VARIATION IN WIND VELOCITY OVER WATER

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ABSTRACT

Starting from the equations of motion and continuity, a theoretical model is deduced in this paper for the variation in wind velocity over water caused by abrupt changes in surface roughness and temperature when air flows from land to water, based on the consideration that the turbulent exchange coefficient varies with height and distance from the upwind edge. According to the computation of this model, the variation in wind velocity over water, as the drift of air is from land to water, occurs mainly in the first few kilometers from the upwind edge. The wind velocity over water increases to a maximum when the air over land is stable, it tends to moderate when neutral condition is reached, and least variation is shown in unstable condition. And when the air over land is unstable the wind velocity is less over water than over land in strong winds, but somewhat greater in light winds.

I. INTRODUCTION

When air flow travels from land to water, or vice versa, the wind velocity is varied owing to the abrupt changes in surface roughness and temperature. The study of the variation in wind velocity over different underlying surfaces, especially over land–water surfaces, is not only of theoretical importance but also of great practical significance. For this very reason, quite a number of studies (see Gandin, 1952; Zaisev, 1963; Nadejdina, 1964; Panofsky, 1964; Townsend, 1965; Peterson, 1969, 1980; Nickerson, 1968; Taylor, 1969, 1970; Panchev, 1971; Shir, 1972) have been made of the wind velocity variation caused by the abrupt change of underlying surfaces over the past 30 years, of which many are concerned with the wind velocity variation when the drift of air is from land to water. Some of the studies have represented the turbulent exchange coefficient k over water as a function z only, ignoring its variation with distance from the upwind edge and have solved the partial differential equation by Shwetz's (1949) method; some have, based on experimental data, considered the variation of exchange coefficient k with distance from the upwind edge or represented indirectly the herizontal variation of k by the relationship between exchange coefficient k and wind velocity u but neglected the vertical velocity, i. e., the continuity equation; and still some have calculated the contour change in a few specific cases by using the numerical method for the motion, continuity, heat conduction and energy balance equations.

This paper shows that: since the variation in the velocity of airflow passing over different underlying surfaces is mainly due to the changes in surface roughness and turbulent exchange strength which is related to the thermal stratification and wind velocity, it is necessary, in establishing a theoretical model of the airflow variation over a new underlying surface, to consider the variations of exchange coefficient k with thermal stratification and wind velocity and get the effects of surface roughness on wind velocity reflected in the boundary conditions or differential equation. In addition, since the pronounced horizontal variation in the airflow velocity over a new underlying surface is inevitably accompanied by upward and downward currents, the vertical velocity must be taken into account and the continuity equation can never be ignored. From this point the basic equation of air motion and its linearization will now be considered.

II. BASIC EQUATION

If the water body is not very large, the difference in pressure over land and water is negligible and the effects due to the earth's rotation can be neglected. Thus, in a steady case, the equations of motion and continuity over water can be written as

$$u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}=\frac{\partial}{\partial z}\left(k\frac{\partial u}{\partial z}\right), \tag{1}$$

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0, \tag{2}$$

where x is the coordinate in the wind direction measured from the upwind edge; z the vertical coordinate; u and w the horizontal velocity and vertical velocity over water; and k the turbulent coefficient over water.

Letting z_0 denote the surface roughness of water and u_0 the wind velocity over the land upwind of the water, the corresponding boundary conditions are

$$\left.\begin{array}{lll} u=u_0 & \text{on} & x=0 \\ u=0 & \text{on} & z=z_0 \\ u=u_0 & \text{as} & z\to\infty \end{array}\right\}. \tag{3}$$

Integrating Eq. (2) with respect to z and then substituting the value of w in Eq. (1), we get the governing equation

$$u\frac{\partial u}{\partial x}-\frac{\partial u}{\partial z}\int_0^z\frac{\partial u}{\partial x}dz=\frac{\partial}{\partial z}\left(k\frac{\partial u}{\partial z}\right). \tag{4}$$

In order to linearize differential equation (4), we have the approximate expression

$$u=u_{z_1}\left(\frac{z}{z_1}\right)^p, \tag{5}$$

where p is a parameter relating to the thermodynamic stratification of the atmosphere and the roughness length of the underlying surface. The second term on the left-hand of Eq. (4) can be simplified as follows:

$$\frac{\partial u}{\partial z}\int_0^z\frac{\partial u}{\partial z}dz=\frac{pu_{z_1}}{z_1^p}z^{p-1}\frac{\partial}{\partial x}\int_0^z\frac{u_{z_1}}{z_1^p}z^p dz=\frac{pu_{z_1}}{z_1^p}z^{p-1}\frac{\partial}{\partial x}\left(\frac{u_{z_1}}{p+1}\frac{z^{p+1}}{z_1^p}\right)=\frac{p}{p+1}u\frac{\partial u}{\partial x}.$$

Then Eq. (4) becomes

$$\frac{1}{1+p}u\frac{\partial u}{\partial x}=\frac{\partial}{\partial z}\left(k\frac{\partial u}{\partial z}\right). \tag{6}$$

The turbulent exchange coefficient k may be, in general, expressed by Lahetman formula

$$k=\frac{\varepsilon\kappa^2 z_0^{2\varepsilon}u_{z_1}}{(1-\varepsilon)^2(z_1^\varepsilon-z_0^\varepsilon)}z^{1-\varepsilon}, \tag{7}$$

where u_{z_1} denotes the wind velocity at height z_1, ε the parameter relating to the thermodynamic stratification, and κ the Karman constant. Using expression (5), formula (7) can be written as

$$k=f(z)u, \tag{8}$$

where

$$f(z)=\frac{\varepsilon\kappa^2 z_0^{2\varepsilon} z_1^p}{(1-\varepsilon)^2(z_1^\varepsilon-z_0^\varepsilon)} z^{1-p-\varepsilon}. \tag{9}$$

Letting

$$U=u^2, \quad U_0=u_0^2, \quad V=U-U_0,$$

and using (8), Eq. (6) becomes

$$z^2\frac{\partial^2 V}{\partial z^2}+2(1-\alpha)z\frac{\partial V}{\partial z}-c^2 z^{2\alpha}\frac{\partial V}{\partial x}=F(z), \tag{10}$$

where

$$\alpha=\frac{1+P+\varepsilon}{2}, \quad C^2=\frac{(1-\varepsilon)^2(z_1^\varepsilon-z_0^\varepsilon)}{\varepsilon\kappa^2(1+P)z_0^{2\varepsilon}z_1^P},$$

$$F(z)=-\left[z^2\frac{\partial^2 U_0}{\partial z^2}+2(1-\alpha)z\frac{\partial U_0}{\partial z}\right]. \tag{11}$$

If the profile of wind velocity over land is expressed by Yugin-Shwetz (Gandin, 1955) model, then

$$U_0=\begin{cases}\left(\frac{k_1}{\kappa^2}\ln\frac{z}{z_{00}}\right)^2, & z_{00}\leqslant z\leqslant h\\ V_g^2+\frac{k_1^3}{\lambda h\kappa^4}e^{-2(z-h)\sqrt{\frac{\lambda}{2k_1h}}}-2V_g\frac{k_1}{\kappa^2}\sqrt{\frac{k_1}{\lambda h}}e^{-(z-h)\sqrt{\frac{\lambda}{2k_1h}}} & \\ \times\cos\left[(z-h)\sqrt{\frac{\lambda}{2k_1h}}+\frac{\pi}{4}-\theta\right], & z\geqslant h\end{cases} \tag{12}$$

where z_{00} is the surface roughness; V_g the velocity of geostrophic wind; $\lambda=2\omega\sin\varphi$ (ω being the angular velocity of the earth's rotation, and φ the latitude of the observation site); α the angle between the surface and geostrophic winds over land; and h the height of the surface boundary layer, which can be determined by

$$\frac{k_1}{\kappa^2}\ln\frac{h}{z_{00}}=V_g(\cos\alpha-\sin\alpha),$$

where k_1 is the coefficient of turbulent exchange at height z_1 over land. According to (7), it may be expressed as

$$k_1=\frac{\varepsilon_0\kappa^2 z_{00}^{2\varepsilon_0} z_1^{1-\varepsilon_0} u_{0z_1}}{(1-\varepsilon_0)^2(z_1^{\varepsilon_0}-z_{00}^{\varepsilon_0})}$$

where u_{0z_1} is the wind velocity at height z_1 over land and ε_0 the parameter related to the thermodynamic stratification over land.

Substituting (12) into (11), we get

$$F(z)=\begin{cases}0, & z<z_{00}\\ \frac{2k_1^2}{\kappa^4}\left[(2\alpha-1)\ln\frac{z}{z_{00}}-1\right], & z_{00}\leqslant z\leqslant h\\ \frac{2k_1^2}{\kappa^4h^2}z\left(\frac{1-\alpha}{a}-z\right)e^{-2a(z-h)}+\frac{2\sqrt{2}k_1V_g}{\kappa^2h}ze^{-a(z-h)} & \\ \times\{(az+\alpha-1)\sin[a(z-h)+\Phi]-(1-\alpha)\cos[a(z-h)+\Phi]\}, & z\geqslant h\end{cases} \tag{13}$$

where

$$a=\sqrt{\frac{\lambda}{2k_1h}}, \quad \Phi=\frac{\pi}{4}-\theta.$$

The boundary conditions (3) now becomes

$$\left.\begin{array}{l} v=0 \text{ on } x=0 \\ v=0 \text{ on } z=z_0 \\ v=0 \text{ as } z\to\infty \end{array}\right\}. \tag{14}$$

III. THE GENERAL SOLUTION OF THE PROBLEM

By using the Laplace transform formula

$$\bar{V}(s,z)=\int_0^\infty e^{-sx}V(x,z)dx, \tag{15}$$

it is easy to transform the partial differential equation (10) into an ordinary differential equation

$$z^2\frac{d^2\bar{V}}{dz^2}+2(1-a)z\frac{d\bar{V}}{dz}-C^2z^{2a}s\bar{V}=\frac{1}{s}F(z), \tag{16}$$

and the boundary conditions (14) are correspondingly transformed into

$$\left.\begin{array}{l} \bar{V}=0 \text{ on } z=z_0 \\ \bar{V}=0 \text{ as } z\to\infty \end{array}\right\}. \tag{17}$$

The first condition in (14) has been used in carrying out Laplace-transform, and Eq. (16) will satisfy itself automatically.

Eq. (16) is a boundary-value problem of the non-homogeneous equation with homogeneous boundary conditions. To attack the equation we introduce the Green function $G(y,z)$ for satisfying the homogeneous equation

$$\frac{d^2}{dz^2}(z^2G)-\frac{d}{dz}[2(1-a)zG]-C^2z^{2a}sG=0 \tag{18}$$

and the homogeneous boundary conditions

$$\left.\begin{array}{l} G=0 \text{ on } z=z_0 \\ G=0 \text{ as } z\to\infty \end{array}\right\}. \tag{19}$$

Eq. (18) is the conjugate equation of homogeneous equation

$$z^2\frac{d^2\bar{V}}{dz^2}+2(1-a)z\frac{d\bar{V}}{dz}-C^2z^{2a}s\bar{V}=0. \tag{20}$$

Multiplying (16) by G and (18) by $\bar{V}$, subtracting the latter from the former, integrating it from $z=z_0$ to $z=\infty$, and replacing the independent variable z with y, we obtain after some simple calculus

$$\left[Gy^2\frac{d\bar{V}}{dy}-\bar{V}\left(2ayG+y^2\frac{dG}{dy}\right)\right]_{y=z_0}^{y=z_{-0}}+\left[Gy^2\frac{d\bar{V}}{dy}-\bar{V}\left(2ayG+y^2\frac{dG}{dy}\right)\right]_{y=z_{+0}}^{y=\infty}$$
$$=\frac{1}{s}\int_{z_0}^\infty FGdy. \tag{21}$$

Now, assume that $G(y,z)$ is continuous throughout the interval $(z_0\leqslant y<\infty)$ and dG/dZ has a jump at $y=z$ with its degree $1/z^2$, namely,

$$[G(y,z)]_{y=z_{+0}}-[G(y,z)]_{y=z_{-0}}=0, \tag{22}$$

$$\left[\frac{dG}{dy}\right]_{y=z_{+0}}-\left[\frac{dG}{dy}\right]_{y=z_{-0}}=\frac{1}{z^2}. \tag{23}$$

Besides, $\bar{V}$ is continuous throughout the interval $(z_0\leqslant z<\infty)$, having derivatives no less than the second order and satisfying homogeneous conditions (17). Thus we obtain

$$\bar{V}=\frac{1}{s}\int_{z_0}^{\infty} F(y)G(y,z)dy. \tag{24}$$

Now we have come to work out the expression of $G(y,z)$ from the characters of the Green function stated above. Replacing the independent variable z with y, Eq. (19) can be written as

$$y^2\frac{d^2G}{dy^2}+2(1+a)y\frac{dG}{dy}+(2a-C^2sy^{2a})G=0. \tag{25}$$

This is the Bessel equation, its two solutions being

$$y^{-\frac{2a+1}{2}}I_\nu(Y) \text{ and } y^{-\frac{2a+1}{2}}K_\nu(Y),$$

where

$$\nu=\frac{2a-1}{2a},\ Y=\frac{C\sqrt{s}\,y^a}{a}, \tag{26}$$

$I_\nu(Y)$ is the Bessel function of pure argument, and $K_\nu(Y)$ the modified Bessel function of the second kind.

According to the characters of the Green function, we may assume

$$G(y,z)=\begin{cases} a_1(z)y^{-\frac{2a+1}{2}}I_\nu(Y)+b_1(z)y^{-\frac{2a+1}{2}}K_\nu(Y), & z_0\leqslant y\leqslant z \\ a_2(z)y^{-\frac{2a+1}{2}}I_\nu(Y)+b_2(z)y^{-\frac{2a+1}{2}}K_\nu(Y), & z\leqslant y<\infty \end{cases} \tag{27}$$

Having determined the integral constants a_1, b_1 and a_2, b_2 from conditions (20), (22) and (23), we obtain the expression of G as follows:

$$G(y,z)=\begin{cases} z^{\frac{2a-1}{2}}y^{-\frac{2a+1}{2}}v(Z,Y), & z_0\leqslant y\leqslant z \\ z^{\frac{2a-1}{2}}y^{-\frac{2a+1}{2}}v(Y,Z), & z\leqslant y<\infty \end{cases} \tag{28}$$

where

$$V(\xi,\eta)=\frac{K_\nu(\xi)[K_\nu(z_0)I_\nu(\eta)-I_\nu(z_0)K_\nu(\eta)]}{aK_\nu(z_0)} \tag{29}$$

$$z=\frac{C\sqrt{s}\,z^a}{a},\ z_0=\frac{C\sqrt{s}\,z_0^a}{a}. \tag{30}$$

Thus, from Eq. (24), the solution of Eq. (10) can be written as

$$\bar{V}(s,z)=z^{\frac{2a-1}{2}}\Big[\int_{z_0}^{z} y^{-\frac{2a+1}{2}}F(y)\bar{\varphi}_1(s,y,z)dy +\int_{z}^{\infty} y^{-\frac{2a+1}{2}}F(y)\bar{\varphi}_2(s,y,z)dy\Big], \tag{31}$$

where

$$\left.\begin{aligned} \bar{\varphi}_1(s,y,z)&=\frac{V(Z,Y)}{s} \\ \bar{\varphi}_2(s,y,z)&=\frac{V(Y,Z)}{s} \end{aligned}\right\}. \tag{32}$$

$\bar{\varphi}_j(s,y,z)(j=1,2)$ in (31) may be considered as the Laplace transform of function $\varphi_j(x,y,z)$. According to the inversion formula of the Laplace transform, the original function may be expressed as

$$\varphi_j(x,y,z)=\frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty}e^{sx}\bar{\varphi}_j(s,y,z)\,ds. \tag{33}$$

Now we have come to the inversion problem of $\bar{\varphi}_j(s,y,z)$ expressed by (33), viz, the determination of the original function $\varphi_j(x,y,z)$ of $\bar{\varphi}_j(s,y,z)$. In order to get a one-value solution we make a cut along the negative real axis. It is proved that $\bar{\varphi}_j$ are regular functions in the whole complex plane except at $s=0$. Thus, as shown in Fig. 1, we may add an integral line σ (KA) on the contour AB...HK, and regard ABC and GHK of the contour as infinitely distant, and the radius of circle DEF as infinitely small.

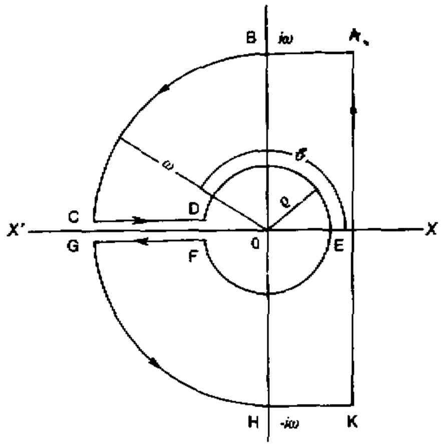

Fig. 1. Configuration of integral contour.

Since there are no single points within the contour, we may transform the integral path on the right side of (33) into AB...CK. According to Cauchy's theorem, the integral of regular function $e^{sx}\bar{\varphi}_j(s,y,z)$ along this closed contour is equal to zero.

Now, let us consider the case of the integral on different sections of the contour in Fig.1 ($x>0$) as the radius ω approaches to ∞ and the radius ρ of the small circle approaches to 0.

When variable z is large, $I_\nu(z)$ and $K_\nu(z)$ can be expressed by the asymptotic expansions

$$I_\nu(z)\approx\sqrt{\frac{1}{2\pi z}}e^{z},\qquad K_\nu(z)\approx\sqrt{\frac{\pi}{2z}}e^{-z}. \tag{34}$$

Now, it is easy to prove that $\bar{\varphi}_j$, just like $e^{-c\sqrt{s}}$, will approach to 0 when $|s|\to\infty$, and therefore the integrals along arcs BC and GH and along lines AB and HK will all approach to 0 with $\omega\to\infty$ when $x\to 0$.

When z is small, $I_\nu(z)$ and $K_\nu(z)$ may be represented by the approximate expressions

$$\left.\begin{aligned}&I_\nu(z)\approx\frac{z^\nu}{2^\nu\Gamma(1+\nu)}\\&K_\nu(z)\approx\frac{\Gamma(\nu)}{2}\left(\frac{z}{2}\right)^{-\nu}\left[1-\frac{\Gamma(1-\nu)}{\Gamma(1+\nu)}\left(\frac{z}{2}\right)^{2\nu}\right]\end{aligned}\right\}, \tag{35}$$

where $\Gamma(n)$ is the gamma function.

Placing $s=\rho e^{i\theta}$ and changing the rectangular coordinates into polar coordinates, it is found that the integral along the small circle DEF is M_j $(j=1,2)$ when its radius becomes infinitely small and that

$$\left.\begin{aligned} M_1 &= (y^{2a-1}-z_0^{2a-1})/(2a-1)\, y^{\frac{2a-1}{2}}\, z^{\frac{2a-1}{2}} \\ M_2 &= (z^{2a-1}-z_0^{2a-1})/(2a-1)\, y^{\frac{2a-1}{2}}\, z^{\frac{2a-1}{2}} \end{aligned}\right\}. \tag{36}$$

Now, let us consider the integral along lines CD and FG

$$\int_{-\omega}^{0} e^{sx}\overline{\varphi}_j ds \text{ and } \int_{0}^{\omega} e^{sx}\overline{\varphi}_j ds.$$

As CD and FG are assumed infinitely to approach to the negative real axis OX', we can put $s=e^{\pm\pi i}r^2$, $\sqrt{s}=e^{\pm\frac{\pi i}{2}}r$, with the argument of s being $+\pi$, CD above OX'and the argument being $-\pi$, FG below OX'. Thus, the sum of integrals along CD and FG is

$$\lim_{\omega\to\infty}\frac{1}{2\pi i}\int_{-\omega}^{0} e^{sx}\overline{\varphi}_j(s,y,z)ds+\lim_{\omega\to\infty}\frac{1}{2\pi i}\int_{0}^{-\omega} e^{sx}\overline{\varphi}_j(s,y,z)ds$$

$$=\frac{1}{\pi i}\int_{0}^{\infty} e^{-r^2x}\{[\overline{\varphi}_j]_{\sqrt{s}=ir}-[\overline{\varphi}_j]_{\sqrt{s}=-ir}\}rdr.$$

Finally, when $\omega\to\infty$ the integral along line KA is

$$\lim_{\omega\to\infty}\frac{1}{2\pi i}\int_{\sigma-i\omega}^{\sigma+i\omega} e^{sx}\overline{\varphi}_j(s,y,z)ds=\frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty} e^{sx}\overline{\varphi}_j(s,y,z)ds=\varphi_j(s,y,z).$$

Hence the integral along the contour AB...KA is

$$\lim_{\substack{\omega\to\infty\\ \rho\to 0}}\frac{1}{2\pi i}\int_{AB\cdots KA} e^{sx}\overline{\varphi}_j(s,y,z)ds=M_j+\frac{1}{\pi i}\int_{0}^{\infty}e^{-r^2x}$$
$$\times\{[\overline{\varphi}_j]_{\sqrt{s}=ir}-[\overline{\varphi}_j]_{\sqrt{s}=-ir}\}rdr+\varphi_j(x,y,z)=0.$$

It follows that

$$\varphi_j(x,y,z)=-M_j-\frac{1}{\pi i}\int_{0}^{\infty}e^{-r^2x}\{[\overline{\varphi}_j]_{\sqrt{s}=ir}-[\overline{\varphi}_j]_{\sqrt{s}=-ir}\}rdr. \tag{37}$$

By use of the relationships for the Bessel function

$$\left.\begin{aligned} K_\nu(\pm iy) &= -\frac{\pi}{2}e^{\mp\frac{1}{2}\nu\pi i}[N_\nu(y)\pm iJ_\nu(y)] \\ I_\nu(\pm iy) &= e^{\pm\frac{1}{2}\nu\pi i}J_\nu(y) \end{aligned}\right\}, \tag{38}$$

where $J_\nu(y)$ is the Bessel function and $N_\nu(y)$ the Neumann function, we get

$$\left.\begin{aligned} \varphi_1(x,y,z) &= -M_1-\frac{1}{a}\int_{0}^{\infty}e^{-r^2x}\Phi(y,z,r)dr \\ \varphi_2(x,y,z) &= -M_2-\frac{1}{a}\int_{0}^{\infty}e^{-r^2x}\Phi(y,z,r)dr \end{aligned}\right\}, \tag{39}$$

where

$$\Phi(y,z,r)=\frac{\left[J_\nu\left(\frac{Crz_0^a}{a}\right)N_\nu\left(\frac{Cry^a}{a}\right)-N_\nu\left(\frac{Crz_0^a}{a}\right)J_\nu\left(\frac{Cry^a}{a}\right)\right]}{\left[N_\nu^2\left(\frac{Crz_0^a}{a}\right)\right.}$$

$$\frac{\left[J_\nu\left(\frac{Crz_0^a}{a}\right)N_\nu\left(\frac{Crz^a}{a}\right)-N_\nu\left(\frac{Crz_0^a}{a}\right)J_\nu\left(\frac{Crz^a}{a}\right)\right]}{+J_\nu^2\left(\frac{Crz_0^a}{a}\right)\Big]r} \tag{40}$$

According to (31) we obtain the original function of $\bar{V}(s,z)$

$$V(x,z)=z^{\frac{2a-1}{2}}\left[\int_{z_0}^{z} y^{-\frac{2a+1}{2}}F(y)\varphi_1(x,y,z)dy+\int_z^\infty y^{-\frac{2a+1}{2}}F(y)\varphi_2(x,y,z)dy\right]. \tag{41}$$

Thus

$$u(x,z)=[u_0^2(z)+V(x,z)]^{1/2},$$

or

$$\frac{u(x,z)}{u_0(z)}=\left[1+\frac{V(x,z)}{u_0^2(z)}\right]^{1/2}. \tag{42}$$

IV. THE POSSIBLE MAXIMUM VELOCITY OF AN AIRFLOW PASSING OVER WATER

According to the Laplace-transform formula, a larger x is corresponding to a smaller s, and hence $s\to 0$ as $x\to\infty$. In view of the character of function $K_\nu(y)$, the expression of $F(z)$ and the values of p,ε and a, which might appear in reality, it is seen by careful analysis that the principal value of integral $\int_z^\infty y^{-\frac{2a+1}{2}}F(y)\bar{\varphi}_2(s,y,z)dy$ is determined by the value of the integral from z to H (H is a limited height), and that the variable $\frac{C\sqrt{s}\,y^a}{a}$ in function $K_\nu\left(\frac{C\sqrt{s}\,y^a}{a}\right)$ is practically very small when s is infinitesimal. Thus, the Bessel functions $K_\nu(z)$ and $I_\nu(z)$ can be represented by the approximate expression (35) when x is very large, i. e., s is very small. In this case, after some calculus, Eq. (31) can be written as

$$\bar{V}(s,z)=\frac{1}{2a-1}\Big[\int_{z_{00}}^{z}F(y)(y^{-1}-z_0^{2a-1}y^{-2a})\bar{X}_1(s,z)dy$$

$$+(z^{2a-1}-z_0^{2a-1})\int_z^\infty F(y)y^{-2a}\bar{X}_2(s,y)dy\Big], \tag{43}$$

where

$$\bar{X}_1(s,z)=\frac{1}{s}-b(z^{2a-1}-z_0^{2a-1})\frac{1}{s^{1-\nu}}-b^2z_0^{2a-1}z^{2a-1}\frac{1}{s^{1-2\nu}},$$

$$\bar{X}_2(s,y)=\frac{1}{s}-b(y^{2a-1}-z_0^{2a-1})\frac{1}{s^{1-\nu}}-b^2z_0^{2a-1}y^{2a-1}\frac{1}{s^{1-2\nu}},$$

$$b=\frac{\Gamma(1-\nu)}{\Gamma(1+\nu)}\left(\frac{C}{2a}\right)^{2\nu}.$$

By use of the corresponding relationship of the Laplace-transformation

$$s\bullet\!\!-\!\!\circ\, 1,\quad \frac{1}{s^{1-\nu}}\bullet\!\!-\!\!\circ\,\frac{x^{-\nu}}{\Gamma(1-\nu)},\quad \frac{1}{s^{1-2\nu}}\bullet\!\!-\!\!\circ\,\frac{x^{-2\nu}}{\Gamma(1-2\nu)},$$

we get at once the original functions of $\bar{X}_j(s,z)$ $(j=1,2)$

$$\left.\begin{aligned}X_1(x,z)&=1-b(z^{2a-1}-z_0^{2a-1})\frac{x^{-\nu}}{\Gamma(1-\nu)}-b^2z_0^{2a-1}z^{2a-1}\frac{x^{-2\nu}}{\Gamma(1-2\nu)}\\X_2(x,y)&=1-b(y^{2a-1}-z_0^{2a-1})\frac{x^{-\nu}}{\Gamma(1-\nu)}-b^2z_0^{2a-1}y^{2a-1}\frac{x^{-2\nu}}{\Gamma(1-2\nu)}\end{aligned}\right\}, \tag{44}$$

and hence the original function of $\bar{V}(s,z)$ is

$$V(x,z)=\frac{1}{2\alpha-1}\Big[X_1(x,z)\int_{z_{00}}^{z}F(y)\,(y^{-1}-z_{00}^{2\alpha-1}y^{-2\alpha})\,dy$$

$$+(z^{2\alpha-1}-z_0^{2\alpha-1})\int_{z}^{\infty}F(y)\,y^{-2\alpha}X_2(x,y)\,dy\Big]. \tag{45}$$

Since $x_1=x_2=1$ as $x\to\infty$, the possible maximum velocity $u(\infty,z)$ of an airflow passing over water is determined by

$$V(x,z)=\frac{1}{2\alpha-1}\Big[\int_{z_{00}}^{z}F(y)\,(y^{-1}-z_{00}^{2\alpha-1}y^{-2\alpha})\,dy+(z^{2\alpha-1}-z_0^{2\alpha-1})\int_{z}^{\infty}F(y)\,y^{-2\alpha}dy\Big]. \tag{46}$$

V. THE RESULTS OF COMPUTATION

It is well-known that there are remarkable changes in the thermal stratification of the air over land as a result of the great daily change in the temperature near the ground surface. During the daytime heating of ground surface, the air over land is, in general, unstable and $\varepsilon_0<0$; during nighttime when the surface is cooled, the air over land is stable and $\varepsilon_0>0$. Because of the greatly reduced range of temperature over water surface, the diurnal change of air stability over water is not so marked and $\varepsilon\approx0$. In addition, the roughness z_0 of water surface is a function of wind velocity (Yan, 1982), and the stronger the wind, the larger the roughness. On this account, we have, by using formula (42), computed the variations of wind velocity with distance x when an airflow moves over water for different ε_0, z_0 and z_{00} (taking $\varphi=30$, $\theta=24$, $h=30$ m, and $p=0.09$ as $z_0=0.01$ cm, $p=0.13$ as $z_0=0.6$ cm). The results are shown in Figs. 2—8.

It is seen from Figs. 2 and 3 that, in neutral equilibrium ($\varepsilon_0=0$), the increase in airflow velocity over water occurs mainly in the 2—3 km from the upwind edge. It gradually becomes moderate with increased distance and the change is imperceptible beyond 10 km. It is also found that the smaller the distance to the water surface and the weaker the wind (thus the smaller the roughness z_0), the steeper the increase in wind velocity over water. When the roughness z_{00} of the ground surface is 3 cm, the wind velocity at $x=10$ km and $z=1$ m may increase by more than 50% in light winds ($z_0<0.01$ cm) over water than over land, but only by 14% in strong winds ($z_0>0.8$ cm); at the height of 10 m above the water surface, the increase in wind velocity is less than 15% even in light winds. With $z_{00}=9$ cm, the wind velocity at 1 m above the water surface may increase by more than 80% in light winds, and by about 40% in strong winds. These figures are in good agreement with the observations of ours in the upper basin of the Changjiang River, where, with the surface roughness z_{00} of the land surface being about 3 cm on the average, the increasing variation in wind velocity is found mainly in the first two kilometers of the distance covered by the airflow over water, and the stationary wind velocity at 1 m above the water surface may increase by 55—60% over land in light winds.

Fig. 4 shows that, when the thermal stratification is stable over land, both the increase in wind velocity and the distance of its variation over water are much greater than at neutral stability. For $z_{00}=3$ cm, the wind velocity over water at $x=10$ km and $z=1$ m may increase 3.8 times as much as that over land, and even at 10 m above the water surface the wind velocity is still 60-90% greater than that over land. The increment has a larger value in strong winds (large z_0) than in light winds (small z_0), for in high winds the strong turbulent exchange is more favourable to the transfer of momentum of the fast-moving air from an upper layer to a lower layer, thus making the effect of increasing the wind velocity at a distance above the water surface exceed the effect of decreasing the wind velocity due to the increase of water surface roughness. All the above calculations also coincide with the observations made at Dazong Lake near Xinghua conuty of Jinagsu Province in October, 1982.

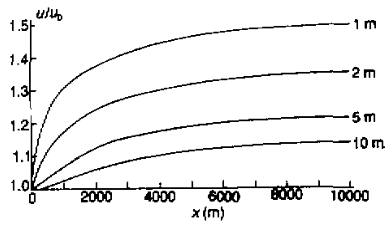

Fig. 2. Variation of wind velocity over water with distance x from the upwind edge when $\varepsilon_0=0$, $z_0=0.01$ cm and $z_{00}=3$ cm.

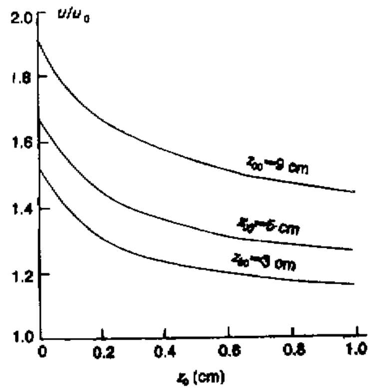

Fig. 3. Variation of wind velocity over water with z_0 and z_{00} at $x=10$ km and $z=1$ m when $\varepsilon_0=0$.

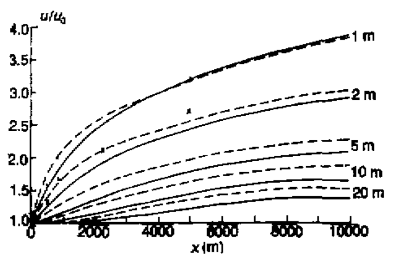

Fig. 4. Variation of wind velocity over water with distance x from the upwind edge when $\varepsilon_0=0.4$.
Solid lines—$z_0=0.01$ cm, $z_{00}=3$ cm; Dashed lines—$z_0=0.6$ cm, $z_{00}=3$ cm. The signs dot and asterisk denote the observational data at 1 and 2m above the water surface of Dezong Lake respectively when $\varepsilon_0=0.5$, $z_0=0.66$ cm, $z_{00}=3.2$ cm.

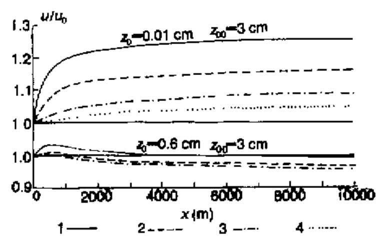

Fig. 5. Variation of wind velocity over water with distance x from the upwind edge when $\varepsilon_0=-0.1$.
Curves 1, 2, 3, and 4 represent the cases for z being 1, 2, 5 and 10 m, respectively.

It is shown in Fig. 5 that, when the thermal stratification over land is unstable, in the case of light winds ($z_0=0.01$ cm) over a water surface whose small roughness leads to an increase in wind velocity exceeding the disadvantageous retarding effect of the thermal stratification, the velocity of an airflow moving over water also increases somewhat, but the distance of wind variation is much smaller than that in neutral stability, mainly within 1—2 km. With $z_{00}=3$ cm and $x=10$ km, the wind velocity may be 25% greater over water than over land at the height of 1 m, and only 5% greater at 10 m. In strong winds ($z_0=0.6$ cm), however, the velocity of

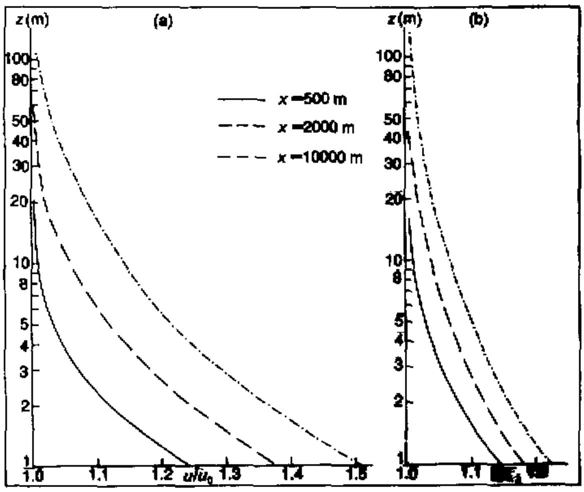

Fig. 6. Vertical variation of wind velocity over water when $\varepsilon_0=0$ (in semi-logarithmic system of coordinates)
(a) $z_0=0.01$ cm, $z_{00}=3$ cm; (b) $z_0=0.6$ cm, $z_{00}=3$ cm.

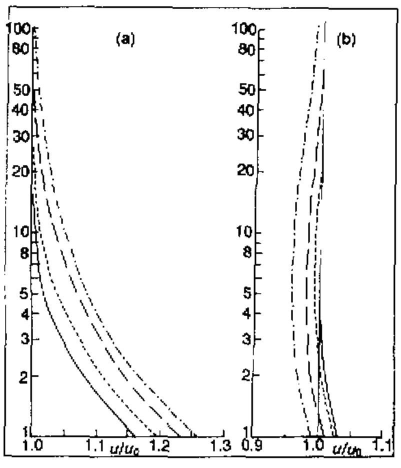

Fig. 7. Vertical variation of wind velocity over water when $\varepsilon_0=0.4$ (in semi-logarithmic system of coordinates).
(a) $z_0=0.01$ cm, $z_{00}=3$ cm;
(b) $z_0=0.6$ cm, $z_{00}=3$ cm.

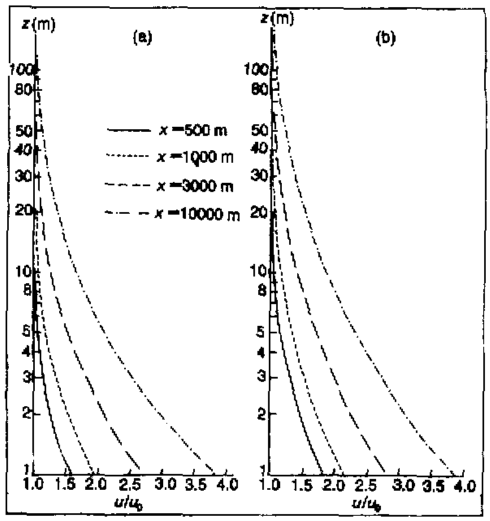

Fig. 8. As in Fig. 7, except for $\varepsilon_0=-0.1$.

airflow passing over water decreases slowly because the effect of thermal stratification exceeds that of water surface roughness. With $z_{00}=3$cm and $x=10$ km, the wind velocity at heights of 2—10 m may decrease by 3—4% than that over land, but at the height of 1 m it still increases somewhat with x in the distance of 500 m and becomes much smaller than over land after covering a distance of 5 km.

Figs. 6—8 show that the effect of water bodies on wind velocity decreases sharply with heights above the water surface, the most pronounced variation occurring below the level of 10—20 m. It is found that the more stable the atmosphere remains and the farther the airflow has moved over water, the greater the water effect will become.

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