On the Multiple Equilibrium of the Development of Tropical Cyclone in Nonlinear CISK Model

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ABSTRACT

By using non-static atmosphere equations including basic current, heating force and friction, we discuss the balance amplitude of vertical motion in the conditions of constant heating force, linear and non-linear interaction between large-scale ascending motion and diabatic heating force. In the non-linear condition, the multiple equilibrium feature of the tropical cyclone development is discussed and the reason of the sudden varying of tropical cyclone intensity is studied preliminarily.

I. INTRODUCTION

A major conceptual breakthrough in understanding tropical cyclone was provided by Ooyama (1964) and Charney and Eliassen (1964) when they demonstrated that cyclone development could occur by a kind of secondary instability involving the cooperative interaction of two scale of motion, the cumulus scale and the cyclone scale. The newly discovered instability was called conditional instability of the second kind (CISK). By CISK mechanism the major features of tropical cyclone development could be well explained.

Undoubtedly, CISK theory is significant to understand the physical mechanism of tropical cyclone development. However, not all tropical disturbances can develop to typhoon. It is found from observations that some tropical disturbances can develop to typhoon, and others can not. Gray (1982) carried out diagnostic studies of composite tropical cyclone over the Atlantic and Pacific, and Rosenthal (1978) simulated the development of tropical cyclone with a discernible cumulus-scale grid model. From the results, they indicated that CISK mechanism played little role in the initial stage of tropical cyclone development. Therefore, it is reasonable to consider that CISK mechanism has not efficient effect until the disturbance reaches a critical state. Whether an initial weak disturbance can intensify to the critical state or not depends on the intensity and size of the disturbance, basic state static and inertial stabilities, and so on. Only when they dispose well enough, can the disturbance develop to the critical state.

CISK is in essence a process in which vortex and cumulus interact and intensify cooperatively. This process can only be depicted well by nonlinear model. But nonlinear model is quite complicated, the earliest CISK theoretical studies were based on linearization, by which some satisfactory basic results can be obtained. Usually, it was supposed that the diabatic heating is directly proportional to the geostrophic vorticity at the top of Ekman layer or is proportional to the vertical velocity at a certain layer. The bigger proportional coefficient denotes the stronger feedback.

Based on CISK theory, a series of numerical modeling experiments of occurrence and development of typhoon have been performed by using various models for more than twenty years, and good results have been obtained. By using the parameterization schemes mentioned previously, the development of tropical cyclone could be simulated, but sudden burst of tropical cyclone intensity could not be simulated well (Chen Lianshou, Ding Yihui, 1979). In the case that heating source was directly proportional linearly to large–scale ascending motion, burst can not be simulated by even increasing feedback coefficients (Zhang Ming, Zeng Qingcun, 1983), Zhang and Zeng used axisymmetric five–level primitive equation model in cylindrical coordinate to simulate burst development of tropical cyclone, and pointed out that burst development of tropical cyclone may be induced by strong nonlinear relationship between cumulus convective condensation heating and large–scale flow field, and sudden varying may be a strong nonlinear phenomenon and is restrained by inherent nonlinear mechanism; therefore, nonlinear parameterization scheme must be used. Although their nonlinear models were very simple and there were something artificial, they simulated successfully the important characteristics of burst increase and verified the above viewpoint. In this paper, we firstly imitate formula of nonlinear interaction between the heating and vertical motion according to observations and analysed data, and then discuss burst increase and multiple equilibrium feature of tropical cyclone under above nonlinear interactions.

II. IMITATION OF OBSERVATIONAL AND ANALYTICAL DATA

Gray (1981) analysed composite tropical cyclones over the Atlantic and Pacific and found that in different stages of the tropical cyclone development although there were rainfall differences between inner and outer regions, total average precipitation in rainfall area was a constant approximately. Table 1 gives radial distribution of rainfall of composite storms over Atlantic and Pacific.

Table 1. Radial Distribution of Rainfall of Composite Storms over Atlantic and Pacific (cm / d) (data from Gray, 1981)

	0—2°	2—4°	4—6°	6—8°	8—10°	Area average
Pre–hurricane	1.9	1.5	0.9	0.5	0.5	0.76
Hurricane	6.3	2.0	0.7	0.3	0.4	0.86
Pre–typhoon	3.4	2.6	1.5	0.7	0.7	1.20
Typhoon	8.3	2.3	0.1	0.8	0.7	1.10

According to Gray's analysis, we suppose total heating rate in typhoon area $\int_0^{z_t}\int_0^{2\pi}\int_0^{r_\infty} Q(r,\theta,z)\ rdrd\theta dz$ is a constant and

$$Q(r,\theta,z)=\begin{cases}0 & 0<r<r_1\\ \hat{Q}\cdot\sin(\frac{\pi}{z_t}z) & r_1<r<r_2\\ 0 & r_2<r<r_\infty\end{cases}$$

where r_1 and r_2 are radii of eye wall and inner region respectively, $\hat{Q}$ is average heating amplitude between r_1 and r_2 and is not related to θ. Therefore, $\hat{Q}\cdot(r_2^2-r_1^2)$ = constant = $10.0^{\circ}\mathrm{C}\cdot d^{-1}\cdot(250\ km)^2$.

Above constants are selected from analysis of a typical tropical cyclone by Schubert and Hack (1982). They analysed the typical values of parameters r_1 and r_2 as well as potential temperature variability in different stages of the tropical cyclone development (five stages: A, B, C, D and E; A–B for tropical depression, C for tropical storm, and D–E for hurricane), and gave following table.

Table 2. Various Parameter Values of Typical Tropical Cyclone under Different Stages (after Schubert and Hack, 1982)

Stage	r_1 (km)	r_2 (km)	$\left.\frac{\partial\theta}{\partial t}\right\|_{r=0}$ (℃ · d^{-1})	$\hat{Q}$ (℃ · d^{-1})	$\left.\frac{\partial\theta/\partial t}{\hat{Q}}\right\|_{r=0}$
A	0	250—350	0.58	6.94	0.083
B	0	200—300	1.67	10.00	0.167
C	0	150—250	3.69	15.63	0.236
D	0—30	100—200	12.67	27.78	0.456
E	0—30	50—150	43.63	62.50	0.698

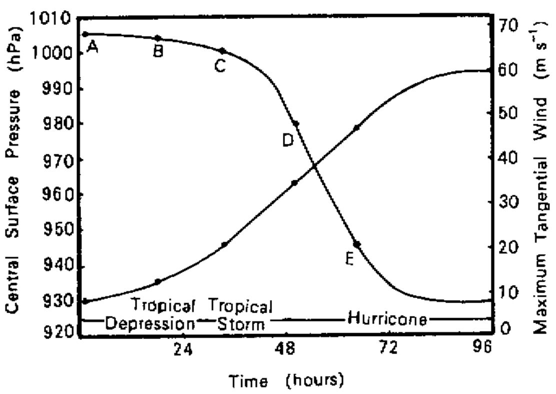


Fig. 1 (a) The changes of central surface pressure and tangential velocity maximum of typical tropical cyclone with time.

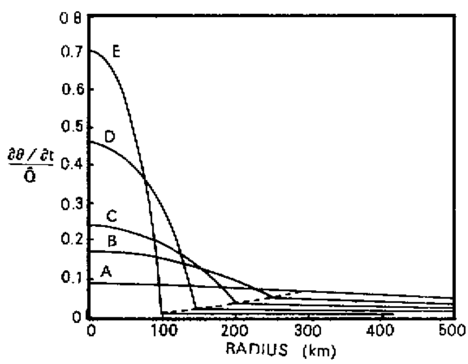


Fig. 1 (b) Radial distributions of $\frac{\partial\theta}{\partial t}/\hat{Q}$ in five stages A–E (after Schubert and Hack, 1982).

In the tropics, atmospheric state is quasi–barotropic, thermodynamic equation can be written as $\frac{\partial\theta}{\partial t}=Q-\frac{\theta_0}{g}\cdot N^2\cdot W$.

We can see from Table 2 and Fig. 1 that the radial distribution of $\partial\theta/\partial t$ in inner region $(r<r_2)$ is cosine and in outer region is linear with small slope. If amplitude of cosine in inner region $\left.\frac{\partial\theta}{\partial t}\right|_{r=0}$ is stood for as $\frac{\partial\hat{\theta}}{\partial t}$, the relation between amplitudes of upward motion in inner region $\hat{W}$ and heating $\hat{Q}$ can be expressed as $\frac{\hat{W}}{\hat{Q}}=(1-\frac{\partial\hat{\theta}}{\partial t}/\hat{Q})\cdot\frac{g}{\theta_0 N^2}$.

Table 3. Amplituds of Vertical Motion in Different Stages Calculated by Thermodynamic Equation from Table 2 ($\bar{N}=0.6\cdot10^{-2}\ s^{-1}$, $\theta_0=298K$)

Stage	$\hat{Q}$ (10^{-5} ℃s^{-1})	$\hat{W}/\hat{Q}$ · (10^2 m/℃)	$\hat{W}$ (ms^{-1})
A	8.03	8.38	0.067
B	11.57	7.61	0.088
C	18.09	6.98	0.126
D	32.15	4.97	0.160
E	72.34	2.76	0.200

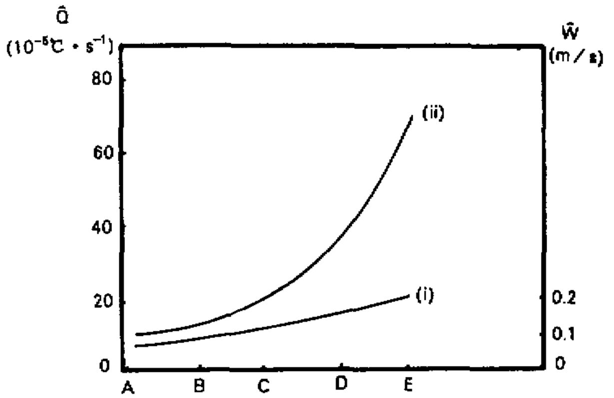


Fig. 2 Changes of vertical motion (i) and heating (ii) with time.

We can see that during early stages of the tropical cyclone development, ratio of $\hat{W}$ and $\hat{Q}$ is a constant approximately (i.e. the relation between $\hat{W}$ and $\hat{Q}$ is approximately linear), and that during late stages, the value of $\hat{W}/\hat{Q}$ decreases rapidly, heating rate increases at higher rate than vertical motion, and both are nonlinear. Therefore, during early stages, we may give following formula:

$$\hat{Q}_1=a_1\cdot\left(\hat{W}/W_0\right)^3+b_1\cdot\left(\hat{W}/W_0\right)^2+c_1\cdot\left(\hat{W}/W_0\right)+d_1 \qquad (1)$$

where W_0 is characteristic scale of vertical motion, a_1, b_1, c_1 and d_1 are interactive coefficients. To examine weak nonlinear interaction in early stages of the tropical cyclone development, we suppose $a_1=b_1=0.1\cdot c_1$, and coefficients c_1 and d_1 (unit: ℃ · S^{-1}) are determined by least square method; therefore, $a_1=4.16\cdot10^{-5}$, $b_1=4.16\cdot10^{-5}$, $c_1=4.16\cdot10^{-4}$, $d_1=-7.79\cdot10^{-5}$.

Examing nonlinear interaction between the heating and upward motion during all stages of tropical cyclone development, we suppose

$$\hat{Q}_2=a_2\cdot\hat{W}^3+b_2\cdot\hat{W}^2+c_2\cdot\hat{W}+d_2 \qquad (2)$$

where a_2, b_2, c_2, and d_2 are nonlinear interaction coefficients and are determined by least square method. Fig.3a gives above two kinds of interaction curves. At can be seen from

Fig.3a that when vertical velocity is less than a critical value W_c (W_c=0.037 m / s for l_1, W_c = 0.042 m / s for l_2), heating is negative, above two formulae can not be used properly, and CISK does not exist. However, when $\hat{W}$ is more than W_c, CISK is important, which agrees with Gray's diagnostic analysis.

We suppose $a_1, b_1 \ll c_1$ in (1). If we delete this restriction and use least square method directly, we can deduce the formula of nonlinear interaction between the heating and upward motion in early stages of tropical cyclone development which is

$$\hat{Q}_3 = a_3 \cdot \hat{W}^3 + b_3 \cdot \hat{W}^2 + c_3 \cdot \hat{W} + d_3 = \\ -0.1392\hat{W}^3 + 0.0397\hat{W}^2 - 0.0019\hat{W} + 7.43 \cdot 10^{-5}. \quad (3)$$

Fig.3b gives above this nonlinear interaction curve. We can see that this curve has a minimum when $\hat{W}$=0.029 m / s and returns to initial heating value when $\hat{W}$=0.059 m / s. We consider that the feature in Fig.3b shows unimportance of CISK in initial stage of disturbance development. As we know, that CISK is unimportant means that in initial stage of disturbance development, scattered and disorganized convective condensational heating is not strong enough to offset the adiabatic cooling so that the disturbance developed slowly. Meanwhile, upward current made cold and warm air at high and low layers exchange and stabilized stratification. As long as cumulus begin to organize and release a lot of latent heat, CISK intensifies disturbance rapidly. When slow increase of N^2 is not considered in the model, N^2 is regarded as a constant, corresponding to weaker diabatic heating during this stage.

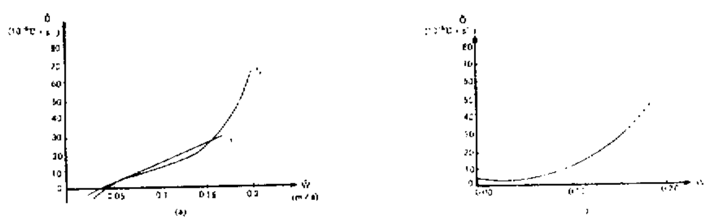


Fig. 3. (a) The interaction curves of the quasi-linear in early stages (l_1) and the nonlinear in all stages (l_2); (b) The nonlinear interaction curve in early stages.

III. MODEL

As the first step, we use non-static dynamic equations including heat source and dissipation under Boussinesq hypothesis, and basic flow field is as follows

$$\bar{u} = \bar{u}_y \cdot y,$$

where $\bar{u}_y$ is not related to y and is only a slow-changing function of time. For convenience, the change of $\bar{u}$ with z is not considered in the model. Brunt-Vaisala frequency N was defined as

$$N^2 = -g \cdot \frac{\partial}{\partial z} \ln\bar{\rho},$$

and suppose N^2 does not change with y and z, and is a slow–changing variable of time.

Two–dimensional linearized Boussinesq equations are written as

$$\begin{cases} \left(\dfrac{\partial}{\partial t}+k\right)u'=(f-\bar{u}_y)\cdot v' \\ \left(\dfrac{\partial}{\partial t}+k\right)v'=-f\cdot u'-\dfrac{\partial p'}{\partial y} \\ \left(\dfrac{\partial}{\partial t}+k\right)w'=-g\cdot\rho'-\dfrac{\partial p'}{\partial z} \\ \left(\dfrac{\partial}{\partial t}+k'\right)g\cdot\rho'=N^2\cdot w'-Q \\ \dfrac{\partial v'}{\partial y}+\dfrac{\partial w'}{\partial z}=0, \end{cases} \tag{4}$$

where $(u',v',w')=\bar{\rho}(z)\cdot(u,v,w)$, k is Reyleigh frictional coefficient, and k' is Newton cooling coefficient. k and k' should be considered as slow–changing variables of time. For convenience, we only discuss two situations: (1) $k'=k=0$; (2) $k'=k\neq 0$. Thus equations (4) become:

$$L(w)=Q_{yy}, \tag{5}$$

where $$L=\left[\left(\frac{\partial}{\partial t}+k\right)^2+I^2\right]\cdot\frac{\partial^2}{\partial z^2}+\left[\left(\frac{\partial}{\partial t}+k\right)^2+N^2\right]\cdot\frac{\partial^2}{\partial y^2},$$

and $$I^2=f\cdot(f-\bar{u}_y),$$

Q is the forcing term of diabatic heating. As one knows, over tropical ocean, especially over the ITCZ region, cumuli are very active, and upward motion makes a lot of vapor release condensational latent heat and the ITCZ becomes the forcing source of tropical disturbance. Suppose forcing source as follows:

$$Q=\hat{Q}(t)\cdot\cos(my)\cdot\sin(nz), \tag{6}$$

where $m=\pi/2r_0$, $n=\pi/H$, r_0 is radius of disturbance, H is thickness. Above heating distribution means that heating maximum is located at $y=0$ in horizontal direction, and $H/2$ in vertical direction.

Suppose the solution of (5) is

$$w=\hat{W}(t)\cdot\cos(my)\cdot\sin(nz). \tag{7}$$

Substituting (6) and (7) into (5), we have

$$\left(1+\frac{n^2}{m^2}\right)\hat{W}''+2k\left(1+\frac{n^2}{m^2}\right)\hat{W}'+\left[\left(k^2+I^2\right)\frac{n^2}{m^2}+k^2+N^2\right]\cdot\hat{W}=\hat{Q}(t). \tag{8}$$

IV. ANALYSIS AND DISCUSSIONS

As mentioned previously, CISK was not important during the initial stage of disturbance development and only when disturbance developed to reach a critical value, could CISK process be produced. Suppose CISK was produced after amplitude of vertical motion reached a critical amplitude W_c, we first investigated a condition under which disturbance could develop to critical value, i.e. under real initial heating condition, how to dispose horizontal and vertical sizes of the heating as well as static and inertial stabilities facilitated disturbance to develop to a critical value.

Suppose $\hat{Q}(t)=Q_{00}=$ constant, $k=0$ (8) can be written as

$$\left(1+\frac{n^2}{m^2}\right)\cdot \hat{W}''+\left(I^2\cdot\frac{n^2}{m^2}+N^2\right)\cdot \hat{W}=Q_{00}. \tag{9}$$

Multiply $\hat{W}'$ and integrate on both sides of (9), and suppose integrating constant is zero, we obtain

$$\frac{1}{2}\left(1+\frac{n^2}{m^2}\right)\cdot\left(\hat{W}'\right)^2=-\frac{1}{2}\left(I^2\cdot\frac{n^2}{m^2}+N^2\right)\cdot\hat{W}^2+Q_{00}\cdot\hat{W}. \tag{10}$$

Let $\hat{W}'=0$, balanced amplitude $\overline{W}$ of disturbance was deducted as

$$\overline{W}=2\cdot Q_{00}\Big/\left(I^2\cdot\frac{n^2}{m^2}+N^2\right). \tag{11}$$

Generally, the intensity of the initial heat source is small and assumed to be 1℃ / d, thus

$$Q_{00}=\frac{g}{\theta_0}\cdot\frac{1}{24\times 3600}=3.8\cdot 10^{-7}(m\cdot s^{-3}).$$

Therefore

$$I^2\cdot\frac{n^2}{m^2}+N^2\leqslant\frac{2\cdot Q_{00}}{\hat{W}_c}. \tag{12}$$

It was the condition satisfied by the disturbances developing to a critical value. Northeastward trade wind is in north of the ITCZ and southwestward monsoon in south of the ITCZ, and $\bar{u}_y<0$, $I^2>0$, therefore, if $\hat{W}_c=0.05$ m / s, when $N^2<1.52\cdot 10^{-5}s^{-2}$, disturbances may develop to the critical state.

Resonance Brunt–Vaisala frequency is

$$N^2\big|_{res}=-\frac{n^2}{m^2}\cdot I^2. \tag{13}$$

If $k\neq 0$, (5) becomes

$$\hat{W}''+2k\hat{W}'+\sigma^2\cdot\hat{W}=Q_{00}\Big/\left(1+\frac{n^2}{m^2}\right). \tag{14}$$

$$\hat{W}=e^{-kt}\cdot(A_0\cdot\cos\sqrt{\sigma^2-k^2}\,t+B_0\sin\sqrt{\sigma^2-k^2}\,t)+Q_{00}/\sigma^2\Big/\left(1+\frac{n^2}{m^2}\right), \tag{15}$$

where $\sigma^2=\left[(k^2+I^2)\cdot\frac{n^2}{m^2}+k^2+N^2\right]\Big/\left(1+\frac{n^2}{m^2}\right)$ let $t\to\infty$, balanced amplitude of disturbance is

$$\overline{W}=Q_{00}\Big/\left[(k^2+I^2)\cdot\frac{n^2}{m^2}+k^2+N^2\right]. \tag{16}$$

In this situation, the condition that vertical motion amplitude which can reach to the critical amplitude $\hat{W}_c$ may be presented as

$$I^2\cdot\frac{n^2}{m^2}+N^2\leqslant\frac{Q_{00}}{\hat{W}_c}-\left(1+\frac{n^2}{m^2}\right)\cdot k^2. \tag{17}$$

Obviously, due to introduction of friction, higher requirement of environmental condition is needed.

Now, we discuss equilibrium of disturbance development by by CISK effect. First of all, we give the linear interaction relation between the heating and upward motion similar to that in previous CISK work, i.e. $\hat{Q}(t)=Q_{00}+A\cdot\hat{W}$ (A is the heating coefficient, $A>0$). Thus the balanced amplitude of disturbance is

$$\overline{W}=Q_{00}\Big/\left[(k^2+l^2)\cdot\frac{n^2}{m^2}+k^2+N^2-A\right]. \tag{18}$$

Obviously, the balanced amplitude of disturbance reaches a higher value due to the interaction between disturbance and forcing source. Because $\overline{W}>0$, we obtain

$$A<(k^2+l^2)\cdot\frac{n^2}{m^2}+k^2+N^2=A_{\rm cri}. \tag{19}$$

This means that heating heating coefficient A can not be determined arbitrarily and must be less than a critical value, and linear CISK only can decrease static stability and can not produce multiple equilibrium. When CISK effect is very strong and heating coefficient A approaches $A_{\rm cri}$, resonance occurs. Resonance Brunt–Vaisala frequency is

$$N^2\big|_{\rm res}=A-l^2\cdot\frac{n^2}{m^2}-\left(1+\frac{n^2}{m^2}\right)\cdot k^2. \tag{20}$$

Now, we discuss the features of the balanced amplitude of disturbance when the heating and upward motion interact nonlinearly. We use (3)

$$\hat{Q}(t)=Q_{00}+\hat{Q}_3=Q_{00}+a_3\cdot\hat{W}^3+b_3\cdot\hat{W}^2+c_3\cdot\hat{W}+d_3, \tag{21}$$

where Q_{00} is initial heat source, a_3,b_3,c_3 and d_3 are nonlinear interactive coefficients. Considering in real tropical cyclone development, the heating and upward motion do not interact completely by (3), so we do not think that d_3 is a constant as (3), we regard d_3 as a slow–changing coefficient of time to vary slowly with disturbance development.

When we do not consider friction dissipation, (8) becomes

$$\left(1+\frac{n^2}{m^2}\right)\hat{W}''+\left(l^2\cdot\frac{n^2}{m^2}+N^2\right)\hat{W}=Q_{00}+a_3\cdot\hat{W}^3+b_3\cdot\hat{W}^2+c_3\cdot\hat{W}+d_3. \tag{22}$$

Multiply $\hat{W}'$ and integrate on both sides of (22), and suppose integrating constant equals to zero, thus we can get the balanced amplitude equation:

$$a_3\cdot\overline{W}^3+\frac{4}{3}b_3\cdot\overline{W}^2+2\left(c_3-l^2\cdot\frac{n^2}{m^2}-N^2\right)\overline{W}+4(Q_{00}+d_3)=0. \tag{23}$$

Suppose $\overline{W}=X-\dfrac{4b_3}{9a_3}$, we get $X^3+pX+q=0$,

where $p=-2.203\cdot10^{-2}+4.369\cdot10^2\left(N^2+l^2\cdot\dfrac{n^2}{m^2}\right)$

$$q=-5.292\cdot10^{-4}+5.536\cdot10\left(N^2+l^2\cdot\frac{n^2}{m^2}\right)+f_{00}\quad f_{00}=\frac{4}{a_3}(Q_{00}+d_3).$$

Use $N^2=1.98\cdot10^{-5}S^{-2}$, $\bar{u}_y=-f$, $l^2=f(f-\bar{u}_y)=2f^2$

$=0.284\cdot10^{-8}S^{-2}$, $\dfrac{n^2}{m^2}=\left(\dfrac{2r_0}{H}\right)^2=3600$, $Q_{00}=1℃d^{-1}$, we can obtain

$$\Delta=\frac{1}{4}q^2+\frac{1}{27}p^3=\frac{1}{4}(d_3+0.185\cdot10^{-5})(d_3+0.132\cdot10^{-5}).$$

When $d_3>-0.185\cdot10^{-5}$ ms^{-3} or $d_3<-0.132\cdot10^{-5}$ ms^{-3}, $\Delta>0$, only one real root exists (the magnitude of d_3 is 2–3℃ / d).

When $-0.185\cdot10^{-5}$ms$^{-3}<d_3<-0.132\cdot10^{-5}$ms^{-3}, $\Delta<0$, three real roots exist.

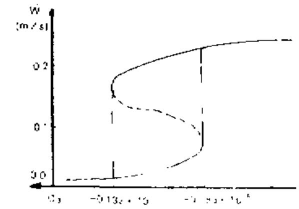
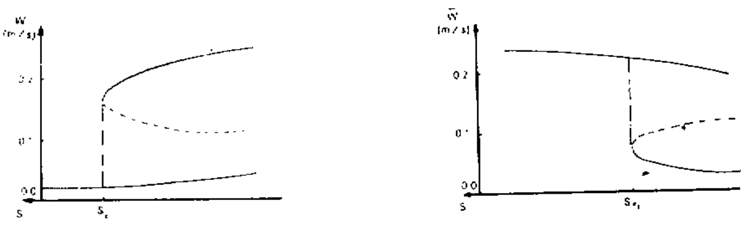


Fig.4. The changing curve of the balanced amplitude of vertical motion with d_3 (solid line denotes stable equilibrium, dashed line denotes unstable equilibrium).

Fig.4 gives the change of the balanced amplitute of vertical motion with interactive coefficient d_3 . It is shown that with slow and continuous change of d_3, discontinuous sudden jump does occur in vertical motion field. When tropical cyclone intensifies, the vertical motion increases suddenly from 0.069 m / s to 0.216 m / s, then enhances slowly and continuously in new equilibrium, and when tropical cyclone decays, the vertical motion decreases suddenly from 0.167 m / s to 0.021 m / s, then decays smoothly. This explains sudden change of tropical cyclone intensity preliminarily.

A lot of observations and numerical simulation studies have shown that the thickness and maximum wind radius of tropical cyclone as well as static and inertial stabilities change in the process of tropical cyclone development. Therefore, it is necessary to investigate multiple equilibrium feature of tropical cyclone development when static and inertial stabilities (N^2 and I^2) as well as tropical cyclone structure parameter n^2/m^2 change slowly with time.



(a) (b)

Fig.5. The change curves of the balanced amplitude of the vertical motion with $S=N^2+I^2\cdot n^2/m^2$ (a: $d_3=-0.132\cdot10^{-5}$, b:$d_3=-0.185\cdot10^{-5}$).

We have discussed multiple equilibrium feature of tropical cyclone development when the heating and upward motion interact nonlinearly by (3). We see that when the nonlinear interactive coefficient d_3 changes slightly, discontinuous sudden jump of the system occurs, but the change of linear interactive coefficient A can not show the above feature. This is just one of the most important features of nonlinear interaction. When we adopt the nonlinear interactive relation (2), we can also obtain the above important feature (figure omitted).

V. CONCLUSIONS

Tropical cyclone is a subsynoptic system, which is controlled by large–scale basic field and feedbacked by small–scale cumulus convection. Undoubtedly, the relation of interaction between the heating and vortex motion is very complicated, and is not the same in different stages of tropical cyclone development. It is not complete that such an interaction is only expressed by the forms which do not change with time. In this paper we have constructed the nonlinear interactive relations between the heating and upward motion by imitating observational and analysed data. Unlike original linear CISK model, we obtained cubic interactive form of the heating and upward motion. Due to the limitation of data, the imitation is coarse, and real situation is more complicated. However, even this kind of simple model indicates that nonlinear CISK effect produces important dynamic features differing from those in linear CISK situation.

The theoretic and observative studies showed that the sudden varying of intensity exists in tropical cyclone development. It was considered that the sudden varying was induced by the nonlinear interaction among a variety of waves of the system, i.e. by nonlinear advection effect. Indeed, the nonlinear advective effect is an important factor to produce sudden varying, but we think it is not the unique one. Especially in early stages of the tropical cyclone development when nonlinear advective effect is still not strong, the nonlinear CISK effect is a main factor to produce the sudden varying. In late stages of the tropical cyclone development, although the nonlinear advective effect becomes more and more important, the nonlinear CISK effect also intensifies continuously. Therefore, it is significant and necessary to study the nonlinear CISK mechanism.

As preliminary discussion, we have investigated multiple equilibrium feature of tropical cyclone devlopment with nonlinear force in the linear system, and further work will be done in the near future.

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