A Regional Spectral Nested Shallow Water Equation Model

Liao Dongxian (廖洞贤)
State Meteorological Administration, Beijing 100081
Received November 18,1988

ABSTRACT

A method to expand meteorological elements in terms of finite double Fourier series in a limited–region and a spectral nested shallow water equation model based upon the method with conformal map projection in rectangular coordinates, have been proposed. and computational stability and efficiency of time integration have been discussed.

I. INTRODUCTION

In recent years the spectral models have been widely used in NWP. No matter whether short–and–medium–range forecasts or long–range forecasts are made by them, they are drawing more and more forecasters' and meteorologists' attention. These models, however, are nearly all global or hemispheric ones under certain conditions. Up to now, for forecasts with computational domain smaller than the hemisphere, no predicted model has been proposed, except few spectral limited–area models, such as that given by Tatsumi (1985). According to his numerical experiments, although the CPU time taken by the model is about 1.2–1.3 times as much as that by the grid point model 12L–FLM to complete the same forecast period, the locations of the predicted systems with the former model are in general more accurate than those with the latter.

In this paper the method for expanding meteorological elements in terms of Fourier series in one–dimensional case in a limited interval given by the author in 1978 is modified and generalized so that it can be used in two–dimensional case. Then, a regional spectral nested shallow water equation model and related numerical techniques are suggested.

II. SPECTRAL EXPANSION

Assume a limited domain $\hat{R}$ with fine grid to be nested in a domain R with coarse grid (Fig.1) and let F represent a meteorological element. Then the spectral expansion of F in two–dimensional case can be reduced to two successive one–dimensional expansions, such as expanding F in x direction and then in y direction.

At first we expand F in x direction.

Let F be expressed by

$$F(x,y)=\hat{F}(x,y)+\hat{\hat{F}}(x,y), \tag{1}$$

where

$$\hat{F}(x,y)=[F(x_M,\ y)-F(0,y)]\frac{x}{L_x}. \tag{2}$$

It can be seen that $\hat{\hat{F}}$ is identically equal to $F(0,y)$ at the boundary points (x_M,y) and $(0,y)$. Thus $\hat{\hat{F}}$ may be expressed by

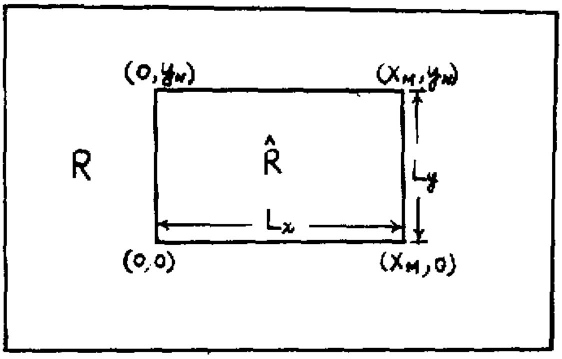

Fig.1. The computational domains R and $\hat{R}$.

$$\hat{\hat{F}}(x,y) = \sum_{m=-l_m}^{l_m} F_m(y) e^{i\hat{m}x}, \tag{3a}$$

where l_m is the largest wave number in x direction, $\hat{m} = 2m\pi / L_x$.

$$F_m(y) = \frac{1}{L_x}\int_0^{L_x} \hat{\hat{F}}(x,y) e^{-i\hat{m}x} dx. \tag{3b}$$

For convenience, the summation symbol on the right hand side of the above expression is replaced by the symbol $\sum_m$ in the below.

Now we expand $F_m(y)$ in y direction. Let

$$F_m(y) = F_m^*(y) + F'_m(y), \tag{4}$$

where

$$F_m^*(y) = [F_m(y_n) - F_m(0)]\frac{y}{L_y}. \tag{5}$$

Like the case in x direction, F'_m can be expressed by

$$F'_m(y) = \sum_{n=-l_n}^{l_n} F_{mn} e^{i\hat{n}y} = \sum_n F_{mn} e^{i\hat{n}y}, \tag{6a}$$

where l_n is the largest wave number in y direction, $\sum_n = \sum_{-l_n}^{l_n}$, $\hat{n} = 2n\pi / L_y$,

$$F_{mn} = \frac{1}{L_y}\int_0^{L_y} F'_m(y) e^{-i\hat{n}y} dy. \tag{6b}$$

Combining the expressions (1)–(6) gives

$$F(x,y) = \hat{F}(x,y) + \sum_m F_m^*(y) e^{i\hat{m}x} + \sum_m\sum_n F_{mn} e^{i(\hat{m}x+\hat{n}y)}. \tag{7}$$

However, the expression (7) can only be used within the domain $\hat{R}$. At the boundary $x=0$ and $x=x_M$, spectral expression is still needed. For this reason, the manners similar to the above may be applied. The resulting expressions read as follows:

$$F(0,y) = \hat{F}(0,y) + \sum_n F_n(0) e^{i\hat{n}y} \tag{8a}$$

and

$$F(x_M,y)=\hat{F}(x_M,y)+\sum_n F_n(x_M)e^{i\tilde{n}y}, \tag{8b}$$

where

$$\hat{F}(0,y)=[F(0,y_N)-F(0,0)]\frac{y}{L_y},$$

$$\hat{F}(x_M,y)=[F(x_M,y_N)-F(x_M,0)]\frac{y}{L_y}.$$

The expressions (7) and (8) are the basic ones in this paper. In the following sections all the meteorological elements involved will be expanded according to the basic expressions.

III. SHALLOW WATER EQUATIONS

According to Robert (1982), the shallow water equations in rectangular coordinates with conformal map projection can be written in the form

$$\frac{\partial U}{\partial t}=-\frac{\partial\varphi}{\partial x}+N_u, \tag{9}$$

$$\frac{\partial V}{\partial t}=-\frac{\partial\varphi}{\partial y}+N_v, \tag{10}$$

$$\frac{\partial\varphi'}{\partial t}=-\varphi_0 D+N_\varphi, \tag{11}$$

where $U=u/l, V=v/l$, l is the magnification factor of conformal map projection, φ_0 is the average depth of the fluid surface, $\varphi'=\varphi-\varphi_0$,

$$N_u=-S(U\frac{\partial}{\partial x}+V\frac{\partial}{\partial y})U-K\frac{\partial s}{\partial x}+fV,$$

$$N_v=-S(U\frac{\partial}{\partial x}+V\frac{\partial}{\partial y})V-K\frac{\partial s}{\partial y}+fU,$$

$$N_\varphi=-S(U\frac{\partial}{\partial x}+V\frac{\partial}{\partial y})\varphi'-\varphi'D,$$

$$D=S(\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}),$$

$$S=l^2,$$

$$K=\frac{1}{2}(U^2+V^2),$$

the others are conventional symbols used in meteorology.

In the above expressions the N_u, N_v and N_φ represent nonlinear terms. However, for the convenience of computation, the Coriolis terms are also included in N_u and N_v .

IV. DIFFERENTIAL–DIFFERENCE EQUATIONS AND SPECTRAL EQUATIONS

1. *The Explicit Case*

(1) *Differential–difference equations*

Express Eqs. (9)–(11) in explicit leapfrog form; then

$$U^{\tau+1}=U^{\tau-1}-2\left(\frac{\partial\varphi}{\partial x}\right)^{\tau}\Delta t+2(N_u)^{\tau}\Delta t, \tag{12}$$

$$V^{\tau+1}=V^{\tau-1}-2\ (\frac{\partial\varphi}{\partial x})^{\tau}\Delta t+2(N_v)^{\tau}\Delta t \tag{13}$$

and

$$\varphi^{\tau+1}=\varphi^{\tau-1}-2\varphi_0 D^{\tau}\Delta t+2(N_{\varphi})^{\tau}\Delta t. \tag{14}$$

Note that the superscripts of φ have been neglected.

(2) *Spectral equations*

In the following the transform method will be used. At first replacing F in (7) by U, V, φ and S respectively and expanding them in the domain $\hat{R}$, we can find the values of D, N_u, N_v and N_{φ} at the gridpoints. Then we expand them. Substituting the expressions expanded into Eq.(12), we have

$$\hat{U}^{\tau+1}+\sum_m U_m^{*,\tau+1}e^{i\hat{m}x}+\sum_m\sum_n U_{mn}^{\tau+1}e^{i(\hat{m}x+\hat{n}y)}$$
$$=\hat{U}^{\tau-1}+\sum_m \hat{U}_m^{*,\tau-1}e^{i\hat{m}x}+\sum_m\sum_n U_{mn}^{\tau+1}e^{i(\hat{m}x+\hat{n}y)}$$
$$+\hat{A}_1^{\tau}+\sum_m A_m^{*,\tau}e^{i\hat{m}x}+\sum_m\sum_n A_{mn}^{\tau}e^{i(\hat{m}x+\hat{n}y)},$$

where

$$\hat{A}_1^{\tau}=2\Delta t[-(\partial\hat{\varphi}/x)^{\tau}+(\hat{N}_u)^{\tau}],$$
$$A_m^{*,\tau}=2\Delta t[-i\hat{m}\varphi_m^{*,\tau}+(\hat{N}_u)_m^{*,\tau}],$$
$$A_{mn}^{\tau}=2\Delta t[-i\hat{m}\varphi_{mn}^{\tau}+(\hat{N}_u)_{mn}^{\tau}].$$

Multiplying the above expression by $e^{-i(\hat{m}x+\hat{n}y)}$ and integrating it over the computational domain $\hat{R}$ leads to

$$U_{mn}^{\tau+1}=U_{mn}^{\tau-1}+A_{mn}^{\tau}+\tilde{U}_{mn}+\tilde{U}_{mn}^{*}, \tag{15}$$

where

$$\tilde{U}_{mn}=-\frac{1}{4mn\pi^2}\Big\{U^{\tau-1}(x_M,y_N)-U^{\tau-1}(x_M,0)-U^{\tau-1}(0,y_N)+U^{\tau-1}(0,0)$$
$$-U^{\tau-1}(x_M,y_N)+U^{\tau+1}(x_M,0)+U^{\tau+1}(0,y_N)-U^{\tau+1}(0,0)$$
$$+2\Delta t\Big[(N_u)^{\tau}(x_M,y_N)-(N_u)^{\tau}(x_M,0)-(N_u)^{\tau}(0,y_N)+(N_u)^{\tau}(0,0)\Big]\Big\},$$
$$-\frac{1}{2m\pi i}\Big[U_n^{\tau-1}(x_M)-U_n^{\tau-1}(0)-U_n^{\tau+1}(x_M)+U_n^{\tau+1}(0)+2\Delta t\ ((N_u)_n^{\tau}(x_M)$$
$$-(N_n)_n^{\tau}(0))\Big],$$

$$\tilde{U}_{mn}^{*}=-\frac{1}{2n\pi i}\Big[U_m^{\tau-1}(y_N)-U_m^{\tau-1}(0)+U_m^{\tau+1}(0)-U_m^{\tau+1}(y_N)$$
$$+2\Delta t\cdot\ ((N_u)_m^{\tau}(y_N)-(N_u)_m^{\tau}(0)\Big]+\frac{2m\Delta t}{nL_x}\Big[\varphi_m^{\tau}(y_N)-\varphi_m^{\tau}(0)\Big],$$

$$U_n(x_M)=\frac{1}{L_y}\int_0^{L_y}\Big[U(x_M,y)-\hat{U}(x_M,y)\Big]e^{-i\hat{n}y}dy,$$

and $U_n(0)'S$ has similar form.

With the manners similar to that used in deriving (15) the spectral equation

$$V_{mn}^{\tau+1} = V_{mn}^{\tau-1} + B_{mn}^{\tau} + \tilde{V}_{mn} + \tilde{V}_{mn}^{*} \tag{16}$$

and

$$\varphi_{mn}^{\tau+1} = \varphi_{mn}^{\tau-1} + C_{mn}^{\tau} + \tilde{\varphi}_{mn} + \tilde{\varphi}_{mn}^{*} \tag{17}$$

can be obtained. Here

$$B_{mn}^{\tau} = 2\Delta t\left[-i\hat{n}\varphi_{mn}^{\tau} + (N_v)_{mn}^{\tau}\right],$$

$$\begin{aligned}\tilde{V}_{mn} = &-\frac{1}{4mn\pi^2}\Big\{V^{\tau-1}(x_M, y_N) - V^{\tau-1}(x_M, 0) - V^{\tau-1}(0, y_N) + V^{\tau-1}(0,0)\\ &- V^{\tau+1}(x_M, y_N) + V^{\tau+1}(x_M, 0) + V^{\tau+1}(0, y_N) - V^{\tau+1}(0,0) + 2\Delta t\cdot\\ &\left[(N_v)^{\tau}(x_M, y_N) - (N_v)^{\tau}(x_M, 0) - (N_v)^{\tau}(0, y_N) + (N_v)^{\tau}(0,0)\right]\Big\}\\ &-\frac{1}{2m\pi i}\Big[V_n^{\tau-1}(x_M) - V_n^{\tau-1}(0) - V_n^{\tau+1}(x_M) + V_n^{\tau+1}(0)\\ &+ 2\Delta t\Big((N_v)_n^{\tau}(x_M) - (N_v)_n^{\tau}(0)\Big)\Big],\end{aligned}$$

$$\begin{aligned}\tilde{V}_{mn}^{*} = &-\frac{1}{2n\pi i}\Big[V_m^{\tau-1}(y_N) - V_m^{\tau-1}(0) - V_n^{\tau+1}(y_N) + V_m^{\tau+1}(0)\\ &+ 2\Delta t\Big((N_v)_m^{\tau}(y_N) - (N_v)_m^{\tau}(0)\Big)\Big],\end{aligned}$$

$$C_{mn}^{\tau} = 2\Delta t\left[-\varphi_0 D_{mn}^{\tau} + (N_\varphi)_{mn}^{\tau}\right],$$

$$\begin{aligned}\tilde{\varphi}_{mn} = &-\frac{1}{4mn\pi^2}\Big\{\varphi^{\tau-1}(x_M, y_N) - \varphi^{\tau-1}(x_M, 0) - \varphi^{\tau-1}(0, y_N) + \varphi^{\tau-1}(0,0)\\ &- \varphi^{\tau+1}(x_M, y_N) + \varphi^{\tau+1}(x_M, 0) + \varphi^{\tau+1}(0, y_N) - \varphi^{\tau+1}(0,0)\\ &+ 2\Delta t\left[\Phi(x_M, y_N) - \Phi(x_M, 0) - \Phi(0, y_N) + \Phi^{\tau}(0,0)\right]\Big\}\\ &-\frac{1}{2m\pi i}\Big[\varphi_n^{\tau-1}(x_M) - \varphi_n^{\tau-1}(0) - \varphi_n^{\tau+1}(x_M) + \varphi_n^{\tau+1}(0)\\ &+ 2\Delta t\Big(\Phi_n^{\tau}(x_M) - \Phi_n^{\tau}(0)\Big)\Big],\end{aligned}$$

$$\begin{aligned}\tilde{\varphi}_{mn}^{*} = &-\frac{1}{2n\pi i}\Big[\varphi_m^{\tau-1}(y_N) - \varphi_m^{\tau-1}(0) - \varphi_m^{\tau+1}(y_N) + \varphi_m^{\tau+1}(0)\\ &+ 2\Delta t\Big(\Phi_m^{\tau}(y_N) - \Phi_m^{\tau}(0)\Big)\Big],\end{aligned}$$

$$\Phi = -\varphi_0 D + N_\varphi .$$

2. *The Semi–implicit Case*

(1) *Differential–difference equations*

The semi–implicit differential–difference forms of Eqs. (9)–(11) are

$$U^{\tau+1}=U^{\tau-1}-\left[\left(\frac{\partial\varphi}{\partial x}\right)^{\tau+1}+\left(\frac{\partial\varphi}{\partial x}\right)^{\tau-1}\right]\Delta t+2N_{n}^{\tau}\Delta t, \tag{18}$$

$$V^{\tau+1}=V^{\tau-1}-\left[\left(\frac{\partial\varphi}{\partial y}\right)^{\tau+1}+\left(\frac{\partial\varphi}{\partial y}\right)^{\tau-1}\right]\Delta t+2N_{v}^{\tau}\Delta t, \tag{19}$$

and

$$\varphi^{\tau+1}=\varphi^{\tau-1}-\varphi_0(D^{\tau-1}+D^{\tau-1})\Delta t+2N_{\varphi}\Delta t. \tag{20}$$

Eliminating $U^{\tau+1}$ and $V^{\tau+1}$ from the above equations gives

$$(\nabla^2-a^2)\varphi^{\tau+1}=G, \tag{21}$$

where

$$G=\frac{2}{\Delta t}\delta^{\tau-1}-2a^2\Delta tN_{\varphi}^{\tau}-(\nabla^2+a^2)\varphi^{\tau-1}+2\left(\frac{\partial N_n^{\tau}}{\partial x}+\frac{\partial N_v^{\tau}}{\partial y}\right),$$

$$\delta=\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}.$$

$$a^2=1/s\varphi_0\Delta t.$$

(2) *Spectral equations*

If U,V and φ are expanded as F in (7) and substituted into (21), by means of the orthogonality of complex exponential functions, we have

$$\varphi_{mn}^{\tau+1}=-\frac{1}{\hat{m}^2+\hat{n}^2+a^2}(G_{mn}+\tilde{\varphi}_{mn}+\tilde{\varphi}_{mn}^{*}), \tag{22}$$

where

$$\tilde{\varphi}_{mn}=-\frac{1}{4mn\pi^2}\Big\{G(x_M,y_N)-G(x_M,0)-G(0,y_N)+G(0,0)-H^{\tau+1}(x_M,y_N)$$
$$+H^{\tau+1}(x_M,0)+H^{\tau+1}(0,y_N)-H^{\tau+1}(0,0)\Big\},$$

$$\tilde{\varphi}_{mn}^{*}=-\frac{1}{2n\pi i}\Big\{G_m(y_N)-G_m(0)+(\hat{m}^2+a^2)(\varphi_m(y_n)-\varphi_m(0))\Big\},$$

$$H^{\tau+1}=(\nabla^2-a^2)\varphi^{\tau+1}.$$

The expression of G in Eq.(21) can be rewritten as

$$G=\hat{G}+\sum_m G_m^{*}e^{i\hat{m}x}+\sum_m\sum_n G_{mn}e^{i(\hat{m}x+\hat{n}y)}, \tag{23}$$

where

$$\hat{G}=\frac{2}{\Delta t}\hat{\delta}^{\tau-1}-2a^2\Delta t\hat{N}_{\varphi}^{\tau}-(\nabla^2+a^2)\hat{\varphi}^{\tau-1}+2\left(\frac{\partial\hat{N}_n^{\tau}}{\partial x}+\frac{\partial\hat{N}_v^{\tau}}{\partial y}\right),$$

$$G_m^{*}=\frac{2}{\Delta t}\delta_m^{*,\tau-1}-2a^2\Delta t(N_{\varphi})_m^{*,\tau}+(\hat{m}^2-a^2)\varphi_m^{*,\tau-1}+4m\pi i(N_n)_m^{*,\tau}$$
$$+\frac{2}{L_y}[(N_v)_m(y_N)-(N_v)_m(0)],$$

$$G^{mn}=\frac{2}{\Delta t}\delta_{mn}^{\tau-1}-2a^2\Delta t(N_{\varphi})_{mn}^{\tau}+(\hat{m}^2+\hat{n}^2-a^2)\varphi_{mn}^{\tau-1}+4\pi i[m(N_n)_{mn}^{\tau}+n(N_v)_{mn}^{\tau}].$$

After having obtained $\varphi_{mn}^{\tau+1}$, from (22) $\overline{\varphi}_{mn}^{t}[=(\varphi_{mn}^{\tau+1}+\varphi_{mn}^{\tau-1})/2]$ can be found. With the help of boundary values and replacing φ^{τ}_{mn} in Eq.(15) and Eq.(16) by $\overline{\varphi}^{t}_{mn}$ we obtain the values of $U^{\tau+1}_{mn}$ and $V^{\tau+1}_{mn}$ immediately.

3. *Drtermination of the Largest Truncated Wave Number*

As is known the largest truncated wave number may be greater than $L_x / 2d$ (or $L_y / 2d$) in computing nonlinear terms. According to Machenhauer et al. (1979), in order to avoid aliasing and estimate the value of spectral coefficients accurately, the inequalities

$$l_m \leqslant \frac{1}{3}(\frac{L_x}{d} - 1) \tag{24}$$

and

$$l_n \leqslant \frac{1}{3}(\frac{L_y}{d} - 1) \tag{25}$$

should be satisfied.

V. COMPUTATIONAL STABILITY AND TIME INTEGRATION

1. *Computational Stability*

(1) *The explicit case*

Consider the linearized case with the basic current in x direction at speed $\overline{U}$ as the basic state. Then it is sufficient to take the computational stability of a single weve into account. The wave may be assumed in the form

$$X = \begin{bmatrix} U \\ V \\ \varphi \end{bmatrix} = \begin{bmatrix} A_u \\ A_v \\ A_\varphi \end{bmatrix} e^{i(\hat{m}x + \hat{n} + \omega t)}. \tag{26}$$

If the boundary values are prescribed with periods L_x and L_y in x and y directions, f=const and $l \equiv 1$, then substituting the above expression in Eqs.(12)—(14), we have

$$H_1 X = 0, \tag{27}$$

where

$$H_1 = \begin{bmatrix} 2i(\sin\omega\Delta t + \overline{U}\hat{m}\Delta t) & -2f\Delta t & 2i\Delta t\hat{m} \\ 2f\Delta t & 2i(\sin\omega\Delta t + \overline{U}\hat{m}\Delta t) & 2i\Delta t\hat{n} \\ 2i\varphi_0\Delta t\hat{m} & 2i\varphi_0\Delta t\hat{n} & 2i(\sin\omega\Delta t + \overline{U}\hat{m}\Delta t) \end{bmatrix}.$$

If the homogeneous equation (27) has nontrival solutions, it is necessary that

$$|H_1| = 0.$$

Thus

$$\sin\omega\Delta t = 0$$

and

$$\sin\omega\Delta t = \pm\left\{\varphi_0(\hat{m}^2+\hat{n}^2)+f^2\right\}^{1/2}\Delta t - \overline{U}\hat{m}\Delta t.$$

Hence, stable computation should be subject to the criterion

$$\Delta t \leqslant \left\{\overline{U}\hat{m} + [\varphi_0(\hat{m}^2+\hat{n}^2)+f^2]^{1/2}\right\}^{-1}. \tag{28}$$

Given $\overline{U}=50$ m/s, $\varphi_0/g=9\times10^3$ m, $g=9.8$ m/s^2, $f=1.031\times10^{-4}$s^{-1}, $L_x=L_y=5.0\times10^3$ km, $d=100$ km, the dependence of the permitted largest time step Δt_{max} upon $\Lambda(=\sqrt{m^2+n^2})$ is shown in Table 1. It will be seen from the table that in the case of $\Lambda\leqslant7$, $\Delta t_{max}>5$ minutes, which is far greater than the time step selected according to the largest truncated wave number.

Table 1. The Dependence of Δt_{max} upon Λ

Λ	1	3	5	7	9
Δt_{max} (min)	36.7	12.7	7.5	5.3	4.2
Λ	11	13	15	17	19
Δt_{max} (min)	3.5	2.8	2.5	2.2	2.0

(2) *The semi-implicit case*

Like the manners used in the explicit case, substituting (26) into Eqs.(18)–(20) gives

$$H_2X=0, \tag{29}$$

where

$$H_2=\begin{bmatrix} 2i(\sin\omega\Delta t+\overline{U}\hat{m}\Delta t) & -2f\Delta t & 2i\Delta t\hat{m}\cos\omega\Delta t \\ 2f\Delta t & 2i(\sin\omega\Delta t+\overline{U}\hat{m}\Delta t) & 2i\Delta t\hat{n}\cos\omega\Delta t \\ 2i\varphi_0\hat{m}\Delta t\cos\omega\Delta t & 2i\varphi_0\hat{n}\Delta t\cos\omega\Delta t & 2i(\sin\omega\Delta t+\overline{U}\hat{m}\Delta t) \end{bmatrix}.$$

Hence, if there exist nontrivial solutions, it is necessary that

$$|H_2|=0. \tag{30}$$

Then

$$\sin\omega\Delta t = -\overline{U}\hat{m}\Delta t, \tag{31}$$

and

$$(\sin\omega\Delta t+\overline{U}\hat{m}\Delta t)^2-\varphi_0(\hat{m}^2+\hat{n}^2)\Delta t^2\cos^2\omega\Delta t-f^2\Delta t^2=0. \tag{32}$$

From (32) the computational stability criterion is given by

$$\Delta t \leqslant \frac{1}{\overline{U}\hat{m} + [f^2 + \varphi_0(\hat{m}^2 + \hat{n}^2)\cos^2 \omega\Delta t]^{1/2}} .$$

It follows that

$$\frac{1}{\overline{U}\hat{m} + f} \geqslant \Delta t \geqslant \Delta t_{ex} , \tag{33}$$

where Δt_{ex} is the permitted largest time step from (28) in the explicit case. For the wave $m = 16$, based upon the figures given previously, $1 / (f + \overline{U}\hat{m}) \approx 15$ minutes. In practice Δt should be determined from experiments under the guidance of (33).

2. *Time Integration*

In the above the spectral equations in the explicit and semi–implicit cases, and their related computational stability criteria have been given. It can be seen that the amount of computation needed in the explicit case for integrating the spectral equations is far less than that in the semi–implicit case, and the time step selected in the former case is generally far less than that in the latter case. Therefore, in time integration appropriate schemes should be taken according to practical situation. However, in order to guarantee higher accuracy and save the amount of computation the following two ways offer a choice to be taken.

One way is adopting the semi–implicit scheme in time integration completely. The time step, however, should not be taken too large. The ideal step, we think, might be the step that its accuracy is acceptable as compared with those in the explicit case and the computational amount needed is economical.

The other way is to adopt the explicit technique for longer waves and the semi–implicit technique for shorter waves. For example, by taking Δt to be 5 minutes, the explicit technique is used in the case $\Lambda \leqslant 7$ and the semi–implicit technique in the case $\Lambda > 7$. It should be pointed out that Burridge (1975) had suggested similar strategy for a 10–level gridpoint model.

VI. DISCUSSION

The above–mentioned model, in principle, may be put into test. However, there are a lot of questions to be answered, such as the comparison of the model with that given by Tatsumi (1985), the consistency between the coarse grid and the fine grid, how to extend the model suggested in the above to a regional spectral multilevel primitive equation model, and whether there exist other proper basis functions, and so on. In the following they will be discussed successively.

In substance, the above model and Tatsumi's model are the same. The differences between them lie in the basis functions used and the integration limits selected. In the above, the suggested model uses double complex exponential functions as basis functions and the integration limits being zero and 2π , while Tatsumi's model uses different trigonometric series for different predicted variables and 0, π as its integration limits. Therefore, in Tatsumi's model, the integrals of some basis functions are not vanished, and soms basis functions are not orthogonal. Thus extra computational amount would be needed as compared with the above–suggested model.

Besides, in evaluating nonlinear terms in the suggested model, the largest wave number may be in excess of l_m or l_n . Although adoption of inequalities (24) and (25) can avoid aliasing, such treatment is different from the case that L_x / d and L_y / d are allowed to be equal to $2l_m$ and $2l_n$ respectively under the condition of mean square error being minimum. In practice, such treatment is equivalent to filtering out some short waves. Therefore, the fitting accuracy decreases. However, if other proper orthogonal functions, such as Walsh–Hadamard functions, are adopted as basis functions, the difficulties encountered in the above might be overcome, because the products of those functions still belong to the basis functions selected initially, unlike the case of using trigonometric functions.

As for the connection of the coarse and the fine grid, current treatments used in grid models, such as relaxation technique and so on, may be used in order to eliminate the inconsistency between the two different grids due to their different truncation errors in the vicinity of the boundaries.

Finally, how to extend the above model to a multilevel spectral nested primitive equation model is a complex problem. As is known, however, a multilevel primitive equation model can be reduced to several sets of shallow water type equations characterized by different equivalent depths by means of normal modes. In this way integration of the multilevel primitive equation model is equivalent to integrating sets of shallow water type equations. Therefore, all the techniques proposed in the above sections can be used. This problem will be discussed and analyzed in other papers later.

REFERENCES

Burridge, D. M. (1975), A split semi–implicit reformulation of the Bushby–Timpson 10–level model, *Quart. J. R. Meteor. Soc.*, **101**: 777–792.

Liao Dongxian, (1978), CAS Working Group on Weather Prediction Research (Subgroup on Short and Medium–Range Forecasting), 2nd Session, Peking, 27 November–2 December 1978, WMO, Appendix D, p.3.

Machenhauer, B. (1979), Numerical Methods Used in Atmospheric Models II, Chapter 3, GARP Publication Series, 17, WMO,124–275.

Robert, A. J. (1892), *J. Met. Soc. Japan*, **60**: 319–325.

Tatsumi, Y. (1985), JMA / NPD Tech. Report, **3**: 1–71.