The Solitary Wave of Barotropic Atmosphere on a Sphere

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Received March 12, 1988

ABSTRACT

In this paper, we have investigated large scale disturbance in barotropic atmosphere on a sphere. It demonstrates that: considering nonlinear effects of interaction between waves in barotropic vorticity equation, the wave packet of the disturbance is governed by the famous equation—nonlinear Schrodinger equation. For the solitary wave, two factors are very important: one is spherical effect of the disturbance and the other is meridional shear of blocking high. In comparison with the results of local Cartesian coordinates, the former factor is the individuality of spherical soliton.

I. INTRODUCTION

Blocking high generally forms in high or middle latitude zone. Sometimes it can keep up one and a half months. Its lasting time is long enough to show that the energy dissipation of blocking high is very weak. Yeh (1949) pointed out that the energy dispersion of Rossby waves is proportional to Rossby parameter β. So in relatively high latitude zone, the energy dispersion is less. On the other hand, in relatively low latitude zone, Coriolis parameter f is small. and weather systems which intrude into low latitude zone are easily broken, so that strong systems are very difficult to keep up long time. In addition, blocking high is always accompanied by meridional shear of basic wind field in the atmosphere. We have a question that whether blocking high makes jet stream bifurcate or the branches of the jet stream which leads to the meridional shear of basic wind field cause the formation of blocking high. From this paper, we can see that meridional shear is a key to the formation of the high.

Meteorologists have investigated blocking high as solitary wave for a long time. Just due to this idea, solitary wave theory has got a great progress in meteorology. Long (1964) used simple barotropic atmosphere model to involve soliton in the westerlies, and got KdV soliton. Following his pioneering work, Redekopp (1977) found that the amplitude of moving Rossby wave group in meridional shear zonal wind field is governed by KdV or MKdV (modified KdV) equation, in last case the air density is of stratification. Meanwhile, based upon the suggestion made by Redekopp and Maxworthy (1976) about the Great Red Spot (GRS) and some other features in the Jovian atmosphere, Redekopp (1978) discussed the interaction between solitary Rossby wave propagating in zonal shear flow and the flow. All these researches are the fundament of large scale solitary wave theory, i.e., the soliton theory of blocking high. We had introduced nonlinear Schrodinger equation (hereafter referred to as NSE), a well-known equation of quantum mechanics, discussed meteorological problems, and got very satisfactory results in the past. In fact, scientists had interested in this equation long before. They used spectral method (Zakharov,1968) and multiscale analysis method to solve the aspects of nonlinear modulation fluid wave. They discovered that: due to nonlinear modulation, the disturbance which approximates to a single harmonic wave is governed by NSE

(Tanitui et al., 1968; Asano et al., 1969; Karpman et al.,1969; Zakharov et al., 1972; Dewar, 1972). In addition, Zakharov and Shabat(1972) investigated the traits of NSE solution under given conditions, and discovered that: any initial shape of wave packet eventually evolves into a number of "envelope solitons" and a dispersive tail. We also have used NSE to explain the formation and maintenance of blocking high in local Cartesian coordinates and strong mesoscale systems in the atmosphere, for instance, cold surge etc., and have got some suitable explanations. We have pointed out that: it is meridional shear of basic wind field that plays an important role in blocking highs, in case of no vertical shear of basic wind field (i.e., barotropic atmosphere), the mesoscale solitary waves are not easy to form.

Zeng (1979) studied small amplitude disturbance on a sphere from linear hand, it is not necessary to repeat his work here. In condition that the disturbance is small (i.e., perturbation), considering nonlinear effects of the interaction between waves, we investigate the evolution of the disturbance and deduce governed wave packet equation of Rossby wave group on a sphere in following sections. First, in Section II we research linear Rossby wave and introduce linear Schrodinger equation (hereafter referred to as LSE). And then in Section III, we investigate evolution of the disturbance on the sphere systematically. At last, in Section IV some conclusions will be given out.

II. THE PACKET OF LINEAR ROSSBY WAVE GROUP

The theory of Rossby wave was established by Rossby himself in 1939. Under the efforts of Kuo (1949), Charney (1947), Yeh (1949), Eady (1949) and other meteorologists, the theory has made a great progress. It has become the fundament of researching nonlinear long wave today.

Holton (1979) gave out the wave length of maximum instability (corresponding to this length, the wave number is $k = 2^{\frac{1}{4}}\lambda$, $\lambda = f_0 / \sqrt{\sigma}\Delta p$). From routine weather map, we know that a Rossby wave, of which velocity is about 10 m / s or so, is predominant. Apart from this wave, the other Rossby waves, including rapidly propagating eastern waves, quasi-stationary wave or slowly moving western waves, are trivial in energy. So we can assume that the energy is controlled by Guassian distribution at the initial, and $k = k_0$ the wave has maximum energy. For this wave, its corresponding frequency is ω_0, wave velocity C_0 and so on.

Introducing two-dimensional barotropic nondivergent vorticity equation on β-plane, we have:

$$\left(\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x}\right)\nabla^2\psi' + \beta\frac{\partial\psi'}{\partial x} = 0 \tag{1}$$

Where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, $\bar{u}$ is zonal mean wind speed. In the zonal channel on β-plane, assuming that:

$$\psi' = \tilde{A}\sin l_1 y \cdot e^{ik(x - c_1 t)} \quad . \tag{a}$$

Then we have:

$$\frac{\partial D}{\partial t} + \left(\bar{u} - \frac{\beta(l_1^2 - k_0^2)}{(k_0^2 + l_1^2)^2}\right)\frac{\partial D}{\partial x} = i\frac{2k_0\beta l}{(k_0^2 + l_1^2)^3}\frac{\partial^2 D}{\partial x^2} \quad , \tag{2}$$

where D is the envelope of Rossby wave group. (2) is recognized as LSE, and the group velocity C_{gl} is:

$$C_{g1}=\bar{u}-\frac{\beta(l_1^2-k_0^2)}{(k_0^2+l_1^2)^2}\ . \tag{3}$$

In next section, we shall investigate nonlinear disturbance problem and compare the results with equation (2).

III. THE NONLINEAR DISTURBANCE ON A SPHERE

Just as emphasized in the previous section: spherical effect of disturbance contributes significantly to the formation and maintenance of blocking high. We concentrate our attention to the evolution of modulating wave of large scale disturbance in high and middle latitude zone. For barotropic atmosphere on sphere, based on following equation (conservation equation of absolute vorticity):

$$\left(\frac{\partial}{\partial t}+\bar{v}_\theta\frac{\partial}{a\cdot\partial\theta}+\bar{v}_\lambda\frac{\partial}{a\sin\theta\partial\lambda}\right)\left(2\Omega\cos\theta+a^{-2}\Delta\psi\right)=0\ , \tag{4}$$

where a is radius of the earth, λ is longitude, $\varphi=90^\circ-\theta$, is latitude. ψ is stream function, Δ is Laplacian operator in the sphere coordinates, and:

$$v_\theta=-\frac{\partial\psi}{a\sin\theta\partial\lambda}\ , \tag{5}$$

$$v_\lambda=\frac{\partial\psi}{a\partial\theta}\ . \tag{6}$$

Substituting $\varphi=90^\circ-\theta$ into (4), we obtain:

$$\left(\frac{\partial}{\partial t}+\frac{\partial\psi}{a\cos\varphi\partial\lambda}\cdot\frac{\partial}{a\partial\varphi}-\frac{\partial\psi}{a\partial\theta}\frac{\partial}{a\cos\varphi\partial\lambda}\right)\left(2\Omega\sin\varphi+a^{-2}\Delta\psi\right)=0\ , \tag{4$'$}$$

where

$$\Delta=\left(\frac{\partial}{\partial\varphi}(\cos\varphi\frac{\partial}{\partial\varphi})+\frac{1}{\cos\varphi}\frac{\partial^2}{\partial\lambda^2}\right)/\cos\varphi=\frac{\partial^2}{\partial\varphi^2}-\mathrm{tg}\varphi\frac{\partial}{\partial\varphi}+\frac{1}{\cos^2\varphi}\frac{\partial^2}{\partial\lambda^2}\ . \tag{7}$$

Taking $u=v_\lambda=-\frac{\partial\psi}{a\partial\varphi}$, $v=v_\theta=-\frac{\partial\psi}{a\cos\varphi\partial\lambda}$ represents zonal and meridional wind respectively. We divide the stream into mean basic stream field and perturbation stream field (denoted by prime) and assume meridional circulation very weak, i.e. $\bar{v}=0$, so that

$$\psi=\bar{\psi}(\varphi)+\psi'(\lambda,\varphi,t)\ . \tag{8}$$

Substituting (8) into (4)$'$ and assuming that mean stream function also satisfies equation (4)$'$, then we obtain perturbation vorticity equation in the form of perturbation stream function formed by spherical Rossby wave which we shall discuss in the following.

$$\left(\frac{\partial}{\partial t}-\frac{\partial\bar{\psi}}{a\partial\varphi}\frac{\partial}{a\cos\varphi\partial\lambda}\right)\left(a^{-2}\Delta\psi'\right)+\frac{\partial\psi'}{a\cos\varphi\partial\lambda}\frac{\partial}{a\partial\varphi}\left(2\Omega\sin\varphi+a^{-2}\Delta\bar{\psi}\right.$$
$$\left.+\frac{\partial\psi'}{a\cos\varphi\partial\lambda}\frac{\partial}{a\partial\varphi}(a^{-2}\Delta\psi')-\frac{\partial\psi'}{a\partial\varphi}\frac{\partial}{a\cos\varphi\partial\lambda}(a^{-2}\Delta\psi')=0\ . \tag{9}$$

In above equation, we have assumed that $\psi'\ll\bar{\psi}$, i.e. the perturbation is at the stage of small amplitude. Expanding ψ' in ε series ($\varepsilon>0$ is small parameter), we have the following form:

$$\psi'=\varepsilon\psi_0+\varepsilon^2\psi_1+\varepsilon^3\psi_2+\cdots \tag{10}$$

According to multiscale analysis method, we introduce slowly varying spacious and time variables as follows: τ, T; λ_1, λ_2 and introducing transformations:

$$\tau = \varepsilon t, \quad T = \varepsilon^2 t; \quad \lambda_1 = \varepsilon\lambda, \quad \lambda_2 = \varepsilon^2\lambda. \tag{11}$$

So that time derivative, zonal derivative, meridional derivative in (4) are transformed into:

$$\begin{aligned} &\frac{\partial}{\partial t} \to \frac{\partial}{\partial t} + \varepsilon\frac{\partial}{\partial \tau} + \varepsilon^2\frac{\partial}{\partial T}\ , \\ &\frac{\partial}{\partial \lambda} \to \frac{\partial}{\partial \lambda} + \varepsilon\frac{\partial}{\partial \lambda_1} + \varepsilon^2\frac{\partial}{\partial \lambda_2}\ , \\ &\frac{\partial}{\partial \varphi} \to \frac{\partial}{\partial \varphi}\ . \end{aligned} \tag{12}$$

Since we only discuss the evolution of the disturbance in strong meridional shear of basic wind field, it is reasonable to assume that $\bar{\psi}$ is rapidly varying variable in meridional direction and in zonal direction, the stream function ψ' is also rapidly varying variable in φ direction.

Substituting (10), (12) into (9), we obtain:

$$\begin{aligned} &\varepsilon a^{-2}\left[\frac{\partial}{\partial t} + \varepsilon\frac{\partial}{\partial \tau} + \varepsilon^2\frac{\partial}{\partial T} + \frac{\bar{u}}{a\cos\varphi}\left(\frac{\partial}{\partial \lambda} + \varepsilon\frac{\partial}{\partial \lambda_1} + \varepsilon^2\frac{\partial}{\partial \lambda_2}\right)\right]\cdot\left[\frac{\partial^2}{\partial \varphi^2} - \mathrm{tg}\varphi\frac{\partial}{\partial \varphi}\right. \\ &\left. + \frac{1}{\cos^2\varphi}\left(\frac{\partial^2}{\partial \lambda^2} + 2\varepsilon\frac{\partial^2}{\partial\lambda\partial\lambda_1} + 2\varepsilon^2\frac{\partial^2}{\partial\lambda\partial\lambda_2} + \varepsilon^2\frac{\partial^2}{\partial\lambda_1^2}\right)\right](\psi_0 + \varepsilon\psi_1 + \varepsilon^2\psi_2) \\ &+ \varepsilon\left[\frac{1}{a^2\cos\varphi}\frac{\partial}{\partial \varphi}(2\Omega\sin\varphi + a^{-2}\Delta_1\bar{\psi})\right] \\ &\cdot\left(\frac{\partial}{\partial \lambda} + \varepsilon\frac{\partial}{\partial \lambda_1} + \varepsilon^2\frac{\partial}{\partial \lambda_2}\right)(\psi_0 + \varepsilon\psi_1 + \varepsilon^2\psi_2) + \frac{\varepsilon^2}{a^4\cos\varphi}\left[\left(\frac{\partial}{\partial \lambda}\right.\right. \\ &\left. + \varepsilon\frac{\partial}{\partial \lambda_1} + \varepsilon^2\frac{\partial}{\partial \lambda_2}\right)(\psi_0 + \varepsilon\psi_1 + \varepsilon^2\psi_2)\frac{\partial}{\partial \varphi}\left(\frac{\partial^2}{\partial \varphi^2} - \mathrm{tg}\varphi\frac{\partial}{\partial \varphi}\right. \\ &\left.\left. + \frac{1}{\cos^2\varphi}\left(\frac{\partial^2}{\partial \lambda^2} + 2\varepsilon\frac{\partial^2}{\partial\lambda\partial\lambda_1} + 2\varepsilon^2\frac{\partial^2}{\partial\lambda\partial\lambda_2} + \varepsilon^2\frac{\partial^2}{\partial\lambda_1^2}\right)\right)\right. \\ &\cdot(\psi_0 + \varepsilon\psi_1 + \varepsilon^2\psi_2) - \frac{\partial}{\partial \varphi}(\psi_0 + \varepsilon\psi_1 + \varepsilon^2\psi_2) \\ &\cdot\left(\frac{\partial}{\partial \lambda} + \varepsilon\frac{\partial}{\partial \lambda_1} + \varepsilon^2\frac{\partial}{\partial \lambda_2}\right)\left(\frac{\partial^2}{\partial \varphi^2} - \mathrm{tg}\varphi\frac{\partial}{\partial \varphi} + \frac{1}{\cos^2\varphi}\left(\frac{\partial^2}{\partial \lambda^2}\right.\right. \\ &\left.\left.\left. + 2\varepsilon\frac{\partial^2}{\partial\lambda\partial\lambda_1} + 2\varepsilon^2\frac{\partial^2}{\partial\lambda\partial\lambda_2} + \varepsilon^2\frac{\partial^2}{\partial\lambda_1^2}\right)\right)\left(\psi_0 + \varepsilon\psi_1 + \varepsilon^2\psi_2\right)\right] = 0\ , \end{aligned} \tag{13}$$

where

$$\Delta_1 = \frac{\partial^2}{\partial \varphi^2} - \mathrm{tg}\varphi\frac{\partial}{\partial \varphi}\ . \tag{14}$$

According to the principles of multiscale analysis method, we separate out equal powers of ε and get following approaches:

$O(\varepsilon^1)$:

$$a^{-2}\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{a\cos\varphi\partial\lambda}\right)\left(\frac{\partial^2}{\partial\varphi^2}-\mathrm{tg}\varphi\frac{\partial}{\partial\varphi}+\frac{1}{\cos^2\varphi}\frac{\partial^2}{\partial\lambda^2}\right)\psi_0$$
$$+\frac{1}{a^2\cos\varphi}\frac{\partial}{\partial\varphi}(2\Omega\sin\varphi+a^{-2}\Delta_1\bar{\psi})\frac{\partial\psi_0}{\partial\lambda}=0\ . \tag{15}$$

$O(\varepsilon^2)$:

$$a^{-2}\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{a\cos\varphi\partial\lambda}\right)\left(\frac{\partial^2}{\partial\varphi^2}-\mathrm{tg}\varphi\frac{\partial}{\partial\varphi}+\frac{1}{\cos^2\varphi}\frac{\partial^2}{\partial\lambda^2}\right)\psi_1$$
$$+a^{-2}\left(\frac{\partial}{\partial t}+\frac{\bar{u}}{a\cos\varphi}\frac{\partial}{\partial\lambda}\right)\left(\frac{2}{\cos^2\varphi}\frac{\partial^2\psi_0}{\partial\lambda\partial\lambda_1}\right)+a^{-2}\left(\frac{\partial}{\partial\tau}+\frac{\bar{u}}{a\cos\varphi}\frac{\partial}{\partial\lambda_1}\right)$$
$$\cdot\left(\frac{\partial^2}{\partial\varphi^2}-\mathrm{tg}\varphi\frac{\partial}{\partial\varphi}+\frac{1}{\cos^2\varphi}\frac{\partial^2}{\partial\lambda^2}\right)\psi_0$$
$$+\left[\frac{1}{a^2\cos\varphi}\frac{\partial}{\partial\varphi}(2\Omega\sin\varphi+a^{-2}\Delta_1\bar{\psi})\right]\left(\frac{\partial\psi_1}{\partial\lambda}+\frac{\partial\psi_0}{\partial\lambda_1}\right)+\frac{1}{a^4\cos\varphi}$$
$$\cdot\left[\frac{\partial\psi_0}{\partial\lambda}\frac{\partial}{\partial\varphi}\left(\frac{\partial^2}{\partial\varphi^2}-\mathrm{tg}\varphi\frac{\partial}{\partial\varphi}+\frac{1}{\cos^2\varphi}\frac{\partial^2}{\partial\lambda^2}\right)\psi_0\right.$$
$$\left.-\frac{\partial\psi_0}{\partial\varphi}\frac{\partial}{\partial\lambda}\left(\frac{\partial^2}{\partial\varphi^2}-\mathrm{tg}\varphi\frac{\partial}{\partial\varphi}+\frac{1}{\cos^2\varphi}\frac{\partial^2}{\partial\lambda^2}\right)\psi_0\right]=0\ . \tag{16}$$

And $O(\varepsilon^3)$:

$$a^{-2}\left[\left(\frac{\partial}{\partial t}+\frac{\bar{u}}{a\cos\varphi}\frac{\partial}{\partial\lambda}\right)\left(\frac{\partial^2}{\partial\varphi^2}-\mathrm{tg}\varphi\frac{\partial}{\partial\varphi}+\frac{1}{\cos^2\varphi}\frac{\partial^2}{\partial\lambda^2}\right)\psi_2\right.$$
$$+\left(\frac{\partial}{\partial t}+\frac{\bar{u}}{a\cos\varphi}\frac{\partial}{\partial\lambda}\right)\frac{2}{\cos^2\varphi}\frac{\partial^2\psi_1}{\partial\lambda\partial\lambda_1}+\left(\frac{\partial}{\partial t}+\frac{\bar{u}}{a\cos\varphi}\frac{\partial}{\partial\lambda}\right)$$
$$\cdot\frac{2}{\cos^2\varphi}\frac{\partial^2\psi_0}{\partial\lambda\partial\lambda_2}+\left(\frac{\partial}{\partial\tau}+\frac{\bar{u}}{a\cos\varphi}\frac{\partial}{\partial\lambda_1}\right)\left(\frac{\partial^2}{\partial\varphi^2}-\mathrm{tg}\varphi\frac{\partial}{\partial\varphi}\right.$$
$$\left.+\frac{1}{\cos^2\varphi}\frac{\partial^2}{\partial\lambda^2}\right)\psi_1+\left(\frac{\partial}{\partial\tau}+\frac{\bar{u}}{a\cos\varphi}\frac{\partial}{\partial\lambda_1}\right)\frac{2}{\cos^2\varphi}\frac{\partial^2\psi_0}{\partial\lambda\partial\lambda_1}$$
$$\left.+\left(\frac{\partial}{\partial T}+\frac{\bar{u}}{a\cos\varphi}\frac{\partial}{\partial\lambda_2}\right)\left(\frac{\partial^2}{\partial\varphi^2}-\mathrm{tg}\varphi\frac{\partial}{\partial\varphi}+\frac{1}{\cos^2\varphi}\frac{\partial^2}{\partial\lambda^2}\right)\psi_0\right]$$
$$+\left[\frac{1}{a^2\cos\varphi}\frac{\partial}{\partial\varphi}(2\Omega\sin\varphi+a^{-2}\Delta_1\bar{\psi})\right]\left(\frac{\partial\psi_0}{\partial\lambda_2}+\frac{\partial\psi_1}{\partial\lambda_1}+\frac{\partial\psi_2}{\partial\lambda}\right)$$
$$+\frac{1}{a^4\cos\varphi}\left\{\frac{\partial\psi_0}{\partial\lambda}\frac{\partial}{\partial\varphi}\left[\frac{\partial^2\psi_1}{\partial\varphi^2}-\mathrm{tg}\varphi\frac{\partial\psi_1}{\partial\varphi}+\frac{1}{\cos^2\varphi}\frac{\partial^2\psi_1}{\partial\lambda^2}\right.\right.$$
$$\left.+\frac{2}{\cos^2\varphi}\frac{\partial^2\psi_0}{\partial\lambda\partial\lambda_1}\right]+\left(\frac{\partial\psi_1}{\partial\lambda}+\frac{\partial\psi_0}{\partial\lambda_1}\right)\frac{\partial}{\partial\varphi}\left(\frac{\partial^2}{\partial\varphi^2}-\mathrm{tg}\varphi\frac{\partial}{\partial\varphi}\right.$$
$$\left.+\frac{1}{\cos^2\varphi}\frac{\partial^2}{\partial\lambda^2}\right)\psi_0-\left[\frac{\partial\psi_0}{\partial\varphi}\frac{\partial}{\partial\lambda}\left(\frac{\partial^2}{\partial\varphi^2}-\mathrm{tg}\varphi\frac{\partial}{\partial\varphi}+\frac{1}{\cos^2\varphi}\frac{\partial^2}{\partial\lambda^2}\right)\psi_1\right.$$

$$+\frac{\partial\psi_0}{\partial\varphi}\frac{2}{\cos^2\varphi}\frac{\partial^3\psi_0}{\partial\lambda_1\partial\lambda^2}+\frac{\partial\psi_0}{\partial\varphi}\frac{\partial}{\partial\lambda_1}\left(\frac{\partial^2}{\partial\varphi^2}-\mathrm{tg}\varphi\frac{\partial}{\partial\varphi}\right.$$

$$\left.+\frac{1}{\cos^2\varphi}\frac{\partial^2}{\partial\lambda^2}\right)\psi_0+\frac{\partial\psi_1}{\partial\varphi}\frac{\partial}{\partial\lambda}\left(\frac{\partial^2}{\partial\varphi^2}-\mathrm{tg}\varphi\frac{\partial}{\partial\varphi}+\frac{1}{\cos^2\varphi}\frac{\partial^2}{\partial\lambda^2}\right)\psi_0$$

$$\left.\left.+\frac{\partial\psi_0}{\partial\varphi}\frac{2}{\cos^2\varphi}\frac{\partial^3\psi_0}{\partial\lambda_1\partial\lambda^2}\right]\right\}=0\ . \tag{17}$$

Letting

$$B=\frac{1}{a^2\cos\varphi}\frac{\partial}{\partial\varphi}(2\Omega\sin\varphi+a^{-2}\Delta_1\bar{\psi}) \tag{18}$$

and introducing operators:

$$G=a^{-2}\left(\frac{\partial}{\partial t}+\frac{\bar{u}}{a\cos\varphi}\frac{\partial}{\partial\lambda}\right)\left(\frac{\partial^2}{\partial\varphi^2}-\mathrm{tg}\varphi\frac{\partial}{\partial\varphi}+\frac{1}{\cos^2\varphi}\frac{\partial^2}{\partial\lambda^2}\right)+B\frac{\partial}{\partial\lambda}\ . \tag{19}$$

We get:

$$G(\psi_0)=0, \tag{20}$$

$$G(\psi_1)=F_1, \tag{21}$$

$$G(\psi_2)=F_2, \tag{22}$$

where F_1 and F_2 are forceful terms which can be got from (16) and (17). Before we get the evolution equation of the perturbation, we must separate the effects of rapidly varying variables from slowly varying variables on the stream function. In accordance with general roles, it is suitable to assume that wave packet is only the function of slowly varying variables. To make the solutions have physical meaning, Maslowe (1977) developed a method of separating the effects of rapidly varying variables from slowly varying variables on the function in discussing weakly nonlinear stability of the viscous free shear layer. According to his method, we assume that:

$$\psi'=\varepsilon(\varphi_0e^{ik(\lambda-ct)}+*)+\varepsilon^2(\varphi_1e^{2ik(\lambda-ct)}+*)+\varepsilon^3\psi_2+\cdots\ . \tag{23}$$

Where $*$ is the complex of corresponding terms preceding it, c is similar to wave phase velocity, and k is also analogous to wave number, but c and k all have some differences from general meaning.

Further letting:

$$\psi_0=\varphi_0e^{ik(\lambda-ct)}+*=\Phi(\varphi)A(\tau,\ T;\ \lambda_1,\ \lambda_2)e^{ik(\lambda-ct)}, \tag{24}$$

$$\psi\ =\varphi_1e^{2ik(\lambda-ct)}+*=\Phi(\varphi)A^2(\tau,\ T;\ \lambda_1,\ \lambda_2)e^{2ik(\lambda-ct)}+*, \tag{25}$$

the forms of (24), (25) are also based on Maslowe's work. He also got an amplitude equation which is similar to NSE. In (24) and (25), A is the packet of perturbation, and we assume that A is only the function of slowly varying variables.

Substituting (23), (24) into (20)—(22), in accordance with the principles of multiscale analysis method, i.e. if the solution is convergent, the long time terms should be cancelled. As a matter of fact, in the forms of ψ_1, we have already introduced this idea, and so we can obtain a series of equations as follows:

$$\cos^2\varphi\frac{d^2\Phi_0}{d\varphi^2}-\sin\varphi\cos\varphi\frac{d\Phi_0}{d\varphi}+\left(\frac{a^3B\cos^3\varphi}{\bar{u}-ca\cos\varphi}-k^2\right)\Phi_0=0\ , \tag{26}$$

$$2a(\bar{u}-ca\cos\varphi)\left(\frac{d^2\Phi_1}{d\varphi^2}-\mathrm{tg}\varphi\frac{d\Phi_1}{d\varphi^2}-\frac{4k^2}{\cos^2\varphi}\Phi_1\right)=\Phi_0\frac{d}{d\varphi}\left(\frac{aB\cos\varphi}{\bar{u}-ca\cos\varphi}\right). \tag{27}$$

And:

$$\frac{2k^2\Phi_0}{a^2\cos^2\varphi}\left(\frac{\bar{u}}{a\cos\varphi}-C\right)\frac{\partial A}{\partial\lambda_2}-ik\frac{2\Phi_0}{a^2\cos^2\varphi}\left(\frac{\partial}{\partial\tau}+\frac{\bar{u}}{a\cos\varphi}\frac{\partial}{\partial\lambda_1}\right)\frac{\partial A}{\partial\lambda_1}$$
$$-\frac{1}{a^2}\left(\frac{d^2\Phi_0}{d\varphi^2}-\frac{k^2}{\cos^2\varphi}\Phi_0-\mathrm{tg}\varphi\frac{d\Phi_0}{d\varphi}\right)\left(\frac{\partial}{\partial T}+\frac{\bar{u}}{a\cos\varphi}\frac{\partial}{\partial\lambda_2}\right)A$$
$$+B\Phi_0\frac{\partial A}{\partial\lambda_2}+ik\frac{1}{a^4\cos\varphi}\left[\Phi_0^*\frac{d}{d\varphi}\left(\frac{d^2\Phi_1}{d\varphi^2}-\mathrm{tg}\varphi\frac{d\Phi_1}{d\varphi}-\frac{4k^2}{\cos^2\varphi}\Phi_1\right)\right.$$
$$+2\Phi_1\frac{d}{d\varphi}\left(\frac{d^2\Phi_0^*}{d\varphi^2}-\mathrm{tg}\varphi\frac{d\Phi_0^*}{d\varphi}-\frac{k^2\Phi_0^*}{\cos^2\varphi}\right)+2\frac{d\Phi_0^*}{d\varphi}\left(\frac{d^2\Phi_1}{d\varphi^2}\right.$$
$$\left.\left.-\mathrm{tg}\varphi\frac{d\Phi_1}{d\varphi}-\frac{4k^2}{\cos^2\varphi}\Phi_1\right)-\frac{d\Phi_1}{d\varphi^2}\left(\frac{d^2\Phi_0^*}{d\varphi^2}-\mathrm{tg}\varphi\frac{d\Phi_0^*}{d\varphi}-\frac{k^2\Phi_0^*}{\cos^2\varphi}\right)\right]=0\ . \tag{28}$$

Although we have given emphasis that the disturbance must be small amplitude (i.e. weakly nonlinear problem), it only means that the amplitude A of the disturbance is small compared to the amplitude of long waves, not A itself. That is, the disturbance is not very narrow (for blocking high). Further more in high latitude zone, Rossby parameter β varies quickly with the increase of φ, in other words, we must consider spherical effect of the disturbance, so that we would not discuss the aspects on β–plane. It is clear that the coefficients of equation (28) can be counted in condition that the boundary conditions of Φ_0, Φ_1 are given, and then matching with (26), (27) forms an eigenvalue problem. After getting the coefficients of Φ_0, Φ_1, we can solve the problem and obtain Φ_0 and Φ_1, at last the coefficients of equation (28) are set. Generally speaking, we can assume fixing boundary at φ_1 and φ_2, i.e. $\Phi_0(\varphi_1)=\Phi_0(\varphi_2)=0$. There is no restriction on φ_1 and φ_2 provided that the disturbance does not violate the assumption of small amplitude, so in this case, $\varphi_2-\varphi_1$ can come relatively large to some extent. Based on these assumptions, equation (28) has only one variable A—the modulating wave amplitude. To make the equation's physical meaning more clear, multiplied equation (28) by $a^2\cos^2\varphi\frac{d\Phi_0}{d\varphi}$, then integrate the equation upon $[\varphi_1, \varphi_2]$ and use the boundary conditions of Φ_0 and Φ_1 , noting that A is independent of φ, we have:

$$\frac{\partial A}{\partial T}+Cg\frac{\partial A}{\partial\lambda_2}+ikl\frac{\partial^2 A}{\partial\lambda_1^2}+ikm|A|^2A=0\ . \tag{29}$$

Equation (29) is recognized as the famous equation of quantum mechanics—nonlinear Schrödinger equation.

Where

$$Cg=\frac{\int_{\varphi_1}^{\varphi_2}2k^2\Phi_0\left(\frac{\bar{u}}{a\cos\varphi}-C\right)\frac{d\Phi_0}{d\varphi}d\varphi-\int_{\varphi_1}^{\varphi_2}\frac{\bar{u}\cos^4\varphi}{\bar{u}-Ca\cos\varphi}a^2B\Phi_0\frac{d\Phi_0}{d\varphi}d\varphi}{\int_{\varphi_1}^{\varphi_2}\frac{a^3B\cos^3\varphi}{\bar{u}-Ca\cos\varphi}\Phi_0\frac{d\Phi_0}{d\varphi}d\varphi}$$

$$-\frac{\int_{\varphi_1}^{\varphi_2}B\Phi_0a^2\cos^2\varphi\frac{d\Phi_0}{d\varphi}d\varphi}{\int_{\varphi_1}^{\varphi_2}\frac{a^3B\cos^3\varphi}{\bar{u}-Ca\cos\varphi}\Phi_0\frac{d\Phi_0}{d\varphi}d\varphi}, \tag{30}$$

$$l=-\frac{\int_{\varphi_1}^{\varphi_2}\frac{2\bar{u}}{a\cos\varphi}\Phi_0\frac{d\Phi_0}{d\varphi}d\varphi}{\int_{\varphi_1}^{\varphi_2}\frac{a^3B}{\bar{u}-Ca\cos\varphi}\Phi_0\frac{d\Phi_0}{d\varphi}d\varphi}, \tag{31}$$

$$m=\frac{\int_{\varphi_1}^{\varphi_2}2a^2\cos^2\varphi\Phi_1\Phi_0^*\frac{Ba^3\cos\varphi}{\bar{u}-Ca\cos\varphi}d\varphi}{\int_{\varphi_1}^{\varphi_2}\frac{Ba^3\cos^3\varphi}{\bar{u}-Ca\cos\varphi}\Phi_0\frac{d\Phi_0}{d\varphi}d\varphi}$$

$$+\frac{\int_{\varphi_1}^{\varphi_2}2a^2\cos^2\varphi\frac{d\Phi_0^*}{d\varphi}\left(\frac{d^2\Phi_1}{d\varphi^2}-\mathrm{tg}\varphi\frac{d\Phi_1}{d\varphi}-\frac{4k^2}{\cos^2\varphi}\Phi_1\right)d\varphi}{\int_{\varphi_1}^{\varphi_2}\frac{Ba^3\cos^3\varphi}{\bar{u}-Ca\cos\varphi}\Phi_0\frac{d\Phi_0}{d\varphi}d\varphi}$$

$$+\frac{\int_{\varphi_1}^{\varphi_2}a^2\cos^2\varphi\frac{\Phi_0^*Ba^3\cos\varphi}{\bar{u}-Ca\cos\varphi}\frac{d\Phi_1}{d\varphi}d\varphi}{\int_{\varphi_1}^{\varphi_2}\frac{Ba^3\cos^3\varphi}{\bar{u}-Ca\cos\varphi}\Phi_0\frac{d\Phi_0}{d\varphi}d\varphi}. \tag{32}$$

From above discussion, we know that Cg, l, m all have fixed values. Supposing that: when $x=t=0$, $A=A_0$ (A_0 is an initial amplitude of the disturbance). It is easy to get the solution of equation (29) as follows:

$$A=A_0\mathrm{Sech}[\varepsilon A(2m/l)^{\frac{1}{2}}(\lambda-Cg\cdot t)]e^{-i\nu_0 t}, \tag{33}$$

where

$$\nu_0=2\varepsilon^2\mathrm{km}A_0^2. \tag{34}$$

When large scale disturbance is at the stage of small amplitude, considering interaction between waves, the disturbance stream function is about (first order approximation):

$$\psi' \simeq \varepsilon\psi_0 = \varepsilon A_0 \mathrm{Sech}\left[\varepsilon A_0 (2m/l)^{\frac{1}{2}}(\lambda - Cg \cdot t)\right] e^{ik[\lambda - (C + 2\varepsilon^2 m A_0^2)t]} + * \quad . \tag{35}$$

Let C be a given value, for example, $C=0$, then it can be counted. Due to the effects of the interaction between waves, the modulated wave velocity changes into:

$$C_A = C + 2\varepsilon^2 m A_0^2 \ . \tag{36}$$

When coefficient $m>0$, the modulated wave phase velocity C_A is greater than non–modulated wave velocity C; otherwise when $m<0$, C_A is smaller than C. The change caused by the nonlinear interaction between waves is $2\varepsilon^2 m A_0^2$, which is proportional to square of strength of interaction effect parameter ε (noting that: substituting (10) into (9), we can see that ε is a strength order of interaction effect) and square of initial amplitude. We find that the interaction has very great effect on the modulated wave phase velocities. From above discussion, it is clear that Cg is the velocity of modulating wave (the wave packet). In addition, the relationship between coefficients m and l decides the characters of NSE: when $m/l<0$, equation (29) represents envelope–hole soliton; otherwise when $m/l>0$, it represents propagating solitary wave. The detailed description can be seen from Mei (1984).

From above discussion, we believe that: although the characteristics of linear and nonlinear Rossby wave packet seem to be different in form, it is unified in essence.

IV. CONCLUSION REMARKS

In Section II, under the assumption of existing predominant wave at initial time, we have investigated large scale linear disturbance on β–plane, and obtained LSE. As for large scale linear disturbance on sphere, we introduce absolute vorticity conservation equation from Zeng (1979) in the case of K = 0 (nondivergent),

$$\frac{\partial}{\partial t}(\Delta\psi) + 2\Omega\sin\theta \cdot \frac{\partial\psi}{a\sin\theta\partial\lambda} = 0, \tag{1$'$}$$

where:

$$\Delta = \frac{1}{\sin\theta}\left(\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial}{\partial\theta}) + \frac{1}{\sin\theta}\frac{\partial^2}{\partial\lambda^2}\right). \tag{7$'$}$$

Since (1)$'$ is similar to (1) exactly, we can also find similar evolution equation of wave packet as (2), here is not repeated. Generally speaking, for large nonlinear perturbation on sphere, considering the nonlinear interaction between waves, modulating wave is soliton, and the packet is governed by NSE. Two important factors of the solitary wave formation can find expression in the equation, one is spherical effect on the disturbance and the other is meridional shear of basic wind field. Due to the existence of the first factor, even basic wind field is a constant (i.e. $\frac{d\bar{u}}{dy} = 0$), it cannot lead to any coefficient coming to zero or infinite for ever. That is: spherical effect of the disturbance is good for the formation of solitary wave. In this point the result is different from the local Cartesian coordinates' one.

Yeh (1949) pointed out the importance of spherical effect to the formation and maintenance of blocking high. He showed that the energy dispersion is proportional to Rossby parameter β. Because of spherical effect, the higher of the latitude, the less of the disturbance energy dispersion. So that in high latitude zone, blocking high is easy to form and maintain. This result coincides with our conclusion. As for the second factor, the meridional shear of basic wind field is good for blocking high. This point can be proved from routine weather maps. Its effect on blocking high is the same as the results of local Cartesian coordinates'.

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