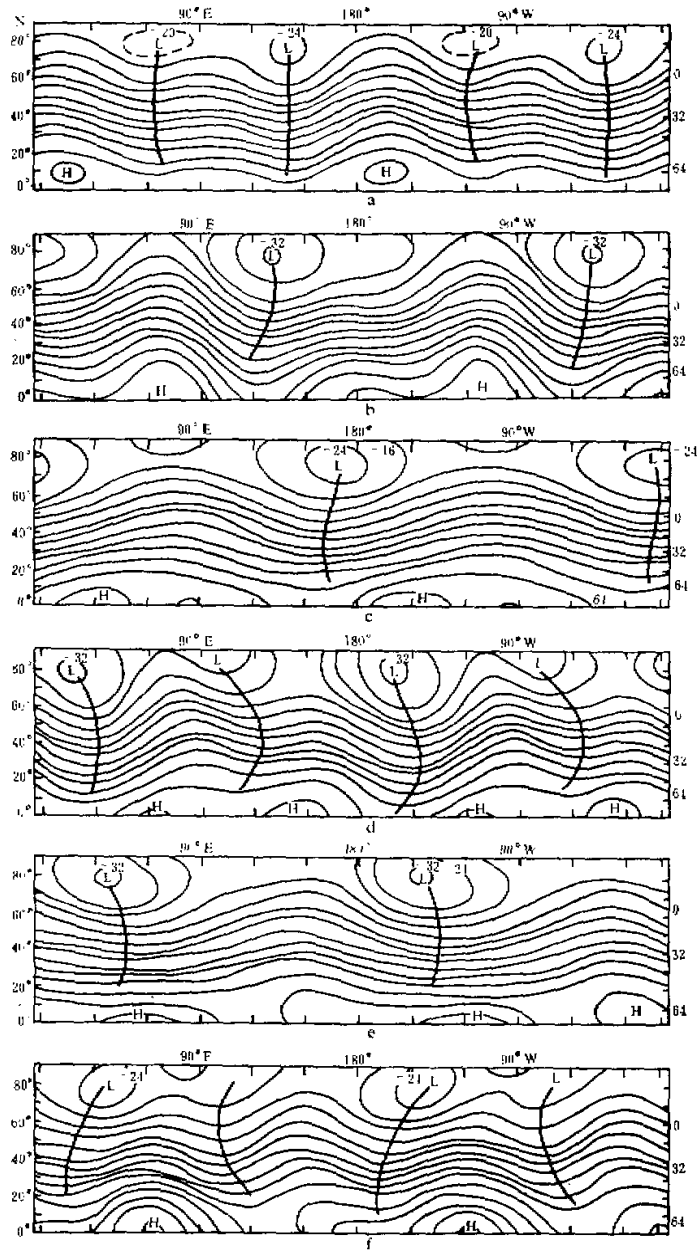


Fig. 3. The predicted  $\phi$ , (geopotential height) field with effect of nonlinear advection in different time stages.  
 (a) The initial field, (b) 5-day, (c) 7-day, (d) 10-day,  
 (e) 15-day, and (f) 20-day.



the wave in the south is a trailing one (the trough line is directed to NE-SW), then  $\frac{\partial \overline{u'v'}}{\partial y} < 0$ , making  $\bar{u}$  increase. Therefore the eddy kinetic energy is transformed into mean zonal kinetic energy. On the contrary, when the wave in the north of westerly jet stream is a trailing one and the wave in the south of jet stream is a leading one, then  $\frac{\partial \overline{u'v'}}{\partial y} > 0$ , making  $\bar{u}$  decrease. Therefore the vacillation of tilt of trough line would make the transformation between the kinetic energy of basic current  $K_z$  and eddy kinetic energy  $K_e$  possess quasi-periodic feature. That is a sort of mechanism which creates the index cycle of general circulation. The vacillation of tilt of trough line can only be created by considering the nonlinear interaction between the zonal currents and waves.

(4) *The vacillation in amplitudes of various scale waves*

Pfeffer's experiment<sup>[11]</sup> of rotational tank has shown that there are the vacillations not only in the tilt of trough and ridge, but also in their amplitudes.

Using Case 4 as an initial field, consisting of the zonal current, ultra-long waves (wavenumber 1-3) and long waves (wavenumber 4-6), and taking account of the nonlinear interaction between them, we calculate the variations of amplitudes of ultra-long waves and long waves shown in Fig. 4 for waves 2-6. It is seen from the figure that the vacillation in amplitudes shows a quasi-periodic feature.

The longer the wavelength, the longer its period is. The periods of waves as listed in Table 1 are in agreement with the real situation.

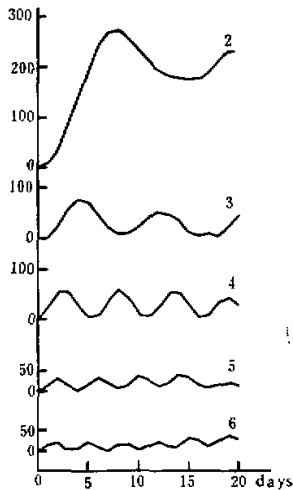


Fig. 4. The predicted "amplitude" changes of waves 2-6 for Case 4.

Table 1. Wavenumber ( $m$ ) and Vacillation Period ( $T$ )

$m$	2	3	4	5	6
$T$ (days)	14	8	5-6	4	3-4

The periods of vacillation of wave are related to initial fields. Using Cases 1 and 2 as initial fields, the obtained periods of amplitude vacillation are smaller. The period of

wave 2 is 7—12 days, wave 4 only 3—4 days, and their quasi-periodic features are very apparent. The change of amplitude for wave 2 corresponds to that of circulation index. When ultra-long waves develop, the circulation has an inclination to low index. Therefore, we may have an idea that the nonlinear interaction between different wave components would create the vacillations of wave amplitude and, in particular, the vacillation of ultra-long wave amplitude is another important mechanism for the formation of index cycle of general circulation. Naturally, the vacillation of amplitude of long wave may cause the variation of circulation index. However, the joint effect of vacillations of different wave amplitudes makes the variation of circulation index much complicated and varied. It makes the vacillation period of circulation index time-varying, becoming quasi-periodic and also makes the vacillation with longer period about two weeks embody shorter periodic vacillations.

(5) *The quasi-periodicity of adjustment of large-scale circulation.*

The evolution of actual flow pattern not only presents an increase or decrease of meridional extension, i. e. the change of wave amplitude, but also shows the change of wavenumber, which is usually known as the adjustment of circulation. Undoubtedly, the adjustment is closely connected with the nonlinear interaction between different wave components (of course, the forcing sources are also important), because the interaction enables the originally weaker wave to strengthen and yet to create new waves. The variation of wavenumber of circulation is very clearly shown in Fig. 3 and appears to be quasi-periodic feature. In the initial field, there are four vortexes over high latitudes. Five days later, the flow pattern is transformed into two-vortex type, and then returns to the circulation of four-vortex type. For Case 1, the period of this sort of process is about 8 days, while for Case 2, it is about 12 days.

It is interesting to analyse the evolution of the ultra-long waves (waves 1—3 composite map) and long waves (waves 4—6 composite map). When the deep low on ultra-long scale develops, the low on synoptic scale damps. At that time, the flow pattern of bi-polar vortex dominates in high latitudes. Several days later, the long wave develops and the ultra-long wave damps, resulting in the dominance of four-vortex type in high latitudes. In a word, the adjustment of circulation is due to the nonlinear interactions between the zonal current and waves, and between the ultra-long waves and long waves.

#### IV. CONCLUSIONS

From the results calculated above, we come to the following conclusions.

(1) The process of index cycle of general circulation with a period about two weeks is attributed mainly to the nonlinear interaction between different wave components (including zonal current). So it is believed that the index cycle is a quasi-periodic vacillation inherent to the atmosphere itself.

(2) The tendency of circulation index variation at high latitudes is opposite to that at low latitudes. It is insufficient to explain this phenomenon simply by the nonlinear interaction. To be more precise, it is mainly due to the existence of two (or more) wave belts, or a couple of high pressure and low pressure in the meridional direction of hemisphere range.

(3) The nonlinear interaction between the zonal current and waves may cause the vacillation of the orientation of trough-line of planetary wave.

(4) For the quasi-periodic vacillation in different wave amplitudes and in adjustment

of wavenumber of general circulation, the nonlinear interaction among the zonal current, ultra-long waves and long waves is the main mechanism.

It is seen from the investigation presented above that the quasi-periodic vacillation of circulation index with about 24-day period has not been obtained yet. Our model does not take the effects of heat sources and topography into account. It is thought that they are also important for the intensity and period of index cycle. The effects of the heat source and topography on index cycle remain to be further investigated.

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