A Regional Spectral Nested Multilevel Primitive Equation Model

Liao Dongxian (廖洞贤)
State Meteorological Administration, Beijing 100081
Received November 18, 1988

ABSTRACT

By means of vertical normal modes a regional nested multilevel primitive equation model can be reduced to several sets of shallow water equations characterized by various equivalent depths. Therefore, time integration of the model in spectral form can be performed in the manner similar to those used in the spectral nested shallow water equation model case.

I. INTRODUCTION

In a previous paper, the author proposed a scheme for integrating a regional spectral nested shallow water equation model, in which a method of spectral expansion in a limited area without any additional conditions, and a set of spectral equations were given.

This paper is a continuation of the previous one, with a view of extending its scheme to a multilevel primitive equation model case. For this purpose the multilevel primitive equation model has been reduced to several sets of shallow water type equations by means of vertical normal modes, in order to use the techniques in the previous paper to carry out the corresponding time integration. In the previous paper, however, the mean depth of the fluid surface, equivalent to the height of the homogeneous atmosphere, is a constant; the time step selected in the explicit case is quite small under the constraint of the linear computational stability criterion. In this paper, on the contrary, the possibility of selecting a time step larger than usual one greatly increases, owing to the equivalent characteristic depths of the shallow water type equations, except few cases, being far less than the height of the homogeneous atmosphere. For this reason, the scheme given here might save much computation time and keep reasonable accuracy for a multilevel primitive equation model.

II. VERTICAL COORDINATE, VERTICAL DISCRETIZATION AND VERTICAL BOUNDARY CONDITION

1. *Vertical Coordinate*

In the vertical the σ–coordinate defined as

$$\sigma=\frac{p}{p_s} \tag{1}$$

is adopted, where p is the pressure; p_s the surface pressure.

2. *Vertical Discretization of the Model Atmosphere*

Usually the vertical discretization of the model atmosphere can be determined from the object of prediction and objective conditions. Now we discuss a general case. It is assumed that the atmosphere is divided into K layers (see Fig. 1). At each integer level the wind components u and v, the temperature T and the geopotential height φ are predicted, while the vertical velocity $\dot{\sigma}$ is computed at each half level.

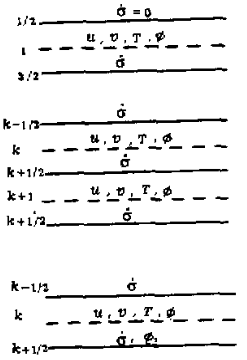

Fig. 1. Discretization of the model atmosphere.

However, the thickness of each layer $\Delta\sigma_k(=\sigma_{k+1/2}-\sigma_{k-1/2})$ may vary with k, but $\sum_{k=1}^{k}\Delta\sigma_k=1$.

3. *Vertical Boundary Condition*

At $\sigma=0$ and 1, the homogeneous boundary condition

$$\dot{\sigma}=0 \tag{2}$$

is adopted.

III. DYNAMIC AND THERMODYNAMIC EQUATIONS

At σ_k–level the dynamic and thermodynamic equations in the adiabatic and inviscid case may be written as

$$\frac{\partial U_k}{\partial t}=-\frac{\partial G_k}{\partial x}+fV_k+(N_u)_k \tag{3}$$

$$\frac{\partial V_k}{\partial t}=-\frac{\partial G_k}{\partial y}-fU_k+(N_v)_k \tag{4}$$

$$\frac{\partial T}{\partial t}=-\frac{R\overline{T}_k}{c_p}(\sigma\frac{\partial}{\partial\sigma}\frac{\dot{\sigma}_L}{\sigma}+D)_k-\dot{\sigma}_L\frac{\partial\overline{T}}{\partial\sigma}+(N_T)_k \tag{5}$$

$$\frac{\partial P}{\partial t}=Ad_k(p)-D_k-(\delta_\sigma\dot{\sigma})_k \tag{6}$$

$$(\frac{\partial\varphi}{\partial\ln\sigma})_k=-RT_k \tag{7}$$

Utilizing the vertical boundary condition (2) and summing up the continuity equation (6) from $k=1$ to K in the vertical, we have

$$\frac{\partial P}{\partial t}=\sum_{k=1}^{K}Ad_k(p)\Delta\sigma_k-\sum_{k=1}^{K}D_k\Delta\sigma_k \tag{8}$$

where $T=\overline{T}+T'$, $\overline{T}=\overline{T}(\sigma)$, $G=\varphi+R\overline{T}P$, $P=\ln p_s$, $C_{kj}=\Delta\sigma_j/\sigma_k$, $U=u/l$, $V=v/l$, l is the magnification factor of conformal map projection, $Ad(F)=-S(U\frac{\partial}{\partial x}+V\frac{\partial}{\partial y})F$, $S=l^2$, F a certain meteorological element, $D=S(\partial U/\partial x+\partial V/\partial y)$, $\delta_\sigma\dot{\sigma}=(\dot{\sigma}_{k+1/2}-\dot{\sigma}_{k-1/2})/\Delta\sigma_k$, $K=(U^2+V^2)/2$,

$$N_u=Ad(U)-K\frac{\partial s}{\partial x}-RT'\frac{\partial P}{\partial x}-\dot{\sigma}\frac{\partial U}{\partial\sigma}\ ,\tag{9}$$

$$N_v=Ad(V)-K\frac{\partial s}{\partial y}-RT'\frac{\partial P}{\partial y}-\dot{\sigma}\frac{\partial V}{\partial\sigma}\ ,\tag{10}$$

$$N_T=Ad(T)-\dot{\sigma}\frac{\partial T'}{\partial\sigma}-\dot{\sigma}_N\frac{\partial\overline{T}}{\partial\sigma}+\frac{R\overline{T}}{c_p}(-\sigma\frac{\partial}{\partial\sigma}\frac{\dot{\sigma}_N}{\sigma})+\frac{RT'}{c_p}(-\sigma\frac{\partial}{\partial\sigma}\frac{\dot{\sigma}_L}{\sigma}-D)\tag{11}$$

and

$$\dot{\sigma}_{k+1/2}=(\dot{\sigma}_L)_{k+1/2}+(\dot{\sigma}_N)_{k-1/2}$$
$$(\dot{\sigma}_L)_{k+1/2}=-\sum_{j=1}^{k}D_j\Delta\sigma_j+\sigma_{k-1/2}\sum_{j=1}^{K}D_j\Delta\sigma_j,$$
$$(\dot{\sigma}_N)_{k+1/2}=\sum_{j=1}^{k}Ad_j(P)\Delta\sigma_j-\sigma_{k+1/2}\sum_{j=1}^{K}Ad_j(P)\Delta\sigma_j\tag{12}$$

IV. SETS OF SHALLOW WATER TYPE EQUATIONS

1. *The Thermodynamic Equation and the Hydrostatic Equation in Matrix Form*

If the vertical advection term is approximately replaced by

$$(\dot{\sigma}\frac{\partial E}{\partial\sigma})_k\approx\frac{1}{2\Delta\sigma_k}\left[\dot{\sigma}_{k+1/2}(E_{k+1}-E_k)+\dot{\sigma}_{k-1/2}(E_k-E_{k-1})\right]\tag{13}$$

then the thermodynamic equation (5) can be expressed in a matrix form

$$\frac{\partial T}{\partial t}=-FD+N_T\tag{14}$$

by use of Eqs. (6) and (7). Here E represents U_k, V_k, T_k and so on, $T=(T_1, T_2,\cdots T_k)^T$, $D=(D_1, D_2,\cdots D_k)^T$, $N_T=((N_T)_1, (N_T)_2,\cdots(N_T)_k)^T$, the superscript T represents a transpose, F a matrix whose elements are functions of $\Delta\sigma_k$, C_{kj} and $\overline{T}$.

For the hydrostatic equation (12), the difference scheme developed by ECMWF is adopted (1980), namely

$$\varphi_k=\varphi_s+R\sum_{j=k}^{K}B_{kj}T_j\tag{15}$$

where B_{kj} is only related to $\ln[(\sigma_{j+1/2})/(\sigma_{j-1/2})]$, $\ln[(\sigma_{k+1/2})/(\sigma_{k-1/2})]$, σ_1 and zero. In this way, the above equation in the matrix form is

$$\varphi=\varphi_s+BT\tag{16}$$

where $\varphi=(\varphi_1, \varphi_2,\cdots\varphi_k)^T$, $\varphi_s=(\varphi_s, \varphi_s,\cdots\varphi_s)^T$, B is a matrix with $B_{kj}'s$ as its elements.

2. *Shallow Water Type Equations*

From Eqs. (8), (14) and (16), we have

$$\frac{\partial G}{\partial t} = -CD + N_G \tag{17}$$

where $G = \varphi + R\overline{T}P$, $C = BF + R\overline{T}\Delta^T$, $\Delta^T = (\Delta\sigma_1, \Delta\sigma_2, \cdots \Delta\sigma_K)$, $N_G = BN_T + R\overline{T}\Delta^T Ad(P)$, $Ad(P) = (Ad_1(P), Ad_2(P), \cdots Ad_K(p))^T$.

Let the latent values of the matrix C be λ_1, λ_2, $\cdots$ λ_K, ordered from the largest to the smallest, and their corresponding characteristic vectors be W_1, W_2, $\cdots$ W_K,

$$\Lambda = \mathrm{diag}(\lambda_1, \lambda_2, \cdots \lambda_K)$$

and

$$W = (W_1, W_2, \cdots W_K)$$

then

$$W^{-1}CW = \Lambda$$

Making the transformation

$$\hat{E} = W^{-1}E \tag{18}$$

Eqs. (3), (4) and (17) become

$$\frac{\partial \hat{U}}{\partial t} = -\frac{\partial \hat{G}}{\partial x} + f\hat{V} + \hat{N}_u \tag{19}$$

$$\frac{\partial \hat{V}}{\partial t} = -\frac{\partial \hat{G}}{\partial y} - f\hat{U} + \hat{N}_v \tag{20}$$

$$\frac{\partial \hat{G}}{\partial t} = -\Lambda\hat{D} + \hat{N}_G \tag{21}$$

respectively, where E represents U, V, G, D, N_U or N_G, $U = (U_1, U_2, \cdots U_K)^T$, $V = (V_1, V_2, \cdots V_K)^T$, $N_u = ((N_u)_1, (N_u)_2, \cdots (N_u)_K)^T$, $N_v = ((N_v)_1, (N_v)_2, \cdots (N_v)_K)^T$.

Eqs. (19)–(20) are the shallow water type equations characterized by various latent values (or equivalent characteristic depth) λ_j ($j = 1, 2, \cdots K$).

As for P and T, they can be rewritten as

$$\frac{\partial P}{\partial t} = -\Delta^T W\hat{D} + \Delta^T Ad \tag{22}$$

and

$$\frac{\partial T}{\partial t} = -FW\hat{D} + N_T \tag{23}$$

V. SPECTRAL EXPANSION AND SPECTRAL EQUATIONS

1. *Spectral Expansion*

Based upon the results of the previous paper, any function $F(x, y)$ defined in the domain $\hat{R}(0 \leqslant x \leqslant x_M, 0 \leqslant y \leqslant y_N)$ can be expanded as

$$F(x,y) = \hat{F}(x,y) + \sum_m F_m^*(y)e^{i\hat{m}x} + \sum_m\sum_n F_{mn}e^{i(\hat{m}x + \hat{n}y)}, \tag{24}$$

where $\hat{n} = 2m\pi / L_x$, $\hat{n} = 2n\pi / L_y$, $L_x = x_M$, $L_y = y_N$,

$\hat{F}(x, y) = x[F(x_M, y) - F(0, y)] / L_x$,

$F_m^*(y) = y[F_m(y_N) - F_m(0)] / L_y$,

$$F_m(y)=\frac{1}{L_x}\int_0^{L_x}[F(x,\ y)-\hat{F}(x,\ y)]e^{-i\hat{m}x}dx,$$

$$F_{mn}=\frac{1}{L_y}\int_0^{L_y}[F_m(y)-F_m^*\quad(y)]e^{-i\hat{n}x}dy,$$

(24) is the basic expression in this paper. In the following all the meteorological elements concerned will be expanded by using it in $\hat{R}$.

2. *Spectral Equations*

(1) *The explicit case*

With the help of replacing the time derivatives by the central differences, Eqs. (19)–(21) in a component form are given by

$$\hat{U}_j^{\tau+1}=\hat{U}_j^{\tau-1}-2(\frac{\partial\hat{G}}{\partial x})_j^\tau\Delta t+2f\hat{V}_j\Delta t+2(\hat{N}_u)_j^\tau\Delta t, \tag{25}$$

$$\hat{V}_j^{\tau+1}=\hat{V}_j^{\tau-1}-2(\frac{\partial\hat{G}}{\partial y})_j^\tau\Delta t-2f\hat{U}_j\Delta t+2(\hat{N}_v)_j^\tau\Delta t, \tag{26}$$

$$\hat{G}_j^{\tau+1}=\hat{G}_j^{\tau-1}-2\lambda_j\hat{D}_j\Delta t+2(\hat{N}_G)_j^\tau\Delta t. \tag{27}$$

In the below the transform method will be used to obtain the numerical solutions of the dynamic and thermodynamic equations. For this purpose, at first replacing F in (24) by U_k, V_k, G_k, T_k, P and s respectively, and expanding them in $\hat{R}$, we can find the values of D_k, $(N_u)_k$, $(N_v)_k$, and $(N_T)_k$ at the gridpoints. Then by means of the transformation (18), the values of $\hat{U}_j$, $\hat{V}_j$, $\hat{G}_j$, $\hat{D}_j$, $(\hat{N}_u)_j$, $(\hat{N}_v)_j$ and $(\hat{N}_G)_j$ can be found. Thus expanding them and substituting those expansions in Eqs. (25)–(27) and neglecting their superscripts and subscripts, we can obtain the spectral equations.

$$U_{mn}^{\tau+1}=U_{mn}^{\tau-1}+A_{mn}^\tau+\tilde{U}_{mn}+\tilde{U}_{mn}^* \tag{28}$$

$$V_{mn}^{\tau+1}=V_{mn}^{\tau-1}+B_{mn}^\tau+\tilde{V}_{mn}+\tilde{V}_{mn}^* \tag{29}$$

$$G_{mn}^{\tau+1}=G_{mn}^{\tau-1}+C_{mn}^\tau+\tilde{G}_{mn}+\tilde{G}_{mn}^* \tag{30}$$

where

$$A_{mn}^\tau=2\Delta t[-i\hat{m}G_{mn}^\tau+(N_u)_{mn}^\tau],$$

$$\tilde{U}_{mn}=\frac{1}{L_xL_y}\int_0^{L_x}\int_0^{L_y}\Big\{\hat{U}^{\tau-1}-\hat{U}^{\tau+1}+2\Delta t[-\frac{1}{L_x}(G(x_{M,\ y})$$

$$-G(0,\ y)^\tau)+(\hat{N}_u)^\tau]\Big\}e^{-i(\hat{m}x+\hat{n}y)}dxdy,$$

$$\tilde{U}_{mn}^*=\frac{1}{L_y}\int_0^{L_y}\Big\{U_m^{*,\tau-1}-U_m^{*,\tau+1}+2\Delta t[-i\hat{m}G_m^{*,\tau}+(N_u)_m^{*,\tau}]\Big\}$$

$$\cdot e^{-i\hat{n}y}dy$$

$$B_{mn}^\tau=2\Delta t[-i\hat{n}G_{mn}^\tau+(N_v)_{mn}^\tau],$$

$$\tilde{V}_{mn}=\frac{1}{L_xL_y}\int_0^{L_x}\int_0^{L_y}\Big\{\hat{V}^{\tau-1}-\hat{V}^{\tau+1}+2\Delta t\Big[-\frac{\partial\hat{G}^\tau}{\partial y}+(\hat{N}_v)^\tau\Big]\Big\}$$

$$\cdot e^{-i(\hat{m}x+\hat{n}y)}dxdy$$

$$\tilde{V}^*_{mn}=\frac{1}{L_y}\int_0^{L_y}\left\{V_m^{*,\tau-1}-V_m^{*,\tau+1}+2\Delta t\left[-\frac{\partial G_m^{*,\tau}}{\partial y}+(N_v)_m^{*,\tau}\right]\right\}$$

$$\cdot e^{-i\hat{n}y}dy$$

$$C^\tau_{mn}=2\Delta t[-\lambda D^\tau_{mn}+(N_G)^\tau_{mn}],$$

$$\tilde{G}_{mn}=\frac{1}{L_xL_y}\int_0^{L_x}\int_0^{L_y}\left\{\hat{G}^{\tau-1}-\hat{G}^{\tau+1}+2\Delta t\left[-\lambda\hat{D}^\tau+(\hat{N}_G)^\tau\right]\right\}$$

$$\cdot e^{-i(\hat{m}x+\hat{n}y)}dxdy$$

$$\tilde{G}^*_{mn}=\frac{1}{L_y}\int_0^{L_y}\left\{G_m^{*,\tau-1}-G_m^{*,\tau+1}+2\Delta t\left[-\lambda D_m^{*,\tau}+(N_G)_m^{*,\tau}\right]\right\}$$

$$\cdot e^{-i\hat{n}y}dy.$$

It can readily be seen that in the integral expressions of $\tilde{U}_{mn}$'s the integrands in the braces are all related to boundary values. Thus they may be taken as known and can be found analytically. It can also be seen that the spectral equations (28)–(30) are similar to those of the shallow water equations in the explicit case, if we replace φ_0 by λ and φ by G. Therefore, from the above equations we can obtain $U^{\tau+1}_{mn}$, $V^{\tau+1}_{mn}$ and $G^{\tau+1}_{mn}$ from the values of such quantities at the instants τ and $\tau-1$, and then obtain U, V and G by using the boundary values provided by the coarse grid prediction.

(2) *The semi–implicit case*

In the semi–implicit case the component equations (19)–(21) in differential–difference form are as follows.

$$\hat{U}_j^{\tau+1}=\hat{U}_j^{\tau-1}-\left[\left(\frac{\partial\hat{G}_j}{\partial x}\right)^{\tau+1}+\left(\frac{\partial\hat{G}_j}{\partial x}\right)^{\tau-1}\right]\Delta t+2(\hat{N}_u)_j^\tau\Delta t \tag{31}$$

$$\hat{V}_j^{\tau+1}=\hat{V}_j^{\tau-1}-\left[\left(\frac{\partial\hat{G}_j}{\partial y}\right)^{\tau+1}+\left(\frac{\partial\hat{G}_j}{\partial y}\right)^{\tau-1}\right]\Delta t+2(\hat{N}_v)_j^\tau\Delta t \tag{32}$$

$$\hat{G}_j^{\tau+1}=\hat{G}_j^{\tau-1}-\lambda_j(\hat{D}_j^{\tau+1}+\hat{D}_j^{\tau-1})\Delta t+2(\hat{N}_G)_j\Delta t \tag{33}$$

Neglecting the carets and the subscripts, and eleminating $D^{\tau+1}$ from the above equations immediately gives

$$(\Delta-a^2)G^{\tau+1}=H \tag{34}$$

where

$$H=\frac{2}{\Delta t}\delta^{\tau-1}-2a^2\Delta tN_G^\tau-(\Delta+a^2)G^{\tau-1}+2\left(\frac{\partial N_u^\tau}{\partial x}+\frac{\partial N_v^\tau}{\partial y}\right)$$

$$\delta=\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}$$

$$a^2=\frac{1}{S\lambda\Delta t}$$

$$\Delta=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y}\ .$$

With manners similar to those used in the above, the spectral equations below are given by

$$G_{mn}^{\tau+1} = -\frac{1}{\hat{m}^2 + \hat{n}^2 + a^2}(H_{mn} + \tilde{H}_{mn} + \tilde{H}_{mn}^*), \tag{35}$$

where

$$\tilde{H}_{mn} = \frac{1}{L_x L_y}\int_0^{L_x}\int_0^{L_y}[\hat{H} - (\Delta - a^2)\hat{G}^{\tau+1}]e^{-i(\hat{m}x+\hat{n}y)}dxdy,$$

$$\tilde{H}_{mn}^* = \frac{1}{L_y}\int_0^{L_y}\left\{H_m^* + (\hat{m}^2 + a^2)G_m^{*,\tau+1}\right\}e^{-i\hat{n}y}dy.$$

After obtaining $G_{mn}^{\tau+1}$ from (35), we can get $\overline{G}_{mn}^{\tau}$ by use of $G_{mn}^{\tau-1}$ and boundary values. Replacing G_{mn}^{τ} in the spectral Eqs. (28) and (29) by $\overline{G}_{mn}^{\tau}$, we obtain $U_{mn}^{\tau+1}$ and $V_{mn}^{\tau+1}$, where $\overline{G}_{mn}^{\tau} = (G_{mn}^{\tau+1} + G_{mn}^{\tau-1})/2$.

As for T and P, if they are expanded as F in the expression (24), then from Eqs. (22) and (23) the corresponding spectral equations are given by

$$P_{mn}^{\tau+1} = P_{mn}^{\tau-1} + \Delta^T[2(Ad)_{mn}^{\tau} - W(\hat{D}_{mn}^{\tau+1} + \hat{D}_{mn}^{\tau-1})\Delta t + \tilde{P}_{mn} + \tilde{P}_{mn}^*, \tag{36}$$

and

$$T_{mn}^{\tau+1} = T_{mn}^{\tau-1} - FW(\hat{D}_{mn}^{\tau+1} + \hat{D}_{mn}^{\tau-1})\Delta t + 2\Delta t(N_T)_{mn}^{\tau}\Delta t + \tilde{T}_{mn} + \tilde{T}_{mn}^* \tag{37}$$

where

$$Ad_{mn} = \Big((Ad_{mn})_1,\ (Ad_{mn})_2,\ \cdots(Ad)_n)_K\Big)^T,$$

$$\hat{D}_{mn} = \Big((\hat{D}_{mn})_1,\ (\hat{D}_{mn})_2,\ \cdots(\hat{D}_{mn})_K\Big)^T,$$

$$\tilde{P}_{mn} = \frac{1}{L_x L_y}\int_0^{L_x}\int_0^{L_y}\Big\{\hat{P}^{\tau-1} - \hat{P}^{\tau+1} + \Delta^T[2\hat{A}d^{\tau} - W(\hat{D}^{\tau+1} + \hat{D}^{\tau-1})]\Delta t\Big\}e^{-i(\hat{m}x+\hat{n}y)}dxdy,$$

$$\tilde{P}_{mn}^* = \frac{1}{L_y}\int_0^{L_y}\Big\{P_m^{*,\tau-1} - P_m^{*,\tau+1} + \Delta^T[2Ad_m^* - W(\hat{D}_m^{*,\tau+1} + \hat{D}_m^{*,\tau-1})]\Delta t\Big\}e^{-i\hat{n}y}dy,$$

$$\tilde{T}_{mn} = \frac{1}{L_x L_y}\int_0^{L_x}\int_0^{L_y}\Big\{\hat{T}^{\tau-1} - \hat{T}^{\tau+1} - FW(\hat{D}^{\tau+1} + \hat{D}^{\tau-1})\Delta t + 2(\hat{N}_T)^{\tau}\Delta t\Big\}e^{-i(\hat{m}x+\hat{n}y)}dxdy,$$

$$\tilde{T}_{mn}^* = \frac{1}{L_y}\int_0^{L_y}\Big\{T_m^{*,\tau-1} - T_m^{*,\tau+1} - FW(\hat{D}_m^{*,\tau+1} + \hat{D}_m^{*,\tau-1})\Delta t + 2(N_T)_m^{*,\tau}\Delta t\Big\}e^{-i\hat{n}y}dy,$$

$$\hat{A}d = (\hat{A}d_1,\ \hat{A}d_2,\ \cdots\ \hat{A}d_K)^T,$$

$$\hat{D}=\left((\hat{D}_1,\ \hat{D}_2,\ \cdots\ (\hat{D}^*_m)_K\right)^T,$$

$$\hat{D}^*_m=\left((\hat{D}^*_m)_1\ (\hat{D}^*_m)_2,\ \cdots(\hat{D}^*_m)_K\right)^T,$$

$$Ad^*_m=\left((Ad^*_m)_1,\ (Ad^*_m)_2,\ \cdots\ (Ad^*_m)_K\right)^T,$$

$$(\hat{N}_T)_{mn}=\left((\hat{N}_T)_{mn,1},\ (\hat{N}_T)_{mn,2},\ \cdots\ (\hat{N}_T)_{mn,K}\right)^T,$$

$$(\hat{N}_T)=\left((\hat{N}_T)_1,\ (\hat{N}_T)_2,\ \cdots\ (\hat{N}_T)_K\right)^T,$$

$$(N_T)^*_m=\left((N_T)^*_{m,1},\ (N_T)^*_{m,2},\ \cdots\ (N_T)^*_{m,K}\right)^T,$$

VI. TIME INTEGRATION

In the previous paper the computational stability of the linearized spectral shallow water equations was discussed under periodic conditions. The results are as follows.

(1) *In the explicit case*

The inequality

$$\Delta t \leqslant \left\{\bar{u}\hat{m}+[\varphi_0(\hat{m}^2+\hat{n}^2)+f^2\right\}^{-1} \tag{38}$$

holds, where $\bar{u}$ is the speed of the basic current and φ_0 the average depth of the fluid surface.

(2) *In the semi-implicit case*

$$\frac{1}{\bar{u}\hat{m}+f}\geqslant \Delta t \geqslant \Delta t_{ex} \tag{39}$$

Table 1. Latent Values and Characteristic Velocities

j	λ_j / g(m)	C_j(m / s)
1	9760.0	309.3
2	2676.5	162.0
3	545.6	73.1
4	181.2	42.1
5	62.1	24.7
6	19.8	13.9
7	7.2	8.4
8	2.4	4.8
9	0.5	2.2

holds, where Δt_{ex} is the permitted largest time step obtained from the inequality (38).

It can be seen by comparing the shallow water equations with the component equations (19)–(21) that the latent value λ_j is equivalent to φ_0.

According to the computations made by Yao (1988), when $\bar{T} = 300K$ and $K = 9$, λ_j / g and its characteristic velocity $C_j (= \sqrt{\lambda_j})$ are shown in Table 1. It will be seen from the table that the latent value rapidly decreases with j ; except the first few values, most characteristic velocities have the same magnitude as the wind velocity. Considering the computation time taken in time integration in the explicit case less than that in the semi–implicit case, and the stable integration with a large time step by use of the semi–implicit technique achieved at the expanse of large errors, in order to keep reasonable accuracy and save computation time, we may adopt the strategy suggested by Burridge (1975), namely, to make integration by using the explicit technique for the equations with smaller latent values and by using the semi–implicit technique for the equations with larger ones. Of course, in practice, the selection of the time step and the integration technique should still be determined by tests. However, the above results may be taken as a preliminary guidance.

REFERENCES

A.P.M. Baede et al. (1979), Adiabatic Formulation and Organization of ECMWF's Spectral Model, Tech. Report, No. 15, ECMWF.

Arakawa, A. and V. R. Lamb (1976), *Methods in Computational Physics*, **17**: 174–265.

Burridge, D.M. (1975), A Split Semi–Implicit Reformulation of the Bushby– Timpson 10–Level Model, *Q. J. R. M. S.*, **101**: 777–792.

Kasahara, A. (1976), Normal Modes of Ultralong Waves in the Atmosphere, *Mon. Wea. Rev.*, **104**: 669–690.

Liao Dongxian (1989), A Regional Spectral Nested Shallow Water Equation Model, *Advances in Atmospheric Sciences*, Vol. 6, No. 4, 393–402.

Machenhauer, B. (1979), *Numerical Methods Used in Atmosphere Models* II, Chapter 3, GARP Publications Series 17, WMO.

Tatsumi, Y. (1985), A Spectral Limited–Area Model with Time–Dependent Lateral Boundary Conditions and its Application to a Multilevel Primitive Equation Model, JMA / NPD Tech. Report, No. 3.

Temperton, C. et al. (1981), Normal Mode Initialization for a Multilevel Grid–Point Model, Part 1: Linear Aspects, *Mon. Wea. Rev.*, **109**: 729–743.

Yao Jianguo (1989), A Global Spectral Baroclinic Primitive Equation Model Suitable for Parallel Time Integration, Master Thesis, AMS (to be published).