# The Effect of Spatial Structure Character of Heat Source on the Ray Path and the Evolution of Wave Energy of Meridional Wave Train

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Received April 21, 1990; revised July 22, 1990

ABSTRACT

This paper studies correlations between the spatial structure character of thermal forcing and deformation and the amplitude of rays of meridional wave train. It is shown that if thermal forcing appears a meridional linear variation the rays of quasi–stationary planetary wave may propagate along oblique lines and if the meridional variability of heat source has second order term the rays show distinct deformation as a great circular route. Additionally, the inhomogeneous distribution may cause lower frequency oscillations in mid– and low–latitudes. The combination of zonal and meridional wave numbers and distributive character of heat source may form an inverse mechanism of variational trend of generized wave energy, reflecting in some degree the physical process of transition between meridional and zonal flow patterns.

## I. INTRODUCTION

In recent years, some substantial advances on research of teleconnection of the general circulation have been made. Wallace and Gutzler (1981) found that if there is anomalous heating (El Nino) in equatoral eastern and middle Pacific a Pacific–North America(PNA) teleconnection pattern will appear in areas far away from the forcing in the Northern Hemisphere. Additionally, the western Pacific (WP) and western Atlantic (WA) teleconnection patterns are also suggested in the Northern Hemisphere circulation. The teleconnection analysis indicates that the distribution of correlation coefficient in those teleconnections appears as a propagating wave train along a special path. Hoskins and Koroly (1981) proposed a great circular route to explain the phenomenon. Huang (1984) pointed out that quasi–stationary planetary wave in a baroclinic atmosphere may propagate into the troposphere in middle and high latitudes in summer.

Fig.1 shows the ray path of quasi–stationary planetary waves forced by the thermal forcing near the Philippines (Huang et al., 1988).

It is attempted in this paper to investigate the physical mechanism of the formation of great circular route of quasi–stationary planetary wave from the spatial distribution of thermal source.

As pointed out by Nitta (1986) and Maruyama et al. (1986) when the sea temperature in tropical western Pacific increases anomalously, the convections over the western Pacific and the Philippines will be enhanced and there is a strong heating source over the Philippines and the South China Sea. The heating anomaly over the western Pacific is distinctly correlated with the anomalies of general circulation over East Asia and climatological state in areas of North China and Japan.

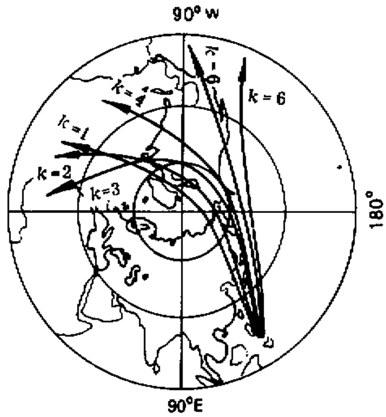


Fig.1. Propagation path of the quasi–stationary planetary wave forced by the thermal forcing near the Philippines.

By using WKB approximation, this paper studies relations between characters of spatial structure of heating source simalar with the abovementioned and deformation of propagation path of meridional wave train and amplitude variations in order to explore the evolutions of propagation path of the quasi–stationary planetary waves and disturbance intensity forced by the tropical thermal source.

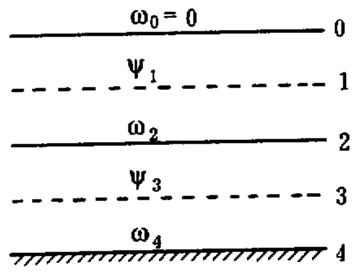


Fig.2. Rays along straight lines (dashed lines) and great circular route (solid lines).

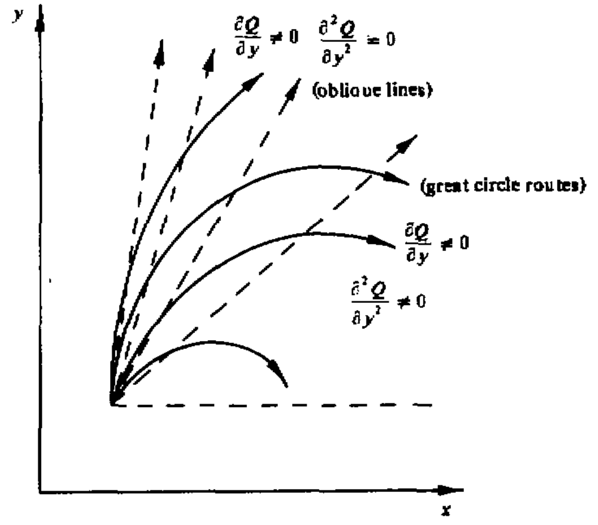


Fig.3. Vertical representation of a two-layer model.

The results of this research show that if the thermal forcing varies linearly ( $\partial Q / \partial y$ = constant) in meridional direction, rays of quasi-stationary planetary wave propagate along oblique lines and if the meridional variability of the thermal forcing has second order term ($\partial^2 Q / \partial y^2 \neq 0$) rays of the quasi-stationary planetary wave show a remarkable deformation and propagate along parabolic arch paths (Fig.2).

II. SIMPLIFIED MATHEMATICAL MODEL

Thermal forcing are included in a two-layer quasi-geostrophic model and its vertical representation is schematically shown in Fig.3. The kinetic and thermal equations can be written as:

$$\frac{\partial \zeta_3}{\partial t} + \vec{V}_3 \cdot \nabla(\zeta_3 + f) = f_0 \frac{\partial \omega_3}{\partial p}, \tag{1}$$

$$\frac{\partial \zeta_1}{\partial t} + \vec{V}_1 \cdot \nabla(\zeta_1 + f) = f_0 \frac{\partial \omega_1}{\partial p}, \tag{2}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial \psi}{\partial p}\right)_2 + \vec{V}_2 \cdot \nabla\left(\frac{\partial \psi}{\partial p}\right)_2 = -\frac{c_s^2}{f_0 p_2^2}\omega_2 - \frac{R\dot{Q}}{c_p p_2 f_0}, \tag{3}$$

where $\dot{Q}$ is diabatic heating rate, $c_s^2 = \alpha_0 RT$, $\alpha_0 = \frac{R}{g}(\Gamma_d - \Gamma)$, $\psi = \frac{\varphi}{f_0}$. The top and bottom conditions are respectively $\omega_0 = 0$ ($z = H$ at the top of the atmosphere) and $\omega_4 = -\lambda' \zeta_3$ ($z = 0$ at the bottom boundary, $\lambda' = \rho_0 g(\nu / 2f_0)^{1/2}$),

$$\text{setting} \quad \begin{bmatrix} \psi_1 \\ \psi_3 \end{bmatrix} = \begin{bmatrix} \bar{\psi} + \psi'_1 \\ \bar{\psi} + \psi'_3 \end{bmatrix}. \tag{4}$$

$$\begin{bmatrix} \psi^* \\ \bar{\psi} \end{bmatrix} = \begin{bmatrix} (\psi_1 + \psi_3)/2 \\ (\psi_1 - \psi_3)/2 \end{bmatrix} = \begin{bmatrix} \bar{\psi} + (\psi'_1 + \psi'_3)/2 \\ (\psi_1 - \psi'_3)/2 \end{bmatrix}. \tag{5}$$

$$\begin{bmatrix} \zeta_1 \\ \zeta_3 \end{bmatrix} = \nabla^2 \begin{bmatrix} \psi_1 \\ \psi_3 \end{bmatrix}. \tag{6}$$

$$\frac{\partial \omega_k}{\partial p} = (\omega_{k+1} - \omega_{k-1})/2\Delta p \qquad (k = 1,2,3). \tag{7}$$

After substituting (4)–(7), the top and bottom boundary conditions and rearranging, Eqs.(1)–(3) merge into one as:

$$\frac{\partial}{\partial t}(\nabla^2 - \tilde{\lambda}^2)\bar{\psi} + J(\psi^*,\ \nabla^2 \bar{\psi}) + J(\bar{\psi},\ \nabla^2 \psi^*) - \tilde{\lambda}^2 J(\psi^*, \bar{\psi})$$
$$+ \beta \frac{\partial \bar{\psi}}{\partial x} = -\frac{\tilde{\lambda}^2 R}{2 c_p f_0} \dot{Q} + \frac{f_0 g}{R T_4} \sqrt{\frac{v}{2 f_0}}\, \nabla^2 (\psi^* - \bar{\psi}), \tag{8}$$

where $\tilde{\lambda}^2 = f_0^2 / 2\sigma(\Delta p)^2$, $\sigma = c_\alpha^2 / p_2^2$ , and

$$J(A,B) = \frac{\partial A}{\partial x}\frac{\partial B}{\partial y} - \frac{\partial A}{\partial y}\frac{\partial B}{\partial x}. \tag{9}$$

If the basic flow is only the founction of $y$, i.e.

$$\bar{\psi} = \bar{\psi}(y). \tag{10}$$

therefore

$$\begin{bmatrix} \bar{u} \\ \bar{v} \end{bmatrix} = \begin{bmatrix} -\dfrac{\partial \bar{\psi}}{\partial y} \\ 0 \end{bmatrix}, \tag{11}$$

$$\begin{bmatrix} u' \\ v' \end{bmatrix} = \begin{bmatrix} -\dfrac{\partial \psi'}{\partial y} \\ \dfrac{\partial \psi'}{\partial x} \end{bmatrix}. \tag{12}$$

Eq.(8) can be linearized as:

$$\left(\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x}\right)\left(\nabla^2 - \tilde{\lambda}^2\right)\tilde{\psi}' + \left(\beta - \frac{d^2 \bar{u}}{d y^2}\right)\frac{\partial \tilde{\psi}'}{\partial x} = -\frac{1}{2}\frac{\tilde{\lambda}^2 R}{c_p f_0}\dot{Q}. \tag{13}$$

where $\tilde{\psi}' = (\psi'_1 - \psi'_3)/2$, and the term associated with the friction in the boundary layer is omitted.

Eq.(13) shows the correlation among thermal forcing, $\beta$ factor, characters of the westerly profile,stratification stability with the evolution of disturbances in the atmospheric thickness field.

If the diabatic heating $\dot{Q}$ is mainly constituted of meridional in homogeneous distribution of thermal forcing and its relevant meridional advection,namely, $\dot{Q} \approx -c^* \tilde{v}\frac{\partial Q}{\partial y}$, where $\tilde{v} = \frac{\partial \tilde{\psi}'}{\partial x}$. Substituting the relation into Eq.(13), we yield

$$\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)\left(\nabla^2-\tilde{\lambda}^2\right)\tilde{\psi}'+\left(\beta-\bar{u}''-\tilde{\lambda}^{*2}\frac{\partial Q}{\partial y}\right)\frac{\partial\tilde{\psi}'}{\partial x}=0, \tag{14}$$

where $\tilde{\lambda}^{*2}=\frac{\tilde{\lambda}^2 Rc^*}{2c_p f_0}$, $c^*$ is a constant coefficient, Eq.(14) is similar to the integral form of quasi–geostrophic vorticity equation in Lu's paper (1981).

## III. MERIDIONAL DISTRIBUTION OF THERMAL SOURCE AND PROPAGATION OF ENERGY

In the propagation problem of quasi–stationary planetary waves, amplitudes of waves can be assumed to be a slowly varied quantity, and therefore $\tilde{\psi}'$ may be expressed in the form by multi–scale method, namely

$$\tilde{\psi}'=A(X,Y,T)e^{i\theta/\varepsilon} \tag{15}$$

where $X=\varepsilon x$, $Y=\varepsilon y$, $T=\varepsilon t$, $\theta=kX+lY-\omega T$, and $\omega$, $k$, $l$ are local transient frequency and wave numbers.

$$\begin{bmatrix}\omega\\ k\\ l\end{bmatrix}=\begin{bmatrix}-\frac{\partial\theta}{\partial T}\\ \frac{\partial\theta}{\partial X}\\ \frac{\partial\theta}{\partial Y}\end{bmatrix}, \tag{16}$$

and

$$\begin{bmatrix}\frac{\partial k}{\partial Y}\\ \frac{\partial l}{\partial T}\\ \frac{\partial\omega}{\partial X}\end{bmatrix}=\begin{bmatrix}\frac{\partial l}{\partial X}\\ -\frac{\partial\omega}{\partial Y}\\ -\frac{\partial k}{\partial T}\end{bmatrix}. \tag{17}$$

Substituting (15) and (16) into Eq.(14) gives

$$\begin{aligned}&\left(-i\omega+\varepsilon\frac{\partial}{\partial T}+i\bar{u}k+\varepsilon\bar{u}\frac{\partial}{\partial X}\right)\left[\varepsilon^2\left(\frac{\partial^2 A}{\partial X^2}+\frac{\partial^2 A}{\partial Y^2}\right)+i\varepsilon\left(2k\frac{\partial A}{\partial X}+2l\frac{\partial A}{\partial Y}\right.\right.\\&\left.\left.+A\frac{\partial k}{\partial x}+A\frac{\partial l}{\partial Y}\right)-(k^2+l^2+\tilde{\lambda}^2)A\right]+\left(\beta-\bar{u}''-\tilde{\lambda}^{*2}\frac{\partial Q}{\partial y}\right)\\&\left(ikA+\varepsilon\frac{\partial A}{\partial x}\right)=0.\end{aligned} \tag{18}$$

According to small parameter method,the amplitude $A(X,Y,T)$ can be expressed in the form

$$A(X,Y,T)=A_0(X,Y,T)+\varepsilon A_1(X,Y,T)+\varepsilon^2 A_2(X,Y,T)+\cdots\cdots. \tag{19}$$

Substituting (19) into Eq.(18), we obtain zero order approximation equation

$$[(\bar{u}k-\omega)(k^2+l^2+\tilde{\lambda}^2)-(\beta-\bar{u}''-\tilde{\lambda}^{*2}\frac{\partial Q}{\partial y})k]A_0=0. \tag{20}$$

Because $A_0\neq 0$, therefore

$$\omega=\bar{u}k-\frac{(\beta-\bar{u}''-\tilde{\lambda}^{2}Q^{*})k}{(k^{2}+l^{2}+\tilde{\lambda}^{2})} \tag{21}$$

where $\omega$ is local transient wave frequency and $k$, $l$ are local transient wave unmbers in $X,Y$ direction respectively, therefore $X,Y$ components of quasi–stationary planetary waves are as follows:

$$\begin{aligned} C_{gX}&=\bar{u}-\frac{(\beta-\bar{u}''-\tilde{\lambda}^{2}Q^{*})}{(k^{2}+l^{2}+\tilde{\lambda}^{2})}+\frac{2(\beta-\bar{u}''-\tilde{\lambda}^{2}Q^{*})k^{2}}{(k^{2}+l^{2}+\tilde{\lambda}^{2})^{2}} \\ &=\bar{u}+(k^{2}-l^{2}-\tilde{\lambda}^{2})(\beta-\bar{u}''\tilde{\lambda}^{2}Q^{*})/(k^{2}+l^{2}+\tilde{\lambda}^{2})^{2}, \\ C_{gY}&=\frac{2(\beta-\bar{u}''-\tilde{\lambda}^{2}Q^{*})kl}{(k^{2}+l^{2}+\tilde{\lambda}^{2})^{2}}. \end{aligned} \tag{22}$$

where $Q^{*}$ is $\frac{\partial Q}{\partial y}$, i.e. the meridional gradient of diabatic heating.

It can be seen from Eqs.(21) and (22) that the thermal forcing may affect the frequency $\omega$ of local transient wave and the character $(C_{gX},C_{gY})$, of energy propagation of planetary waves, especially, Eq.(21) indicates that if $\partial Q/\partial y<0$, frequency $\omega$ will be reduced, vice versa. The existence of the tropical high temperature zone makes the condition $\partial Q/\partial y<0$, satisfy in lower and middle latitudes, resulting the lower frequency oscillation. Therefore,the meridional inhomogeneity of diabatic heating is also one of the factors which cause the lower frequency oscillation in the lower and middle latitudes. Additionally, it is known from $C_{gY}$ representative that the planetary waves may make distinct meridional propagation near the diabatic heating source, namely

$$\begin{aligned} &\text{if } Q^{*}>0,\ \text{i.e. } \frac{\partial Q}{\partial y}>0,\ \text{favourable to } C_{gY}<0, \\ &\text{if } Q^{*}<0,\ \text{i.e. } \frac{\partial Q}{\partial y}<0,\ \text{favourable to } C_{gY}>0. \end{aligned} \tag{23}$$

If the basic flow $\bar{u}$ is omitted and only $\beta$ and diabatic heating factor are considered, the first order approximation equation has the form

$$\begin{aligned} \frac{\partial}{\partial T}[(k^{2}+l^{2}+\tilde{\lambda}^{2})A_{0}]-\omega\left(2k\frac{\partial A_{0}}{\partial X}+2l\frac{\partial A_{0}}{\partial Y}+A_{0}\frac{\partial k}{\partial X}+A_{0}\frac{\partial l}{\partial Y}\right) \\ -(\beta-\tilde{\lambda}^{2}Q^{*})\frac{\partial A_{0}}{\partial X}=0. \end{aligned} \tag{24}$$

multiplying Eq.(24) by $2A_{0}\tilde{K}^{2}$ yields

$$\begin{aligned} \frac{\partial}{\partial T}(\tilde{K}^{4}A_{0}^{2})-\tilde{K}^{2}\cdot 2\omega\left(k\frac{\partial A_{0}^{2}}{\partial X}+l\frac{\partial A_{0}^{2}}{\partial Y}\right)-\tilde{K}^{2}\cdot 2\omega A_{0}^{2}\left(\frac{\partial k}{\partial X}+\frac{\partial l}{\partial Y}\right) \\ -\tilde{K}^{2}(\beta-\tilde{\lambda}^{2}Q^{*})\frac{\partial A_{0}^{2}}{\partial X}=0 \end{aligned} \tag{25}$$

where $\tilde{K}^{2}=k^{2}+l^{2}+\tilde{\lambda}^{2}$.

Because

$$\frac{\partial}{\partial X}(\tilde{K}^{4}A_{0}^{2}C_{gX})=-\tilde{K}^{2}\frac{\partial A_{0}^{2}}{\partial X}(\beta-\tilde{\lambda}^{2}Q^{*})+2k^{2}(\beta-\tilde{\lambda}^{2}Q^{*})\frac{\partial A_{0}^{2}}{\partial X}$$

$$
\begin{aligned}
&-2A_0^2(\beta-\tilde{\lambda}^{*2}Q^*)\left(k\frac{\partial k}{\partial X}+l\frac{\partial l}{\partial X}\right)\\
&+4A_0^2(\beta-\tilde{\lambda}^{*2}Q^*)k\frac{\partial k}{\partial x}+\tilde{K}^2A_0^2\tilde{\lambda}^{*2}\frac{\partial Q^*}{\partial X}\\
&-2k^2A_0^2\frac{\partial Q^*}{\partial X}\tilde{\lambda}^{*2}\,,
\end{aligned}
\tag{26}
$$

$$
\begin{aligned}
\frac{\partial}{\partial Y}(\tilde{K}^4A_0^2C_{gY})=&\,2lk(\beta-\tilde{\lambda}^{*2}Q^*)\frac{\partial A_0^2}{\partial Y}+2A_0^2\Big[(\beta-\tilde{\lambda}^{*2}Q^*)l\frac{\partial k}{\partial Y}\\
&+(\beta-\tilde{\lambda}^{*2}Q^*)k\frac{\partial l}{\partial Y}+kl\left(\frac{\partial\beta}{\partial Y}-\tilde{\lambda}^{*2}\frac{\partial Q^*}{\partial Y}\right)\Big]
\end{aligned}
$$

Eq.(26) can be written in the following form by using relations (17)

$$
\begin{aligned}
\frac{\partial\tilde{q}_0}{\partial T}+\nabla_h\cdot(\tilde{q}_0\vec{C}_g)=&\,(\tilde{K}^2A_0^2\tilde{\lambda}^{*2}-2k^2A_0^2\tilde{\lambda}^{*2})\frac{\partial Q^*}{\partial X}\\
&+2lkA_0^2\left(\frac{\partial\beta}{\partial Y}-\tilde{\lambda}^{*2}\frac{\partial Q^*}{\partial Y}\right).
\end{aligned}
\tag{27}
$$

where $\tilde{q}_0=\tilde{K}^4A_0^2$ is the density of generalized wave energy.

Integrating (27) in a fixed volume and using the divergence theorem, we obtain the local rate of generalized wave energy

$$
\begin{aligned}
\int_v\frac{\partial\tilde{q}_0}{\partial T}dv=&\int_v(\tilde{K}^2A_0^2\tilde{\lambda}^{*2}-2k^2A_0^2\tilde{\lambda}^{*2})\frac{\partial Q^*}{\partial x}dv\\
&+\int_v 2lkA_0^2\left(\frac{\partial\beta}{\partial Y}-\tilde{\lambda}^{*2}\frac{\partial Q^*}{\partial Y}\right)dv.
\end{aligned}
\tag{28}
$$

As known from (28), the tendency of generalized wave energy density is correlated with the spatial distributions ($\partial Q^*/\partial X$, $\partial Q^*/\partial Y$) of diabtic term and $\beta$ factor. If the spatial distribution of diabatic term shows a South–high and North–low trend, i.e. $\partial Q^*/\partial Y<0$, then there will be an enhancing tendency of eddy of generalized wave energy in the south of the strong baroclinic zone (frontal zone) caused by diabatic factors, i.e. $\partial\tilde{q}_0/\partial T>0$; it is not favourable to the enhancement of generalized wave energy in the north of the baroclinic zone, i.e. $\partial\tilde{q}_0/\partial T<0$.

Omitting the stability factor $\tilde{\lambda}^{*2}$ in right hand terms of (28), we obtain

$$
\frac{\partial\tilde{q}_0}{\partial T}\propto(l^2-k^2)A_0^2\tilde{\lambda}^{*2}\frac{\partial Q^*}{\partial X}
\tag{29}
$$

Eq.(29) reflects the relations between the tendency of generalized wave energy and zonal wave number, the meridional wave number and spatial distribution of $Q^*$, namely

$$
l^2-k^2\leqslant 0,\ \left(\frac{\partial Q^*}{\partial x}>0\right)\ \frac{\partial\tilde{q}_0}{\partial T}\leqslant 0.
\tag{30}
$$

If the initial flow is assumed to be a zonal pattern, i.e. $k\approx 0$, and it is during the transition from zonal pattern to meridional pattern, i.e. $k$ is increasing, when $l^2-k^2>0$ becomes $l^2-k^2<0$ (meridional patterns dominate), then the eddy tendency of generalized wave energy will change inversely, i.e. the tendency sign—becomes opposite, although the distribution of $Q^*$ field is not varied. This suggests that under the condition of the external thermal forcing being constant, the eddy tendency of wave energy in circumstances of the formation and decay of blocking is closely related with the characters of flow pattern ($l$, $k$ relation). During the stages before and after the formation of blocking, the thermal forcing may play an opposite self—adjustment role due to changes of flow pattern.

## IV. EFFECT OF THERMAL FORCING ON THE AMPLITUDE OF TELECONNECTION MERIDIONAL WAVE TRAIN

Setting

$$\tilde{\psi}' = \psi^*(y) e^{i(kx - \omega t)}, \tag{31}$$

and substituting (31) into Eq.(14) yields

$$\psi^{*''}(y) + \left[\frac{\beta(y) - \bar{u}'' - \tilde{\lambda}^{2*}\dfrac{\partial Q}{\partial y}}{\bar{u}(y)(1 - \omega / \bar{u}(y)k)} - k^2\right]\psi^*(y) = 0, \tag{32}$$

Namely

$$\psi^{*''}(y) + v^2(y)\psi^*(y) = 0, \tag{33}$$

where

$$v^2(y) = \frac{\beta - \bar{u}'' - \tilde{\lambda}^{2*}\dfrac{\partial Q}{\partial y}}{\bar{u}(1 - \omega / \bar{u}k)} - k^2. \tag{34}$$

Using WKB approximation, we obtain

$$\psi(x,y,t) = \tilde{C} v^{-\frac{1}{2}}(y) e^{i(kx + \int^y v(y)dy - \omega t)} \tag{35}$$

Therefore, the relation of thermal forcing and the amplitude of teleconnection disturbances are as follow:

$$|\psi|_{\text{amplitude}} \propto v^{-\frac{1}{2}}(y) \propto \left(\frac{\bar{u}}{\beta - \bar{u}'' - \tilde{\lambda}^{2*}\dfrac{\partial Q}{\partial y} - k^2\bar{u}}\right)^{-1/4}. \tag{36}$$

As known from the above relation, the amplitude of teleconnection wave train is related with $\beta, \bar{u}, k$ and the spatial distribution of diabatic heating, generally the higher the latitude, the small the $\beta$ and the greater the amplitude of teleconnection wave train. Furthermore, it is also related with characters of westerly profiles and the diabatic heating field. If $\partial Q / \partial y < 0$, the eddy amplitude of meridional wave train will decay and if $\partial Q / \partial y > 0$, it will result in the enhancement of the eddy amplitude. Therefore the amplitude of meridional wave train, during the propagation from the lower to the higher latitudes, is not only affected by the $\beta$ factor and the westerly profile character, but the effect of distribution character of diabatic heating is also not negligible, especially, the combination relation of $\beta$, $\bar{u}$, $\bar{u}''$, $k$, and $\partial Q / \partial y$ must be considered in accounting for the amplitude variation.

## V. CORRELATIONS BETWEEN THE DISTRIBUTION CHARACTER AND MERIDIONAL PROPAGATION PATH OF QUASI—STATIONARY PLANETARY WAVES

After neglecting the meridional profile and stability parameter and introducing the abovementioned thermal forcing $\vec{Q}$, the ray equation of quasi—stationary planetary wave can be written as:

$$\frac{C_{gY}}{C_{gX}}=\frac{1}{k}\sqrt{\frac{\beta-\bar{\lambda}^{2}\frac{\partial Q}{\partial y}}{\bar{u}}-k^{2}}\,, \tag{37}$$

$$\frac{dY}{dX}=\frac{\varepsilon dy}{\varepsilon dx}=\frac{dy}{dx}. \tag{38}$$

The effects of $\beta$ factor and the meridional distribution characters of heat source are not negligible for the teleconnection wave train which propagates through a long distance from the lower latitudes to the higher latitudes, for example, when the thermal source shows an elliptical distribution or meridional parabolic variations.

The Taylor expansion approximation is taken here, and the meridional variation functions of $\beta$ and thermal source are

$$\begin{bmatrix} f(y) \\ Q(y) \end{bmatrix}=\begin{bmatrix} f_0 \\ Q(y_0) \end{bmatrix}+\begin{bmatrix} \beta y+\frac{1}{2}\frac{d\beta}{dy}y^2+\cdots\cdots \\ \frac{\partial Q_0}{\partial y}y+\frac{1}{2}\frac{\partial^2 Q_0}{\partial y^2}y^2+\cdots\cdots \end{bmatrix}, \tag{39}$$

where $f_0$ and $Q(y_0)$ are constants, which are related with the Coriolis parameter in the initial location and initial state of heat source.

Assuming $\beta$, $\frac{\partial Q_0}{\partial y}$, $\frac{\partial^2 Q_0}{\partial y^2}$, being constant and taking the first order term in the meridional rate of heat source as $\frac{\partial Q_0}{\partial y}\neq 0$ and $\frac{\partial^2 Q_0}{\partial y^2}=0$ the integral form of ray equation can be written as:

$$\int_{y_0}^{y}\frac{k\,dy}{\sqrt{\frac{\beta-\bar{\lambda}^{2}\frac{\partial Q_0}{\partial y}}{\bar{u}}-k^2}}=x-x_0, \tag{40}$$

If $\bar{u}>0$, $\beta-\bar{\lambda}^{2}\frac{\partial Q_0}{\partial y}>k^2\bar{u}$ or $\bar{u}<0$, $\beta-\bar{\lambda}^{2}\frac{\partial Q_0}{\partial y}<k^2\bar{u}$ then the wave train may propagate meridionally. This suggests that the meridional propagation of wave train in the easterly or westerly wind may occur only under some special combinations of thermal source and $\beta$ factor. Generally, the wave train will be trapped meridionally in the easterly without special combination of heat source. Under the condition with heat sources, if $\beta<\bar{\lambda}^{2}\frac{\partial Q_0}{\partial y}+k^2\bar{u}$, then the wave train may propagate meridionally, and if $\beta>\bar{\lambda}^{2}\frac{\partial Q_0}{\partial y}+k^2\bar{u}$, then the meridional wave train will be trapped in easterly basic flows.

Under the meridional propagation conditions mentioned above, integrating Eq.(40) yields

$$y - y_0 = tg\alpha(x - x_0), \tag{41}$$

where

$$tg\alpha = \frac{1}{k}\sqrt{\left(\beta - k^2\bar{u} - \bar{\lambda}^2 \frac{\partial Q_0}{\partial y}\right) / \bar{u}} \tag{42}$$

it's values represent a set of slope of oblique line of ray, which are related with zonal number, $\beta$ factor, speed of westerly and heat source distribution. This suggests that although the ray paths appear as oblique lines, they have a close relation with the factors including the heat source distribution. When other factors are fixed, if $\frac{\partial Q_0}{\partial y} > 0$, then

$$\beta_{\min} = k^2\bar{u} + \bar{\lambda}^2 \frac{\partial Q_0}{\partial y} \qquad \text{(slopes being zero)} \tag{43}$$

i.e. rays propagate along parallels of latitudes, and if, $\frac{\partial Q_0}{\partial y} < 0$, then it is easy for rays to propagate meridionally.

If the fisrt three terms of $Q(y)$ in (39) are considered (i.e. $\frac{\partial Q_0}{\partial y} \neq 0$; $\frac{\partial^2 Q_0}{\partial y^2} \neq 0$), the relevant ray integral equation has the form

$$\int_{y_0}^{y} \frac{K dy'}{\sqrt{\dfrac{\beta - k^2\bar{u} - \bar{\lambda}^2 \left(\dfrac{\partial Q_0}{\partial y} + \dfrac{\partial^2 Q_0}{\partial y^2} y\right)}{\bar{u}}}} = x - x_0. \tag{44}$$

solving the Eq.(44) yields

$$y = \frac{1}{\bar{\lambda}^2 \dfrac{\partial^2 Q_0}{\partial y^2}}\left(\beta - k^2\bar{u} - \bar{\lambda}^2 \frac{\partial Q_0}{\partial y}\right) - \frac{\bar{\lambda}^2 \dfrac{\partial^2 Q_0}{\partial y^2}}{4\bar{u}k^2}(x - x_0^*)^2, \tag{45}$$

which represents parabolic arch curves, and

where

$$x_0^* = x_0 + \frac{2\sqrt{\bar{u}}K}{\bar{\lambda}^2 \dfrac{\partial^2 Q_0}{\partial y^2}}\sqrt{\left(\beta - k^2\bar{u} - \bar{\lambda}^2 \frac{\partial^2 Q_0}{\partial y^2}\right) - \bar{\lambda}^2 \frac{\partial^2 Q_0}{\partial y^2} y_0}.$$

Fig.2 shows that if the second order meridional rate is considered, ray paths change from initial oblique lines to parabolic arch curves, i.e.great circular routes. This reveals to some extent that the distribu– tive character of heat source will distinctly affect the meridional propagation route of quasi–stationary planetary wave.

At the turning point of ray, where

$$\left.\frac{dy}{dx}\right|_{y=y_{max}}=0, \tag{46}$$

Eq.(46) can be expressed as:

$$\sqrt{\frac{\beta-\bar{\lambda}^{2}\cdot\left(\frac{\partial Q_0}{\partial y}+\frac{\partial^2 Q_0}{\partial y^2}y\right)_{max}}{\bar{u}}}-k^2=0, \tag{47}$$

and

$$y_{max}=\frac{\beta-k^2\bar{u}-\bar{\lambda}^{2}\cdot\frac{\partial Q_0}{\partial y}}{\bar{\lambda}^{2}\cdot\frac{\partial^2 Q_0}{\partial y^2}}. \tag{48}$$

As known from Eqs.(47) and (48), ray paths may have turning point if the second order term in meridional distribution of heat source is considered. The smaller the zonal wave number, and the greater the $\beta$, the higher the latitude of turning point of ray. Additionally, if heat source shows a south–high and north–low pattern (there is a high value of diabatic heating in the lower latitudes and a low value in polar areas) i.e. $\frac{\partial Q_0}{\partial y}<0$, and the meridional rate of second order of heat source is smaller, then rays will turn at a hige latitude.

Zonal great circular redius of the parabolic arch curve is

$$R_{max}^2=(x-x_0^{\cdot})^2=\frac{4\left(\beta-k^2\bar{u}-\bar{\lambda}^{2}\cdot\frac{\partial Q_0}{\partial y}\right)k^2\bar{u}}{\left(\bar{\lambda}^{2}\cdot\frac{\partial^2 Q_0}{\partial y^2}\right)^2}. \tag{49}$$

As known from(49), zonal great circular radius is also related with characters of heat source distribution, $\beta$ factor and westerly profile, especially with the fisrt and second order meridional rates of heat source. Therefore, the heat source is not a negligible factor to meridional teleconnection wave trains. By using the above simplified mathematical treatment,it becomes easy to discuss the problem directly and reveal the physical mechanism of great circular route of ray and it's affecting factors.

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