THE LOCAL SPLINE VERTICAL INTERPOLATION METHOD OF TEMPERATURE AND GEOPOTENTIAL HEIGHT FIELDS AND THE TIME-DEPENDENT DIFFERENCE FORM OF THE HYDROSTATIC EQUATION

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ABSTRACT

In numerical weather prediction (NWP), the accuracy of vertical interpolation of the initial data is a problem which is greatly concerned by people. In this paper, we specify vertical distributions of the temperature and the geopotential height fields and examine three interpolation methods, i.e. the Lagrangian polynomial interpolation method (thereafter abbreviated to LP method), the linear interpolation method (LN method) and the local spline interpolation method (LS method) proposed by the author. The examination shows that when the vertical resolution of the initial data is high enough, for example, the number of the given data levels N is 10 or more, all the three methods get good accuracy of interpolation, especially, the LP and the LS methods have very little errors almost tending to zero, while the LN method has a little larger errors than the two formers and the errors at various levels have the same sign. When N is reduced to 5, the LP and the LS methods still have quite good accuracy and similar error distributions, while the LN method has less accuracy. If the geopotential height field needs to be adjusted in order to satisfy the hydrostatic equilibrium with the temperature field which is assumed fixed, then the LS method has minimum errors. The examination also indicates that the vertical resolution with at least 5 levels of initial data can keep the interpolation accuracy. Otherwise the accuracy will not be guaranteed no matter which method is used.

It is also pointed out in this paper that the temperature and the geopotential height fields can be given independently in numerical prediction models in order to keep higher interpolation accuracy. However, the hydrostatic equation should be finite differenced in other way which is somewhat different from the conventional one. In other words, the time dependent difference form of the equation should be used, so that the initial interpolation accuracy could have influence on the time integration.

I. INTRODUCTION

Vertical interpolation problem of the temperature and the geopotential height fields exists in all numerical prediction models no matter with or without topography included in the model. This is simply because that the coordinate surfaces of the model atmosphere do not everywhere coincide with the standard observational levels of the real atmosphere. At some special moments of time integration the model produced results at some standard levels are certainly required in order to obtain the weather prediction and to compare the results with real weather processes for evaluation of the model.

The importance of vertical interpolation accuracy is evident. It directly affects the accuracy of the model initial field which in turn is very important for the accuracy of numerical weather prediction. The nonlinear theory of dynamic meteorology shows that the slight differences in the initial fields would result in bifurcation (Chou, 1985). Therefore high similarity between the model and the real atmosphere should be kept as possible in N.W.P.

The author has discussed some methods of vertical interpolation of the temperature and the geopotential height fields in numerical models in some detail (Qian, 1983). It has been indicated that the hypothesis of linear distribution of the temperature with logarithmic pressure and/or with height is adequate to obtaining better interpolation accuracy. However, the interpolation accuracy is affected by the linear distribution hypothesis itself. Janjic (1977) also discussed the vertical distribution pattern of the geopotential height field with logarithmic pressure and pointed out that the linear distribution with square logarithmic pressure can be used in the vertical interpolation of geopotential height field. Mesinger and Janjic (1985) suggested using above linear interpolation method to model atmosphere in order to improve the calculation of pressure gradient force at the steep slope of mountains.

The real distributions of the temperature and the geopotential height fields with logarithmic pressure, of course, are not linear. Therefore, the hypothesis of local linear distribution will result in some errors and probably the whole distribution curve may not be smooth. In order to improve the interpolation accuracy and obtain smooth distribution curve of the temperature or the geopotential height field with logarithmic pressure, polynomial interpolation methods such as the Lagrangian method should be used. As well known, however, the higher the polynomial order, the more the time-consuming for the computation takes, and the interior relationship between the temperature and the geopotential height fields is not considered enough even not completely taken into account. To improve the polynomial interpolation method the author suggests using the so-called local spline interpolation (LS) method in this paper which is equivalent to a third order polynomial of the temperature or the geopotential height field with logarithmic pressure. Because of taking into consideration the interior relationship of the temperature and the geopotential height fields, smooth distribution curves will be obtained by using the geopotential values at only two end points of a vertical pressure interval. To examine the property of the LS method we compare the results obtained with that of the LP and the LN methods, respectively.

Besides the discussions of the interpolation methods and their accuracies we suggest in this paper that a new time dependent difference form of the hydrostatic equation be used in order to keep the hydrostatic equilibrium between the temperature and the geopotential height fields and to improve the quality of N.W.P.

II. THE LOCAL SPLINE INTERPOLATION (LS) METHOD

1. *The LS Method for Geopotential Height Field*

At first let us suppose that in the real atmosphere we only get the geopotential height field ϕ_i at some isobaric levels p_i, where $i=1,2,\ldots,N$, and N is the number of the data levels. We want to obtain the distribution of the geopotentials between arbitrary two isobaric levels. For the convenience of description, here we set $x=\ln(p)$, $d=x_{i-1}-x_i$, then $M=\frac{d^2\phi}{dx^2}$ can be linearly interpolated from M_i and M_{i+1} at p_i and p_{i-1} levels, i.e.

$$M=\frac{d^2\phi}{dx^2}=[M_i(x_{i-1}-x)-M_{i+1}(x-x_i)]/d, \tag{1}$$

after integration of (1) with respect to x, we have

$$\frac{d\phi}{dx}=-\frac{M_i}{2d}(x_{i-1}-x)^2+\frac{M_{i+1}}{2d}(x-x_i)^2+C_1, \tag{2}$$

Further integration gives the vertical distribution of ϕ with respect to $x=\ln(p)$, that is

$$\phi(x)=\frac{M_i}{6d}(x_{i+1}-x)^3+\frac{M_{i+1}}{6d}(x-x_i)^3-C_1x+C_2, \qquad (3)$$

where C_1 and C_2 are the integration constants determined by ϕ_i and ϕ_{i+1} at x_i and x_{i+1} levels:

$$C_1=(\phi_{i+1}-\phi_i)/d-(M_{i+1}-M_i)d/6, \qquad (4)$$

$$C_2=(\phi_i x_{i+1}-\phi_{i+1}x_i)/d-(M_i x_{i+1}-M_{i+1}x_i)d/6. \qquad (5)$$

So far M_i and M_{i+1} remain unknown. From the definition of M, we get

$$M=\frac{d^2\phi}{dx^2}=\frac{d}{d\ln p}\left(\frac{d\phi}{d\ln p}\right)=-R\frac{dT}{d\ln p}=-R\gamma, \qquad (6)$$

where $\gamma=\frac{dT}{d\ln p}$, is the vertical change rate of the temperature. From (6) the M_i and M_{i+1} can be expressed respectively as follows:

$$M_i=-R\gamma_i, \quad M_{i+1}=-R\gamma_{i+1}. \qquad (7a,b)$$

Substituing them into (3), we obtain

$$Z(x)=-\frac{R}{g}[\gamma_i(x_{i+1}-x)^3+\gamma_{i+1}(x-x_i)^3]/6d+C_1'x+C_2', \qquad (3')$$

where $C_1'=C_1/g$, $C_2'=C_2/g$, g is the gravity acceleration.

In order to obtain γ_i and γ_{i+1}, it is necessary to use the mean temperatures in the interval in question and the adjacent intervals above and below. From the hydrostatic equation, it follows that:

$$T_i=(\phi_i-\phi_{i+1})/R(x_{i+1}-x_i) \quad (i=1,2,\cdots,N-1), \qquad (8)$$

then

$$\gamma_i=(T_i-T_{i-1})/(\bar{x}_i-\bar{x}_{i-1}) \quad (i=2,3,\cdots,N-1), \qquad (9)$$

where

$$\bar{x}_i=(x_i+x_{i+1})/2.$$

The temperature interpolation formula can be obtained from the hydrostatic equation again as follows:

$$T(x)=[\gamma_{i+1}(x-x_i)^2-\gamma_i(x_{i+1}-x)^2]/2d-C_1/R. \qquad (10)$$

We can see from the above interpolation formulas that the local spline interpolation method is equivalent to the cubic polynomial interpolation method for the geopotential height field and to the parabolic interpolation for the temperature. The γ values in the uppermost and the lowest layers of the given data set can not be obtained by (8) and (9), however, they can be determined by assuming that they are equal to those in the most adjacent layers or by the linear extrapolation method. Of course, this will result in some errors and affect the interpolation accuracy.

2. *The LS Method for the Temperature Field*

Now we assume that the geopotential height ϕ_i and the temperature T_i at the p_i level are both known, where $i=1,2,\ldots,N$, and N is the level number of the given data set. The temperature and the geopotential height distributions between any two levels are found.

Replacing ϕ with T in the previous paragraph, we can easily obtain that

$$T(x)=\frac{W_i}{6d}(x_{i+1}-x)^3-\frac{W_{i+1}}{6d}(x-x_i)^3-D_1x-D_2, \qquad (11)$$

$$D_1=(T_{i+1}-T_i)/d-(W_{i+1}-W_i)d/6, \tag{12}$$

$$D_2=(T_i x_{i+1}-T_{i+1}x_i)/d-(W_i x_{i+1}-W_{i+1}x_i)d/6, \tag{13}$$

where W is the second derivative of T with respect to x, i.e.

$$W=\frac{d^2T}{dx^2}=\frac{d\gamma}{dx}. \tag{14}$$

In order to get W, γ is needed at first. Because T_i at p_i are known, and from ϕ_i at p_i we can calculate $N-1$ mean temperatures, there are totally $2N-1$ temperatures. Therefore we can get $2(N-1)$ values of γ and the W_i ($i=2,3,\ldots,N-1$) at any levels can be computed. By assuming $W_1=W_2$ and $W_N=W_{N-1}$ or by other methods the total W values can be determined.

Again by using the hydrostatic equation and integrating (11) with respect to x after multiplied by $-R$, we obtain

$$\begin{aligned}\phi(x)=\phi_{i+1}-R[&W_i(x_{i+1}-x)^4/24/d-W_{i+1}(x-x_i)^4/24/d\\&+W_{i+1}d^3/24-D_1(x^2-x_{i+1}^2)-D_2(x-x_{i+1})].\end{aligned} \tag{15}$$

From equations (11) and (15), we see that the interpolation polynomials for the $T(x)$ and the $\phi(x)$ fields are one order higher than that in the Paragraph 1, they become the third and the fourth order polynomials, respectively.

3. *LS Method in the Model Atmosphere*

In time integration of the model equations, the surface pressure p_s, the temperatures in model layers are the prediction quantities, the surface height z_s above the sea level is known fixed, while the geopotentials should be calculated from the temperatures by the hydrostatic relation.

As in Paragraph 2, the LS interpolation formula for temperatures in the model atmosphere is the same as (11), however, the W values can not be obtained by the same procedure as in Paragraph 2. Here again we assume N is the total number of the model layers, then we can obtain only $N-2$ values of W, W_1 and W_N should be determined by extrapolation. The LS formula for ϕ is the same as (15).

Now we assume $W_N=W_{N-1}$, then the ϕ field between the $N-1$ and the N levels can be interpolated by

$$\begin{aligned}\phi(x)=&\phi_N+RW_{N-1}[(x_N-x)^4-(x-x_{N-1})^4+d^4]/24/d\\&-R[D_1(x^2-x_N^2)/2+D_2(x-x_N)]\\=&\phi_N+RW_{N-1}[(x_N^4-x_{N-1}^4)+4x^2(x_N^2-x_{N-1}^2)-4x(x_N^3-x_{N-1}^3)\\&+2x^2(x_N^2-x_{N-1}^2)-4x^3(x_N-x_{N-1})-d^4]/24/d\\&-R[D_1(x^2-x_N^2)/2-D_2(x-x_N)].\end{aligned} \tag{16}$$

We see that the interpolation polynomial reduces to third order, the ϕ_N can be found from the next formula:

$$\phi_N=\phi_s+R[W_{N-1}d^2/12-D_1(x_N+x_s)(x_s-x_N)/2+D_2d], \tag{17}$$

where $\phi_s=gz_s$, $D_1=(T_s-T_N)/d=\gamma_N$ is the temperature change rate in the lowest layer below the N-th level, T_s is the surface temperature extrapolated by (11).

III. THE TIME DEPENDENT DIFFERENCE SCHEME OF THE HYDROSTATIC EQUATION

In the above paragraph we have already seen that the computational accuracy of the ϕ field at the uppermost and the lowest layers of the model atmosphere may be reduced if the ϕ field is calculated by using only the temperatures and the vertical integration of the hydrostatic equation. We have also seen that the ϕ field is only related to the temperatures and the surface pressure p_s at that time and has no explicit relation with the ϕ field itself at the previous time steps. Therefore, only one field (usually the T field) can keep its original interpolation accuracy and the other (usually the ϕ field) will lose the accuracy in spite of how accurate it is at the initial time, if they are originally interpolated independently. It is because that the other field will be immediately changed by the hydrostatic equation at the first step time integration. The weather prediction quality will be influenced. In order to overcome the shortcoming mentioned above, it is necessary to find a method which can keep the interpolation accuracy of both fields at the initial time and the hydrostatic relationship between the two fields of T and ϕ. Such a method is suggested in this paper by using the time dependent difference scheme of the hydrostatic equation.

We still assume that the hydrostatic equation can be written in the following vertical integration form:

$$\phi_K = \phi_{K+1} + \int_{p_K}^{p_{K+1}} RT \cdot d\ln(p), \tag{18}$$

where K is the layer number of the model with larger value downward, T the mean temperature between p_K and p_{K+1} as the function of $\ln(p)$. If we substitute (11) into (18), then we get (15). In most numerical models the T is taken as the predictive temperature in the layer.

At two adjacent time steps t_0 and t_1, where $t_1 = t_0 - \Delta t$ and Δt the time step, we can use (18) to compute the ϕ fields and subtract $\phi_K^{t_0}$ from $\phi_K^{t_1}$, where the upperscripts t_0 and t_1 mean the ϕ fields at t_0 and t_1 time, respectively. Then we get

$$\phi_K^{t_1} - \phi_K^{t_0} = \phi_{K+1}^{t_1} - \phi_{K+1}^{t_0} + \int_{p_K}^{p_{K+1}} R(T^{t_1} - T^{t_0}) \cdot d\ln(p), \tag{16}$$

where we have already assumed the p_K and p_{K+1} do not change with time. We can see from (19) that the geopotential height ϕ_K at the isobaric surface p_K is not only related to the geopotential heights and the mean temperatures at and below that surface, but also related to those at the previous step. It is more important that (19) represents the relationship between the variations of the temperature and the geopotential height fields but not the two fields themselves. Therefore, if there are some systematic errors in the temperature field, for example, the mean temperature between two levels is systematically higher than the real one in the whole time integration, the height field would not be affected in such a situation. Besides if the temperature is not the linear function of $\ln(p)$, the Eq. (18) contains larger errors, however the variation of T field may be the linear function of $\ln(p)$, so Eq. (19) may give smaller errors in the ϕ field.

Now we assume the temperature variation with time is the linear function of $\ln(p)$, then we get

$$\Delta T = (\Delta T_K + \Delta T_{K+1})/2 = (T_K^{t_1} + T_{K+1}^{t_1} - T_K^{t_0} - T_{K+1}^{t_0})/2, \tag{20}$$

where $\Delta T = T^{t_1} - T^{t_0}$. Substituting (20) into (19), it turns out that

$$\phi_K^{t_1} = \phi_K^{t_0} + (\phi_{K+1}^{t_1} - \phi_{K+1}^{t_0}) + R(\Delta T_K + \Delta T_{K+1}) \cdot \ln(p_{K+1}/p_K)/2. \tag{21}$$

In the σ-coordinate system, Eq. (21) changes to the following form because of the time variation of the pressure at the model levels:

$$\phi_K^{t_1} = \phi_K^{t_0} + (\phi_{K+1}^{t_1} - \phi_{K+1}^{t_0}) + R[(T_K^{t_1} + T_{K+1}^{t_1}) \cdot \ln(p_{K+1}^{t_1}/p_K^{t_1})$$
$$- (T_K^{t_0} + T_{K+1}^{t_0}) \cdot \ln(p_{K+1}^{t_0}/p_K^{t_0})]/2. \tag{22}$$

The experimental results of the above two Eqs. are given in the other paper (Chen and

Qian, 1986). Here we would like to point out that in 1975 we used a similar equation as (22) in the two-layer model as follows:

$$\frac{\partial z_5}{\partial t} = R[\ln(1-0.002p_s^*)\frac{\partial T_7}{\partial t}/(p_s^*+500)/\frac{\partial p_s^*}{\partial t}]/g, \tag{23}$$

where z_5 is the height of the 500-hPa level, $p_s^* = p_s - 500$, p_s the surface pressure, T_7 the temperature at $\sigma = 1/2$ surface (Qian et al., 1978). Forecasting experiments told us that Eq. (23) was specially suitable to the models with coarser vertical resolution. In the B model of the Beijing Meteorological Centre and other models outside of China similar equations as (23) are also being used to predict the ϕ field, however, with more complicated forms. It is easy to see that using Eqs. (21) and (22) is better than using equations with prediction form as (23), because Eqs. (21) and (22) are the diagnostic equations with no connection to the time integration schemes and thus save computer time too.

IV. THE TEST RESULTS OF INTERPOLATION SCHEMES

The local spline interpolation method is no doubt simpler and takes less time than the Lagrangian polynomial interpolation method or the whole interval spline method. However, whether its computational accuracy can be comparable with the latters and to meet the accuracy requirements remains unknown. The answer should be given by testing check. In order to do this, at first we select an ideal vertical distribution of the temperature field and the vertical distribution of the geopotential height field can be obtained by using the hydrostatic equation. Then we use the three schemes to interpolate the temperature and the geopotential height fields at given levels and compare them with the ideal ones and with one another, thus we can check the advantages or disadvantages of the three methods.

1. *The Ideal Vertical Distributions of the Temperature and Geopotential Height Fields*

An ideal vertical distribution function of the temperature field should not be a polynomial with finite terms, otherwise the testing experiments may not get the reliable results. Therefore we select the vertical distribution of the temperature T as follows:

$$T(x) = T_0 + [DT + ET \cdot \exp(x-x_0)](x-x_0), \tag{24}$$

where DT and ET are parameters, $x = \ln(p)$, $x_0 = \ln(p_0)$, p_0 is the reference pressure surface. T_0 the reference temperature at p_0. We see that (24) represents an unlimited series.

According to the hydrostatic equation, the geopotential height field $\phi(x)$ at any pressure surface can be found when ϕ_0 at p_0 surface is known, that is

$$\phi(x) = \phi_0 - R\{[T_0 + DT \cdot (x-x_0)/2 - ET \cdot \exp(x-x_0)](x-x_0) + ET \cdot [\exp(x-x_0)-1]\}, \tag{25}$$

where $\phi_0 = gz_0$, z_0 the reference height at p_0 surface.

2. *The Testing Results of the Three Methods*

By giving the different values of the parameters DT, ET, x_0, T_0 and z_0 in Eqs. (24) and (25), we can get the distribution patterns of T and ϕ with x. Fig. 1 is the vertical distribution patterns of T and ϕ with x when $DT = 30$, $ET = 25$, $x_0 = \ln(500)$, $z_0 = 580$ (decameters) and $T_0 = 273$ K.

In the testing experiments we use different numbers of data levels in order to check the effect of the vertical data resolution on the interpolation accuracy. The computational

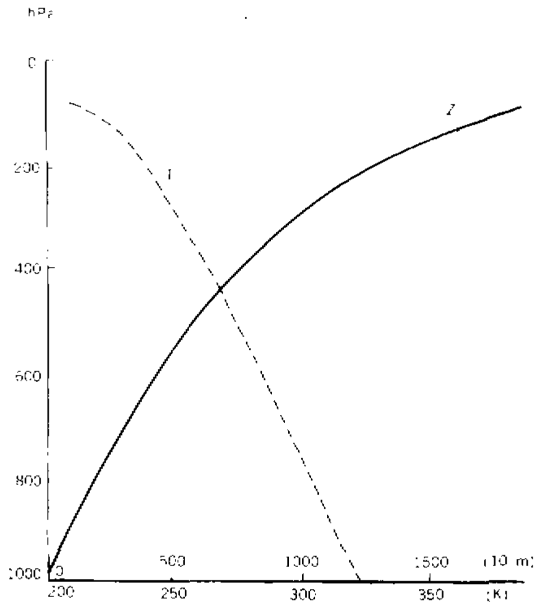

Fig. 1. The vertical distribution patterns of T (dashed line) and ϕ (solid line) with $\ln(P)$, when $DT=30$, $ET=25$, $p_0=500$ hPa, $z_0=580$ and $T_0=273$ K.

results show that the accuracy of all the three methods can be similar to one another in the interpolation intervals when the vertical data resolution is high enough (for example 10 levels of data are used). The LP and the LS methods have almost the same negligible errors at all levels. This means that in the interpolation intervals with two fixed end values a third order polynomial is good enough for interpolation. As well known, when the number of the vertical levels $N=10$, the LP method uses a polynomial with 9-th order, therefore its interpolation accuracy should be better than that of the LS method. However, the fact that we obtain very similar errors in the two schemes indicates that the coefficients of the terms higher than the third order of the polynomial are very small and the property of the polynomial is mainly dependent on the first four terms. The interpolation errors at all levels in the LN method are also small, however, with the same sign, the interpolation curve is therefore not smooth. In the extrapolation intervals with only one end value fixed (the 880 hPa-1000 hPa interval is taken as the extrapolation interval), the LP method has the smallest errors while the LN method has the largest errors. This perhaps is related to the distribution pattern of T.

Figures. 2a,b,c and d are the error distributions of the three methods when $N=3$ and $N=5$. The solid curves show the errors in the geopotential height field while the dashed curves show the errors of the temperature field. Comparison shows that the LN method not only has the largest errors, but also gets nonsmooth interpolation curves. For example, the errors in the ϕ field are all negative and those in the T field all positive at all vertical levels except for the node levels where the errors in either fields are zero certainly determined by the interpolation method. On the contrary, the errors in the LP and the LS methods have very similar vertical distribution and the error curves are smooth.

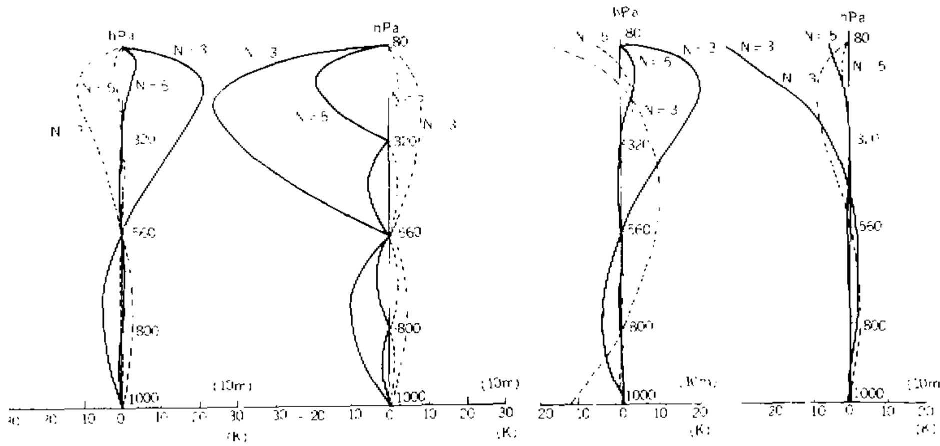

Fig. 2 The error distributions of ϕ (solid line) and T (dashed line) in the three methods. a. the LP method, b. the LN method, c. the LS method (I) for the ϕ field and d. the LS method (II) for the T field.

The errors of the T and the ϕ fields given above in the LP and the LN methods are independent of each other, i.e. the errors of the ϕ field have nothing to do with that of

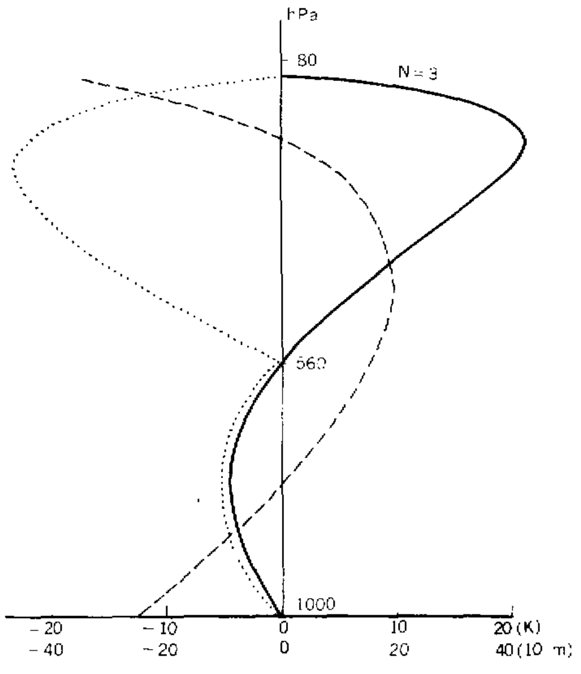

Fig. 3. The ϕ (solid or dotted line) and the T (dashed line) interpolation errors of the three methods, when N=3. The dotted line represents the LN method, the solid line the LP and the LS methods (I) and (II).

the T field. Therefore the initial T and the ϕ fields should be given at the same time, but the interpolated T and ϕ fields may not satisfy the hydrostatic equation with difference form. As pointed out in the previous paragraphs, the LS method can also produce independent T and ϕ interpolations by using Eqs. (3) (the LS method I) and (11) (the LS method II) respectively, thus we should take the ϕ errors in Fig. 2c and the T errors in Fig. 2d to compare them with the same type errors in the other two methods. If so, we find that the errors and their vertical distributions in the LS and the LP methods are more approximate to each other.

Now we are going to discuss the situation with the ϕ field given only. In this case the T field should be computed from the ϕ field by using the hydrostatic equation as a conventional procedure in N.W.P.

Figure. 3 is the error distribution with $N=3$. We see that the interpolated T fields have the same errors in all the three methods. This is because that when $N=3$ we can only obtain two mean temperature, the three methods all turn to linear interpolation formulas. However, the interpolated ϕ fields are still different, the LP and the LS methods have the same results while the LN method has totally different error distribution type, its errors again are the largest and have the same sign.

Figure. 4 is the error distributions when $N=5$. We see that the ϕ errors in the LP method are a little smaller than that in the LS method, while the T errors in the latter are

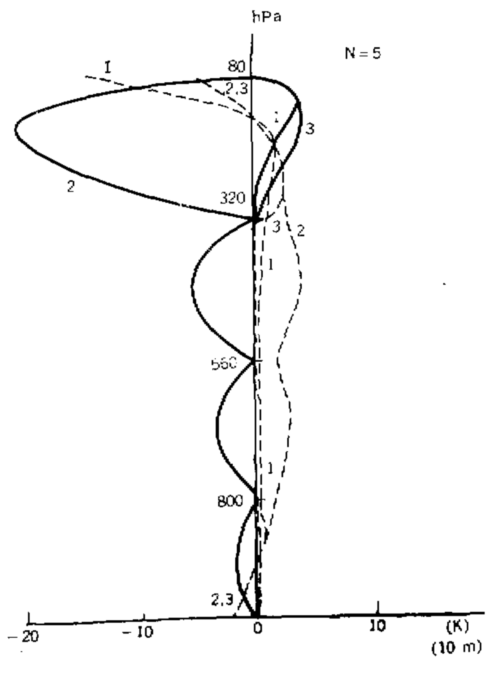

Fig. 4. The error distributions in the three methods when $N=5$ and only the ϕ field is given. The numbers next to the lines represent different schemes, where 1 represents the LP method, 2 the LN and 3 the LS(I) methods, respectively. The solid line is for the ϕ error (decameter) and the dashed line for the T error (K).

in turn a little smaller than that in the former. The LN method has the largest errors in both fields.

In the numerical prediction models, there is a consistent problem between the T and the ϕ fields. That is, when the T and the ϕ fields are independently interpolated, only one field can remain to be of no change, the other should be adjusted according to the hydrostatic constrain, unless the prediction model uses the hydrostatic equation with forms (21) or (22) in the time integration. Now we assume the T field is taken as fixed, though it is obtained from the ϕ field. Then the ϕ field is adjusted according to the following formula:

$$\phi(x_K) = \phi_s - \sum_{i=K}^{N} R(T_i - T_{i+1}) \ln(p_{i+1}/p_i)/2, \qquad (26)$$

where K is the level number of the model and $\phi_s = gz_s$, z_s is the surface height above the mean sea level. Eq. (26) is valid when the T field is linear with respect to $\ln(p)$ in a certain interval, it is only approximate for the LP and the LS methods. However, we use it to adjust the ϕ field in all the three methods for simplicity. The ϕ errors are shown in Fig. 5. We see that the errors in the adjusted ϕ fields are the smallest in the LS method with an exception in the lower atmosphere where the LP method has the smallest errors. The ϕ errors in the LN method are still the largest because of the same sign in the T errors.

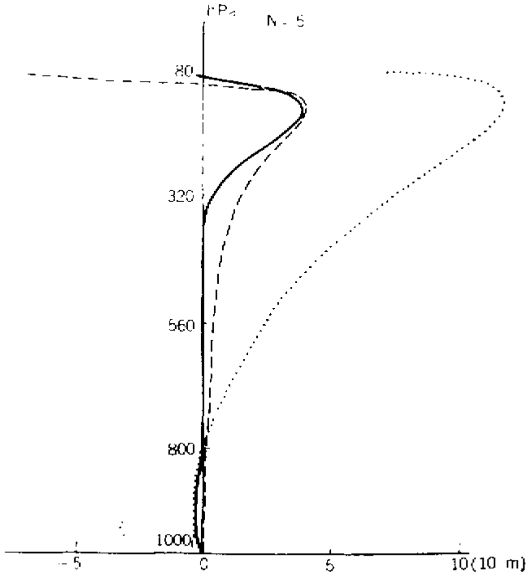

Fig. 5 The ϕ errors in the three methods. The dashed line is for the LP method, the solid line for the for the LS method and the dotted line for the LN method. The T field is obtained from the ϕ field and then taken as unchanged.

The above discussions are verified only by the ideal distributions of the T and the ϕ fields. *Their distributions in the real atmosphere have their own properties, for example,* the temperature changes fast near the troposphere, therefore the conclusions obtained in this paper have to be verified by the real atmosphere. However they are universal to some extent because we have changed the parameters and obtained the different distributions of the T and the ϕ fields, but got the similar error patterns. As far as the usage of (21) or (22) in the five layer primitive equation model (Qian, 1985) is concerned, the readers can refer to (Chen and Qian, 1986) for the preliminary results.

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