Atmosphere–Ocean Coupling Schemes in a One–Dimensional Climate Model

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ABSTRACT

In this paper, the coupling schemes of atmosphere–ocean climate models are discussed with one–dimensional advection equations. The convergence and stability for synchronous and asynchronous schemes are demonstrated and compared.

Conclusions inferred from the analysis are given below. The synchronous scheme as well as the asynchronous–implicit scheme in this model are stable for arbitrary integrating time intervals. The asynchronous explicit scheme is unstable under certain conditions, which depend upon advection velocities and heat exchange parameters in the atmosphere and oceans. With both synchronous and asynchronous stable schemes the discrete solutions converge to their unique exact ones. Advections in the atmosphere and ocean accelerate the rate of convergence of the asynchronous–implicit scheme. It is suggusted that the asynchronous–implicit coupling scheme is a stable and efficient method for most climatic simulations.

I. INTRODUCTION

A comprehensive climate model usually consists of the atmosphere, oceans and other components. One of the important problems of climate simulation is coupling strategy between both fluids: the atmosphere and oceans. The atmospheric model usually takes much longer to compute per unit simulated time than the oceanic model does, however, the latter takes more time to reach the equilibrium state. In the past, several investigators (e.g., Manabe et al., 1979; Washington et al.; 1980) have proposed coupling strategies in their climate simulation models. First ,Manabe et al. (1979) presented the problem. Later Schlesinger (1979) reviewed these coupling strategies. Schneider and Thompson (1981) and Bryan et al. (1982) studied the transient response to atmospheric CO_2 with a coupled atmosphere–ocean model. The theoretical investigation of this subject was made by Dickinson (1981). In his analysis the stability and rate of convergence for various coupling schemes was discussed with a zero–dimensional climate model in which the advection in either medium was ignored .

The present paper intends to analyze the same issue using a one–dimensional model, i.e., inclusion of horizontal advection. We will investigate the effect of the horizontal "motion" on coupling strategy. Some fundamental problems are presented to us, such as which coupling scheme is stable when the procedure is carried out? Does it converge? If so, what is the climate equilibrium state? Is the equilibrium state unique or does it depend on which scheme is used?

II. AIR–SEA COUPLING MODEL

As an introduction to the basic concepts involved in the air–sea coupling schemes, a simple set of linear equations with constant coefficients, usually referred to as the advection equation, will be considered. First, we propose a thermodynamical system in which the heat exchange between the atmosphere and oceans is included. It is assumed that the solar radiation passes through the atmosphere, as a transparent media, and is absorbed by sea water. Heat transfers from ocean to atmosphere, and then emits to the space by infrared radiation. Under the balance between the incident radiation and outgoing radiation,there exists an equilibrium state–climatic state.

Generally speaking, the coupling schemes between the atmospheric and oceanic model can be simplified as in Fig.1. The atmospheric model is integrated from t_1 to $t_2(=t_1+\Delta t)$ and its boundary conditions are taken to be the ocean temperature as calculated by the oceanic model. From t_2 to t_3 $(=t_2+\Delta t_s)$ the oceanic model is integrated, the upper boundary conditions on the ocean model are taken to be the variables calculated by the atmospheric model. The entire cycle, from t_1 to t_3, is repeated until equilibrium is achieved.

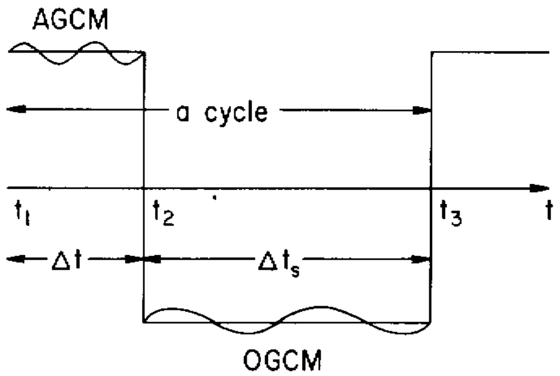

Fig.1. Schematic representation of coupling methodology between AGCM and OGCM.

The heat exchange fluxes (sensible, latent and radiative) between both fluids, in linear form, can be taken in terms of air–sea temperature difference with different coefficients. The advection velocities, in the atmosphere and ocean, U, U_s are assumed constants.

The equations of the model are

$$\frac{\partial T}{\partial t}+U\frac{\partial T}{\partial x}=\alpha_{as}(T_s-T)-\alpha_r T, \tag{1}$$

$$\frac{\partial T_s}{\partial t}+U_s\frac{\partial T_s}{\partial x}=\alpha_{sa}(T-T_s)+Q_r e^{i\omega_0 t}. \tag{2}$$

T, T_s are the departures of atmospheric and upper oceanic temperature from climatic means. Q_r is the amplitude of a periodic external forcing such as changes in solar radiation. We can also discuss the response of the system to the other forcings in the atmosphere or oceans. $\omega_0\equiv 2\pi/p$. p is the period of forcing. α_r is the parameter related to outgoing radiation at the top of the atmosphere and equals λ/C_a. From the work on the global surface temperature response to an increase in atmospheric carbon dioxide concentration $\lambda=1.7\pm0.8$ W $m^{-2}K^{-1}$ (NAS,1979), and C_a is heat capacity of the atmosphere of unit area, about 10^7 W m^{-2} s K^{-1}. Then $\alpha_r\approx1.7\times10^{-7}s^{-1}$, but for the local problem we assume $\alpha_r\approx5\times10^{-7}s^{-1}$. The first term on the right side of (1) may be considered the total heat transfer from the ocean to the atmosphere and α_{as} equals λ_{am}/C_a where λ_{am} is approximately 45 W m^{-2} K^{-1} as evaluated by Dickinson (1981). Hence α_{as} becomes $4.5\times 10^{-6}s^{-1}$. In a similar way this represents transfer of heat to ocean from atmosphere, thus α_{sa} should be $C_a\alpha_{as}/C_s$ where C_s is heat capacity of the upper ocean, which is about 3.24 $\times 10^8$ W m^{-2} s K^{-1} , thus $\alpha_{sa}\approx 1.4\times 10^{-7}$s. $Q_r = R/C_s \approx 1.05\times 10^{-6}$K s^{-1}. Here *the depth of upper ocean is 80 m.For these processes,the time scales are*

$$0(t_{as})\approx \alpha_{as}^{-1}\approx 2.2\times 10^5 s(2.6 \text{ days })$$
$$0(t_{sa})\approx \alpha_{sa}^{-1}\approx 7\times 10^6 s(81 \text{ days }) \tag{3}$$
$$0(t_r)\approx \alpha_r^{-1}\approx 2\times 10^6 s(23 \text{ days }),$$

respectively, where δT_s is a variation of ocean temperature about 4 °. In general, the horizontal scales of large–scale atmospheric and oceanic motion L, $L_s\approx 3\times 10^6$ m. The mean flows $U\approx 10$ m s^{-1}in the atmosphere and $U_s\approx 0.1$ m s^{-1} in the oceans. Hence the advection time scales of atmospheric and oceanic motion are

$$0(t)\approx L/U \approx 3\times 10^5 \; s \text{ (3.5 days)}$$
$$0(t_s)\approx L_s/U_s \approx 3\times 10^{-7} \; s \text{ (350 days)}. \tag{4}$$

In the atmosphere, advection and heat transfer from the ocean have nearly the same time scales, i.e., about 3 days .Relatively, the solar and outgoing radiation are somewhat slower processes. But due to the huge heat capacity of the ocean, the ocean temperature changes very slowly by heat transfer from the atmosphere, and has a time scale on the order of months. In this sense, the ocean temperature is approximately steady on the atmospheric advection time scale $0(t)$, i.e.,

$$\frac{\partial T_s}{\partial t}\approx 0, \qquad T_s = T_s(x,t_s). \tag{5}$$

On the other hand, when the ocean temperature varies on the time scale $0(t_s)$, the air temperature will adjust quickly to oceanic conditions, then

$$U\frac{\partial T}{\partial x} + (\alpha_{as}+\alpha_r)T = \alpha_{as}\cdot T_s. \tag{6}$$

Taking into consideration these two time scales, the atmospheric and oceanic models in the coupled system can be integrated separately.

As discussed by schlesinger (1979) and Dickinson (1981). the coupling scheme between atmospheric general circulation model (AGCM) and oceanic general circulation model (OGCM) can be mainly divided into two groups. We will give them the exact definitions.

a) synchronous scheme: a time step is contained in each integrating time interval of AGCM and OGCM ,then

$$\Delta t = \delta t, \qquad \Delta t_s = \delta t_s. \tag{7}$$

where $\delta t, \delta t_s$ are the time step of AGCM and OGCM respectively. Because the time step is much shorter than the time scales of motion in corresponding fluid, therefore , the integration of system approaches to the exact one.

b) asynchronous scheme: a number of time step are contained in each integrating time interval then

$$\Delta t = N\cdot \delta t$$
$$\Delta t_s = N_s\cdot \delta t_s. \tag{8}$$

Where N and N_s are integers.

In order to reach an equilibium state, the integration of climatic model (at least including the atmosphere and oceans) should take a long time ($10^2 \sim 10^3$years). With synchronous scheme, it will expend plenty of computer time. Thus most climatic models so far employ the asynchronous schemes.

III. ATMOSPHERIC MODEL

In the previous section , the analysis of the time scales has shown that the processes in the atmosphere are one to two orders faster than those in the ocean. When the atmospheric model is integrated, T_s as a boundary condition changes slowly. Let us assume that T_s varies only due to horizontal advection

$$T_s = Ae^{i\mu(x - U_s t)}, \tag{9}$$

where U_s represents the advection speed of a perturbation. A is the amplitude of ocean temperature and μ is a wavenumber.

The equation for the atmosphere by substituting (9) into (1) is

$$\frac{\partial T}{\partial t} + U\frac{\partial T}{\partial x} + (\alpha_{as} + \alpha_r)T = \alpha_{as} Ae^{i\mu(x - U_s t)}, \tag{10}$$

and initial condition is assumed as $T(x,0) = Fe^{i\mu x}$. The corresponding solution Δt later is

$$T(x,t) = \frac{\alpha_{as} A}{\alpha_{as} + \alpha_r + i\mu U(1 - U_s / U)}\left[e^{i\mu(x - U_s\Delta t)} - e^{i\mu(x - U\Delta t) - (\alpha_{as} + \alpha_r)\Delta t}\right] + Fe^{i\mu(x - U\Delta t) - (\alpha_{as} + \alpha_r)\Delta t}. \tag{11}$$

The second term on the right side of (11) contains a decaying wave resulting from initial fluctuation, and the first term comes from boundary condition T_s . When the atmospheric model has been integrated for a long time, $\Delta t \gg (\alpha_{as} + \alpha_r)^{-1} = 2 \times 10^5$ s

$$T = \frac{\alpha_{as} A}{\alpha_{as} + \alpha_r + i\mu U(1 - U_s / U)} e^{i\mu(x - U_s\Delta t)}. \tag{12}$$

The atmospheric temperature changes along with the ocean temperature, or say, have adjusted to T_s . This is an exact solution of atmospheric state.

When using the asynchronous coupling scheme, oceanic temperature does not change with time, $U_s = 0$, then

$$T = a_x Ae^{i\mu x}, \tag{13}$$

where

$$a_x \equiv \frac{\alpha_{as}}{\alpha_{as} + \alpha_r + i\mu U}.$$

In fact, the ocean temperature always varies. Compared to the analytic solution (12), the relative error of the this case for $\Delta t \gg (\alpha_{as} + \alpha_r)^{-1}$ and $U_s / U \ll 1$ is

$$E = \frac{T_1 - T_2}{T_2} \simeq e^{i\mu U_s \Delta t} - 1.$$

If $\Delta t \ll (\mu U_s)^{-1}$, which is for time scales much shorter than advection time scales of ocean, $|E| \approx 0$; but when $\mu U_s \Delta t \approx \pm \pi$, $E \approx 0(1)$. Therefroe, an appropriate integrating time interval for the atmospheric model with asynchronous scheme should be taken

$$(\alpha_{as} + \alpha_r)^{-1} \ll \Delta t \ll (\mu U_s)^{-1}. \tag{14}$$

The slower the current is, the smaller the error and the larger the limitation on the interval for Δt . For large-scale motions in the atmosphere and oceans , $\mu \approx 10^{-6}$ m^{-1}.

$$2 \times 10^5 \text{ s (2.4 days)} \ll \Delta t \ll 10^7 \text{ s (116 days)}.$$

This means that the atmospheric state will adjust to the oceanic boundary conditions with a small error in this interval of time .

IV. OCEANIC MODEL

In this section, we will investigate the integration of oceanic modle in more detail. Several coupling schemes have been used in climatic simulations.

a. Synchronous scheme

As indicated in section 2, in this case atmospheric temperature T adjusts to T_s at any moment and is expressed by $T_s, T = T(T_s)$ .The corresponding heat exchange term is proportional to

$$T_s - T(T_s). \tag{15}$$

At the initial time, as the assumption in Section III, the air and sea temperature are

$$T^{(0)} = Fe^{i\mu x}, \qquad T_s^{(0)} = Ae^{i\mu x}. \tag{16}$$

Integrating the atmospheric model (10) ,we have

$$T^{(1)} = a_\infty T_s^{(0)}.$$

The superscripts represent the order of cycles, then, the equation for $T_s^{(1)}$ is

$$(\frac{\partial}{\partial t} + U_s \frac{\partial}{\partial x})T_s + \alpha_{sa}(1 - a_\infty)T_s = Q_r \cdot e^{i\omega_0 t}. \tag{17}$$

After this procedure has been carried out N times, we have

$$T_s^{(N)} = T_s(x, N\Delta t_s) = C^N T_s^{(0)} + Q_r d \sum_{K=0}^{N-1} (G^k e^{+i\omega_0 \Delta t_s (N-K)}), \tag{18}$$

where

$$C \equiv e^{-\Delta t_s (i\mu U_s + \alpha_{sa}(1 - a_\infty))} \tag{19}$$

$$d \equiv \frac{1}{i\omega_0 + \alpha_{sa}(1 - a_\infty)} \left[1 - e^{-[\alpha_{sa}(1 - a_\infty) + i\omega_0]\Delta t_s} \right] \tag{20}$$

$$G \equiv \left[\frac{\alpha_{as}}{\alpha_{as} + \alpha_r} + \frac{\alpha_r}{\alpha_{as} + \alpha_r} e^{-\alpha_{sa}(1 - a_\infty)\Delta t_s} - a_\infty \right] \frac{1}{1 - a_\infty}. \tag{21}$$

The variation of sea temperature includes two parts. One resulting from initial distribution of T_s is a transient wave. The other results from radiation heating. As the procedure continues, the air-sea coupling system should approach an equilibrium state. But it depends on the time interval Δt_s advection (currents) and parameters.

When the series of $T_s^{(N)}$ converges as $N \to \infty$ the ocean temperature would approach a steady state; if the series diverges, $T_s^{(N)}$ would increase infinitely and is unstable. But if

$$|C| \leqslant 1, \qquad |G| < 1, \tag{22}$$

then $T_s^{(N)}$ would converge (note that $|e^{-i\omega_0 \Delta t_s}| = 1$).

We are going to look for the conditions satisfying (22). let $a_\infty = a + bi$,where

$$a = \frac{\alpha_{as}(\alpha_{as} + \alpha_r)}{(\alpha_{as} + \alpha_r)^2 + (\mu U)^2} < 1, \qquad b = \frac{-\mu U \alpha_{as}}{(\alpha_{as} + \alpha_r)^2 + (\mu U)^2} < 1,$$

C becomes

$$C = e^{i(\mu U + b)\Delta t_s} \cdot e^{-a\Delta t_s}.$$

similarly,

$$|e^{i(\mu U - b)\Delta t_s}| = 1.$$

Therefore, the condition $|C| < 1$ inquires ($a > 0$)

$$\Delta t_s > 0. \tag{23}$$

It indicates that the part related to the initial condition is always stable for any time interval $\Delta t_s > 0$ (Fig.2a,2c). On the other hand, absolute $|G| < 1$ means

$$\left| \frac{\alpha_{as}}{\alpha_{as} + \alpha_r} + \frac{\alpha_r}{\alpha_{as} + \alpha_r} e^{-\alpha_{sa}(1 - a_f)\Delta t_s} - a_f \right| < |1 - a_f|. \tag{24}$$

There are two limiting cases: i) when $\Delta t_s \ll (\alpha_{sa})^{-1}$, which is the time interval much shorter than the characteristic time of heat exchange between ocean and atmosphere $(\alpha_{sa})^{-1}$, then

$$\frac{2(\alpha_r + \alpha_{as})}{\alpha_r \cdot \alpha_{as}} > \Delta t_s > 0.$$

ii) When $\Delta t_s \gg (\alpha_{sa})^{-1}$, then

$$e^{-\alpha_{sa}(1 - a_f)\Delta t_s} \approx 0$$

(24) becomes

$$\left| \frac{\alpha_{as}}{\alpha_{as} + \alpha_r} - a_f \right| < |1 - a_f|. \tag{25}$$

Because

$$\frac{\alpha_{as}}{\alpha_{as} + \alpha_r} < 1,$$

(25) is obviously true.

It is shown than the part related to radiation forcing is also stable for any $\Delta t_s > 0$ (Fig.2b,d). In summary, the synchronous scheme in a coupled atmosphere –ocean GCM is always stable for any integrating time interval. The stability of this scheme is independent on mean advection velocity in the ocean because of the fact that the atmosphere always adjusts quickly to ocean temperature changes and is not influenced by annual variation of solar radiation. The annual cycle of radiation affects the evolution of ocean temperature only. This scheme can be carried out until equilibrium is reached. But as mentioned above, a synchronous scheme requires that both T and T_s must adjust to each other at any time. Due to the fact that the atmospheric model takes a great deal of computer time for a synchronous model, it is only practical for climate sensitivity experiments in which only the upper layers of the ocean are involved.

b. Asynchronous schemes

The few experments performed so far have used the asynchronous schemes. The differences among them are the treatments of exchangs of heat, water vapor, momentum and integrating time interval. The problem is which scheme is the most efficient and accurate, and under what conditions it is stable and convergent? In this model, the coupling between ocean and atmosphere is only considered through the term of heat exchange term. This term can be calculated by several methods. The implicit and explicit methods have been tested (see Dickinson, 1981).

1) Implicit asynchronous

The air temperature retains its value at the end of Δt when the oceanic model runs, which means the atmosphere is not responding to the oceanic temperature change within Δt_s , $T=T(t_2)$, and the heat exchange term becomes

$$T_s - T(t).$$

The variation of ocean temperature for the Nth cycle is governed by

$$\frac{\partial T_s}{\partial t} + U_s \frac{\partial T_s}{\partial x} + \alpha_{sa} T_s = Q_r e^{i\omega_0 t} + \alpha_{sa} T_s^{(N)}. \tag{26}$$

It begins to run from the same initial conditions as those in the synchronous case, then (here $T_s(N) = a_x T_s^{(N-1)}$)

$$T_s^{(N)} = C_i^N T_s^{(0)} + Q_r d_i \sum_{K=0}^{N} (G_i^k e^{+i\omega_0 \Delta t_s (N-K)}), \tag{27}$$

where c_i , d_i and G_i are destabilized factors which depend on time interval Δt_s , exchange coefficient and advection (see Appendix). $T_s^{(N)}$ retains limited when $N \to \infty$, if

$$|c_i| \leqslant 1 \qquad |c_i| < 1. \tag{28}$$

Obviously, the latter in (28) holds for any $\Delta t_s > 0$. With the increase of Δt_s , the second term of c_i decays quickly, while the first term is always less than 1. But for $\Delta t_s \ll \alpha_{sa}^{-1}$, absolute $|c_i| \leqslant 1$, results in

$$\Delta t_s \lessapprox \frac{2(\alpha_{sa} + \mu U_s)}{\alpha_{sa}^2 + (\mu U_s)^2} \approx 10^7 \text{ s} \quad (116 \text{ days}) \approx 0(\alpha_{sa}^{-1}, (\mu U_s)^{-1}). \tag{29}$$

Obviously, it is true. The results shown above (see Fig.3) indicate that an asynchronous–implicit scheme is stable for any integration time interval. Its degree of stability depends on the parameters of heat exchange and advection. Because the heat flux changes with ocean temperature, even Δt_s is very long and T , T_s are always adjusted to reach a balance state.

2). Explicit asynchronous

For this scheme, the heat exchange term is determined by the T , T_s at the end of Δt and remains constant when the oceanic model runs. The variation of $T_s^{(N)}$ is governed by

$$\frac{\partial T_s}{\partial t} + U_s \frac{\partial T_s}{\partial x} + \alpha_{sa}(T_s^{(N-1)} - T^{(N-1)}) = Q_r e^{i\omega_0 t}. \tag{30}$$

The initial values of $T_s^{(0)}, T^{(0)}$ are the same for the synchronous, (30) yields

$$(\frac{\partial}{\partial t} + U_s \frac{\partial}{\partial x}) T_s = Q_r e^{i\omega_0 t} - \alpha_{sa}(1 - a_\infty) T_s^{(N-1)}. \tag{31}$$

The solution for the Nth cycle is

$$T_s^{(N)} = c_e^N T_s^{(0)} + d_e Q_r \sum_{K=0}^{N-1} (e^{+i\omega_0 \Delta t_s \cdot (N-K)} \cdot G_e^k), \tag{32}$$

where c_e, d_e and G_e are destabilized factors (see Appendix).

In contrast with the two cases above, the coefficient c_e does not decay

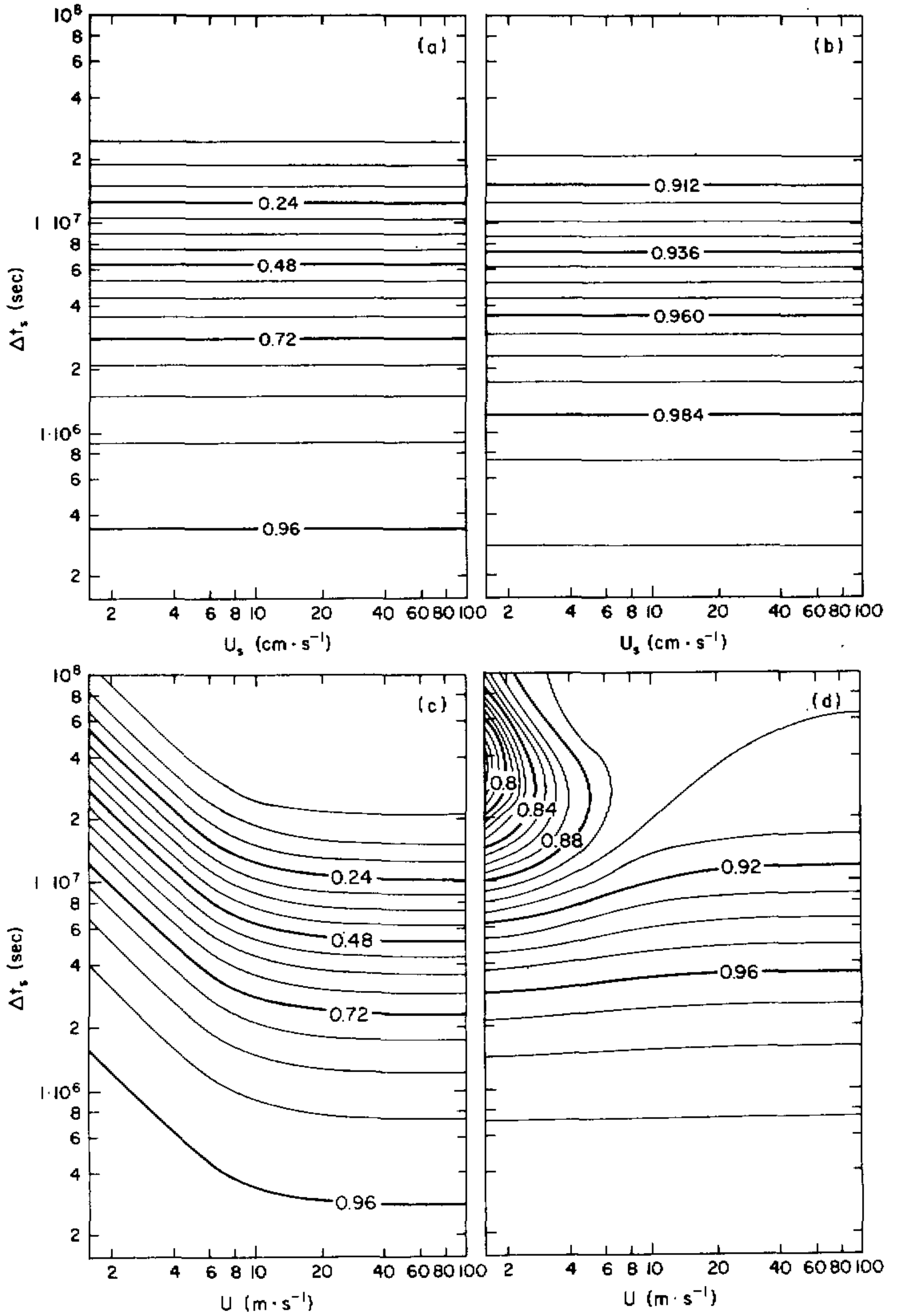
(a)
(b)
(c)
(d)
0.24
0.48
0.72
0.96
0.912
0.936
0.960
0.984
0.8
0.84
0.88
0.92
Δt_s (sec)
U_s (cm·s^{-1})
U (m·s^{-1})

Fig.2a Absolute value of C for synchronous scheme as a function of U_s and Δt_s .

2b. As in Fig.2a except for G .

2c. Absolute value of C for synchronous scheme as a function of U and Δt_s .

2d. As in Fig.2c except for G .

monotonously with the increase of Δt_s (as shown in Fig.4a,b). The other condition keeping series $T_s^{(N)}$ bounded is

$$|G_e| < 1, \tag{33}$$

we obtain the limitation for Δt_s

$$0 < \Delta t_s < \frac{2(\alpha_{as} + \alpha_r)}{\alpha_{sa} \cdot \alpha_r}. \tag{34}$$

Because the initial value does not disappear as $N \to \infty$, this scheme is unstable unless the time interval must be less than a certain value (e.g., 2 months in Fig.4a,b). The temperature would increase infinitely if heat exchange between atmosphere and ocean is held fixed too long. For practical purposes the scheme is indeed stable since it would be unreasonable to hold the heat exchange term constant for two months.

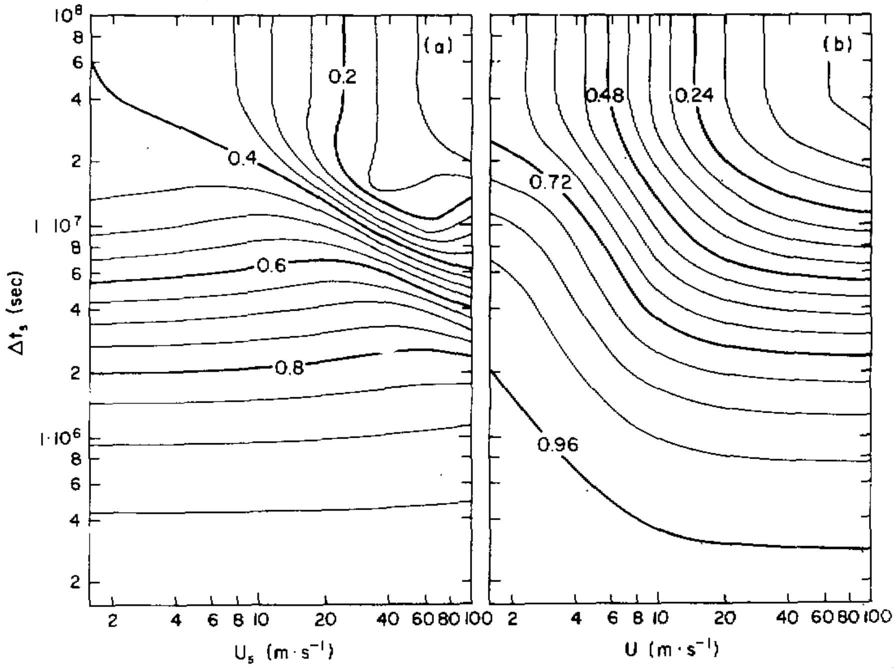

Fig.3a. Absolute value of C_e for asynchronous–explicit scheme as a function of U_s and Δt_s 3b. As in Fig.3a except for U .

V. RATES OF CONVERGENCE

Since we have discussed the stability of various schemes, it will be useful to investigate the rates of convergence and the limit of convergence. Following Dickinson (1981), the ratio can be examined:

$$R_a = \frac{T_s^{(N+1)} - T_s^{(N)}}{T_s^{(N)} - T_s^{(N-1)}}. \tag{35}$$

Solutions (20), (27), (32) have similar forms and consist of two parts: an attenuated wave resulting from initial conditions and the other one coming from radiation forcing. The two separate aspects can be examined separately.

First, assume the radiation vanishes, as $\Delta t_s \to 0$ for the synchronous (Syn):

$$R_{aI} = c = 1 - \Delta t_s[i\mu U_s + \alpha_{sa}(1-a_s)] + \frac{1}{2}\Delta t_s^2[i\mu U_s + \alpha_{sa}(1-a_s)]^2 + 0(\Delta t_s^3).$$

Second, assume the initial and ω_0 vanishes

$$R_{aII} = G = 1 - \frac{\alpha_{sa}\alpha_r}{\alpha_{as}+\alpha_r}\Delta t_s + \frac{\alpha_{sa}^2\alpha_r}{2(\alpha_{as}+\alpha_r)}(1-a_s)\Delta t_s^2 + 0(\Delta t_s^3)$$

Similarly, for the asynchronous–implicit (Asyn–I):

$$R_{aI} = c_i = 1 - \Delta t_s[i\mu U_s + \alpha_{sa}(1-a_s)] + \frac{1}{2}\Delta t_s^2[i\mu U_s + \alpha_{sa}(1-a_s)](i\mu U_s + \alpha_{sa}) + 0(\Delta t_s^3)$$

$$R_{aII} = G_i = 1 - \frac{\alpha_{sa}\alpha_r}{\alpha_{as}+\alpha_r}\Delta t_s + \frac{\alpha_r\alpha_{sa}^2}{2(\alpha_{as}+\alpha_r)}\Delta t_s^2 + 0(\Delta t_s^3)$$

and, for the asynchronous–explicit (Asyn–E):

$$R_{aI} = c_e = 1 - \Delta t_s[i\mu U_s + \alpha_{sa}(1-a_s)] + \frac{1}{2}\Delta t_s^2[i\mu U_s + \alpha_{sa}(1-a_s)](i\mu U_s) + 0(\Delta t_s^3).$$

$$R_{aII} = G_e = 1 - \frac{\alpha_{sa}\alpha_r}{\alpha_{as}+\alpha_r}\Delta t_s.$$

From the above formulas all R_a approach to 1 as $\Delta t_s \to 0$. It means that the three coupling schemes are all convergent to the solutions of governing equations.

Let us examine the rate of convergence. First compare the R_{aI} of the schemes, in which the terms of $0(\Delta t_s)$ have the same form, therefore, for the first degree of approximation, the rates of convergence of three schemes are identified. The coupling schemes converge, if

$$\Delta t_s < \alpha_{sa}\left(1 - \frac{\alpha_{as}(\alpha_{as}+\alpha_r)}{(\alpha_{as}+\alpha_r)^2 + (\mu U)^2}\right).$$

For a given value of Δt_s, the more large the advection U is, the more fastly the schemes converge. But if the advections vanish (i.e., $U=0$, $U_s=0$), then

$$a_s = 1, \qquad a_s = \alpha_{as} / \alpha_{as} + \alpha_r,$$

(a_s see Appendix) which yields

$$R_{aI} = -\begin{cases} 1 - \alpha\cdot\Delta t_s + \frac{1}{2}\alpha^2\Delta t_s^2 & \text{Syn} \\ 1 - \alpha\cdot\Delta t_s + \frac{1}{2}\alpha\alpha_{sa}\Delta t_s^2 & \text{Asyn}-\text{I} \\ 1 - \alpha\cdot\Delta t_s & \text{Asyn}-\text{E}. \end{cases} \tag{36}$$

where $\alpha \equiv (\alpha_{sa}\alpha_r)/(\alpha_{as}+\alpha_r)$. These formulas are similar to those given by Dickinson (1981). [See (24) in his paper and $\beta \approx 20$ for Asyn–I, $\beta = 0$ for Asyn–E.] The terms of $0(\Delta t_s)$ in three formulas are identified. But as to the terms of $0(\Delta t_s^2)$, since

$\alpha = \alpha_{sa}\alpha_{r} / (\alpha_{as} + \alpha_{l}) < \alpha_{sa}$, the Syn–scheme converges more rapidly than Asyn–I and more slowly than Asyn–E, if the latter is stable. Comparing both cases with and without advection indicates that the mean flows in the atmosphere and oceans make the convergence of the coupling schemes go faster.

In a case without advections, a_s , a_∞ are real. T varies in phase with T_s . But in a case with advection, a_s, a_∞ are complex. after adjusting T to T_s , T varies out of phase with T_s . at that time, generally the temperature difference $T_s - T$ is larger than that in a case without advection. The strength of heat flux between the atmosphere and ocean depends on $T_s - T$. The larger temperature defference, the stronger heat exchange and then the faster the procedure approaches to equilibrium climatic state.

Secondly, by comparing the R_a II . the Syn–scheme converges more rapidly than Asyn–I, but slower than Asyn–E.

In general the rates of convergence depend mainly upon the parameters related to thermodynamical processes: heat exchanges and radiation. However, the advections in the atmosphere and oceans would change the rates in some cases and influence the asymptotic behavior (oscillation) of procedures from one cycle to the next.

When the coupled system runs for a long time, what is the equilibrium climatic state which the procedure is approaching? As $N \to \infty$, the effectiveness of the initial conditions eventually disappear and sea temperature has a limit from (20), (27), and (32)

$$T_s^{(\infty)} = dQ_r / (1 - Ge^{-i\omega_0 \Delta t_s}) \qquad \text{Syn} \qquad (37)$$

$$T_s^{(\infty)} = d_i Q_r / (1 - G_i e^{-i\omega_0 \Delta t_s}) \qquad \text{Asyn} - \text{I} \qquad (38)$$

$$T_s^{(\infty)} = d_e Q_r / (1 - G_e e^{-i\omega_0 \Delta t_s}) \qquad \text{Asyn} - \text{E.} \qquad (39)$$

(39) is obtained as Δt_s is less than a certain value (Section IV.b.2). There is a common component in the expressions of d , d_i and d_e , i.e.

$$e^{+i\omega_0 N\Delta t_s},$$

which can be written as $e^{i\omega_0 t}$, $N \to \infty$. Expressions (37)–(39) indicate that when the procedure has been carried out for a long time the ocean temperature will vary with annual variation of radiation and will be almost independent of mean currents.

If $\Delta t_s \ll \alpha_{sa}^{-1} < \omega_0^{-1}$, then we have

$$T_s^{(\infty)} = Q_r e^{i\omega_0 t} / (\alpha + i\omega_0) \qquad (40)$$

for three coupling schemes. Note in spite of the scheme used, the coupled model gives the same climatic state in each case. Thus, we can use any of the air–sea coupling strategies tested in this paper. The amplitudes of $T_s^{(\infty)}$ are determined by the magnitude of atmosphere –ocean heat exchange (and heat capacities of atmosphere and ocean) and solar radiation.

Corresponding to (40) the atmosphere temperature is

$$T^{(\infty)} = Q_r a_\infty e^{i\omega_0 t} / (\alpha + i\omega_0). \qquad (41)$$

It varies also with radiation and is always smaller than ocean temperature because absolute $|a_\infty| < 1$. This means that there always exists heat transfer from oceans to atmosphere in order to balance infrared radiation loss.

VI. INFLUENCE OF ADVECTION IN THE ATMOSPHERE AND OCEAN

In this section we discuss the influence of advection in the atmosphere and oceans and the integrating time interval on the stability and convergence rate. The absolute values of coefficients C , C_i, C_e and G . G_i, G_e in (19), (26) and (32) as a function of advection U , U_s and Δt_s are shown in Figs.2–4.

First, let us examine the synchronous scheme. In Figs.2a–d, G , C are less than unit for whole ranges of U , U_s and Δt_s . This scheme is always stable. Generally speaking, the larger the Δt_s the faster the scheme converges. G , C depend only upon the mean flow in the atmosphere (Figs.2c,d). For larger atmospheric wind speed U , C becomes smaller, but as Δt_s is larger than about 10^7 s (116 days), G becomes larger. Therefore, with an increase of advection U the initial value disappears more quickly and the radiation equilibrium evolves more slowly.

For the asynchronous–implicit scheme, G_i is independent of U , U_s . Advection influences only the C_i (Figs.3a,b). In general for the larger advection and time intervals C_i is smaller. This means the existence of advection only accelerates the disappearance of initial perturbation. This scheme is stable too.

However, for the asynchronous–explicit scheme, the situation is different from the above two schemes. The figures for G_e are not given because it is independent of advection. As shown in Figs. 4a,b, if Δt_s is larger than about 6×10^6 s (69 days), $C_e > 1$. It indicates that the scheme is unstable for both U_s and U . But as Δt_s is less than about 6×10^6 s (69 days) it is stable, and when Δt_s is about 5×10^6 s there is one stable region.

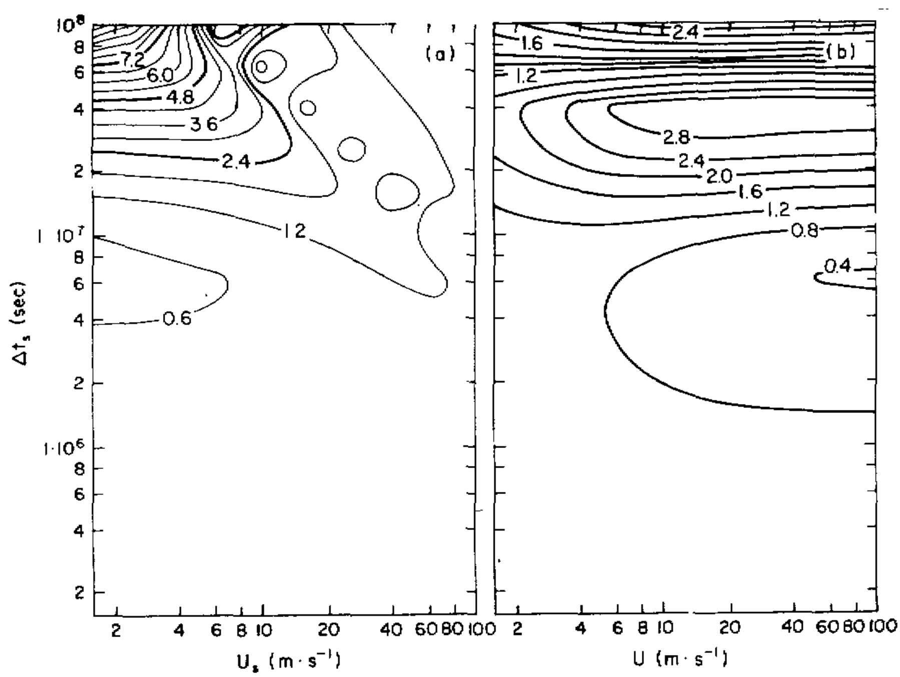

Fig.4a. Absolute value of C_e for asynchronous–explicit scheme as a function of U_s and Δt_s . 4b. As in Fig.4a except for U .

VII. CONCLUDING REMARKS

In this paper we pay special attention to climatic simulation of the final equilibrium state. We have discussed the effect of advection and integrating time interval on the stability and convergence rate.

The coupling model with synchronous–implicit schemes are stable for any advection and integrating time interval. Because in these cases the heat flux between air and sea is not fixed when the oceanic model runs. The ocean temperature rises due to the absorption of solar radiation, then the heat flux from ocean to the atmosphere and the radiative cooling in the atmosphere also increases, so a new balance between them must be reached. These schemes are always stable. On the other hand, for the asynchronous–explicit scheme the heat flux between the atmosphere and ocean is fixed in each cycle. If the oceanic model is integrated for a long time, and radiative heat flux exceeds that between air and sea the ocean temperature could change infinitely. Thus as to the mixed layer model, it may be better that the integrating time interval does not exceed about 6×10^6 s (69 days).

For any coupling scheme, if it is stable that must be convergent, the model approaches the same steady climate state. In some climatic models the equilibrium state obtained from these models with different asychronous–implicit schemes would be the same as that from the model with synchronous scheme, i.e., the exact solution of coupling system.

But, for climate experiments about transient response (e.g., response to increase of atmospheric CO_2 concentration, we want to know not only the equilibrium state, but also time–dependent response. The synchronous scheme is the most suitable scheme, however, it may be too expensive to use for long integrations. The asynchronous scheme may be used if the integrating time interval is very large. The use of an asynchronous scheme will not provide the correct evolution detail.

In regard to the rate of convergence for asynchronous schemes, the asynchronous–implicit scheme is the best compared to others in this advection model. The rate depends mainly on the characteristic parameters (exchange coefficients, heat contents of ocean and atmosphere, etc.) and time interval.

Because the model is linear and interaction between motion and thermodynamical processes is not considered in this study, the influence of horizontal advection on coupling strategies appears only in part.

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APPENDIX

The destabilized factors for the implicit asynchronous scheme in (27) are:

$$C_i = a_\infty a_s + e^{-\Delta t_s(i\mu U_s + \alpha_{sa})}(1 - a_\infty a_s)$$

$$d_i = (1 - e^{-(\alpha_{sa} + i\omega_0)\Delta t_s})/(\alpha_{sa} + i\omega_0)$$

$$G_i = (\alpha_{as} + \alpha_r e^{-\alpha_{ra}\Delta t_s})/(\alpha_{as} + \alpha_r),$$

and those for the explicit asynchronous scheme in (32) are:

$$c_e = \left(1 + \frac{\alpha_{sa}(1 - a_\infty)}{i\mu U_s}\right) e^{-i\mu U_s \Delta t_s} - \frac{\alpha_{sa}(1 - a_\infty)}{i\mu U_s}$$

$$d_e = \frac{1}{i\omega_0}(1 - e^{-i\omega_0 \Delta t_s}), \quad G_e = 1 - \frac{\alpha_{sa}\alpha_r}{\alpha_{as} + \alpha_r}\Delta t_s$$

where

$$a_s = \alpha_{sa} / \alpha_{sa} + i\mu U_s .$$