Sensitivity of Medium–Range Weather Forecasts to the Use of Reference Atmosphere

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ABSTRACT

In this paper, the authors develop the earlier work of Chen Jiabin et al. (1986). In order to reduce spectral truncation errors, the reference atmosphere has been introduced in ECMWF model, and the spectrally–represented variables, temperature, geopotential height and orography, are replaced by their deviations from the reference atmosphere. Two modified semi– implicit schemes have been proposed to alleviate the computational instability due to the introduction of reference atmosphere. Concerning the deviation of surface geopotential height from reference atmosphere, an exact computational formulation has been used instead of the approximate one in the earlier work. To reduce aliasing errors in the computations of the deviation of the surface geopotential height, a spectral fit has been used slightly to modify the original Gaussian grid–point values of orography.

A series of experiments has been performed in order to assess the impact of the reference atmosphere on ECMWF medium– range forecasts at the resolution T21, T42 and T63. The results we have obtained reveal that the reference atmosphere introduced in ECMWF spectral model is generally beneficial to the mean statistical scores of 1000–200 hPa height 10–day forecasts over the globe. In the Southern Hemisphere, it is a clear improvement for T21, T42 and T63 throughout the 10–day forecast period. In the Northern Hemisphere, the impact of the reference atmosphere on anomaly correlation is positive for resolution T21, a very slightly damaging at T42 and almost neutral at T63 in the range of day 1 to day 4. Beyond the day 4 there is a clear improvement at all resolutions.

I. INTRODUCTION

Mountains play directly an important role on the atmospheric motion in different time and space scales. They exert not only a strong barrier effect on the air flow, but also act as a heat source (sink) for atmospheric motion.

As orography was included in forecast models (finite difference or spectral), it became increasingly common for designers of numerical models to use a terrain–following coordinate. With this coordinate the lower boundary condition becomes a relatively simple form, but it is at the expense of steeply sloping coordinate surfaces. This leads however to some problems in numerical computations.

One that has been discussed most by many meteorological scientists is that of the calculation of pressure gradients comprised by two terms. Over steeply sloping orography, the two terms tend to be large with opposite signs. This will give rise to problems in numerical computation.

The second is about the computation of advection of temperature. This consists of two terms (both horizontal and vertical advections), too, which nearly cancel over the steeply sloping orography. This computation would suffer from considerable errors.

For spectral model, there are other problems. In the spectral representation of orography, Gibbs waves occur in the neighbourhood of narrow transition zones with steep slopes between mountains and oceans, and negative values of orography appear on the oceans. Though these Gibbs waves tend to be small, they can trigger noticeable biases in the patterns of precipitation (M. Jarraud, A. J. Simmons and M. Kanamitsu, 1985). Furthermore, in the climatic simulation it will produce spurious high pressure regions at the surface with subsidence, and suppress rainfall, causing an unrealistic splitting of the precipitation area in Northern winter and summer (Ni Yunqi, 1987). In most spectral models in the world, as $\ln(p_s)$ instead of p_s is used as a variable, this violates directly mass conserving. But the use of p_s as a variable without making any adjustments to the basic numerical formulation gives rise to a marked increase in error in the pressure gradient calculation (A. J. Simmons, 1986).

In this paper, attention will be concentrated on solving some of these problems. We introduce a reference atmosphere which is only a function of pressure p in the forecast equations of spectral model. The spectrally represented orography in the divergence equation was replaced by the deviations of geopotential height at the surface from the reference atmosphere. The deviations are much less and smoother than orography itself. Furthermore, on the tilted terrain–following coordinate surface, the deviations of temperature field from the reference atmosphere are smoother than temperature field itself. So the reduction of spectral truncation error of temperature field is possible. Several years ago Chen Jiabin et al. (1986) worked at these problems, and got some preliminary and encouraging computational results. In this paper this work is further developed.

In this paper the basic concept is introduced in Section 2. Section 3 and 4 describe the numerical formulation. The definition of reference atmosphere is given in Section 5. The stability of the modified semi–implicit scheme is described in Section 6. In Section 7 objective verifications for the Northern, Southern Hemisphere and the globe are presented . Particular attention has been paid to improvement of verification scores as a reference atmosphere is introduced into ECMWF spectral model at the resolutions T21, T42 and T63.

II. BASIC CONCEPT

The main idea for the introduction of reference atmosphere in the spectral model is based upon the following concept.

Consider an arbitrary function $f(x)$ and its expansion in Fourier series

$$f(x)=\frac{1}{2}a_0+\sum_{k=1}^{\infty}(a_k\cos kx+b_k\sin kx), \tag{2.1}$$

the sum of its first n terms is regarded as an approximation of $f(x)$

$$f_n(x)=\frac{1}{2}a_0+\sum_{k=1}^{n}(a_k\cos kx+b_k\sin kx), \tag{2.2}$$

and then the truncation error is readily found as follows

$$|f(x)-f_n(x)|\leqslant\frac{\ln(n+\frac{1}{2})}{n+\frac{1}{2}}\max|f'_x(x)|. \tag{2.3}$$

It is evident from the last inequality that there are two sorts of ways to reduce the truncation error. One is to increase truncation wavenumber n. Another is to decrease the maximum derivative f'_x or the slope of the function $f(x)$.

In fact, we can introduce an auxiliary function $\overline{f(x)}$ which has nearly the same derivative (or slope) as the function $f(x)$. The difference

$$f'(x) = f(x) - \overline{f(x)} \tag{2.4}$$

has a smaller slope (or derivative), and hence a smaller truncation error.

In this report, a reference atmosphere which is only a function of pressure p, is introduced as an auxiliary function.

Absolute values of temperature difference and temperature deviation differences between adjacent grid points in the zonal and longitudinal direction along latitude 30.9°N for T42 are presented in Figure 1. The differences can be interpreted as their derivatives (or slope). It is found that over the Himalayas (at about 77°E), the temperature derivative is much larger, but that for temperature deviation is not. Therefore the spectral truncation error will be smaller.

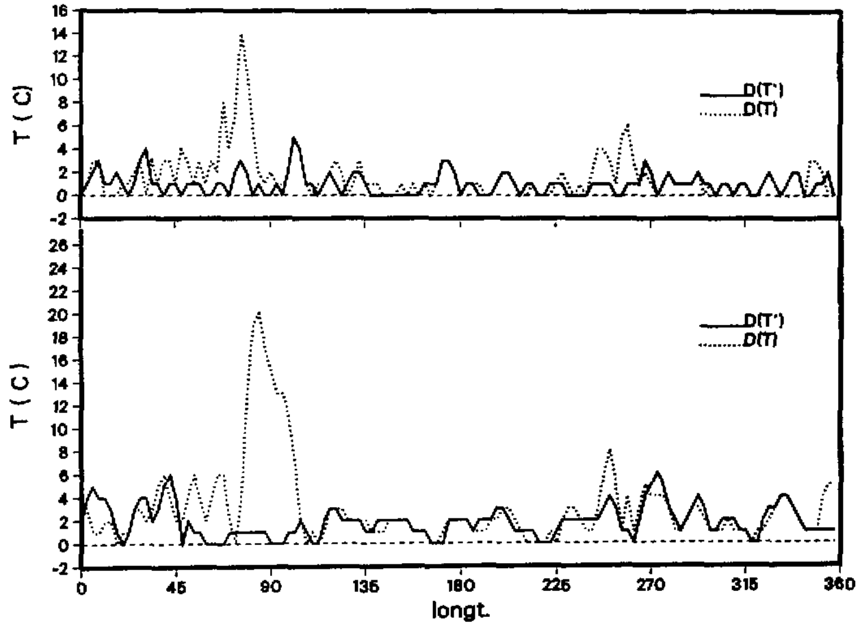

Fig.1. Absolute value of temperature differences $D(T)$ and temperature deviation differences $D(T')$ between adjacent grid points in the zonal-direction (upper panel) and longitudinal-direction (lower panel) along the latitude 30.9°N (T42, Level = 19).

III. THE CONTINUOUS EQUATIONS FOR η-VERTICAL COORDINATE

1. *Split of Variables*

A spectrally represented variable, such as T or Φ, is split into two parts

$$T=\overline{T(p)}+T'(\lambda,\mu,p,t) \tag{3.1}$$

or

$$T_v=\overline{T(p)}+T'_v$$

and

$$\Phi=\overline{\Phi(p)}+\Phi'(\lambda,\mu,p,t).$$

Here T_v is the virtual temperature, λ the longitude, and μ the sine of latitude. This so called reference atmosphere, $\overline{T}(p)$ and $\overline{\Phi}(p)$, is only a function of pressure p, it being considered to fit to hydrostatic relation

$$\frac{\partial\overline{\Phi}}{\partial p}=-\frac{R_d\overline{T}}{p} \tag{3.2}$$

2. *Forecast Equations*

We use a general terrain–following vertical coordinate η (Simmons and Strufing, 1981) as follows

$$\eta=\eta(p,p_s), \tag{3.3}$$

where $\eta(0,p_s)=0$, and $\eta(p_s,p_s)=1$.

Introducing the reference atmosphere, we can directly write the continuous formulation of the forecast equations in the coordinate η as follows

$$\frac{\partial\zeta}{\partial t}=\frac{1}{a(1-\mu^2)}\frac{\partial(F_v+P_v)}{\partial\lambda}-\frac{1}{a}\frac{\partial(F_u+P_u)}{\partial\mu}+K_\zeta \tag{3.4}$$

$$\frac{\partial D}{\partial t}=\frac{1}{a(1-\mu^2)}\frac{\partial(F_u+P_u)}{\partial\lambda}+\frac{1}{a}\frac{\partial(F_v+P_v)}{\partial\mu}+\nabla^2G+K_D \tag{3.5}$$

$$\frac{\partial T'}{\partial t}=-\frac{U}{a(1-\mu^2)}\frac{\partial T'}{\partial\lambda}-\frac{V}{a}\frac{\partial T'}{\partial\mu}-\left(\dot\eta\frac{\partial p}{\partial\eta}\right)\frac{\partial T'}{\partial p}+\left(\frac{R_d}{C_{pd}}T'+\frac{R_d}{C_{pd}}T_q+\frac{C^2}{R_d}\right)\frac{\omega}{p}+P_t \tag{3.6}$$

$$\frac{\partial q}{\partial t}=-\frac{U}{a(1-\mu^2)}\frac{\partial q}{\partial\lambda}-\frac{V}{a}\frac{\partial q}{\partial\mu}-\left(\dot\eta\frac{\partial q}{\partial\eta}\right)\frac{\partial q}{\partial p}+P_q \tag{3.7}$$

$$\frac{\partial}{\partial\eta}\left(\frac{\partial p}{\partial t}\right)+\nabla\cdot\left(\mathbf{v}\cdot\frac{\partial p}{\partial\eta}\right)+\frac{\partial}{\partial\eta}\left(\dot\eta\frac{\partial p}{\partial\eta}\right)=0 \tag{3.8}$$

$$d\Phi'=-R_dT'_v\,d\ln p. \tag{3.9}$$

The rate of change of surface pressure and $\dot\eta$ are obtained by integrating (3.8), using the boundary conditions $\dot\eta=0$ at $\eta=0$ and $\eta=1$;

$$\frac{\partial\ln p_s}{\partial t}=-\frac{1}{p_s}\int_0^1\nabla\cdot\left(\mathbf{v}\cdot\frac{\partial p}{\partial\eta}\right)d\eta \tag{3.10}$$

or

$$\frac{\partial p}{\partial t}=-\int_0^1\nabla\cdot\left(\mathbf{v}\cdot\frac{\partial p}{\partial\eta}\right)d\eta \tag{3.11}$$

and

$$\left(\dot{\eta}\frac{\partial p}{\partial \eta}\right) = -\frac{\partial p}{\partial t} - \int_0^{\eta} \nabla \cdot \left(\mathbf{v} \cdot \frac{\partial p}{\partial \eta}\right) d\eta. \tag{3.12}$$

The pressure coordinate vertical velocity is given by

$$\omega = -\int_0^{\eta} \nabla \cdot (\mathbf{v} \cdot dp) + \mathbf{v} \cdot \nabla p, \tag{3.13}$$

$$\begin{cases} C^2 = \dfrac{R_d}{C_{pd}}\left(R_d \bar{T} - C_{pd} p \dfrac{\partial \bar{T}}{\partial p}\right) \\ F_u = (f+\zeta)V - \left(\dot{\eta}\dfrac{\partial p}{\partial \eta}\right)\dfrac{\partial U}{\partial p} - \dfrac{R_d T'_v}{a}\dfrac{\partial \ln p}{\partial \lambda} \\ F_v = -(f+\zeta)U - \left(\dot{\eta}\dfrac{\partial p}{\partial \eta}\right)\dfrac{\partial V}{\partial p} - \dfrac{R_d T'_v}{a}\dfrac{\partial \ln p}{\partial \mu}(1-\mu^2) \\ G = \Phi' + E \\ E = \dfrac{1}{2(1-\mu^2)}(U^2 + V^2) \\ T'_v = T_v - \bar{T} \\ T_q = qT\dfrac{(\frac{1}{\varepsilon} - 1) - (\delta - 1)}{1 + (\delta - 1)q}. \end{cases} \tag{3.14}$$

C^2 in (3.6) depends on the vertical distribution of reference atmosphere temperature, and generally speaking, it is a function of pressure. And P_u, P_v, P_t and P_q are diffusion terms. Here $\varepsilon = R_d / R_v$ and $\delta = C_{pv} / C_{pd}$. And other symbols have usual meanings in meteorology. This set of equations (3.4)–(3.11) is very similar to that of ECMWF spectral model (ECMWF Research Department, 1988), apart from that the temperature T. geopotential Φ and orography Φ_s are replaced by their deviations T',Φ' and Φ'_s respectively and the modifications for some terms are made. If we use $\partial p_s / \partial t$ in the tendency equation (3.10) instead of $\partial \ln p_s / \partial t$, it will maintain mass conservation, a preferred property for long–term integrations.

IV. THE DISCRETE EQUATIONS

1. *The Hybrid Vertical Representation*

In the ECMWF spectral model, the atmosphere is divided into NLEV layers, which are defined by the pressure of the interfaces between them. The pressure is given by

$$p_{k+\frac{1}{2}} = A_{k+\frac{1}{2}} + B_{k+\frac{1}{2}} p_s \tag{4.1}$$

for $k = 0, 1, \cdots,$ NLEV. p_s is the surface pressure, and the $A_{k+\frac{1}{2}}$ and $B_{k+\frac{1}{2}}$ are constants whose values effectively define the vertical coordinate. Necessary values are

$$A_{\frac{1}{2}} = B_{\frac{1}{2}} = A_{\mathrm{NLEV}+\frac{1}{2}} = 0, \quad B_{\mathrm{NLEV}+\frac{1}{2}} = 1. \tag{4.2}$$

The usual sigma (or pressure) coordinate will be obtained as the special case

$$A_{k+\frac{1}{2}} (\text{or } B_{k+\frac{1}{2}}) = 0, \quad k = 0, 1, \cdots, \text{NLEV}.$$

This form of hybrid coordinate is particularly efficient from the computational viewpoint.

2. *The Vertical Finite Difference and Time Scheme*

Adopting the same vertical finite difference and semi-implicit scheme as that of the ECMWF forecast model, we obtain the following finite difference forms of the forecast equations (3.4)–(3.11). The vertical difference scheme can hold the conservation of angular momentum, and available potential and kinetic energy of the model atmosphere for adiabatic, frictionless motion.

$$\delta_t \zeta = ZT - \frac{\beta_{zQ}}{2a} \frac{U_r(\mu)}{(1-\mu^2)} \frac{\partial}{\partial \lambda} \Delta_{tt} \zeta \quad (4.3)$$

$$\delta_t q = QT - \frac{\beta_{zQ}}{2a} \frac{U_r(\mu)}{(1-\mu^2)} \frac{\partial}{\partial \lambda} \Delta_{tt} q \quad (4.4)$$

$$\delta_t D = DT - \frac{\beta_{DT}}{2} \nabla^2 (R_d \overline{T}_0 \Delta_{tt} \ln p_s + \gamma \Delta_{tt} T') \quad (4.5)$$

$$\delta_t T' = TT - \frac{\beta_{DT}}{2} \frac{C_0^2}{R_d} \tau \Delta_{tt} D \quad (4.6)$$

$$\delta_t \ln p_s = PT - \frac{\beta_{DT}}{2} \nu \Delta_{tt} D \quad (4.7)$$

or

$$\delta_t p_s = PT - \frac{\beta_{DT}}{2} \nu \Delta_{tt} D. \quad (4.8)$$

We define

$$\Delta_{tt} X = [X(t+\Delta t) + X(t-\Delta t) - 2X(t)].$$

Here C_0^2 in the last term on the right–hand side of (4.6) is a constant which comes from linearizing the term $(C^2 / R_d)\omega / p$. And

$$G = \Phi' + E$$

and

$$\Phi' = \Phi'_s + \gamma T'_r. \quad (4.9)$$

The matrices τ and γ, and row vector ν are defined respectively as

$$\gamma = R_d \begin{bmatrix} \alpha_1 & \ln\frac{p^r_{2+1/2}}{p^r_{2-1/2}} & \ln\frac{p^r_{3+1/2}}{p^r_{3-1/2}} & \cdots & \ln\frac{p^r_{NLEV+1/2}}{p^r_{NLEV-1/2}} \\ 0 & \alpha_2 & \ln\frac{p^r_{3+1/2}}{p^r_{3-1/2}} & \cdots & \ln\frac{p^r_{NLEV+1/2}}{p^r_{NLEV-1/2}} \\ 0 & 0 & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & \alpha_{NLEV} \end{bmatrix} \quad (4.10)$$

$$\tau=\begin{bmatrix} \alpha_1 & 0 & 0 & 0 \\ \frac{\Delta p_1^r}{\Delta p_2^r}\ln\frac{p_{2+1/2}^r}{p_{2-1/2}^r} & \alpha_2 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\Delta p_1^r}{\Delta p_{NLEV}^r}\ln\frac{p_{NLEV+1/2}^r}{p_{NLEV-1/2}^r} & \cdots & \frac{\Delta p_{NLEV-1}^r}{\Delta p_{NLEV}^r}\ln\frac{p_{NLEV+1/2}^r}{p_{NLEV-1/2}^r} & \alpha_{NLEV} \end{bmatrix} \tag{4.11}$$

$$\nu=\begin{cases}(\Delta p_1^r,\cdots,\Delta p_{NLEV}^r)/p_r, & \text{if } \frac{\partial \ln p_s}{\partial t} \text{ is used} \\ (\Delta p_1^r,\cdots,\Delta p_{NLEV}^r), & \text{if } \frac{\partial p_s}{\partial t} \text{ is used,}\end{cases} \tag{4.12}$$

where the pressure $p_{k+1/2}^r$ is determined by the form (4.1) in which p_s is replaced by p_r. And here p_r is a constant (1013.2 hPa).

$$\Delta p_k^r = p_{k+1/2}^r - p_{k-1/2}^r$$
$$\alpha_1 = \ln 2$$
$$\alpha_k = 1-\frac{p_{k-1/2}^r}{\Delta p_k^r}\ln\frac{p_{k+1/2}^r}{p_{k-1/2}^r}, \quad (k=2,\cdots,\text{NLEV}).$$

The terms ZT, QT, DT, TT and PT represent those on the right–hand sides of equations (3.4)–(3.11), apart from the diffusion terms, which are neglected here.

Horizontal discretizations of the basic prognostic variables of the model are all the same as ECMWF forecast model.

V. REFERENCE ATMOSPHERE

Phillips (1973, 1974) introduced a reference atmosphere into the forecast equations, but it was assumed to be an adiabatic atmosphere. Zeng Qingcun (1963; 1965; 1979; 1985) first took the standard atmosphere as a reference atmosphere and met with successes in numerical computation of a finite difference model.

The parameters C^2, C_0^2 and the deviation of geopotential height at surface from the reference atmosphere Φ'_s all depend on the definition of the reference atmosphere. In this section we will give the definition and computation of C^2, C_0^2 and Φ'_s which appear in the foregoing equations, and the modification of surface geopotential Φ_s to remove the aliasing errors.

1. *Definition of Reference Atmosphere*

Generally speaking, the temperature of reference atmosphere is formally defined as

$$\bar{T}(p)=[f(p)+f_0]/R_d \tag{5.1}$$

with hydrostatic relation

$$\frac{\partial \overline{\Phi}}{\partial p} = - R_d \overline{T} / p.$$

Here f_0 is a constant. The geopotential height of the reference atmosphere is easily given as follows

$$\overline{\Phi} = f_0 \ln(p_0 / p) + [(F(p_0) - F(P)]. \tag{5.2}$$

Here F is a function of $f(p)$:

$$F = \int f(p) d\ln p \tag{5.3}$$

and p_0 is a constant of 1013.20 hPa.

Using Eq.(5.2) and $\overline{\Phi}(\overline{p}_s) \equiv \Phi_s$, where Φ_s is the orography, we will have the logarithm of surface pressure of reference atmosphere $\overline{p}_s$

$$\ln\overline{p}_s = \ln p_0 - [\Phi_s + F(\overline{p}_s) - F(p_0)] / f_0 . \tag{5.4}$$

C^2 in (3.6) can be represented as

$$C^2 = \frac{R_d}{C_{pd}} \left[f_0 + f(p) - \frac{C_{pd} \partial f(p)}{R_d \partial \ln p} \right] . \tag{5.5}$$

In practice, the temperature of the reference atmosphere can be defined as the special case

$$f_0 + f(p)] / R_d = T_0 + T_1 (p / p_0)^{\chi} \tag{5.6}$$

with

$$\begin{aligned} T_0 &= \overline{T}_0 - T_1 \\ T_1 &= (\gamma C_{pd} / g) \overline{T}_0 \\ p_0 &= 1013.20 \text{ hPa} \\ \overline{T}_0 &= 288 \text{ K} \end{aligned} \tag{5.7}$$

and

$$\gamma = 0.0065 \text{ K} / \text{m}.$$

Here $\chi = R_d / C_{pd}$. There values are chosen such that the temperature of the reference atmosphere $\overline{T}(p)$ matches the following ICAO standard atmosphere at low levels

$$T_{\text{ICAO}} = \overline{T}_0 \left(\frac{p}{p_0} \right)^{\alpha}, \quad \alpha = \gamma R_d / g \quad \text{for} \quad T_{\text{ICAO}} > 216.5 \text{ K}$$

and

$$\overline{T}(p) = T_{\text{ICAO}}, \quad \text{at} \quad p = p_0 .$$

Such defined temperature of the reference atmosphere describes well the mean state of the atmosphere at low levels, except the inversion characteristics of stratification at upper levels. The other definitions of temperature can be done which describe well the mean state of the atmosphere both at lower and upper levels.

Using (5.1)–(5.5), we easily obtain the expressions for $\overline{\Phi}$, $\ln\overline{p}_s$ and C^2 as follows

$$\overline{\Phi}(p) = C_{pd} T_1 [1 - (p / p_0)^{\chi}] - R_d T_0 \ln(p / p_0) \tag{5.8}$$

$$\ln \overline{p}_s = \ln p_0 - [\Phi_s - C_{pd} T_1 (1 - (\overline{p}_s / p_0)^{\chi})] / (R_d T_0) \tag{5.9}$$

$$C^2 = (R_d^2 / C_{pd})(\overline{T}_0 - T_1) = C_0^2. \tag{5.10}$$

Obviously, in view of the foregoing definition of temperature of reference atmosphere, the parameter C^2 in (5.10) is a constant with the value C_0^2.

2. *Computation of* Φ'_s

The pressure gradient term can be represented as

$$\begin{aligned} \nabla \Phi' + R_d T'_v \nabla \ln p &= \nabla \Phi + R_d T_v \nabla \ln p \\ &= \nabla \left(\Phi_s + \int_p^{p_s} \frac{R_d}{p} T'_v dp \right) + R_d T'_v \nabla \ln p \\ &\quad + \nabla \int_p^{p_s} \frac{R_d}{p} \overline{T} dp + R_d \overline{T} \nabla \ln p. \end{aligned}$$

Therefore we have

$$\nabla \Phi' + R_d T'_v \nabla \ln p = \nabla (\Phi'_s + \gamma T'_v) + R_d T'_v \nabla \ln p. \tag{5.11}$$

Here

$$\Phi'_s = \Phi_s + f_0 \ln(p_s / p_0) + [F(p_s) - F(p_0)]. \tag{5.12}$$

Especially when the temperature of reference atmosphere is defined as (5.6), the deviation of geopotential height at surface from the reference atmosphere Φ'_s of (5.12) will become as follows

$$\Phi'_s = \Phi_s + R_d T_0 \ln(p_s / p_0) + C_{pd} T_1 [(p_s / p_0)^{\chi} - 1]. \tag{5.13}$$

The way to compute Φ'_s is different from that used by Chen jiabin at al. (1986), who introduced some approximations in the derivation.

3. *Modification of Surface Geopotential* Φ_s

When either $\ln p_s$ or p_s is used as the spectrally represented variable, the Eq. (5.13) or (5.12) implies that there will be aliasing in the calculation of the spectral components of Φ'_s. When $\ln p_s$ is the prognostic variable, this aliasing is small, since (5.13) may be written as

$$\begin{aligned} \Phi'_s &= \Phi_s + R_d T_0 \ln(p_s / p_0) + C_{pd} T_0 [\exp(\chi \ln(p_s / p_0)^2)) - 1] \\ &= \Phi_s + (R_d T_0 + R_d T) \ln(p_s / p_0) + 0.5 C_{pd} T_0 \chi (\ln p_s / p_0)^2 + O(\varepsilon^3), \end{aligned}$$

where $\varepsilon = \chi \ln p(p_s / p_0)$. Only the $O(\varepsilon^3)$ term will be aliased. Aliasing is much larger when p_s is the prognostic variable, it being transferred from the $R_d T'_v \nabla p / p$ term in the conventional spectral approach to the $\nabla \Phi'_s$ term when temperature is replaced by its deviation from the reference atmosphere as the spectrally represented variable.

Much of the aliasing can in practice be removed by a small modification of Φ_s. Let Φ_{sg} denote the grid-point values of the basic, spectrally represented orography. The equation

$$\Phi_{sg} + f_0 \ln(\bar{p}_{sg} / p_0) + [F(\bar{p}_{sg}) - F(p_0)] = 0$$

or

$$\Phi_{sg} + R_d T_0 \ln(\bar{p}_{sg} / p_0) + C_{pd} T_1 [(\bar{p}_{sg} / p_0)^{\chi} - 1] = 0$$

is solved iteratively for $\bar{p}_{sg}$. The modified values of $\bar{p}_{sg}$, the $\bar{p}_s$ values to be used in Eq. (5.13) or (5.12), are computed by performing a spectral fit of either $\ln\bar{p}_{sg}$ or $\bar{p}_{sg}$, depending on the choice of $\ln\bar{p}_s$ or $\bar{p}_s$ as spectral variable, and then re-evaluating the grid-point values of surface pressure from the spectral fit. Finally the values of Φ_s to be used in Eq. (5.12) or (5.13) are determined as

$$\Phi_s = -R_d T_0 \ln(\bar{p}_s / p_0) - C_{pd} T_1 [(\bar{p}_s / p_0)^{\chi} - 1]$$

or

$$\Phi_s = f_0 \ln(p_0 / \bar{p}_s) + F[(p_0) - [F(\bar{p}_s)].$$

Using the operational T106 ECMWF orography for Φ_{sg}, the modified surface geopotential Φ_s, hardly differs from Φ_{sg}, when $\ln p_s$ is the prognostic variable, with a maximum height difference of under 2 m over the Himalayas. A maximum difference of just under 30 m is formed in some region when p_s is the prognostic variable.

VI. COMPUTATIONAL INSTABILITY

Computational instability may occur when the reference atmosphere is introduced into the primitive equations with use of the semi-implicit method of time integration (Simmons et al. 1978), and the simplest remedy to the computational instability is the use of the modified time averaging as follows

$$X = \frac{\beta_{DT}}{2} [X(t + \Delta t) + X(t - \Delta t)] + (1 - \beta_{DT}) X(t). \tag{6.1}$$

However, with $\beta_{DT} = 2$, stabilization is achieved at the expense of some slowing of the phase speeds. In order to overcome this problem, we design and test the following two modified schemes of the semi-implicit time integration with $\beta_{DT} = 0.75$, to mimic the present semi-implicit scheme of ECMWF model.

1. *Version A*

To mimic the present ECMWF semi-implicit scheme we should treat implicitly the gravity wave term for perturbation about an isothermal basic state with $T = T_r$ and constant surface pressure p_r about which gravity wave oscillations are taking place. Let tilde denote the gravity wave perturbations, we have the gravity wave equations

$$\frac{\partial \tilde{D}}{\partial t} = -\nabla^2(\gamma\tilde{T} + R_d T_r \ln\tilde{p}_s)$$
$$\frac{\partial \tilde{T}}{\partial t} = -\tau\tilde{D}$$
$$\frac{\partial \ln\tilde{p}_s}{\partial t} = -\nu\tilde{D},$$

now

$$T' = T - T_0 - T_1\left(\frac{p}{p_0}\right)^{\chi} = T_r + \tilde{T} - T_0 - T_1\left(\frac{p}{p_0}\right)^{\chi}$$
$$= T_r - T_0 - T_1\left[\left(\frac{p}{p_0}\right)^{\chi}\right]\Bigg|_{p_s=p_r} + \tilde{T} - \chi T_1\left[\frac{1}{p}\left(\frac{p}{p_0}\right)^{\chi}\frac{\partial p}{\partial p_s}\right]\Bigg|_{p_s=p_r} p_r \ln\tilde{p}_s.$$

Denoting the first-order variation of T' by $\tilde{T}'$, we have

$$\tilde{T} = \tilde{T}' + \sigma\ln\tilde{p}_s,$$

where

$$\sigma = p_r \chi T_1\left[\left(\frac{p}{p_0}\right)^{\chi}\frac{\partial \ln p}{\partial p_s}\right]\Bigg|_{p_s=p_r}. \tag{6.2}$$

The linearized gravity wave equation becomes

$$\frac{\partial \tilde{D}}{\partial t} = -\nabla^2(\gamma\tilde{T}' + R_d T_r + \gamma\sigma)\ln\tilde{p}_s \tag{6.3}$$
$$\frac{\partial \tilde{T}'}{\partial t} = -(\tau - \sigma\nu)\tilde{D} \tag{6.4}$$
$$\frac{\partial \ln\tilde{p}_s}{\partial t} = -\nu\tilde{D}, \tag{6.5}$$

Thus, for the semi–implicit scheme, equation (6.3) and (6.4) become

$$\delta_t D = DT - \frac{1}{2}\beta_{DT}\nabla^2[\gamma\Delta_{tt}T' + (R_d T_r + \gamma\sigma)\Delta_{tt}\ln p_s] \tag{6.6}$$

and

$$\delta_t T' = TT - \frac{1}{2}\beta_{DT}(\tau - \sigma\nu)\Delta_{tt}D. \tag{6.7}$$

We replace $\ln\tilde{p}_s$ by $\tilde{p}_s / p_r$ to obtain the scheme when p_s is the spectrally represented variable.

2. *Version B*

Let the deviation of temperature from reference atmosphere be

$$T' = \overline{T'} + T'', \tag{6.8}$$

where $\overline{T'}$ is a constant similar to T_o of ECMWF spectral model. Substituting (6.8) into (4.3) and (4.4), we have divergence and thermodynamic equations as follows

$$\delta_t D = DT - \frac{1}{2}\beta_{DT}\nabla^2[R_d(\overline{T}_0 + \overline{T'})\Delta_{tt}(\ln p_s - \ln\overline{p}_s) + \gamma\Delta_{tt}T'] \tag{6.9}$$

and

$$\delta_t T' = TT - \frac{1}{2}\beta_{DT}\left(\frac{R_d}{C_{pd}}\overline{T'} + \frac{C_0^2}{R_d}\right)\tau\Delta_{tt}D. \tag{6.10}$$

Here $\overline{T'} = 3$ K. Figure 2 indicates the rate of change of kinetic energy in test integration for various values of β_{DT}. It is found that the scheme is stable if $\beta_{DT} \geqslant 0.75$.

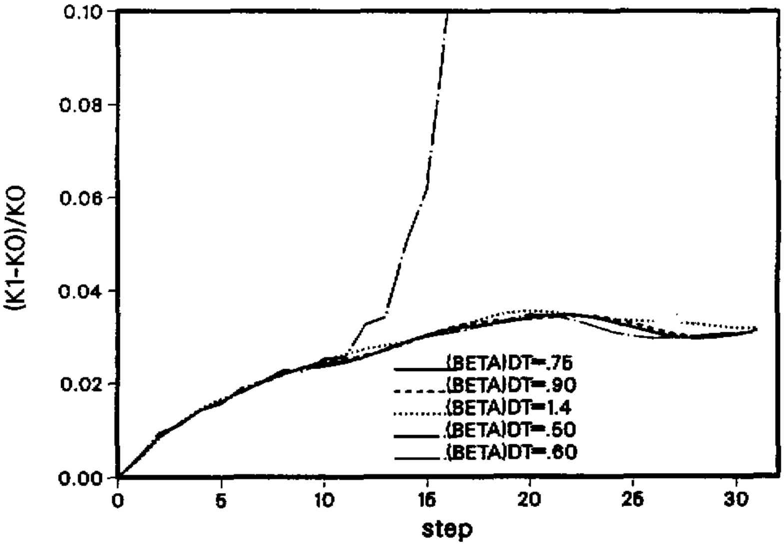

Fig.2. The change of kinetic energy for the modified semi-implicit scheme with different β_{DT}.

VII. OBJECTIVE ASSESSMENT

1. *The Experimental Programme*

In order to get as much confidence as possible in the representativeness of the computational results, 12 cases (12 Z on 15th of each month) were selected spanning 2 years since March 1987. For each case 10–day forecasts were made at three resolutions, T21, T42 and T63 with the standard ECMWF 19–level vertical resolution for the following 4 versions:

ECMWF, PS, RPL and TTRPL3. Here ECMWF represents ECMWF spectral model. In the other 3 versions, a reference atmosphere was used. All versions used the tendency of the logarithm of the surface pressure, except version PS, where the tendency of p_s was used. The specifications are listed in Table 1.

Table 1. The Specification of the Experiments

Version	Reference Atmosphere	Surface Pressure Tendency	β_{DT}	Semi–implicit Scheme
ECMWF	No	$\partial \ln p_s / \partial t$	0.75	Operational
PS	Yes	$\partial p_s / \partial t$	2.00	;
RPL	Yes	$\partial \ln p_s / \partial t$	2.00	;
TTRPL3	Yes	$\partial \ln p_s / \partial t$	0.75	Scheme B

Envelope orography with one standard deviation was used in all experiments. The model envelope orography used in the version PS, RPL and TTRPL3 is slightly different from that of the version ECMWF, as the spectral fit is different. The two kinds of model envelope orography for resolution T21 at selected locations on the earth are shown in Table 2.

Table 2. Comparison of Model Envelope Orography

Location	Model Envelope	Orography Used in
	ECMWF (m)	PS, RPL, TTRPL3 (m)
Himalayas	5860	5810
Greenland	2530	2450
Andes	3200	2950
Rocky	2640	2640
Eastern Africa	1590	1600
Antarctica	4090	4140

The objective verification is based on the anomaly correlation of the height field averaged over the extratropical troposphere. It is the correlations between the observed deviations and the predicted one from climatology.

2. *Examination*

The versions PS, RPL and TTRPL3 are produced by changing the current ECMWF version. In order to get confident in the programming change, we first test whether it is correct.

When the reference atmosphere is considered as an isothermal one with a temperature of 300 K and $p_0 = 800$ hPa, the version RPL should be similar to the ECMWF one. The computational results confirm this. They reveal that all dynamical statistics for a 1–day forecast of version ECMWF and RPL, such as root mean square vorticity and divergence, mean temperature and surface pressure, mean kinetic energy and potential energy, and so on, are absolutely identical. 10–day forecasts with T21 for the two versions were performed. It has been found that the anomaly correlations for the two versions are almost identical. Therefore, we believe our change to the ECMWF programme to be correct.

3. *Global Results*

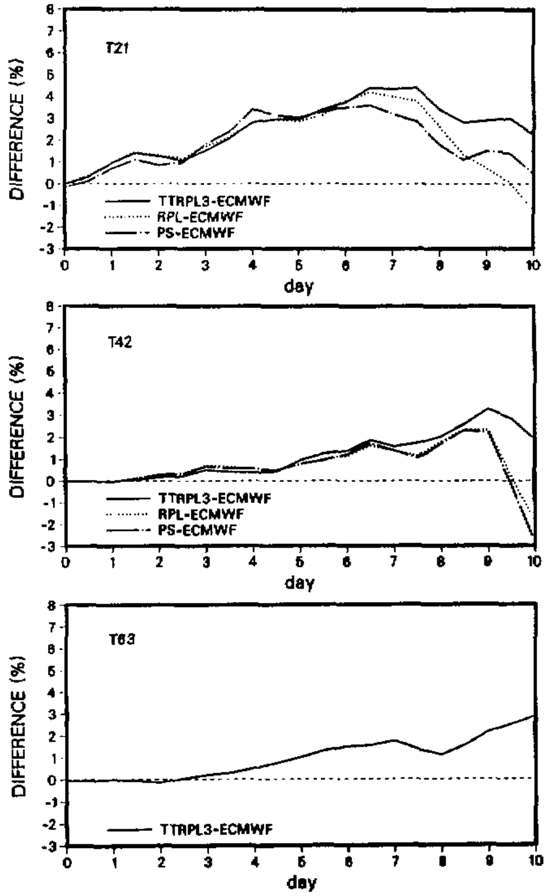

Fig.3. The differences between the anomaly correlation of 1000–200 hPa height field in the extratropical troposphere of globe averaged over the 12 cases for the respective versions TTRPL3, RPL and PS and the version ECMWF.

For the resolutions T21, T42 and T63, in Fig.3 are displayed the differences between the anomaly correlations of 1000–200 hPa height field in the extratropical troposphere of globe averaged over the 12 cases for the individual versions TTRPL3, RPL and PS and the version ECMWF. It can be found from the figure that the anomaly correlation for the version TTRPL3, RPL and PS is almost heigher than that of the version ECMWF, except day 10 for the version RPL at T21 and the version RPL and PS at T42, and day 1 to day 2 for the version TTRPL3 at T63, where the correlations are lower. Therefore for the extratropical troposphere of globe, this means a beneficial overall impact of the reference atmosphere introduced in ECMWF spectral model. These results are in agreement with those reported by Chen Jiabin et al. (1986) who tested version RPL. However, the spectral model used in these early experiments had only truncation wave number of 21 and vertical resolution of 2 levels, and 4–day forecasts of 4 cases were performed.

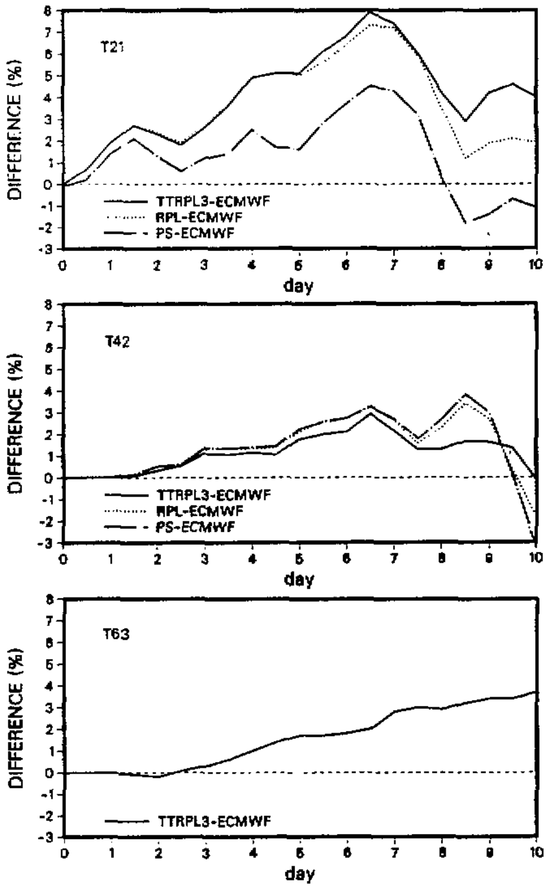

Fig.4. As Fig.3 but for the Southern Hemisphere.

An important fact, as indicated in the same figure, is that the differences in anomaly correlation between the individual version TTRPL3, RPL and PS, and the version ECMWF becomes small, when going from T21 to T63. It seems that the impact of reference atmosphere in the spectral model is sensitive to the change of esolution going from low resolution (T21) to high.

4. *Southern Hemispheric Results*

For the resolutions T21, T42 and T63, Fig.4 displays the differences between the mean anomaly correlations of 1000–200 hPa height in the extratropical troposphere of the Southern Hemisphere for the individual versions TTRPL3, RPL and PS, and the version ECMWF.

As in Figure 3 the correlations of the versions TTRPL3, RPL and PS are better than those of the version ECMWF, except the version PS correlation of which is lower than that of ECMWF after day 8 at T21 and after day 9.5 at T42. The forecast quality in the Southern Hemisphere is improved when a reference atmosphere is introduced.

5. *Northern Hemispheric Results*

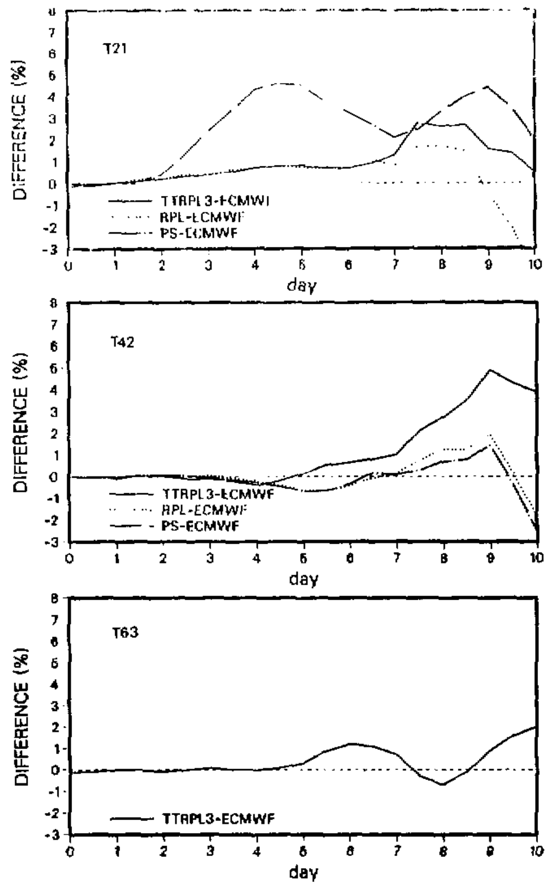

Fig.5. As Fig.3 but for the Northern Hemisphere.

The differences between the mean anomaly correlations of 1000–200 hPa height in the extratropical troposphere of the Northern Hemisphere for the individual versions TTRPL3, RPL and PS and the version ECMWF are shown for resolutions T21, T42 and T63 in Fig. 5. It can be seen in Fig. 5 that for the resolution T21, the reference atmosphere introduced in the ECMWF spectral model produced marked improvement in forecast quality, except the version RPL at day 9 to day 10. However, for resolution T42, as seen in the same figure, the situation becomes more complex in contrast with the Southern Hemisphere. For version PS and RPL, for example, up to day 6 and after day 9, the impact of the reference atmosphere is slightly damaging, and positive only in day 7 to day 9. For the version TTRPL3, up to day 4, the forecast quality becomes slightly worsening, but after that the impact of the reference atmosphere is very evident. For the resolution T63, the impact of reference atmosphere is almost neutral up to day 4, and the improvement is only after day 4.

The forecast maps of 500 hPa height for the version TTRPL3 and ECMWF at T42 are presented for day 8 in Fig. 6. The middle left panel(ID3) is for the version TTRPL3, the middle right one(IBI) for the version ECMWF, and the upper panel for the differences between the two forecasts and corresponding analysis (the lower left).

Comparing the results for Northern and Southern Hemisphere with each horizontal resolution shown in Figs. 4 and 5 it is seen that the improvement of forecast quality is larger in Southern Hemisphere than in Northern Hemisphere, as the reference atmosphere is introduced in ECMWF model.

6. *A Forecast Case*

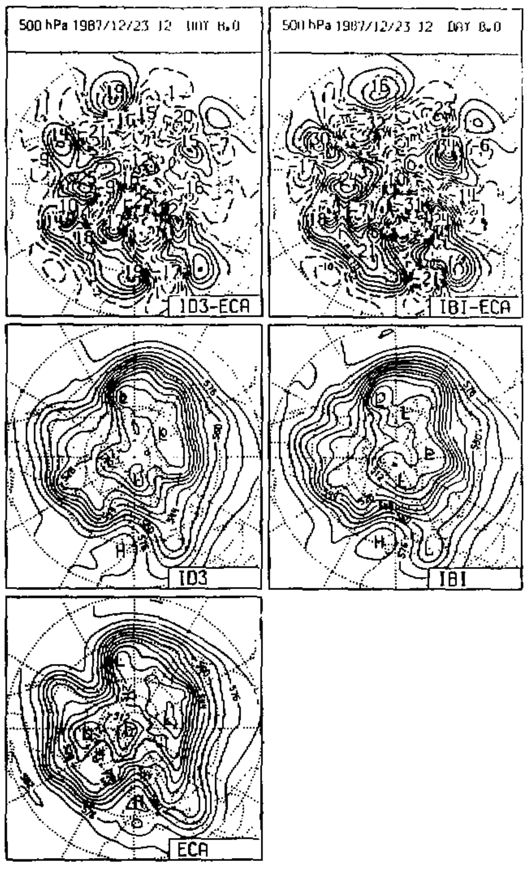

Fig.6. Analyses of 500 hPa height (lower left) and corresponding 8-day forecast maps for version ECMWF (IBI, lower right) and TTRPL3 (ID3, middle left) at T42 from 15 Dec., 1987.

When comparing the forecast charts, we also see improvements. Here we select the case of 15 December 1987 to compare the forecast charts of the version ECMWF and TTRPL3 with the resolution T42.

The figures clearly show the synoptic difference between the forecasts of the two versions. Pronounced differences can be found in two areas. One is in the west of the Rocky Mountains, where the forecast of the version TTRPL3 is closer to the corresponding analysis than that of the version ECMWF. The forecast of the version ECMWF produced spurious trough instead of a weak ridge over the centre of North America in the analysis. Another area is over the Himalayas, where the forecast of the version ECMWF is producing spurious trough.

VIII. CONCLUDING REMARK

In this paper we have introduced a reference atmosphere as a function of pressure in ECMWF spectral model, which aims at reduction of the spectral truncation error, and aliasing error over steep mountains.

The vertical finite difference scheme of the equations can ensure conservation of the sum of the available potential and kinetic energy. Perhaps this is an appropriate integration constraint.

In order to get a stable semi–implicit time scheme as the reference atmosphere is introduced, we mimic the present semi–implicit time scheme of ECMWF model by a modified semi–implicit time scheme. This scheme reveals computational stability, even if the coefficient β_{DT} has a value of 0.75 (time averaging coefficient of the separated gravity wave terms).

A series of experiments has been performed in order to assess the impact of the reference atmosphere on ECMWF medium–range forecasts at the resolution T21, T42 and T63. The results we have obtained reveal that the reference atmosphere introduced in ECMWF spectral model is generally beneficial to mean statistical scores of 1000–200 hPa height 10–day forecasts over the globe. In the Southern Hemisphere, it is a clear improvement for T21, T42 and T63 throughout the 10–day forecast period. In the Northern Hemisphere the situation is quite different. In the range of day 1 to day 4, the impact of the reference atmosphere on anomaly correlation is positive for resolution T21, very slightly damaging at T42 and almost neutral at T63. Beyond day 4 there is a clear improvement at the resolution T21, T42 and T63.

We have performed the computations of 10–day forecasts at T106 resolution. Sensitivity is much less at T106 resolution, but a minor advantage can still be seen in high southern latitudes, and there is a reduction in noise in the vertical velocity field near the Andes at tropical latitudes. All these are referred to the paper by Simmons and Chen Jiabin(1990).

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REFERENCES

Chen Jiabin, Ji Liren and Wu Wanli (1986), Design and test of an improved scheme for global spectral model with reduced truncation error, Short–and Medium–Range Numerical Weather Prediction WMO / IUGG NWP Symposium, Tokyo, 4–8 August 1986, 431–440.

ECMWF Research Department (1988), ECMWF forecast model, adiabatic part, Research manual 2.

Jarraud, M., A. J. Simmons and M. Kanamitsu (1985), Development of the high resolution model, Technical Memorandum, No. 107, ECMWF.

Ni Yunqi (1987), The sensitivity of numerical simulation to orography specification in the lower solution spectral model—part II: impact of the smoothed orography and ripples on simulations, *Advances in Atmospheric Sciences*, **4**: 145—155.

Phillips, N. A. (1974), Application of Arakawa's energy—conserving layer model to operational numerical prediction, US Dept. Commerce, NMC office Note 104, 44pp.

Phillips, N.A. (1973), Principles of large scale numerical weather prediction, *Dynamic Meteorology*, P. Morel (Ed.), 1—96, Dordrecht—Holland, D. Reidel Publishing Comp. 1—96.

Simmons, A. J. (1986), Orography and the development of the ECMWF model, "*observation, theory and modeling of orographic effects*". Seminar / workshop, ECMWF.

Simmons, A. J. and R. Strufing (1981), An energy and angular momentum conserving finite—difference scheme, hybrid coordinates and medium—range weather prediction, Technical Report No. 28, ECMWF.

Simmons, A. J., B. J. Hoskins and D. M. B urridge (1978), Stability of the Semi—implicit method of time integration, *Monthly Weather Review*, **104**: 405—412.

Simmons, A. J., and Chen Jiabin (1990), The calculation of geopotential and the pressure gradient in the ECMWF atmospheric model: Influence on the simulation of the polar atmosphere and on temperature analyses, Submitted to *Quart.J. Roy. Meteor. Soc.*.

Zeng Qingcun (1963), Characteristic Parameter and Dynamical Equation of Atmospheric Motions, *Acta Meteorologica Sinica*, **33**: 472—483 (in Chinese).

Zeng Qingcun (1965), On the Design of Finite Difference Scheme for Solving the Primitive Equations, *Technical Report*, Institute of Geophysics and Meteorology, Academia Sinica (in Chinese).

Zeng Qingcun (1979), — Physical and mathematical foundation for numerical weather forecast (in Chinese), Science Press, 22—25, 543 pp.

Zeng Qingcun et al. (1985), A test for the difference scheme of a general circulation model, *Acta Meteorologica Sinica*, **43**: 441—449.