THE QUANTITATIVE GROWTH LAW OF ICE CRYSTALS AND ITS NEW MODEL

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ABSTRACT

An improved new wedge-shaped chamber of ice thermal diffusion has been used to get a more complete and overall growth law of ice crystals. Based on more than 4,000 data, a quantitative growth law of ice crystals in an ice supersaturation and temperature field has been presented. A new method for quantitatively indicating the form of different kinds of ice crystals has been suggested. The growth rates of ice crystals at different temperatures and ice supersaturations have been studied. A quantitative comparison of static and dynamic experiments for ice crystal growth has also been presented. Finally, the author gives new models of ice crystal growth law in temperature-ice supersaturation or vapor density excess field.

I. INTRODUCTION

In cloud physics and weather modification, the growth and shape variation of ice crystals under different environmental conditions are the most basic and important problems. In atmospheric physics, the behavior development and growth law of ice crystals are relevant to the evolution of cloud and the precipitation process; and they affect the optical radiative and electrical properties of cloud and atmosphere. Because these problems are very important, a number of scientists in atmospheric physics, for example, Nakaya (1954; 1951); Hallett and Mason (1958); Kobayashi (1961); Fukuta (1969); Hobbs (1974); Pruppacher et al. (1978) engaged in investigations of ice crystal growth in laboratories and made considerable progress. Among the above work, the result obtained by Kobayashi (1961) has been frequently quoted. It must be pointed out that certain flaws remain still in the widely recognized Kobayashi-Pruppacher model (Wang Angsheng and Fukuta, 1985b), and we must do some experiments to modify and improve it.

Since 1981, we have been using a cloud chamber and have obtained many results, which have been published in proceedings or journals (Wang Angsheng and Fukuta, 1984a; Wang Angsheng, 1984; Fukuta and Wang Angsheng, 1984; Fukuta, Gong Naifu and Wang Angsheng, 1984; Wang Angsheng and N. Fukuta, 1984b; Wang Angsheng and Fukuta 1985a, 1985b; Wang Angsheng 1985). The chamber has stable environmental conditions and can easily measure 3-dimension ice crystals. The experiments were done under different ice supersaturations (from 0 to 25%), different temperatures (from 0 to –30°C) and longer periods of time (50 minutes or more). Our summation of the above works will be given in this paper.

II. THE QUANTITATIVE CHARACTERISTICS OF ICE CRYSTAL GROWTH

Although Hanajima (1949), Nakaya (1951), Hallett and Mason (1958), Kobayashi (1961) and others have already completed excellent qualitative research on growth characteristics of ice crystals, little quantitative research on ice crystal growth has been done. Thus this work is important in that it would greatly contribute to our understanding of the growth law of ice crystals.

The main difficulties in quantitative research of ice crystals lie in the facts that stable environmental conditions (for example, temperature and supersaturation) are not easy to establish, and that the three-dimensional size of ice crystals could not be measured at the same time in the microscope etc. In our experiments, we had the above conditions and obtained a lot of very useful quantitative data about ice crystal growth. The main results of our work will be given in the following sections.

Let us call the diameter of the longest dimension of the ice crystal basic plane $2a$, and the height of the ice crystal prism plane c. The definitions of $2a$ and c are the same as in previous works. Based on about 500 data of $2a$ and c, we obtained some quantitative characteristics of ice crystal growth (see Figs. 1 and 2).

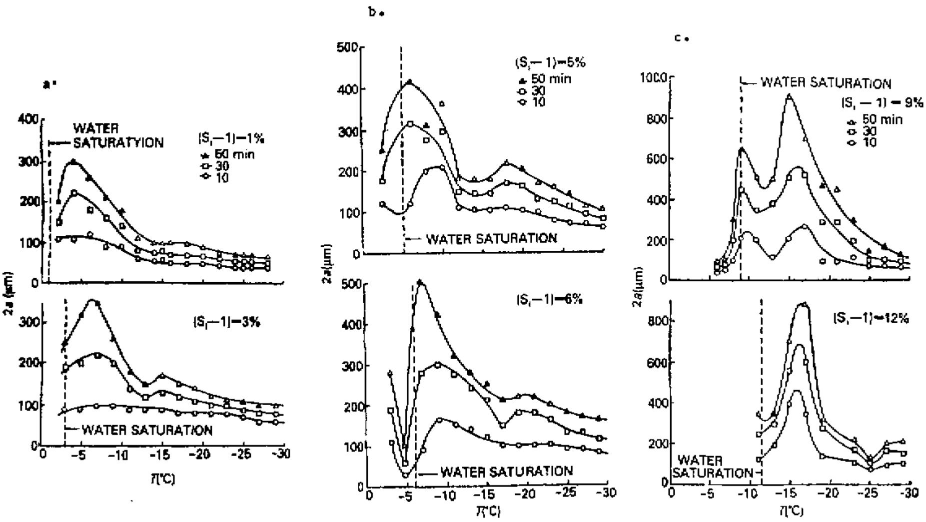


Fig. 1. Ice crystal diameter $2a$ plotted as a function of temperature at different growth periods under (a) 1 and 3%; (b) 5 and 6% ; (c) 9 and 12% ice supersaturation.

1. *The 2a Growth Behavior under Different Temperatures and Ice Supersaturations*

In Fig. 1, we have 6 groups of data, whose growth behaviors of $2a$ were under 1,3,5,6,9 and 12% ice supersaturations respectively. Ice crystal diameter $2a$ is plotted as a function of temperature at different growth periods under different ice supersaturations. Based on these data, we have the following results:

The crystal diameter $2a$ of all ice crystals at different temperatures (from 0 to -30°C) and ice supersaturations (S_i-1) (from 1 to 12%) grows with time, but its growth rates are different. We can find that there are one or two peak values at one ice supersaturation. For example, when ice supersaturation is 1%, the peak value of $2a$ is found at -4°C, and the maximum of $2a$ is 300 μm at 50 min. As the ice supersaturation increases,

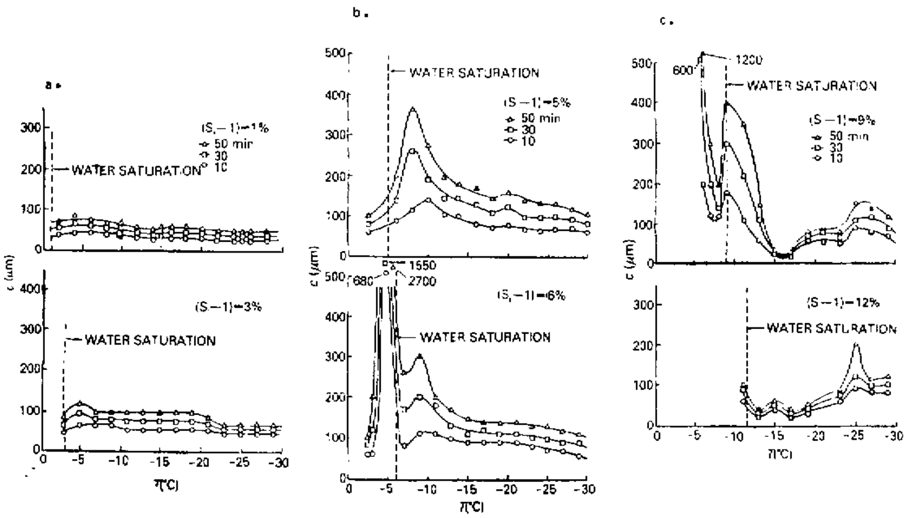


Fig. 2. Ice crystal height c plotted as a function of temperature at different growth periods under (a) 1 and 3%; (b) 5 and 6%; (c) 9 and 12% ice supersaturation.

the peak value of $2a$ increases and its temperature drops down. For example, when ice supersaturation is 6%, the peak value of $2a$ increases to 500 μm and its temperature drops down to -7°C. Generally, when ice supersaturation is lower than 6–7%, the peak value of $2a$ is found near the temperature of water saturation. For example, when (S_i-1) is 5%, the peak value of $2a$ is found at -6°C, which is near the temperature of water saturation -5°C. However, when (S_i-1) is higher, the peak of $2a$ is found near -15°C. For example, at $(S_i-1)=9\%$, the peak of $2a$ is found at -15°C and the peak value of $2a$ is 900 μm.

In Fig. 1, we can see that when both (S_i-1) and temperature are lower, the growth rate is lower. For example, under $(S_i-1)=1$ and 3%, and with the temperature decreasing from 0 to -30°C, the growth rates are the lowest in our data. When (S_i-1) is higher, the growth rate of $2a$ is higher, the peak value of $2a$ is higher, and the width of the peak region is bigger; e.g., when $(S_i-1)=9\%$, the peak of $2a$ is 900 μm and the width of the peak region, in which the value of $2a$ is bigger than 400 μm, is from -8°C to -20.6°C. That is very wide.

2. *The c Growth Behavior under Different Temperatures and Ice Supersaturations*

In Fig. 2, we have 6 groups of data on c growth behavior, which correspond to Fig. 1. The ice crystal height c plotted as a function of temperature at different growth periods under 1, 3, 5, 6, 9 and 12% ice supersaturations has been shown in Fig. 2. Some characteristics of c growth are found as follows:

The crystal height c of all ice crystals at different temperatures (from 0 to -30°C) and different (S_i-1) grows with time, but some crystal heights c grow rapidly under certain conditions [e.g., $(S_i-1)=6\%$ and $T=-5$°C].

When (S_i-1) is 1 to 3% and T is from 0 to -30°C, c grows very slowly, and its curve has no peak (see Fig. 2a). This is an interesting phenomenon that is in close relation with Wulff's law, and will be discussed further below. At $(S_i-1)=5\%$, the peak value of c is 365 μm, T is $-8°C$. When (S_i-1) is 6, 9% or more, c grows rapidly with maximum values bigger than 1,000 μm (time = 50 min), and we can find a second maximum near $T=-10°C$ (see Fig. 2b, bottom; Fig. 2c, top). Another interesting phenomenon is a peak at low temperature ($-25°C$) and another long prism region.

3. *The 2a and c Fields under Different Ice Supersaturations and Temperatures*

Based on the above data of $2a$ and c, and other data (the total number of data at 50 minutes is about 330), $2a$ and c fields are shown in Fig. 3a and b respectively. In these fields, (S_i-1) is from 0 to 25%, and T is from 0 to $-30°C$.

In Fig. 3a, we can see the above results easily. There is a maximum region of $2a$ in Fig. 3a. The maximum value of $2a$ is bigger than 2,000 μm, the temperature region is from -13.5 to $-16.5°C$ and (S_i-1) is larger than 16.5%. There are two minimum regions in the $2a$ field, and their $2a$ values are smaller than 100 μm. One is in lower T and at the lowest (S_i-1); the other region is from -4 to $-7°C$ and at $(S_i-1)>6\%$. In another region, which is between the maximum and minimum regions, $2a$ changes gradually.

In Fig. 3b, the above results for c can also be seen easily. The minimum value is smaller than 50μm, and the maximum value is bigger than 1,000 μm. The minimum region is at about 10% ice supersaturation and from about -12.5 to $-21°C$; and the maximum region of c is at $(S_i-1)>6\%$ and from -4 to $-6°C$. The value of c in other regions changes gradually.

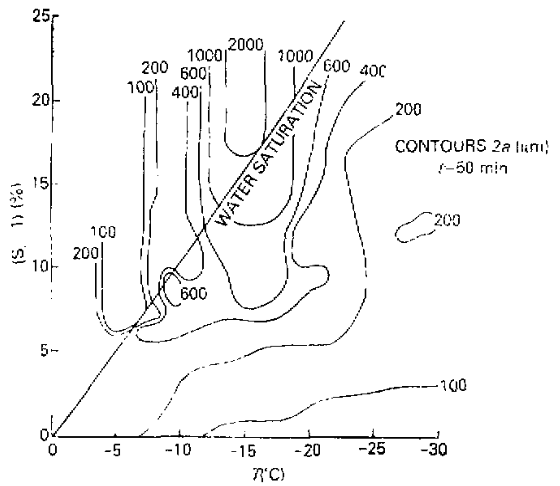
III. THE CHARACTERISTICS OF ICE CRYSTALS IN THE LOW (S_i-1) REGION

Owing to the work of Nakaya (1951, 1954); Mason (1957), and others, qualitative characteristics of ice crystal growth in the higher (S_i-1) region were clearly understood before 1960, when qualitative data in low ice supersaturation regions had not been obtained. Scientists needed to understand more and more complete characteristics of ice crystal growth, and therefore some scientists tried to do experiments in the low ice supersaturation region. In 1961, Kobayashi first finished this kind of work and obtained famous results. But there are still some questions that need to be resolved (see Introduction). Some scientists were engaged in this work, but they failed due to its difficulties. In 1974, Rottner and Vali (1974) successfully obtained a few data in the low ice supersaturation region from -8 to $-24°C$, only their data were qualitative.

In our experiments, we successfully obtained a great number of data in the low ice supersaturation region from 0 to $-30°C$. These are quantitative data. Figures 1a, 2a, 3a and3b show some of the results, and some data obtained in very low ice supersaturations, for example 1 or 3%. We can find that, in low ice supersaturation regions, $2a$ and c grow slowly, especially in lower temperature regions. At 50 minutes, $2a$ is less than 200 μm, c is less than 150 μm when (S_i-1) is lower than 5% and temperature is lower than $-15°C$. That means that in this region, $2a$ and c are close to each other, and their values change with the temperature.

$c/2a$ plotted as a function of temperature under different ice supersaturations(e.g., 1, 3 and 6%) is shown in Fig. 4. We can find that the $c/2a$ value is close to 0.7 when T is from -15 to $-30°C$, and ice supersaturation is from 1 to 6%. According to our data, the $c/2a$ value is near 0.7, which is different from Kobayashi's data, but close to the Wulff's theorem.

It is well known that the ice crystal form is mainly decided by ice crystal growth charac-



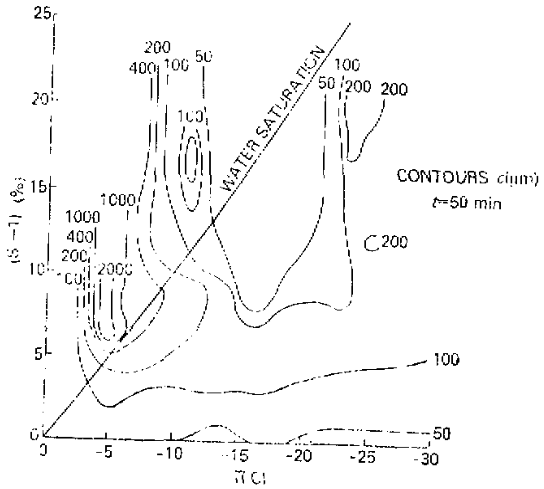


Fig. 3. The variation of ice crystal diameter $2a$ (a, upper) and height c (b, lower) as a function of temperature and ice supersaturation at 50 minutes of growth.

teristics ($2a/c$). According to Wulff's theorem, in an equilibrium, the distance from any crystal surface to the crystal center is proportional to the surface tension at that surface. Thus, for an ice crystal in equilibrium, Wulff s theorem predicts that: $c/2a=0.81$.

Under low ice supersaturation, it does not belong to an equilibrium. Therefore when an ice crystal grows at very lower ice supersaturation, this saturation is close to equilibrium, and the ($c/2a$) value will be close to but not equal to 0.81. Fig. 4 shows that the ($c/2a$) value is close to 0.7. When T is close to -15°C, $c/2a$ is about 0.6; and when T is close to -30°C, $c/2a$ is about 0.7—0.75.

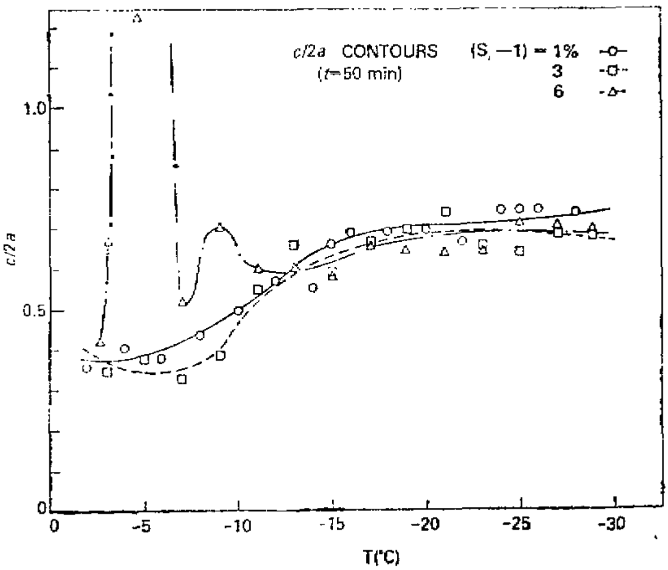


Fig. 4. $c/2a$ plotted as a function of temperature under different lower (S_i-1) regions (1, 3 and 6%).

IV. A QUANTITATIVE COMPARISON BETWEEN STATIC AND DYNAMIC EXPERIMENTS

As mentioned in the Introduction, there are two kinds of experimental studies of ice crystal. They are static and dynamic experiments with different characteristics for studies of ice crystal growth.

In general, static experiments have a wide range of T and (S_i-1) and can be used to get an overall law of ice crystal growth, especially at higher and lower (S_i-1). In static experiments, the conditions are more stable, and thus we can work for a longer time and continually get data, which is very difficult in dynamic experiments. However, for static experiments the ice crystal must be hung in a nylon fiber or spider web, there exist so some differences from the natural situation in this experiment.

The dynamic experiments are closer to natural conditions. Usually, the ice crystal is hung in air flow (updraft). In dynamic experiments the rimed phenomenon of ice crystals can be studied. In this situation, an ice crystal grows rapidly. But we can do dynamic experiments only under water saturation, so that its experimental range is very narrow. Due to the effect of updraft stability, the dynamic experimental time is usually shorter than a few minutes. In our laboratory, longer as it is, the time of dynamic experiment was merely about 20 minutes. In addition, it is difficult to obtain three-dimensional data of ice crystals in laboratory.

Although there are some differences between static and dynamic experiments, many results of ice crystal growth are the same as or close to each other, especially for the ice crystal form. But as few quantitative results of this kind were available, a quantitative comparison could not be made. Now, according to our data and other recent data, we can make some comparisons as follows:

In 1974, Yamashita (1974) obtained some dynamic experimental quantitative results in the region of -4.8 to $-30°C$ under water saturation and at 200 seconds. Figure 5 shows a quantitative comparison of our static data with Yamashita's data. Yamashita's data was obtained under water saturation. Although the growing time is not the same, under the close temperature the ratio of $2a$ to c, which decides the ice crystal form, is close, e.g., in Yamashita's data, $c/2a$ equals 0.67—1.0; our data equals 0.9 (10 min) and 0.62 (50 min) at $-9°C$. That means the results of the above two experiments are close to each other. Of course, due to the conditions, the quantitative and qualitative results have some differences, but the tendency is the same.

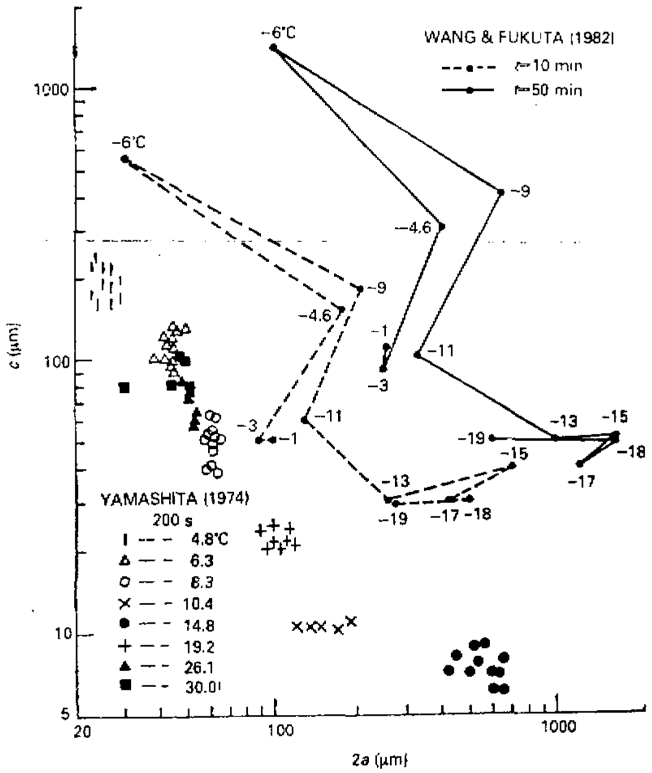


Fig. 5. The quantitative comparison of A.S. Wang and Fukuta's static data (1982, $t=50$ min) with Yamashia's dynamic data (1974).

Figure 6 shows another quantitative comparision. The static experimental data was obtained from Wang Angsheng and Fukuta's results. The dynamic experimental data were obtained from Gong and Fukuta's results. From Fig. 6, we can find that two curves of $\log(2a/c)-T(°C)$ tend unanimously. For example, at $T=-3°C$, $\log(2a/c)$ equals 0.45; the minimum of $\log(2a/c)$ is found near $T=-5$ or $-6°C$; and the maximum of $\log(2a/c)$ is found near $T=-15$ or $-16°C$. Of course, the curves are not unanimous at all points, e.g., there are bigger differences in the region of -17 and $-20°C$, because measuring $2a$ of c or ice crystal is not easy in dynamic experiments in this region.

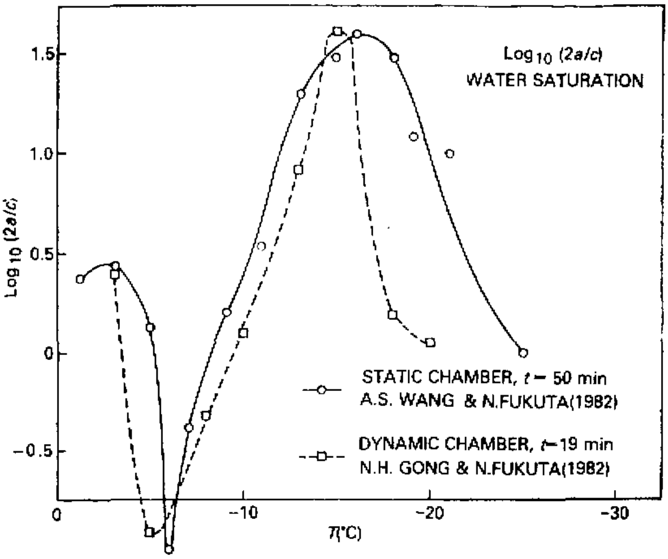


Fig. 6. The quantitative comparison of A.S. Wang and Fukuta's static data (1982) with N.H. Gong and Fukuta's dynamic data (1982) (t =19 minutes).

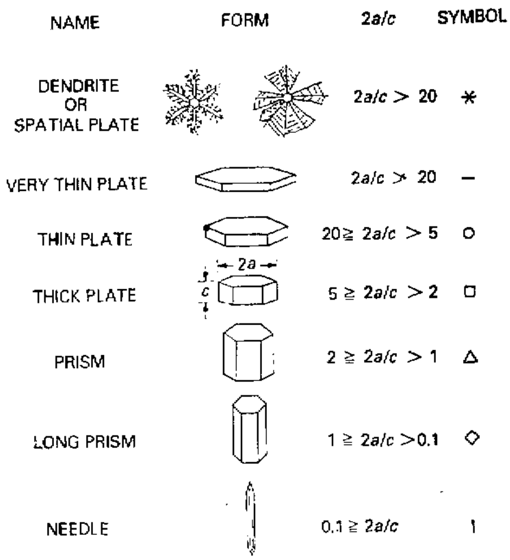


Fig. 7. The ratio of $2a$ and c, form, name and sign of ice crystal for quantitative studies of ice crystal.

In addition, the comparison of static and dynamic experiments shows that the growth rate of ice crystals in the dynamic experiment is higher than that in the static chamber. In our laboratory, the size of ice crystal in the dynamic chamber at 20 min almost equals the size of ice crystal in the static chamber at 50 min. In the dynamic chamber, the growth rate of the ice crystal is evener; but the growth rate of the ice crystal is higher during the first 10 minutes, and is lower during 10-50 minutes in the static chamber.

V. GROWTH LAW OF ICE CRYSTALS IN ICE SUPERSATURATION AND TEMPERATURE FIELD

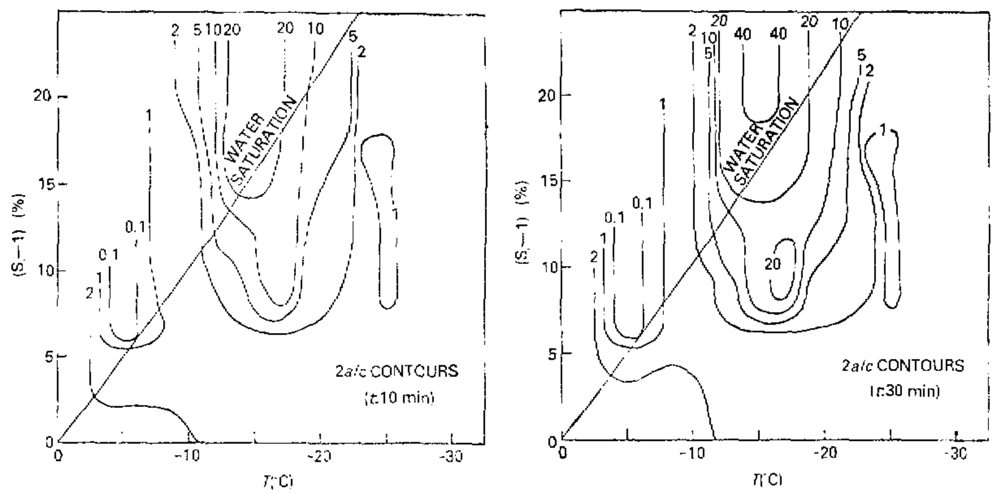
The goal of this work was to get a more complete growth law of ice crystals in the (S_i-1)–T field by using an experimental method to improve formers' work. In our experiment, we had more stable environmental conditions and could observe three-dimension sizes of ice crystal by the use of an ice crystal slide mechanism. Moreover, we have already obtained about 4,000 data under different ice supersaturations, temperatures and times. We had a datum every 1–2°C from 0 to −30°C and every 1–3% of (S_i-1).

Table 1. The Comparison of Form Name of Ice Crystal between A.S. Wang and N. Fukuta's with Former's Data

A.S. Wang & N. Fukuta's data (1982) 2a/c Value	form name	M. Hamajima (1949) and U. Makaya's (1954) data	B.J. Mason (1957) and T. Kobayashi's (1961) data
$2a/c>20$	Dendritic or spatical plate	Dendritic	Dendritic
$2a/c>20$	Very thin plate	Spatical plate	Plate and sector
$20\geq 2a/c>5$	Thin plate	Plate and Sector	
$5\geq 2a/c>2$	Thick plate	Thick plate	Thick plate
$2\geq 2a/c>1$	Prism	Column	Solid column, Very thick plate
$1\geq 2a/c>0.1$	Long Prism	Scroll or cup	Sheath
$0.1\geq 2a/c$	Needle	Needle	Needle

* Except for $2a/c>20$, ice crystals must be dendritic or spatial plate form.

Now, we will introduce a new idea about the quantitative studies of ice crystal, and then we will give the growth law of ice crystals in the (S_i-1)–T field. Figure 7 shows the ratio of $2a$ and c, the form of ice crystal, the names and symbols for quantitative studies of ice crystal. For example, when $20\geq 2a/c>5$, we call the ice crystal "thin plate" and use "○" to indicate it. The names of the different kinds of ice crystal are as follows: dendritic or spatial plate, very thin plate, thin plate, thick plate, prism, long prism and needle. We can use this method



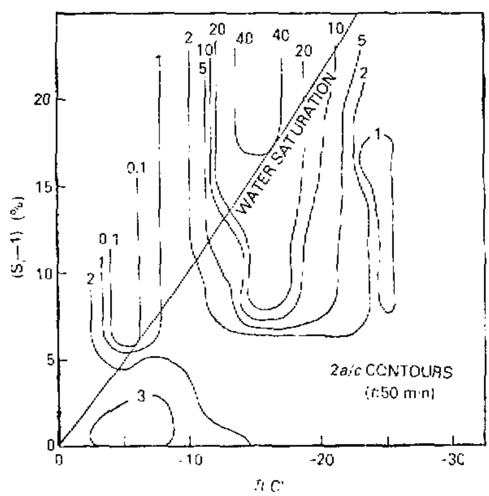


Fig. 8. The variation of $2a/c$ ratio as a function of temperature and ice supersaturation at 10 (a), 30 (b) and 50 (c) minutes of ice crystal growth.

to indicate the result of all fields, but its concept and isopleth are different from quantitative studies. In the qualitative studies, there is no quantitative concept, and the boundary line between different kinds of ice crystals (for example, plate and prism or needle, etc.) is not clear. Care must be taken when you read the following part and compare this paper with previous books or papers. Of course, we have not given more detailed characteristics of ice crystal, e.g., hollow prism or solid prism etc. That is a secondary problem.

In our definition, we decide the form and its name according to the $2a/c$ value only, with an exception that the dendritic or spatial is decided by $2a/c>20$ and its form together. We give a comparative table of the forms and names of ice crystal of A.S. Wang and Fukuta's and the others' data (see Table 1). In the table, we can find that most parts of the data are the same as or close to each other, but some parts are different from each other. As there is no clear boundary line in qualitative data, the concept of ice crystal form is close to but not the same as in Table 1. Since quantitative studies give us an objective standard for judging the form of ice crystals, they will promote our understanding of the growth law of ice crystals.

Now, we will give the growth law of ice crystal in $(S_i-1)-T$ fields. In Fig. 8 there are $2a/c$ contours at time = 10(a), 30(b) and 50(c) minutes respectively (Wang Angsheng, 1984). The main results are as follows.

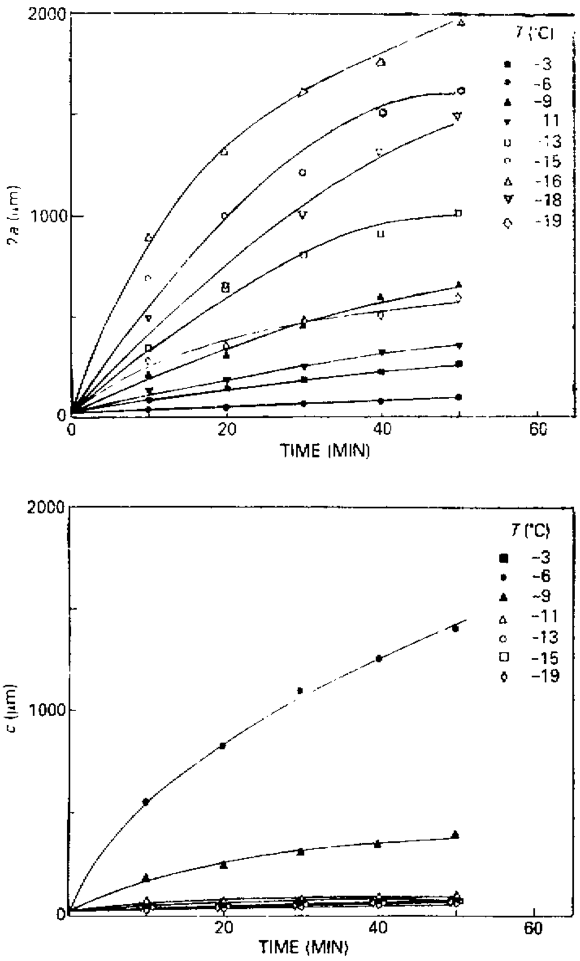


Fig. 9. $2a$ (a) and c (b) plotted as a function of growth time at different temperatures (T) under water saturation.

In an ice supersaturation and temperature field, $2a/c$ contours have a maximum region which is situated near -15°C and over water saturation, the width of T is about 3–4°C. The

$2a/c$ value is bigger than 20, which is the dendritic and spatial plate region. We found the dendritic and spatial plate of ice crystal only in this region. Then the value of $2a/c$ and the area of this region increased with time.

Another minimum region of $2a/c$ value is found near $-5°C$ and over water saturation, with a temperature width of about 2°C, and $2a/c$ less than 0.1. We found the needle form of ice crystal only in this region.

Many more contours of $2a/c$ are distributed along the (S_i-1) axis, which means that the temperature is an important factor for deciding ice crystal form. This is like the qualitative study results.

In the bottom-right part of Figs. 8a, b and c, there is a big rectangular region which has a 1.0–2.0 value of $2a/c$. The region is from about -15 to $-30°C$ and its (S_i-1) is lower than 6%. This is a region in which most of the $2a/c$ values are close to 1.2–1.6, which means $c/2a$ is about 0.83–0.63. Thus in this region ice crystal growth is similar to Wulff's theorem.

Except for the above three regions (e.g., maximum, minimum and lower T–(S_i-1) region), in other parts of the (S_i-1)–T field, the value of $2a/c$ changes gradually from maximum value to minimum, or from maximum to 1–2, or from 1–2 to minimum etc.

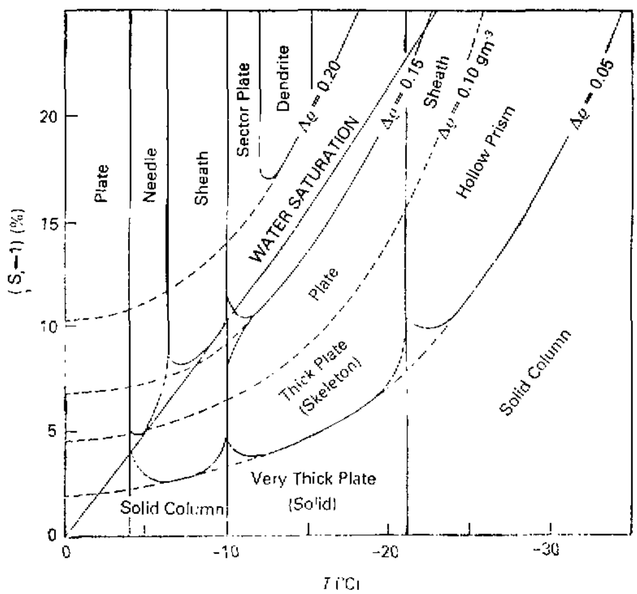
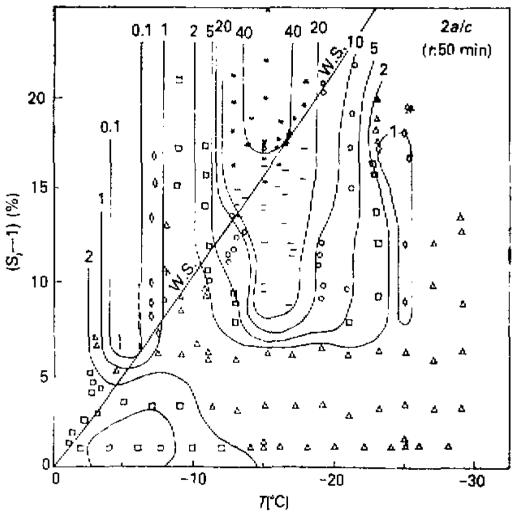


Fig. 10. The form of ice crystal plotted as a function of temperature and ice supersaturation (by Kobayashi, 1961).

We can find from Fig. 8 that the value of $2a/c$ in the (S_i-1)–T field gradually changes with time. When $t=0$ min. (that means the seeding beginning ice crystal is transferred into this (S_i-1)–T field), the values of $2a/c$ are uniform in the field and close to 1.0. But, during the first 10 minutes, great changes of $2a/c$ take place in the (S_i-1)–T field (see Figs. 8 and 9). The maximum ($2a/c$ is bigger than 20) and minimum ($2a/c$ is less than 0.1) occur. After 10 minutes, the value of $2a/c$ continually changes. The maximum value continually increases, and is bigger than 40; the range of maximum value extends too. At the same time, the minimum region of $2a/c$ continually decreases, but the area changes only a little (see Fig. 8). This change in all fields is not easy to find in the qualitative studies or dynamic experiments. So this result is very useful.



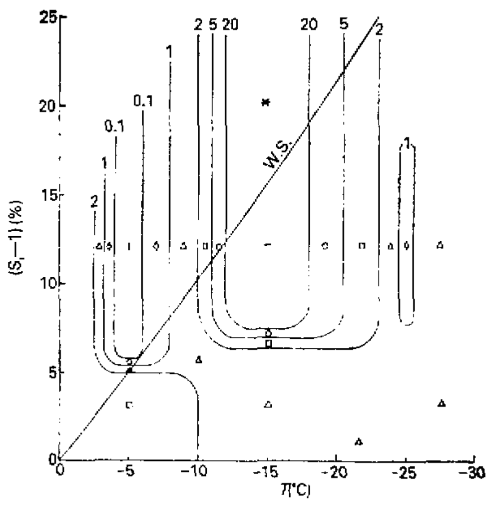


Fig. 11 a. Quantitative experimental results of ice crystal habit variation with T and (S_i-1) at time=50 min.

b. A new model of ice crystal growth in (S_i-1)–T field.

Finally, we will compare our quantitative result with Kobayashi's qualitative result. In Fig. 10, the Kobayashi's result (1961), in which the scale of figure is equal to the scale of Fig. 8, is reproduced. In the comparison of our quantitative result (Fig. 8c) with Kobayashi's result (Fig. 10), we can find that our minimum region corresponds to the needle region of Kobayashi's model; our maximum region nearly corresponds to the dendritic region of Kobayashi's model; and both results are consistent in the high ice supersaturation region. But when we compare Fig. 8c with Fig. 10, in low ice supersaturation, they are clearly different. In the low (S_i-1) region, we have more than 130 data of $2a/c$ at 50 minutes only, while for T, Kobayashi had only a few data and the stability of his experiment was not good. Therefore we believe our measured quantitative data are in accord with the actual situation. Using a few data obtained in the low (S_i-1) region and the isopleth of excess vapor density to decide the form region of ice crystals, is only an inference and not correct. Therefore, our quantitative result provides a complete and overall growth law of ice crystals in the above (S_i-1)–T field.

VI. NEW MODELS OF ICE CRYSTAL GROWTH

Although the models of Nakaya (1954), Masom (1958), Kobayashi (1961) and Pruppacher (1978, 1981) concerning ice crystal growth have been very useful and important, there are some problems in these models (see Introduction). We believe that it is necessary to do more experiments and get a lot of accurate data for improvement in the previous models.

1. *A New Model of Ice Crystal Habit Variation with T and* (S_i-1)

During the last three years, we obtained a great number of accurate data of ice crystal growth by the use of a new wedge-shaped ice thermal diffusion chamber (Fukuta et al., 1982). Based on the data and other conditions we studied the law of ice crystal growth and developed new models.

In Fig. 11a, the quantitative experimental results of ice crystal habit variation with T and (S_i-1) are presented. The isolines of $2a/c$ (e.g., 0.1, 1, 2, 5, 10, 20, 40, etc.) are drawn in Fig. 11a for distinguishing different kinds of ice crystal as in Fig. 7. There are about 130 points in lower ice supersaturation (i.e., below water saturation). The number of data is much greater than the previous author's data, and every $2a/c$ is got from measuring $2a$ and c, so that this result is better than that of previous data.

According to quantitative experimental results shown in Fig. 11a and similar data of at different time (Wang Angsheng and Fukuta, 1984b), a new variational model of ice crystal habit with T and (S_i-1) has been given in Fig. 11b. It is very clear that the new model is similar to Fig. 11a, but it is more ideal and different from previous models in lower ice supersaturation regions. Two centers of this model are the needle ice crystal region [$2a/c \leqslant 0.1$, $T=-5°C$ and (S_i-1) higher than 6%] and the dendrite or spatial plate and very thin plate region [$2a/c \geqslant 20$, T from -12 to $-18°C$, and (S_i-1) higher than 8%]. Around the above two centers, all kinds of ice crystals change gradually. For example, there are thin plate, thick plate, prism and long prism; $2a/c$ changes from about 20 to 10, 5, 2,... to 0.1 gradually, and so on (temperature is from -15 to $-5°C$). We can find that the distribution of ice crystal kinds are symmetric with respect to the two centers of dendrite and needle ice crystal regions. Another characteristic of our model is that there is a Wulff's theory region in the $(S_i-1)=0$–6% region and T from about -15 to $-30°C$. In this region, $2a/c$ is close to about 1.5 or $c/2a$ is about 0.7. This value is close to Wulff's theory, where $c/2a=0.81$. Under this condition, the ice crystal grows when it is approaching an equilibrium situation.

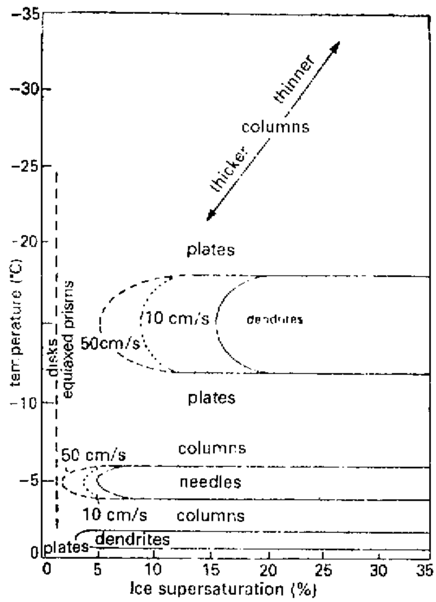


Fig. 12. The shape of ice crystal related to environmental conditions (Hallett, 1984).

When we compare our model results (Fig. 11b) with observational natural results (Magono and Lee, 1966), we find that they are similar in the needle, dendrite, very thin plate, etc. ice crystal regions and their nearby regions; especially in symmetric characteristics of different kinds of ice crystals in the region of -10 and -20°C. In the lower (S_i-1) region (near ice saturation), it is clear that there is only prism or thick plate in Fig. 11 or Magono's work. In the above region, there is a near-equilibrium condition, and the ice crystal grows according to Wulff s theory, so that the form of the ice crystal is prism. In addition, ice crystal form changes gradually from prism to thick plate, thin plate, very thin plate, dendrite, and so on, near -15°C when (S_i-1) increases. We can see a similar phenomenon in the above two figures. Thus we may think that Magono's work supports our model by his natural observation of snow crystal growing. Hallett's (1984) work gives us some new support. His result is given in Fig. 12 where most parts are similar to our work, e.g. -5, -15°C, etc.

2. *A New Model of Ice Crystal Habit Variation with Temperature and Vapor Density Excess*

Based on our data, a new model is given in Figs. 13a and b. Its characteristics are similar to those of Fig. 11. Of course, the isothermohyps are the main factor that decides the ice crystal form, but vapor density excess is a factor too, and it changes with varying temperature, so that most boundaries of different kinds of ice crystal are slopes and are not parallel to vapor density excess isolines. This is a very important result.

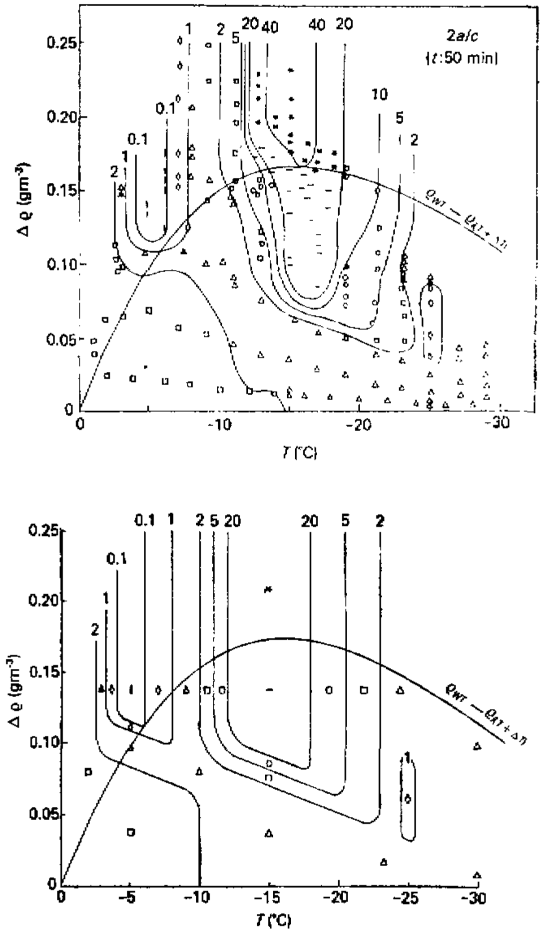


Fig. 13 a. The quantitative result of ice crystal form in temperature and vapor density excess field (time=50 minutes).

b. A new model of ice crystal growth in temperature and vapor density excess field.

VII. SUMMARY

According to our work, the following conclusions might be drawn:

1. The new improved wedge-shaped chamber of ice crystal diffusion used in this study has stable environmental conditions, and three-dimension sizes of ice crystal can be measured easily, which is very important for completing our work.

2. Based on a large number of data, we got the distribution of diameter $2a$ of ice crystal in the (S_i-1) - T field. When $t=50$ min, the maximum value of $2a$ is bigger than 2,000 μm and its region is from about -13.5 to -16.5°C and (S_i-1) is over 16.5%. There are two

minimum regions in the $2a$ field, where the $2a$ values are smaller than 100 μm.

3. At the same time, we got the c distribution. When t=50 min, the maximum of c is much bigger than 1,000 μm, and its region is over 6% and from about −4 to −6°C. The minimum of c is smaller than 50 μm. In the minimum region, (S_i-1) is over 10% while T is from −12.5 to −21°C.

4. Our data show that the characteristics of ice crystal growth in low ice supersaturation region (from 1 to 6%) and lower T region (from −15 to −30°C) are close to Wulff's growth, i.e. $c/2a=0.7$. This proves Wulff's theorem, i.e. $c/2a=0.81$.

5. The quantitative comparison of static and dynamic experiments of ice crystal growth shows that the main tendency of ice crystals is unanimous.

6. The ratio of $2a$ and c, as well as the form, name and sign of ice crystals for quantitative studies have been suggested in this paper.

7. The growth law of ice crystals in the ice supersaturation (from 0 to 25%) and temperature (from 0 to −30°C) field during different time periods (for example, 10, 30 and 50 min) has been given. There is a maximum region of $2a/c$ near −15°C and over water saturation, which corresponds to dendritic and spatial plate form. The minimum region of $2a/c$ has been found near −5°C and over water saturation, and the needle form has been found in this region. The data show that T is an important factor for deciding ice crystal form. Quantitative and overall variation figures of $2a/c$ ratio as a function of T and (S_i-1) at 10, 30 and 50 minutes of growth have been given and compared with Kobayashi's result.

8. A new model of ice crystal habit variation with temperature and ice supersaturation (or model of ice crystal habit variation with temperature and vapor density excess) has been presented in the paper. In lower ice supersaturation, it is different from previons models which have been used for about 20 years. The new model is better than the previous ones, and closer to observational results.

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