A Parameterization of Cooling Rate Calculation under the Non-LTE Condition: Multi-Level Model

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ABSTRACT

Calculations of cooling rate by CO_2 15 μm band in the earth's upper mesosphere and lower thermosphere become very difficult because of the non-LTE. This is primarily due to the nonlinear vibration-vibrational (VV) transition processes between CO_2 molecules in different states. This paper suggests that the non-LTE source function be parameterized as a linear combination of two limiting source functions. One limiting source function neglects the VV transitions while the other limiting source function assumes VV transitions being dominant. These two limiting source functions can be derived by linear models. The parameterization schemes proposed here can be applied to the general circulation models including those non-LTE regions.

1. INTRODUCTION

The state and motion in the earth's middle atmosphere and lower thermosphere (about 10-130 km) depend in a great extent on the radiative heat sources and sinks. There are two heating sources in this region. One is located in the lower thermosphere where molecular oxygen is photodissociated by solar UV radiation (wavelength $\lambda < 0.24$ μm) and the other in the upper stratosphere where ozone absorbs solar UV radiation ($0.2 < \lambda < 0.3$). The main cooling in this region is the infrared cooling by CO_2 15 μm band. In addition to that 9.6μm band cooling by ozone also makes certain contribution in the upper stratosphere (Andrews, et al., 1987; Shi, 1984).

Since it involves integration in frequency over thousands of spectral lines accurate calculation of cooling rate by CO_2 15 μm band requires tremendous amount of computation. The other difficulty in calculating CO_2 15 μm cooling rate comes from that we can not assume local thermodynamic equilibrium (LTE) to specify blackbody radiation as source function above about 70 km. Following the treatment by Curtis and Goody (1956), Houghton (1977) posed a two-level model in matrix form to calculate the CO_2 15 μm band cooling in non-LTE regions. Houghton's formalism has been widely used in many studies to calculate the CO_2 cooling rate (e.g. Wehrbein, et al., 1982; Apruzese, et al., 1984; Fels, 1984). It is known that the two-level model gives only source function for the fundamental band. In the mesosphere, however, since the hot and isotopic bands play important roles in radiative heat exchange (Dickinson, 1973; Leovy, 1984) people recently have paid attention to use multi-level model to calculate CO_2 15 μm band cooling rate in the non-LTE region (Kutepov, et al., 1978; Dickinson, 1984; Lopez-Puertas, et al., 1986; Taylor, 1988). In this paper we propose a multi-level linear parameterization scheme in matrix form to calculate cooling rate by CO_2 15 μm band in the earth's atmosphere including non-LTE region.

II. MODEL

We start from the basic radiative transfer equation

$$\frac{dI_\nu}{ds} = -k_\nu \rho_{co_2}(I_\nu - J_\nu), \tag{1}$$

where I_ν ($Wm^{-2}Sr^{-1}(s^{-1})^{-1}$) is monochromatic radiance, s distance, k_ν ($kg^{-1}m^2$) absorption coefficient, ρ_{co_2} density of absorption gas CO_2, and J_ν the source function. Performing the solid angle and frequency band ($\Delta\nu$) integrations results in the following equation

$$Q = 4\pi S^* \rho_{co_2}(\bar{I}_\nu - J_\nu), \tag{2}$$

where

$$Q = -\int_{\Delta\nu}\int_{4\pi} \frac{dI_\nu}{ds} d\omega d\nu = -2\pi \int_{-1}^{1} \mu d\mu \int_{\Delta\nu} \frac{dI_\nu}{dz} d\nu \tag{3}$$

is the heating rate in unit volume (Wm^{-3}),

$$\bar{I}_\nu = \frac{1}{4\pi S^*}\int_{\Delta\nu}\int_{4\pi} k_\nu I_\nu d\omega d\nu \tag{4}$$

is the radiance averaged with respect to ω and ν, $S^* = Sc$ is frequency integrated band strength while S ($kg^{-1}m$) is wavenumber integrated band strength and c is light speed. In deriving equation (2) we have used assumption that the source function for infrared radiation is isotropic and it is also a slowly varying function with respect to ν comparing to k_ν. Equation (2) gives the relation among the heating rate, averaged radiance and radiative source function at any altitude (k). This relation can be written in the following discrete form for any band (i)

$$\bar{I}_i - J_i = G_i Q_i, \tag{5}$$

in which $\bar{I}_i$, J_i and Q_i are column vectors, G_i is diagonal matrix: $(G_i)_k = (4\pi S_i^* \rho_{co_2})k^{-1}$.

Next we consider to solve equation (1) directly. Assume that the absorb coefficient k_ν is independent on the source function①, J_ν, then equation (1) is a linear equation of I_ν respect to a known function J_ν. By solving I_ν and substituting the solution into (3) we can get a direct relation between the heating rate and the source function. This relation can usually be written discretely in the following matrix form

$$Q_i = C_i J_i + Q_{si}, \tag{6}$$

where C_i is Curtis matrix (1956) and Q_{si} represents the external heating contributed from boundary radiation fluxes. Eliminating Q_i between equations (5) and (6) gives

$$\bar{I}_i = (I + G_i C_i)J_i + G_i Q_{si}, \tag{7}$$

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① This assumption is violated under non-LTE condition. The assumption here is equivalent to neglecting stimulating emission which requires that $e^{-h\nu/kT} \ll 1$. In the problems of calculating cooling rate the above condition is strongly held due to the fact that the wavelength and (translational) temperature approximately satisfy the Wien's displacement law.

in which I is the identity matrix. Hence, under any circumstances, once we know the source function J_i we can immediately obtain the heating rate Q_i and averaged intensity $\bar{I}_i$. In order to calculate the source function we need the following rate equations for the state (i) (Dickinson, 1984; Mihalas, 1978) (Fig. 1):

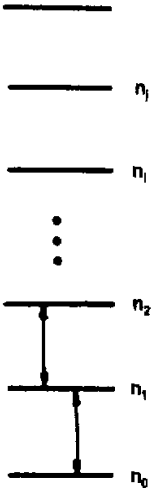

Fig.1. Schematic diagram of the state distribution.

$$n_i(B_{i,i+1}\bar{I}_\nu + A_{i,i-1} + B_{i,i-1}\bar{I}_\nu) + n_i\sum_{j\neq i}(C_{ij} + D_{ij})$$
$$= n_{i+1}(A_{i+1,i} + B_{i+1,i}\bar{I}_\nu) + n_{i-1}B_{i-1,i}\bar{I}_\nu + \sum_{j\neq i} n_j(C_{ji} + D_{ji}), \tag{8}$$

where n_i is the number density of the absorption gas at the state i. The summation of all n_i gives the total number density of CO_2

$$\sum_i n_i = n_{co_2}. \tag{9}$$

The relation between the source function and the number density is given by

$$J_i = \frac{2h\nu^3}{c^2}\left[\frac{n_i g_{i+1}}{n_{i+1}g_i} - 1\right]^{-1} \approx \frac{2h\nu^3}{c^2}\frac{n_{i+1}g_i}{n_i g_{i+1}}. \tag{10}$$

Under the equilibrium condition we have

$$(n_{i+1}g_i / n_i g_{i+1})_{\text{equilibrium}} = e^{-h\nu/kT}, \tag{11}$$

and therefore $J_\nu = B_\nu$. In (8) B_{ij}, B_{ji} and $A_{ji}(j>i)$ are Einstein coefficients for absorption, stimulated emission and spontaneous emission, respectively. The relations between them are

$$A_{ji} = (2h\nu^3 / c^2)B_{ji}, \qquad g_i B_{ij} = g_j B_{ji}. \tag{12}$$

C_{ij} and C_{ji} in (8) are the vibration–translational (VT) collision transitions between CO_2 and air molecules

$$n_i + M \overset{k_{VT}}{\rightleftharpoons} n_j + M.$$

The above reaction equation represents that a CO_2 molecule in state i collides with an air molecule (M represents the number density of molecular nitrogen, oxygen or atomic oxygen) to transit into state j. k_{VT} is reaction rate coefficient (m^3s^{-1}). Relations between

C_{ij} and C_{ji} are $(j>i)$

$$C_{ij} = C_{ji}(g_j / g_i)e^{-h\nu/kT} = C_{ji}(n_j / n_i)_{\text{equilibrium}}, \tag{14}$$

while C_{ji} can be represented as

$$C_{ji} = k_{VT}M. \tag{15}$$

In the rate equation (8) D_{ij} and D_{ji} are the transition terms by vibration–vibration (VV) collisions between CO_2 in different states. For example, if we consider the collisional reaction

$$n_0 + n_2 \overset{k_{VV}}{\rightleftharpoons} n_1 + n_1$$

D_{ij} and D_{ji} will include terms as $k_{VV}n_0, k_{VV}n_2$ and $2k_{VV}n_1$. Since n_i's in these terms are what we want to derive therefore the rate equation (8) is nonlinear. The equation must be solved together with equations (7), (9) and (10) iteratively. For example, in order to get the source function for CO_2 fundamental band Dickinson (1984) performed several dozens of iterations to the order of 100×100 matrix equations. We want to point out one point here: Lopez–Puertas, et al. (1986) showed a multi–level model which looks like linear to treat the non–LTE problem. However, the matrix A in that model still depends on the source function J. So the correct procedures to solve the problem must involve iterations.

In the earth's atmosphere the content of CO_2 (i.e. n_{co_2} / M) is about 3×10^{-4}. Since the reaction (16) is about 4 orders faster than reaction (13) (i.e., $k_{VV} / k_{VT} \sim 10^4$) we expect that the VV transition terms in the rate equation (8) has the same order as the VT transition terms. The main reason that the reaction (13) is much slower than the reaction (16) is as follows: the processing of reaction (13) involves a large amount of energy transferring from vibrational to translational forms while (16) involves almost only energy transfer between the vibrational states (Fig.1). According to this we further know that in (13) we always have $j = i \pm 1$. For $j < i-1$ or $j > i+1$ the reaction rate coefficient will be very much smaller so that it can be neglected in equation (8).

In order to calculate the CO_2 cooling rate by linear model we first consider the limiting case by neglecting the VV transitions ($D_{ij} = D_{ji} = 0$). Under this condition the problem can be transformed into equivalent two–level models. We can write rate equations for state transition ($i \rightarrow i+1$) or the band (i) as

$$n_i(B_{ij}\bar{I}_\nu + C_{ij}) = n_j(A_{ji} + B_{ji}\bar{I}_\nu + C_{ji}), \qquad j = i+1. \tag{17}$$

Substituting equations (11), (12) and (14) into (17) we obtained the source function

$$J_i = \frac{A_{ji}\bar{I} + C_{ji}(2h\nu^3 / c^2)e^{-h\nu/kT}}{A_{ji} + C_{ji}(1 - e^{-h\nu/kT})}. \tag{18}$$

Noting that

$$B_\nu \approx (2h\nu^3 / c^2)e^{-h\nu/kT}, \tag{19}$$

we get

$$J_i = (\bar{I}_i + \varphi_i B_\nu) / (1 + \varphi_i), \tag{20}$$

where $\varphi_i = C_{ji} / A_{ji}$. From the expression for C_{ji} (15) we know that φ_i decreases with increasing altitude. When $\varphi_i \sim 1$, J_ν will depart significantly from B_ν. Further discussions about φ_i and non–LTE are referred to (Curtis and Goody, 1956). Eliminating $\bar{I}_\nu$ from (2) and (20) results in

$$J_i = Q_i / (4\pi S_i^* \rho_{co_2} \varphi_i) + B_\nu. \tag{21}$$

The above equation can be written in the following matrix form (for any band i)

$$\mathbf{J}_i = \mathbf{E}_i \mathbf{Q}_i + \mathbf{B}, \tag{22}$$

where the diagonal matrix $\mathbf{E}_i$ is given by $(\mathbf{E}_i)_k = (4\pi S_i^* \rho_{co_2} \varphi_i) k^{-1}$. Eliminating Q_i from (6) and (22) gives the source function we required for the first limiting case

$$\mathbf{J}_i^{VT} = (\mathbf{I} - \mathbf{E}_i \mathbf{C}_i)^{-1} (\mathbf{B} + \mathbf{E}_i \mathbf{Q}_{si}). \tag{23}$$

Above about 110 km in the earth's atmosphere, the content of atomic oxygen becomes higher. The VT transition reaction (13) with atomic oxygen as M has the rate coefficient k_{VT} only two orders smaller than the rate coefficient k_{VV} for VV transition reaction (16). Under this circumstance the source function is close to this limiting case where D_{ij} and D_{ji} are negligible.

Next, we consider the limiting case when the VV transitions are dominant in determining the state distributions. Now, the populations at each vibrational state of CO_2 are completely determined by very efficient exchanges between vibrational states through the fast reaction (16). All the vibrational temperatures are the same under this condition. In other words, all the source functions for different bands are smoothed to the same value. Such a limiting case appear in the atmospheres of Venus and Mars since the major content of those atmospheres is CO_2(> 95%). Therefore, in the CO_2 riched atmospheres the source functions for different bands are assumed to be the same in calculating the cooling rate (Dickinson, 1972). Furthermore, we notice that the collisional processes which determine the state populations in equation (8) are of local property while the terms including radiation processes are of non–local property. In other words, the dominated VV collisional process only smoothes the source functions for different band at any space point while the value of that smoothed source function must still be determined by the radiation processes. This also means that we can determine the source function formally from (22).Under the assumption that the source functions are same we can derive the source function of the second limiting case from (6) and (22) (for any i):

$$\mathbf{J}_i^{VV} = (\mathbf{I} - \mathbf{E}^* \mathbf{C})^{-1} (\mathbf{B} + \mathbf{E}^* \mathbf{Q}_s), \tag{24}$$

where $\mathbf{C} = \sum_i \mathbf{C}_i$, $\mathbf{Q}_s = \sum_i \mathbf{Q}_{si}$ and $\mathbf{E}^* = (\sum_i \mathbf{E}_i^{-1})^{-1}$, or $(\mathbf{E}^*)_k = (4\pi\rho_{co_2} (\sum_i \mathbf{S}_i^* \varphi_i))_k^{-1}$.

Below 110 km conditions for the earth's atmosphere are between the above two limiting cases. As a first approximation we therefore can consider the actual source function to be a linear combination of above two limiting cases:

$$\mathbf{J}_i = (1 - \beta)\mathbf{J}_i^{VT} + \beta \mathbf{J}_i^{VV}, \tag{25}$$

where the parameter β must satisfy the following condition

$$\beta = \begin{cases} 0, & (\sum D_{ji}) / C_{ji} \to 0, \\ 1, & (\sum D_{ji}) / C_{ji} \to \infty. \end{cases} \tag{26}$$

However, D_{ij} in above expression still contains the unknown state population n_i (i.e., contains the source function J_ν); we therefore assume here that

$$\beta = (2/\pi)\mathrm{arctg}[\alpha(\sum D_{ji}^* / C_{ji})], \tag{27}$$

where D_{ji}^{*} is calculated when n_i in D_{ji} is replaced by the value under the VT limiting condition.

III. RESULTS

We use the approximate Voigt line profile (Zhu, 1988a) and the random model (Zhu, 1988b) to make preliminarily numerical test to the multi-level non-LTE model described

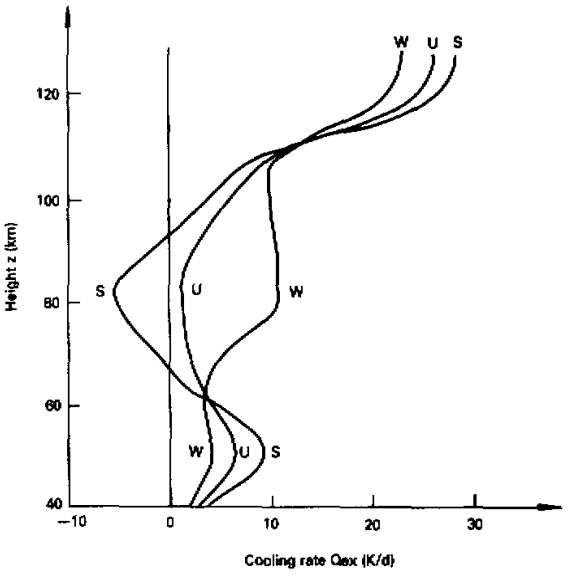

Fig.2. Cooling rates of CO_2 15μm band under winter (W), summer (S) and standard atmospheric (U) conditions calculated by multi-level non-LTE model.

above. We use Dickinson's model (1984) which includes three energy levels and a fundamental band for isotopes. Three temperature profiles which resemble winter, summer and standard atmosphere are used in the tests. The lower and upper boundaries are 40 km and 127 km, respectively. Vertical interval is 3 km. If we denote Q_{EX} as exact solution calculated according to nonlinear iteration and denote Q_{VT} ,Q_{VV} and Q_{β} as the approximate solutions calculated from VT limiting, VV limiting and linear combination then the preliminary results are described as follows. Below 70 km and above 110 km the results from different methods are all very close. The reason for this is that below 70 km the atmosphere is very close to LTE while above 110 km the cooling rate is mainly contributed from the fundamental band and there exists also the condition of $C_{ji} > \sum D_{ji}$ in that region. We show in Fig. 2 the CO_2 cooling rate distributions in winter, summer and standard atmospheric conditions. It is shown in the figure that there exist larger negative values of cooling rate near mesopause in summer. That is mainly contributed by the heating from the stratopause high temperature region(Leovy, 1984), Profiles shown in the Fig. 2 qualitatively consist with the results by other

models (Wehrbein et al., 1982; Dickinson, 1984). In Table 1 we list the cooling rates Q_{VT}, Q_{VV}, Q_{EX} and Q_{β} from 64 km to 115 km for summer condition. Due to the fact that there is a big temperature lapse rate in the mesosphere the maximum difference between Q_{VT} and Q_{VV}, 1.8 K / d, appears near 85 km in summer season. Table 1 shows that Q_{EX} lies within Q_{VT} and Q_{VV}, which means that Q_{VT} and Q_{VV} are two appropriate limitings for Q_{EX}. In present calculations we set $\alpha = 0.25$ in (27). Comparing the last two columns in Table 1 we see that the present model gives a good approximation to exact solution. In the tests we need tens or more than 100 iterations to obtain Q_{EX} while Q_{β} is calculated without iteration.

Table 1. Summer Cooling Rates (64–115 km,kd^{-1}) Calculated according to Different Schemes

z (km)	Q_{VT}	Q_{VV}	Q_{EX}	Q_{β}
115	21.23	21.00	21.22	21.17
112	15.97	15.66	15.95	15.85
109	10.35	9.91	10.32	10.14
106	7.10	6.53	7.05	6.81
103	5.09	4.37	5.02	4.74
100	3.27	2.36	3.16	2.86
97	1.84	0.75	1.66	1.38
94	0.45	−0.84	0.18	−0.05
91	−0.79	−2.23	−1.19	−1.32
88	−2.28	−3.97	−2.85	−2.82
85	−3.93	−5.76	−4.61	−4.40
82	−4.65	−6.29	−5.31	−5.01
79	−4.39	−5.54	−4.88	−4.62
76	−3.70	−4.37	−4.01	−3.84
73	−2.69	−3.00	−2.83	−2.75
70	−1.19	−1.27	−1.23	−1.20
67	0.73	0.75	0.74	0.74
64	2.65	2.70	2.67	2.66

IV. SCHEME

Due to the following two problems we can not apply the above model to general circulation models directly. First, in order to calculate the source functions we need to inverse the matrices (I−EC) in (23) and (24). This involves large amount of computation when levels of the circulation models are large. Noticing that the non−LTE only affects the region

significantly above 70 km we can split the whole matrix into smaller blocks so that only smaller matrices are inversed. This has been done in present SKYHI general circulation model in GFDL (with two-level state model for non-LTE CO_2 cooling). The other problem needs strengthening here is that because of the overlaps among the bands in the multi-level state model it is necessary to appropriately split the matrix into blocks (It is not only for reducing the amount of computation).

When proposing the model and making numerical tests above we made an implicit assumption that there is no overlap among the bands. On this condition the total cooling rate is the summation of the cooling rate for each band $Q=\sum Q_i$. In the real atmosphere the overlaps among different bands are not negligible below about 50 km. Fortunately, in the earth's atmosphere the region where LTE breaks down and the region where overlap is important are well separated. We therefore can apply the above model to the circulation models by slight revision on matching between different regions. In the following we consider the matching treatment where the lower, upper and inner boundaries are surface, 130 km and 70 km, respectively.

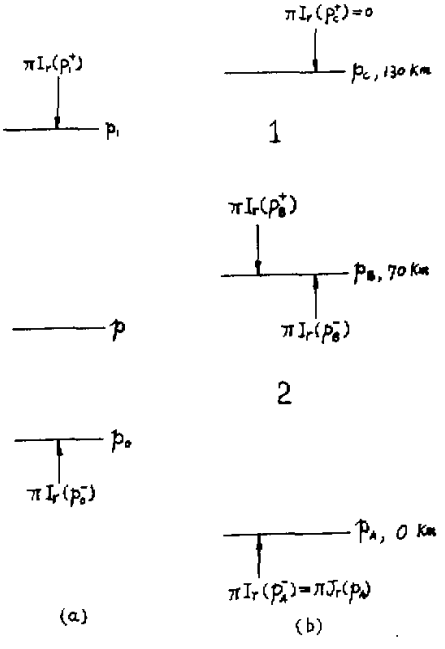

Fig.3. Schematic diagram of matching between region 1 (70 km—130 km) and region 2 (0 km—70 km) through boundary radiation fluxes.

The upward and downward radiative fluxes (Wm^{-2}) at pressure level p are (Andrews, et al. (1987) and Shi (1984) (Fig.3):

$$F_r^{\uparrow}(p) = \pi I_r(p_0^{-})T_f(p_0,p) - \pi\int_p^{p_0} J_r(p')\frac{\partial T_f(p',p)}{\partial p'}dp', \tag{28}$$

$$F_r^{\downarrow}(p) = \pi I_r(p_1^{-})T_f(p_1,p) + \pi\int_{p_1}^{p} J_r(p')\frac{\partial T_f(p',p)}{\partial p'}dp', \tag{29}$$

where $T_f(p',p)$ is flux transmissivity. The cooling rate is given by

$$\frac{dT}{dt} = \frac{Q}{c_p\rho_{air}} = -\frac{1}{c_p\rho_{air}}\frac{\partial}{\partial z}F_r^{net}(p) = (g/c_p)\frac{\partial}{\partial p}[F_r^{\uparrow}(p) - F_r^{\downarrow}(p)]. \tag{30}$$

We notice that if the integrations in (28) and (29) are written appropriately in discrete form $\mathbf{F} \sim \mathbf{C_2 J}$, while the differentiation in (30) is written appropriately in $\mathbf{Q} \sim \mathbf{C_1 F}$, then the Curtis matrix $\mathbf{C} = \mathbf{C_1 C_2}$.

For the whole atmosphere the flux intensities at upper and lower boundaries usually can be assumed as (Fig. 3b)

$$I_r(p_c^{+}) = 0, \qquad I_r(p_A^{-}) = J_r(p_A) = B_r(p_A). \tag{31}$$

However, for the inner boundary at 70 km CO_2 15 μm hot bands and isotope bands have high transpancy in the middle atmosphere (Wehrbein, et al., 1982; Leovy, 1984). The upper region 1 and lower region 2 are closely related through the boundary radiations $I_r(pB^{-})$ and $I_r(pB^{+})$:

$$I_r(pB^{-}) = J_r(pB) + \int_{p_B}^{p_A}\frac{\partial J_r(p')}{\partial p'}T_f(p',pB)dp', \tag{32}$$

$$I_r(pB^{+}) = \int_{p_C}^{p_B} J_r(p')\frac{\partial T_f(p',pB)}{\partial p'}dp'. \tag{33}$$

Through appropriate discretizations these terms can be included in the term $\mathbf{Q}_s$ in the heating rate equation (6).

First, we consider the calculation of heating rate in the lower region 2. There are overlaps of bands in this region we therefore need only one Curtis matrix which includes all the bands. Since the source function in region 1 affects region 2 through the boundary flux $I_r(pB^{+})$ the Curtis matrix must include both regions 1 and 2. The non–LTE effect in region 1 is approximated here by a two–level states model. We split the matrices in blocks in the following forms:

$$\mathbf{Q} = \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix},\ \mathbf{J} = \begin{bmatrix} J_1 \\ J_2 \end{bmatrix},\ \mathbf{C} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix},\ \mathbf{E} = \begin{bmatrix} E_1 & 0 \\ 0 & E_2 \end{bmatrix},\ \mathbf{I} = \begin{bmatrix} I_1 & 0 \\ 0 & I_2 \end{bmatrix}. \tag{34}$$

Also noticing that $\|E_2\| \ll \|E_1\|$, we then have

$$(\mathbf{I} - \mathbf{EC})^{-1} \approx \begin{bmatrix} (I_1 - E_1C_{11})^{-1} & (I_1 - E_1C_{11})^{-1}E_1C_{12} \\ 0 & I_2 \end{bmatrix}, \tag{35}$$

or

$$J_1 = (I_1 - E_1C_{11})^{-1}(B_1 + E_1C_{12}B_2), \tag{36}$$

$$J_2 = B_2. \tag{37}$$

The heating rate in region 2 is given by (note that $Q_s = 0$ in this case)

$$Q_2 = C_{21} J_1 + C_{22} J_2 = C_{21} (I_1 - E_1 C_{11})^{-1} (B_1 + E_1 C_{12} B_2) + C_{22} B_2, \tag{38}$$

in which the first term represents the effect of region 1 to region 2.

Because of the relation (37) the Q_{si} from discrete form of (32) is known. In order to calculate the heating rate in region 1 we need the Curtis matrix C_i which which includes only region 1 for each band. The heating rate in region 1 is therefore

$$\mathbf{Q} = \sum_i \mathbf{Q}_i, \tag{39}$$

where

$$\mathbf{Q}_i = \mathbf{C}_i \mathbf{J}_i + \mathbf{Q}_{si}, \tag{40}$$

$$\mathbf{J}_i = (1-\beta)(\mathbf{I} - \mathbf{E}_i \mathbf{C}_i)^{-1} (\mathbf{B} + \mathbf{E}_i \mathbf{Q}_{si}) + \beta (\mathbf{I} - \mathbf{E}^* \mathbf{C})^{-1} (\mathbf{B} + \mathbf{E}^* \mathbf{Q}_s). \tag{41}$$

The contributions from region 2 to region 1 have been included in the term Q_{si}. The main contributions come from the region above 50 km where the overlaps between the bands are negligible.

In above calculations of the heating rates we need only to inverse matrices whose order depends on the number of the model levels in region 1. This scheme will be considered to apply to the next generation SKYHI general circulation model in GFDL which includes the lower thermosphere.

V. CONCLUDING REMARKS

It is very difficult to calculate CO_2 15μm cooling rate in the upper mesosphere accurately. Such a difficulty mainly comes from the non-LTE effect in the mesosphere. This may also be one of the reasons that we can not directly make remote sounding for the CO_2 cooling rate in mososphere (Liou and Xue, 1988). In this paper we derive the formalism for calculating non-LTE multi-level cooling rate in matrix form by a unified and pedagogic method. This paper also proposes a simple parameterization model for the cooling rate calculation. The preliminary results of numerical test show that the cooling rate calculation scheme based on such a model can be applied to the general circulation models including non-LTE region.

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