Sensitivity of the Multiple Equilibria to Gorverning System, Mode Chosen and Parameter

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I. INTRODUCTION

Multiple equilibria of a forced, dissipative atmosphere system studied by Charney and others have provided a new insight into the dynamics of the atmospheric circulation. But the theoretical results remain some uncertainty for different approaches.

Charney and Devore (1979) obtained two stable equilibria for certain range of external forcing in the barotropic model, one of which is a high–index circulation, and the other a low–index for a blocking state. However Charney and Straus (1980) found that only the low–index (blocking) state is stable and there do not exist multiple equilibria in the baroclinic model.

Goswami (1983) found no existence of multiple equilibria in symmetric 2–layer tropical model with the heating due to a Newtonian cooling with respect to a radiative equilibrium temperature and cumulus convection. But Zhu and Zhao (1987) have found the multiple stable equilibrium solutions in a symmetric barotropic equatorial balance model with the forcing for orography and heating.

Highly truncated model may be appropriate for analysing the basic nonlinear properties of the atmospheric circulation, but they are only the rough approximation of a complete model. So the following two questions are noteworthy:

(1) Is the dynamical insight of the multiple equilibria not an arbitrary one due to highly truncated simplifications?

(2) How about the sensitivity of the multiple equilibria to the governing system, truncations and parameters?

In this paper, a further study is made by using a symmetric tropical model with thermal and orographic forcing to investigate the questions concerned above, especially the second one.

II. SENSITIVITY OF LOW ORDER SYSTEM AND PARAMETERS

1. *Two Low Order Systems*

As in Zhu and Zhao (1987), the thermal and orography forced, dissipation equatorial semi–geostrophic equations can be written as follows

$$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}-\beta y v=-ru,$$
$$\beta y u=-\frac{\partial\varphi}{\partial y}, \tag{1S}$$
$$\frac{\partial v}{\partial y}=-Q+v\frac{\partial h}{\partial y}-\varepsilon\varphi,$$

and the equatorial balance model has the form

$$\frac{\partial^2 u}{\partial t\partial y}+\frac{\partial u}{\partial y}\frac{\partial v}{\partial y}+v\frac{\partial^2 u}{\partial y^2}-\beta v-\beta y\frac{\partial v}{\partial y}+r\frac{\partial u}{\partial y}=0,$$
$$\beta u+\beta y\frac{\partial u}{\partial y}+\frac{\partial^2\varphi}{\partial y^2}=0, \tag{1B}$$
$$\frac{\partial v}{\partial y}=-Q+v\frac{\partial h}{\partial y}-\varepsilon\varphi.$$

Where h is the height of orography, Q is thermal forcing, r and ε are friction and Newtonian cooling coefficients respectively. Taking the following expansions:

$$\begin{bmatrix} u \\ v \\ \varphi \\ h \\ Q \end{bmatrix}=\sum_{n=0}^{N}\begin{bmatrix} u_n \\ v_n \\ \varphi_n \\ h_n \\ Q_n \end{bmatrix}D_n, \tag{2}$$

(D_n is parabolic cylinder function), and substituting (2) into equations (1S) and (1B), we obtain

$$\sum_n a_j\delta_{jn}\frac{\partial u_n}{\partial t}+\sum_n\sum_m d_{jnm}v_n u_m-\sum_n d_{jn}\beta v_n+\sum_n a_j\delta_{nj}u_n=0,$$
$$\sum_n d_{jn}\beta u_n+\sum_n b_{jn}\varphi_n=0, \tag{3S}$$
$$\sum_n b_{jn}v_n-\sum_n a_j\delta_{jn}D_n-\sum_n\sum_m d_{jnm}v_n h_m+\sum_n a_j\delta_{jn}\varepsilon\varphi_n=0;$$

and

$$\sum_n b_{jn}\frac{\partial u_n}{\partial t}+\sum_n\sum_m a_{jnm}v_n u_m-\beta\sum_n c_{jn}v_n-\beta\sum_n a_j\delta_{jn}v_n+r\sum_n b_{jn}u_n=0,$$
$$\beta\sum_n a_j\delta_{jn}u_n+\beta\sum_n c_{jn}u_n+\sum_n a_{jn}\varphi_n=0,$$
$$\sum_n b_{nj}v_n-\sum_n a_j\delta_{jn}Q_n+\sum_n\sum_m d_{jnm}v_n h_m+\varepsilon\sum_n a_j\delta_{jn}\varphi_n=0,$$
$$j=0,1,\ldots N,$$

in which

$$b_{jn}=\langle D_j,\ D_n\rangle,\ a_{jnm}=\langle D_j,\ (D_n D_{mv})_v\rangle.$$

$$
\begin{aligned}
c_{jn} &= \langle D_j ,\ yD_{ny} \rangle,\ a_j \delta_{jn} = \langle D_j ,\ D_n \rangle, \\
a_{jn} &= \langle D_j ,\ D_{n,y} \rangle,\ d_{jnm} = \langle D_j ,\ D_n D_{my} \rangle, \\
\langle A,\ B \rangle &= \int_{-c}^{c} AB dy.
\end{aligned} \tag{4}
$$

If only those terms with $n=1,\ 2$ for $u,\ \varphi,\ h$ and $n=0,1$ for $v,\ Q$ are remained, the two highly truncated low–order systems can be written as

$$
\begin{aligned}
\frac{\partial u_1}{\partial t} &= -\frac{2}{\sqrt{6}} v_0 u_2 + \beta v_0 - r u_1 \\
\frac{\partial u_2}{\partial t} &= \frac{2}{3\sqrt{6}} v_0 u_1 - \frac{1}{9\sqrt{6}} v_1 u_2 + \beta v_1 - r u_2 \\
\varphi_1 &= 2\beta u_1 \\
\varphi_2 &= -2\beta u_2 \\
v_0 &= 2\varepsilon \varphi_1 - 2Q_1 - \frac{4}{\sqrt{6}} v_0 h_2 \\
v_1 &= 2Q_0 + \frac{4}{3\sqrt{6}} v_0 h_1 + \frac{4}{\sqrt{6}} h_2 v_1
\end{aligned} \tag{5S_1}
$$

and

$$
\begin{aligned}
\frac{\partial u_1}{\partial t} &= -\frac{35}{9\sqrt{6}} v_0 u_2 - \frac{8}{9\sqrt{6}} v_1 u_1 + \beta v_0 - r u_1 \\
\frac{\partial u_2}{\partial t} &= \frac{4}{3\sqrt{6}} v_0 u_1 + \frac{8}{9\sqrt{6}} v_1 u_2 + \frac{1}{2} \beta v_1 - r u_2 \\
\varphi_1 &= \frac{2}{3} \beta u_1 \\
\varphi_2 &= \frac{3}{5} \beta u_2 \\
v_1 &= 2\varepsilon \varphi_1 - 2Q_1 - \frac{4}{\sqrt{6}} v_0 h_2 \\
v_1 &= 2Q_0 + \frac{4}{3\sqrt{6}} v_0 h_1 + \frac{4}{\sqrt{6}} v_1 h_2 .
\end{aligned} \tag{5B_1}
$$

Obviously, the coefficients of those two systems are different. In the low order system of semi–geostrophic model (denoted as S_1), the coefficients of the nonlinear terms are smaller, therefore the nonlinear terms become dominant only if the amplitudes of the components of u and v are large enough. In B_1 system, the coefbicients of linear and nonlinear terms are comparable. Therefore the nonlinear terms have larger contribution to nonlinear balance in B_1 system than in S_1 system, although the truncation for those two systems is the same.

2. *Comparison of the Features of Multiple Equilibria*

For convenience, we let $Q_0=0,\ h=0$;

$$V^* = \frac{v_0}{r}, \quad C^* = \frac{\varepsilon\beta^2}{r}, \quad G^* = \frac{2Q_t}{r}. \tag{6}$$

then the steady solution for the two systems can be obtained by the following two sets of equations:

$$\begin{aligned} G^* &= \frac{\alpha_s C^* V^*}{V^{*2} + \mu_s} - V^*, \\ u_1 &= \frac{\mu_s^{\frac{1}{2}} \beta V^*}{V^{*2} + \mu_s} \\ u_2 &= \frac{\mu_s^{\frac{1}{2}} 2}{3\sqrt{6}} \cdot \frac{\beta V^{*2}}{V^{*2} + \mu_s}, \\ \varphi_1 &= \frac{2}{3}\beta u_1, \varphi_2 = \frac{2}{5}\beta u_2; \\ \alpha_s &= 18, \ \mu_s = \frac{9}{2}, \end{aligned} \tag{7S_1}$$

and

$$\begin{aligned} G^* &= \frac{\alpha_B C^* V^*}{V^{*2} + \mu_B} - V^*, \\ u_1 &= \frac{\mu_B^{\frac{1}{2}} \beta V^*}{(V^{*2} + \mu_B)}, \\ u_2 &= \mu_B^{\frac{1}{2}} \frac{4}{3\sqrt{6}} \cdot \frac{\beta V^{*2}}{V^{*2} + \mu_B}, \\ \varphi_1 &= 2\beta u_1, \ \varphi_2 = -2\beta u_2; \ \alpha_B = \frac{54}{35}, \ \mu_B = \frac{81}{70}. \end{aligned} \tag{7B_1}$$

In order to describe the behavior of stationary solutions, we have plotted V^* as a function of G^* for the different systems as shown in Fig.1 a–b. We cen see that there exist three stationary solutions in both systems for certain value of G^*.

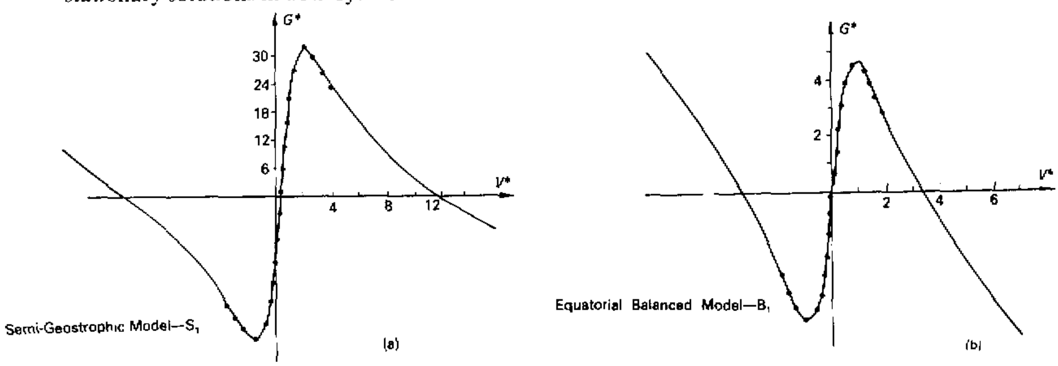

Fig.1. The stationary solutions, (a)for system B_1 and (b) for system S_1. $Q_0 = 0.0$, $h_1 = h_2 = 0.0$, $\varepsilon = 0.3$, $r = 0.2$.

3. *Stability of Multiple Equilibria*

Linearizing system S_1 and B_1 to the stationary solutions, we can determine the stability of these solutions. For S_1, such a linear system can be written as follows:

$$\frac{1}{r}\frac{\partial u_1'}{\partial t} = -u_1' - \frac{2}{\sqrt{6}}\overline{V}^* u_2' + (\beta - \frac{2}{\sqrt{6}}\overline{u}_2)V^{*\prime},$$

$$\frac{1}{r}\frac{\partial u_2'}{\partial t} = \frac{1}{3\sqrt{6}}\overline{V}^* u_1' - u_2' + \frac{2}{3\sqrt{6}}\overline{u}_1 V^{*\prime},$$

$$V^{*\prime} = \frac{4C^*}{\beta} \cdot u', \tag{8S_1}$$

$$\overline{u}_1 = \frac{\beta \overline{V}^*}{(1 + \frac{\overline{V}^{*2}}{\mu_s})},$$

$$\overline{u}_2 = \frac{2}{3\sqrt{6}}\beta\overline{V}^* / (1 + \frac{\overline{V}^{*2}}{\mu_s}),$$

and the corresponding eigenvalue equation is

$$\lambda_{S_1}^2 - T_{S_1}\lambda_{S_1} + D_{S_1} = 0. \tag{9S_1}$$

By the same way, the eigenvalue equation for system B_1 is

$$\lambda_{B_1}^2 - T_{B_1}\lambda_{B_1} + D_{B_1} = 0, \tag{9B_1}$$

where

$$T_j = C_j - 2 - \frac{C_j \overline{V}_j^2}{\overline{V}_j^2 + 1},$$

$$D_j = -(C_j - \frac{C_j \overline{V}_j^2}{1 + \overline{V}_j^2}) + \overline{V}_j^2 + \frac{C_j \overline{V}_j^2}{1 + \overline{V}_j^2}, \tag{10}$$

$$j = S_1, B_1;$$

and

$$\overline{V}_j = \frac{1}{\sqrt{\mu_j}}\overline{V}^*, \; G_j = \frac{1}{\sqrt{\mu_j}}G^*, \; C_j = \frac{\alpha_j}{\mu_j}C^*. \tag{11}$$

Thus a common equation related to equations ($9S_1$) and ($9B_1$) can be obtained.

$$T = C - 2 - \frac{C\overline{V}^2}{1 + \overline{V}^2},$$

$$D = 1 - C + \overline{V}^2 + \frac{2C\overline{V}^2}{1 + \overline{V}^2}. \tag{12}$$

The stationary solutions ($7S_1$)and ($7B_1$) can also be transformed to the following common form by using equation (11).

$$G = -\overline{V} + \frac{C\overline{V}}{\overline{V}^2 + 1}. \tag{13}$$

Equations (12) and (13) then would determine the multiple equilibria and their stabilities for both system S_1 and B_1 respectively. The only distinction is that the definition of the variables and parameters is different as shown in equation (11) in the two systems.Therefore the results of systems S_1 and B_1 can be easily inferred from equations (12) and (13).

When $T<0$, $D>0$, the stationary solution is stable. Thus it can be easily shown that when $C<1$, the stationary solution is always stable. Because $D>0$ for $C<1$, and

$$\frac{\partial G}{\partial V} = -\frac{D}{1+\overline{V}^2} < 0, \tag{14}$$

the stationary solution is singlevalued function of the parameters, i.e. for any given parameters in this case, only one solution exists.

When $1<C<2$, we still have $T<0$, but $D>0$ only if $|V| > v_{c2}$, in which V_{c2} is determined by

$$V_{c2}^2 = \frac{1}{2}\left[\sqrt{(C+2)^2 - 4(1-C)} - (2+C)\right]. \tag{15}$$

Because $\left.\frac{\partial G}{\partial V}\right|_V = \pm V_{c2} = 0$, i.e. there are two zero points for $\partial G / \partial V$, the stationary solution (described by Eg.(13)) is a multivalued function. As shown in Fig.1,three branches of this function exist. The solutions on the middle branch are unstable since $D>0$. Others are stable.

For the case $C>2$, it is necessary that $V>V_{cl}$ for condition $T<0$ and V_{cl} is given by

$$V_{cl} = \sqrt{\frac{C}{2} - 1}. \tag{16}$$

Therefore the stable condition for multiple equilibria is

$$|V| > \max(V_{c1}, V_{c2}) = V_c. \tag{17}$$

As mentioned above, $\pm V$ are the two extreme points of the solution parameter function. Therefore, the existence domains for multiple solutions and single solution on the plane (G,C) can be distinguished by the curve

$$|G| = G(v_{c2})$$

and the domain for two stable equilibria can be determined by

$$|G| = G_c(v_{c1}).$$

The results are shown in Fig.2. It can be seen that the multiple equilibria domain is larger than that for stable domain.

The influence of parameter r, ε and Q on the equilibria and their stabilities can be further explored with Eqs. (7) and (11) as follows:

(1) On the combined parameter space (C, G), when $C>1$, multiple equilibria with two stable solutions can be found in both systems for not too large Q (so for G), but for strong thermal forcing, only one equilibrium can be found.

(2) Strong friction parameter will also destroy the multiple equilibria in both systems.

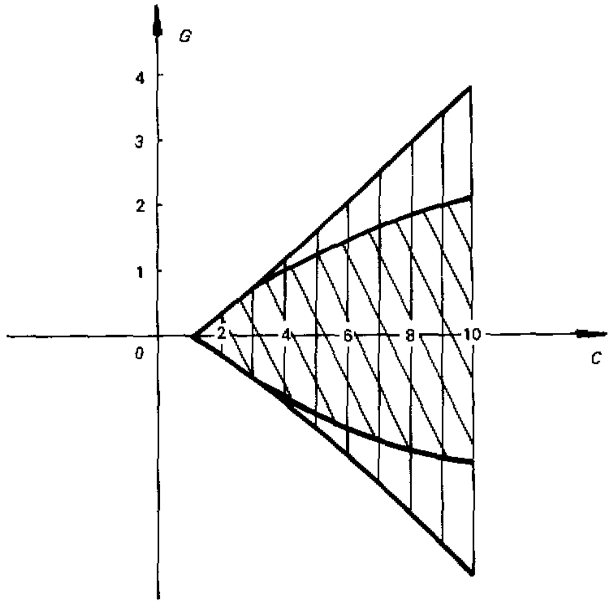

Fig.2. Existence domains for multiple equilibria on parameter space in system S_1 and B_1.

(3) The dynamic behavior of the combined parameter space presented here could be understood as that a large domain of individual parameter values is appropriate for the physical systems.

The above analyses show that the system S_1 and B_1 have the similar qualitative behavior of the multiple equilibria and their stabilities.

However, the coefficients of the nonlinear terms are different, the stationary solutions V have large values in system S_1 than in B_1. Therefore, for the same parameters, the flow patterns of the stationary solutions in S_1 and B_1 are widely different.

4. *The Low Level Wind Structures*

Table 1 and 2 indicate the numerical results of both systems. It can be found that the amplitudes of all components have much large values in S_1 than in B_1. Thus the solutions in the semi–geostrophic system are too strong while the wind structures related to the equatorial balance system are comparable with the observation.

So, we can conclude that the general behavior of the multiple equilibria and their stabilities of the equatorial semi–geostrophic model and the equatorial balance model are qualitatively very similar, but their quantitative properties are sensitive to the two systems. This is due to the incompleteness resulted from highly truncated, although the two systems are mathematically equivalent. Otherwise, the solutions of the two complete spectral expansions would be quite similar.

Table 1. Numerical Results of Multiple Equilibria S_1 ($Q_0=0.04$, $h_1=0.25$, $h_2=h_1/3$, $\varepsilon=0.3$, $\gamma=0.2$)

Q_1	u_1	u_2	V_0	V_1	φ_1	φ_2	λ_{rmax}	λ_i
	0.31	3.36	1.11	0.27	1.45	−15.47	−0.75	0.84
−0.20	−0.19	0.87	−0.12	0.07	−0.89	−4.01	3.71	0.0
	−1.59	2.56	−3.51	−0.46	−7.3	−11.76	0.07	1.87
	0.30	2.92	0.90	0.23	1.38	−13.41	−0.30	0.93
−0.10	−0.09	0.98	−0.04	0.09	−0.41	−4.49	3.47	0.0
	−1.59	2.67	−3.69	−0.49	−7.32	−12.29	−0.04	1.93
	0.27	2.38	0.65	0.20	1.23	−10.94	0.23	0.73
0.00	0.03	1.17	0.07	0.10	0.13	−5.37	3.04	0.0
	−1.59	2.78	−3.87	−0.52	−7.34	−12.80	−0.15	1.99
0.10	−1.60	2.89	−4.06	−0.55	−7.35	13.3	−0.26	2.03
0.20	−1.60	3.00	−4.24	−0.57	−7.36	−13.8	−0.37	2.07

Table 2. Numerical Results of the Multiple Equilibria B_s (with same parameter values as in Table 1)

Q_1	u_1	u_2	V_0	V_1	φ_1	φ_2	λ_{rmax}	λ_i
	−0.08	−1.42	0.42	0.16	0.13	1.30	−0.18	0.56
−0.20	0.87	−1.11	−0.35	0.04	−1.33	1.03	0.03	0.40
	0.57	−0.70	−0.11	0.08	−0.87	0.64	0.74	0.0
	0.19	1.36	0.33	0.14	0.29	1.25	−0.15	0.50
−0.10	0.83	−1.23	0.49	0.01	−1.27	1.13	0.04	0.56
	0.28	−0.62	−0.05	0.08	−0.43	0.57	0.85	0.0
	−0.30	−1.26	0.25	0.13	0.46	1.16	−0.08	0.42
0.00	0.78	−1.30	−0.64	−0.01	−1.20	1.19	−0.08	0.69
	0.0	−0.64	0.0	0.09	0.0	0.59	0.81	0.0
	−0.39	−1.03	0.14	0.11	0.60	0.95	0.06	0.23
0.10	0.75	−1.33	−0.78	−0.03	−1.14	1.23	−0.11	0.81
	−0.31	−0.81	0.07	0.10	0.47	0.74	0.51	0.0
	−0.45	−0.67	0.01	0.09	0.69	0.62	0.73	0.0
0.20	0.73	−1.40	−0.94	−0.06	−1.12	1.29	−0.15	0.96
	−0.45	−0.67	0.01	0.09	0.68	0.62	0.73	0.0

III. SENSITIVITY OF THE MULTIPLE EQUILIBRIA TO TRUNCATIONS

In order to examine the sensitivity of multiple equilibria to different truncations for the same model, another kind of truncations for balance model is chosen as follows:

$$\begin{bmatrix} u \\ \varphi \end{bmatrix} = \sum_{i=0,2} \begin{bmatrix} u_i \\ \varphi_i \end{bmatrix} D_i . \tag{18}$$

The expansion for other variables is the same as in B_1. So we obtain another low order system of the balance model named B_2.

$$\begin{aligned} \frac{\partial u_0}{\partial t} &= \frac{2\times 16}{27\sqrt{6}} u_0 v_1 - \frac{2\times 16}{81\sqrt{6}} u_2 v_1 + \beta v_1 - r u_0 , \\ \frac{\partial u_2}{\partial t} &= \frac{16}{27\sqrt{6}} u_0 v_1 - \frac{88}{81\sqrt{6}} u_2 v_1 + \beta v_1 - r u_2 , \\ \varphi_0 &= \frac{1}{3}\beta(14u_0 + 16u_2), \\ \varphi_2 &= \frac{1}{3}\beta(4u_0 + 2u_2), \\ v_0 &= -\frac{Q_1}{0.5 + \frac{2h_2}{\sqrt{6}}}, \\ v_1 &= (-\varepsilon\varphi_0 - Q_0 + \frac{4}{3\sqrt{6}} h_1 v_0) / (0.5 - \frac{2h_2}{\sqrt{6}}). \end{aligned} \tag{18B_2}$$

It is easy to see that quite different coefficients of the nonlinear terms result from the different choices of the expansion.

The dependence of solution on parameter for system B_2 is plotted in Fig.3. Obviously the functions in Fig.1b and Fig.3 are quite different both quantitatively and qualitatively. The symmetric property of the function in Fig.1a disappears in Fig.3.

Furthermore, the dependence of solutions in B_1 and B_2 to the parameter C is significantly different. In Fig.3, the value of C is almost two orders larger than that in Fig.1b. Thus the results of the two systems may differ completely for the same value of the combined parameter C. Table 3b shows that for even smaller C, the U components of the stationary solutions in B_2 are too strong.

Stabilities of solutions in system B_1 and B_2 are not the same. Table 1 and 3 show that there exist two stable equilibria for a certain range of forcing in B_1, but only one stable equilibrium is found in B_2. Therefore, the results of multiple equilibria are very sensitive to the modes chosen in the low order truncations.

IV. CONCLUSIONS

In this paper, the sensitivities of multiple equilibria to the different controlling models (semi-geostrophic model the balance model) and the different choices of basic functions are discussed with the axisymmetric equatorial β-plane model.

Table 3a. Numerical Results of the Multiple Equilibria B_2 (with same parameter values as in Table 1)

Q_1	u_1	u_2	V_0	V_1	φ_0	φ_2	λ_{rmax}	λ_i
	−15.30	−3.15	3.52	142.13	−202.91	−51.76	2.76	59.43
−2.0	−1.77	1.65	3.52	0.29	1.31	−2.88	2.19	0.0
	0.07	0.07	3.52	0.01	1.72	0.34	−0.20	0.0
	−15.30	−3.15	1.76	141.57	−202.91	−51.76	2.75	59.20
−1.0	−1.84	1.65	−	0.29	0.51	−3.11	2.39	0.0
	0.04	0.04	−	0.00	0.93	0.18	−0.20	0.0
	−15.30	−3.15	0.0	141.02	−202.91	−51.76	2.74	58.96
0.0	−1.91	1.65	−	0.30	−0.30	−3.33	2.59	0.0
	−0.01	−0.01	−	0.00	0.13	0.03	−0.20	0.0
	−15.30	−3.15	−1.76	140.46	−202.91	−51.76	2.73	58.73
1.0	−2.00	1.65	−	0.30	−1.20	−3.56	2.80	0.0
	−0.03	−0.03	−	−0.00	−0.66	−0.13	−0.20	0.0
	−15.30	−3.15	−3.52	139.91	−202.91	−51.76	2.72	58.50
2.0	−2.05	1.64	−	0.31	−1.90	−3.78	3.01	0.0
	−0.06	−0.06	−	−0.01	−1.46	−0.29	−0.20	0.0

Table 3b. Numerical Results of the Multiple Equilibria B_2 (but $\varepsilon=0.01$, $\gamma=0.1$)

Q_1	u_0	u_2	V_0	V_1	φ_0	φ_2	λ_{rmax}	λ_i
−2.0	−15.12	−2.94	3.52	5.61	−198.36	−50.87	0.05	2.32
	−15.09	−2.92	1.76	5.04	−197.76	−50.75	0.05	2.08
−1.0	−0.26	1.52	−	0.10	15.63	1.52	0.06	0.0
	0.45	1.05	−	0.05	17.63	2.97	−0.04	0.0
	−15.06	−2.88	0.0	4.47	−196.98	−50.60	0.04	1.84
0.0	−2.88	1.58	−	0.18	−11.57	−6.42	0.35	0.0
	−0.17	−0.16	−	−0.01	−3.71	−0.75	−0.10	0.0
	−15.01	−2.84	−1.76	3.89	−195.95	−50.39	0.04	1.59
1.0	−4.98	1.26	−	0.23	−38.03	−13.34	0.52	0.0
	−1.32	−0.98	−	−0.04	−26.20	−5.55	−0.12	0.0
	−14.95	−2.78	−3.52	3.30	−194.48	−50.09	0.03	1.33
2.0	−6.94	0.80	−	0.30	−64.72	−20.06	0.63	0.0
	−2.63	−1.69	−	−0.07	−48.89	−10.64	−0.13	0.0

Under the same truncation, the two low order systems derived from equatorial balance model and semi–geostrophic model are qualitatively similar for the existence of the multiple equilibria and their stabilities.

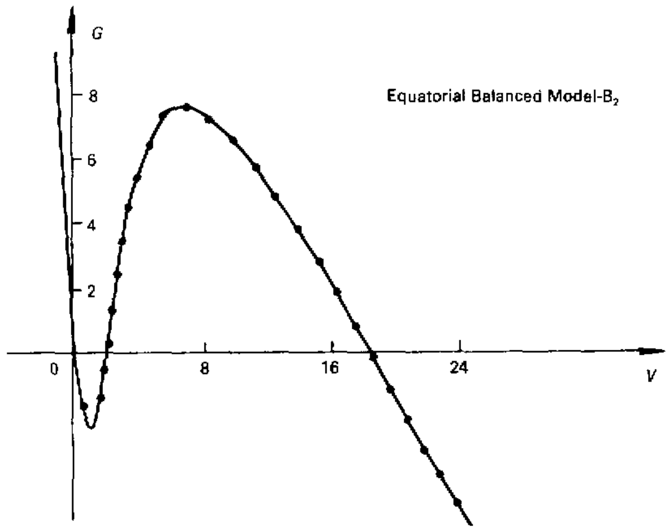

Fig.3. Plot of steady solution as a function of parameter for system B_2.

But the magnitudes of the solutions in these two systems are different by several times. Therefore the folw patterns of the solutions in the low order systems are sensitive to the controlling model. For the same equatorial balance model and the same number of expanding base functions the two low order systems are obtained by choosing different basic functions. In this case, the multiple solutions and their stabilities are essentially different from each other. Therefore the multiple equilibria are very sensitive to the chosen base function in the truncation. Thus, it remains to be studied to ensure that whether the results obtained from such kind of low order model are arbitrary or not.

REFERENCES

Charney, J. G. and Devore, J. G. (1979), Multiple flow equilibria in the atmosphere and blocking, *J. Atmos. Sci.*, **36**: **1205–1216.**

Charney, J. G. and Straus, D. M. (1980),Form drag instability, multiple equilibria and propagating planetary waves in baroclinic, orographically forced, planetary wave systems, *J. Atmos. Sci.*, **37**: **1157–1176.**

Goswami, B.N.(1983). Theoretical study of multiple equilibria in simple axisymmetric tropical circulations, *Tellus*,**35**: **119–135.**

Zhu Baozhen and Zhao Jingxia (1987), Multiple flow equilibria in the tropical circulation and monsoon. *Advances in Atmospheric Sciences*, **4**: 375–384.