A–B Hybrid Equation Method of Nonlinear Bifurcation in Wave–Flow Interaction①

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Received January 15, 1990; revised November 5, 1990

ABSTRACT

In this paper, A–B hybrid equation method is given. This method is different not only from high truncated spectral method, but also from amplitude evolution method. Dynamic problem in the baroclinic atmosphere may be transferred into complex Lorenz system by means of the method. Therefore, this method is an effective tool for studying nonlinear bifurcation in wave–flow interaction. Meanwhile, it is of advantage to use this method, because it can overcome a lot of difficulties existing in high truncated spectral method and amplitude evolution method.

I. INTRODUCTION

The nonlinear stability in wave–flow interaction is a basic problem in the atmospheric dynamics. In this field, many outstanding papers and works have been published. Quite a few well–known meteorologists in the world have made admirable contributions to the study of this field. Some of them have achieved considerable success. When studying two–dimensional convective problem, Lorenz (1963) introduced the Lorenz system and set up a new way for researching the problem of nonlinear stability. When studying the transformation of high–index, low–index and blocking, Charney (1979) systematically illustrated a high truncated spectral method. Once it was thought that the truncated spectral method is a bridge for transferring a physical problem or physical model into Lorenz system. However, when studying some problems by means of truncated spectral method, we met some difficulties, because differently truncated spectra influenced bifurcation points in Lorenz system. Besides, high truncated spectral method itself has two evident faults. The first is that the truncated spectral method, in fact, loses some correlative perturbation terms (namely, Reynolds stress) because of artificial spectral truncation(Pedlosky, 1981). Whereas, the lost correlative perturbation terms are exactly one of the principal factors of changing the basic flow. The second is that choosing spectral function is quite skillful. In addition to making spectral functions satisfying $\nabla^2 F_i = -a^2 F_i$, the spectral functions must still embody mainly dynamic process of the physical problem studied. However, the different research objects are of the different dominant dynamical factors. Hence, the alternative of spectral functions depends on the researcher's experiences. This kind of skill troubles many researchers.

For the sake of overcoming some faults of truncated spectral method, Pedlosky (1979) obtained the amplitude equation of perturbation stream function, using perturbation method. Then the amplitude equation was transferred into Lorenz system via elaborately choosing parameters. The selection of parameters needs quite high skill. It is impossible to

①The Project Supported by National Natural Science Foundation of China

transfer the amplitude equation into Lorenz system if the parameters are not correctly chosen. In short, the selection of parameters also puzzled many meteorologists. In addition, the transformation from amplitude equation into Lorenz system is easily achieved in the barotropic atmosphere rather than in the baroclinic atmosphere.

Obviously, the key problem we faced is whether we can find a method to fulfil the transformation of physical problem into Lorenz system. If so, could this method overcome above shortcomings? In order to answer these questions here, we introduced A–B hybrid equation method.

II. A–B HYBRID EQUATION

Theory of wave–flow interaction indicates that on the one hand, unstable wave must have adjustment heat flux and momentum flux. The convergence of this flux may alter the velocity of the basic flow. On the other hand, the change of the basic flow velocity necessarily gives the feedback to wave disturbance. The supercriticality of the basic flow velocity in the baroclinic atmosphere (it is mainly regarded as the degree of the vertical shear of the basic flow) is a typical example of feedback. In this section, therefore, we derive A–B hybrid equation on the condition of supercriticality. For narrating the method clearly, we adopt the model of f–plane approximation in the baroclinic atmosphere. That is,

$$\left[\frac{\partial}{\partial t}+\frac{\partial\psi_1}{\partial x}\frac{\partial}{\partial y}-\frac{\partial\psi_1}{\partial y}\frac{\partial}{\partial x}\right]\left[\nabla^2\psi_1-\hat{F}(\psi_1-\psi_2)\right]=-\frac{r}{2}\nabla^2\psi_1\ , \tag{1}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial\psi_2}{\partial x}\frac{\partial}{\partial y}-\frac{\partial\psi_2}{\partial x}\frac{\partial}{\partial x}\right]\left[\nabla^2\psi_2-\hat{F}(\psi_2-\psi_1)\right]=-\frac{r}{2}\nabla^2\psi_2\ . \tag{2}$$

For a wave with total wave number K defined by $K=\sqrt{k^2+m^2\pi^2}$, Pedlosky (1987) proved that when this wave becomes instability, the corresponding critical value of the basic flow shear is U_s^*, and the critical value of $\hat{F}$ is F_c. Under the condition of supercritical state, $\hat{F}$ may be written as

$$\hat{F}=F_c+\Delta=\frac{K^2}{2}+\frac{r^2K^2}{2k^2U_s^{*2}}+\Delta, \tag{3}$$

where $\Delta<<F_c$. Suppose we now consider placing a wavelike disturbance on the basic state. The upper and lower velocities of the basic state are U_1 and U_2, and they are anti–symmetric, i.e., $U_1=-U_2$. Each of the critical parameters in the problem can be ordered with respect to a_0, i.e.,

$$\begin{aligned}\Delta&=O(a_0^2)\\ \sigma&=O(a_0)\\ r&=O(a_0)\ ,\end{aligned} \tag{4}$$

where a_0 is a disturbance amplitude, r is a frictional coefficient, Δ is a supercritical value of vertical shear of the basic state, σ is a transformation factor of time scale of amplitude evolution.

We take

$$T=\sigma t, \tag{5}$$

T is a slow time scale of amplitude evolution. By means of these relationships, ψ may be expanded in the form of the asymptotic series with respect to a_0.

$$\psi_n=\psi_n^{(0)}+a_0(\varphi_n^{(1)}+a_0\varphi_n^{(2)}+a_0^2\varphi_n^{(3)}+\cdots) \tag{6}$$

If the flow is within the bounds of $y=0$ and $y=1$ bounded by rigid walls, then the meridional velocity is to vanish. Boundary conditions may be written as

$$\frac{\partial \varphi_n}{\partial x}=0 \qquad y=0,1 \ . \tag{7}$$

When (6) is substituted into (1) and (2), we obtain orders of a_0 equal to the approximation of a sequence of linear problems. The first of which is the $O(a_0)$ problem:

$$\frac{U_s}{2}\frac{\partial}{\partial x}\left[\nabla^2\varphi_n^{(1)}+\frac{K^2}{2}(\varphi_n^{(1)}+\varphi_{n-(-1)^n}^{(1)})\right]=0 \ , \tag{8}$$

where $n=1,2$ (hereafter indications are just the same).
We write

$$U_s=2U_1=-2U_2 \ .$$

The solution of the form (8) may be written as

$$\varphi_n^{(1)}=ReA_n(T)e^{ikx}\sin m\pi y=A_n(T)\frac{e^{ikx}\sin m\pi y}{2}+* \ , \tag{9}$$

where m is any integer, the symbol Re reminds us that only the real part of the following expression is taken, and the symbol " * " refers to the complex conjugate of the preceding term.

Note that the lowest– order horizonal and vertical structures of the wave are determined at this order without regard to the supercriticality, the dissipation and the temporal behavior or the nonlinearity of the wave field.

The $O(a_0^2)$ order approximation yields the following inhomogeneous problem for $\varphi_n^{(2)}$.

$$\frac{U_s}{2}\frac{\partial}{\partial x}\left\{\nabla^2\varphi_n^{(2)}+\frac{K^2}{2}(\varphi_n^{(2)}+\varphi_{n-(-1)^n}^{(2)})\right\}=$$
$$-\frac{\sigma}{a_0}\left(\frac{\partial}{\partial T}+\frac{r}{2\sigma}\right)\nabla^2\varphi_n^{(1)}-J\left\{\varphi_n^{(1)},\nabla^2\varphi_n^{(1)}\right\} \ . \tag{10}$$

The total perturbation stream function up to and including $O(a_0^2)$ is rewritten as

$$\varphi_1=a_0A\frac{e^{ikx}}{2}\sin m\pi y+* \ , \tag{11}$$

$$\varphi_2=a_0\left\{A-a_0\frac{4i}{kU_s}\left(\frac{\sigma}{a_0}\right)\left[\frac{dA}{dT}+\frac{r}{2\sigma}A\right]\right\}\frac{e^{ikx}}{2} \quad \sin m\pi y+* \ . \tag{12}$$

Comparing (9) with (12), we can know that the solution of the first order is a neutral wave. The amplitudes A_1 and A_2 are alternative. A_1 and A_2 may be written as $A_1=A_2=A$.

In the first order, wave–flow interaction does not occur, because the wave is neutral. Hence, it is unnecessary to add the feedback effect of the wave on the basic flow. However, in the $O(a_0^2)$ order, the derivative term of amplitude A with respect to time is comprised in the perturbation stream function φ_2. Therefore, there is wave–flow interaction, and perturbation wave yields enhancement or decrement. In this case, there is the feedback effect of the basic flow on the wave. In order to balance this feedback effect, an additional solution is added into the solution of $O(a_0^2)$ order, i.e.,

$$\varphi_n^{(2)} = \Phi_n^{(2)}(y,T) \ . \tag{13}$$

This solution represents an $O(a_0^2)$ correction to the basic flow and trivially satisfies the homogeneous part of (10), because it is independent of x. Solutions of this type could be added at each stage of the expansion, but as we shall see that such additions are required to balance the basic flow alteration forced by the nonlinear wave fluxes of the wave perturbation.

Collecting terms of $O(a_0^3)$ yields the final problem, i.e.,

$$\begin{aligned}
&(-1)^{n-1}\frac{U_s}{2}\frac{\partial}{\partial x}\left[\nabla^2\varphi_n^{(3)} + \frac{K^2}{2}(\varphi_n^{(3)} + \varphi_{n-(-1)^n}^{(3)})\right] \\
&= -\frac{\sigma}{a_0}\frac{\partial}{\partial T}\left[\nabla^2\varphi_n^{(2)} - \frac{K^2}{2}(\varphi_n^{(2)} - \varphi_{n-(-1)^n}^{(2)})\right] \\
&\quad + (-1)^n\frac{U_s}{2}\frac{\delta}{a_0^2}\frac{\partial}{\partial x}(\varphi_n^{(1)} + \varphi_{n-(-1)^n}^{(1)}) - \frac{r}{a_0}\nabla^2\varphi_n^{(2)} \\
&\quad - J\left[\varphi_n^{(1)}, \nabla^2\varphi_n^{(2)} + \frac{K^2}{2}(\varphi_{n-(-1)^n}^{(2)} - \varphi_n^{(2)})\right] \\
&\quad - J\left[\varphi_n^{(2)}, \nabla^2\varphi_n^{(1)} + \frac{K^2}{2}(\varphi_{n-(-1)^n}^{(1)} - \varphi_n^{(-1)})\right] ,
\end{aligned} \tag{14}$$

where $\delta = \Delta + \frac{r^2}{2}U_s^2\frac{K^2}{k^2}$.

It is found that there is a response in $\varphi_n^{(3)}$ which would be linearly growing in relation to x. Since the x interval is infinite, such solutions for $\varphi_n^{(3)}$ would become large enough to invalidate the ordering implied by (6). For keeping our expansion valid, we need to eliminate secular term, i.e.,

$$\begin{aligned}
&\frac{\partial}{\partial T}\left\{\frac{\partial^2\Phi_n^{(2)}}{\partial y^2} + \frac{K^2}{2}(-1)^n(\Phi_1^{(2)} - \Phi_2^{(2)})\right\} + \frac{r}{2\sigma}\frac{\partial^2\Phi_n^{(2)}}{\partial y_1^2} \\
&= (-1)^{n+1}\frac{K^2}{2}\frac{m\pi}{U_s}\left[\frac{d}{dT}|A|^2 + \frac{r}{\sigma}|A|^2\right]\sin 2m\pi y.
\end{aligned} \tag{15}$$

The left-hand side of (15) represents the rate of the change of the correction to the zonally averaged potential vorticity modified by a loss term due to Ekman friction (Pedlosky, 1987). On the boundary conditions of $y = 0,1$, it is automatically satisfied with

$$\frac{\partial}{\partial T}\left\{\frac{\partial^2\Phi_n^{(2)}}{\partial y^2} + \frac{K^2}{2}(-1)^n(\Phi_1^{(2)} - \Phi_2^{(2)})\right\} + \frac{r}{2\sigma}\frac{\partial^2\Phi_n^{(2)}}{\partial y^2} = 0 \ . \tag{16}$$

In this problem, an additional boundary condition is

$$\frac{\partial^2\Phi_n^{(2)}}{\partial y\partial t} = 0, \qquad y = 0,1 \tag{17}$$

It implies

$$\frac{\partial^2}{\partial y\partial T}(\Phi_1^{(2)} - \Phi_2^{(2)}) \equiv 0, \qquad y = 0,1 \tag{18}$$

Eqs.(17) and (18) indicate that the adjusted meridional velocity caused by the change of zonal flow induced by waves should vanish on the boundary. This boundary condition is corresponding to that posed by Pedlosky (1979) when he studied the nonlinear baroclinic stability.

According to Pedlosky's (1987) research result, waves can only alter the basic flow by inducing a mean meridional circulation. Due to the anti–symmetry of the problem in the vertical direction, this meridional circulation must be with opposite signs in the two layers to satisfy mass conservation, i.e., $\Phi_1^{(2)}$ and $\Phi_2^{(2)}$ should satisfy

$$\Phi_1^{(2)} = -\Phi_2^{(2)} \equiv M\Phi(y,T), \tag{19}$$

where M is a constant to be determined.
Invoking (19), we can write (15) and (17) as

$$\frac{\partial}{\partial T}\left[\frac{\partial^2\Phi}{\partial y^2} - K^2\Phi\right] + \gamma\frac{\partial^2\Phi}{\partial y^2} = \frac{K^2 m\pi}{2MU_s}\left[\frac{d}{dT} + 2\gamma\right]|A|^2\sin 2m\pi y \tag{20}$$

$$\frac{\partial^2\Phi}{\partial y\partial T} \equiv 0\ , \qquad y = 0,1, \tag{21}$$

where $\gamma = \dfrac{\nu}{2\sigma}$.

In order to obtain perturbation amplitude A, only (20) and (21) are not sufficient. If we choose

$$\sigma = kU_s\Delta^{1/2}/2K \tag{22}$$

$$a_0 = \frac{U_s}{Km\pi}\Delta^{1/2}, \tag{23}$$

then by means of $O(a_0^3)$ order, we have

$$\frac{d^2A}{dT^2} + \frac{3}{2}\gamma\frac{dA}{dT} - A + A\int_0^1 \sin 2m\pi y\frac{\partial^2\Phi}{\partial y^2}dy = 0 \tag{24}$$

Eqs.(20), (21) and (24) are the basic equations to be used.

Pedlosky (1987) made a significant contribution to the derivation of the amplitude equation. After getting amplitude equation, however, he concentrated his attention to seeking the methed of salving Φ. His aim is that once Φ is found, the amplitude A could be obtained by substituting Φ into amplitude equation. This is why he did not complete the transformation of the amplitude equation to Lorenz system, although the idea is conforming to common practice.

How can we complete the transformation?

If we define

$$B = \int_0^1 \frac{\partial^2\Phi}{\partial y^2}\sin 2m\pi y\,dy\ , \tag{25}$$

(24) may be rewritten as

$$\frac{d^2A}{dT^2} + \frac{3}{2}\gamma\frac{dA}{dT} - A + AB = 0\ . \tag{26}$$

Multiplying both sides of (20) by $\sin 2m\pi y$, then integrating the equation from 0 to 1 with respect to y, we have

$$\frac{\partial B}{\partial T}+\frac{\gamma}{\left(1+\frac{K^2}{4m^2\pi^2}\right)}B=\frac{K^2 m\pi}{4\left(1+\frac{K^2}{4m^2\pi^2}\right)U_s M}\left[\frac{d}{dT}+2\gamma\right]|A|^2 , \tag{27}$$

where (21) and the methed integration by parts are used.
For convenience, we choose

$$M=\frac{K^2 m\pi}{4\left(1+\frac{K^2}{4m^2\pi^2}\right)U_s}. \tag{28}$$

Therefore, (27) may be rewritten as

$$\frac{\partial B}{\partial T}+\Delta B=\left[\frac{d}{dT}+2\gamma\right]|A|^2, \tag{29}$$

where $\Delta=\frac{\gamma}{\left(1+\frac{K^2}{4m^2\pi^2}\right)}$.

Eqs.(26) and (29) are a group of A–B hybrid equations we concerned mostly. The equations establish the base of transferring a physical problem into Lorenz system.

III. FROM A–B HYBRID EQUATIONS TO LORENZ SYSTEM

In the past, transferring Benard convection problem into Lorenz system was completed by using the truncated spectral method. Although there are some faults in it, we still got some enlightenment from the spectral method. In Benard convection model, there are two equations including two variables ψ^* and θ^*, and Benard convection problem can be transferred into Lorenz system by means of spectral expansions of ψ^* and θ^*. A–B hybrid equations have some analogies with Benard convection equations. There are also two variables A and B in A–B hybrid equation. Their differences are mainly that A and B are independent of space, and ψ^* and θ^* are functions of time and space respectively. Hence, A and B are similar to the coefficients of the spectral expansion. We only consider those factors relating to time, such as $X(T)$, $Y(T)$ and $Z(T)$.

Based on above idea, we take the following transformation.

$$\tau=\Omega T \tag{30}$$

$$X=\sqrt{2}\Omega^{-1}A \tag{31}$$

$$Y=\frac{1}{\gamma}\Omega\dot{X}+X \tag{32}$$

$$Z=\frac{1}{\gamma}\Omega^{-1}B , \tag{33}$$

where Ω is an adjustable parameter to be determined. $\dot{X}$ indicates derivative of X with respect to τ.
Invoking above transformation, (26) may be rewritten as

$$\ddot{X}+\frac{3}{2}\gamma\Omega^{-1}\dot{X}=\Omega^{-2}X-\gamma\Omega^{-1}XZ . \tag{34}$$

Taking derivative of (32) with respect to τ, then substituting the derivative of (32) into (34), we have

$$\dot{Y} = -XZ + \tilde{\gamma}X - Y, \tag{35}$$

where Ω is taken as $\Omega = \frac{1}{2}\gamma$, $\tilde{r} = (1 + 2\gamma^{-2})$.

Invoking (30)–(33), (29) may be rewritten as

$$\dot{Z} = -\frac{\gamma 4m^2\pi^2}{(4m^2\pi^2 + K^2)}\Omega^{-1}Z + \frac{1}{2\gamma}\Omega(X^*\dot{X} + X\dot{X}^*) + XX^* \tag{36}$$

For $Y^* = \frac{1}{\gamma}\Omega\dot{X}^* + X^*$, substituting Y^* into (36), we obtain

$$\dot{Z} = -\tilde{b}Z + \frac{1}{2}(X^*Y + XY^*) , \tag{37}$$

where $\tilde{b} = \dfrac{2}{\left(1 + \dfrac{K^2}{4m^2\pi^2}\right)}$, X^* and Y^* are conjugate terms of X and Y respectively.

Eqs (32),(35) and (37) construct a complex Lorenz system,i.e.,

$$\dot{X} = -\tilde{\sigma}X + \tilde{\sigma}Y \tag{38}$$

$$\dot{Y} = -XZ + \tilde{r}X - Y \tag{39}$$

$$\dot{Z} = -\tilde{b}Z + \frac{1}{2}(X^*Y + XY^*) , \tag{40}$$

where $\tilde{\sigma} = 2$ and $|A|^2 = AA^*$. A^* is conjugate term of A.

Though the transformation of A–B hybrid equation into Lorenz equation has completed, and the behaviors of perturbation amplitude may be studied in phase space, we still need to inspect whether the amplitude of A is real or complex in Lorenz system. When $\dot{X}$ $\dot{Y} = \dot{Z} = 0$, from (38), we have

$$X = Y \tag{41}$$

Substituting (41) into (39), and letting $\dot{Y} = 0$, we have

$$Z = \tilde{r} - 1 \tag{42}$$

Substituting (41), (42) into (40), and letting $\dot{Z} = 0$, we get

$$|X|^2 = \tilde{b}(\tilde{r} - 1) \tag{43}$$

and

$$\tilde{b}(\tilde{r} - 1) = \frac{k^2U_s^2\Delta}{K^2r^2\left(1 + \dfrac{K^2}{4m^2\pi^2}\right)} . \tag{44}$$

As long as $\Delta > 0$, it can be found from (43) that

$$X = \pm[\tilde{b}(\tilde{r} - 1)]^{1/2} , \tag{45}$$

hence X is a real value.

From (42), we obtain

$$Z = (\tilde{r} - 1) = \frac{k^2U_s^2\Delta}{2K^2r^2} . \tag{46}$$

Therefore Z is also a real one.

From (31) and (32), we can know that A and B corresponding to X and Z are also real. It thus is clear that near the balance point, complex Lorenz system may be reduced to real Lorenz system, when discussing the properties of perturbation amplitude.

According to above discussions, Eqs.(38)–(40) may be rewritten as

$$\dot{X} = -\tilde{\sigma}X + \tilde{\sigma}Y \tag{47}$$

$$\dot{Y} = -XZ + \tilde{r}X - Y \tag{48}$$

$$\dot{Z} = -\tilde{b}Z + XY \tag{49}$$

Eqs.(47) and (49) are standard Lorenz equations. $\tilde{\sigma}$ and $\tilde{r}$ correspond to Prandtl number and Rayleigh number respectively. In original Lorenz equations, Prandtl number was chosen as 10. Here $\tilde{\sigma}$ is 2, and the critical value of $\tilde{r}$ is

$$\tilde{r}_c = \frac{\tilde{\sigma}(\tilde{\sigma} + \tilde{b} + 3)}{\tilde{\sigma} - \tilde{b} - 1} = \frac{2(5 + \tilde{b})}{1 - \tilde{b}} . \tag{50}$$

From (50), we can know that when $1 - \tilde{b} > 0$, and $0 < \tilde{r} < 1$, the origin $(X = Y = Z = 0)$ is only a stable balance point,and others are unstable. However, when $1 < \tilde{r} < \tilde{r}_c$, the origin is unstable. Therefore, it is very important to determine $\tilde{r}_c$.

Only when $k \geqslant 7$, can the condition $\left(1 - \tilde{b} = 1 - \dfrac{2}{1 + \dfrac{K^2}{4m^2\pi^2}} > 0\right)$ be correct. Hence in f–plane model, only the evolution of perturbation amplitude of short wave less than synoptic scale is studied. For the researches of long wave and planetary wave, β–plane approximation must be considered.

Note that $\tilde{r}$ depends on frictional coefficient r. The dependence of perturbation amplitude on r may be embodied by $\tilde{r}$. It is clear that $\tilde{r}$ is an important parameter in Lorenz system.

IV. PROBLEMS AND DISCUSSIONS

Above transformation of A–B hybrid equations to Lorenz system is completed via a concrete physical problem. Whether there exists a general transformation method? The answer is certain, as long as we introduce the following general form of A–B hybrid equations (Dodd et al.,1982):

$$\frac{d^2A}{dT_1^2} + \Delta_1 \frac{dA}{dT_1} = \alpha A - \tilde{\alpha} AB \tag{51}$$

$$\frac{dB}{dT_1} + \Delta_2 B = \left(\frac{d}{dT_1} + \Delta_3\right)|A|^2 \tag{52}$$

where Δ_1, Δ_2, Δ_3, α and $\tilde{\alpha}$ are coefficients depending on the concrete physical problem.

Let us introduce the following transformations:

$$\tau = \Omega T_1 \tag{53}$$

$$X = (2\tilde{\alpha})^{1/2}\Omega^{-1}A \tag{54}$$

$$Y = (\tfrac{1}{2}\Delta_3)^{-1}\Omega\dot{X} + X \tag{55}$$

$$Z = (2\tilde{\alpha})\Omega^{-1}\Delta_3^{-1}B \tag{56}$$

Generally speaking, Ω is always regarded as function of Δ_1 and Δ_2. By Utilizing above transformation relations (53)—(56), general A—B hybrid equation can be transferred into Lorenz system.

Here the problem doubted is that when deriving A—B hybrid equation, we consider amplitude A as a complex one. But in the Lorenz system, A is a real one, why? The answer is that if letting

$$A = Re^{i\theta(T)} \tag{57}$$

and substituting (57) into (24), and separating real part from imaginary part, we have

$$\frac{d}{dT}(R^2\dot{\theta}) = -\frac{3\gamma}{2}(R^2\dot{\theta}) \tag{58}$$

$$\frac{d^2R}{dT^2} + \frac{3}{2}\gamma\frac{dR}{dT} - R + R\int_0^1 \sin 2m\pi y \frac{\partial^2\Phi}{\partial y^2}dy = 0, \tag{59}$$

where $\dot{\theta} \equiv \dfrac{d\theta}{dT}$.

From (58), we know

$$R^2\dot{\theta} = D\exp(-\frac{3}{2}\gamma T). \tag{60}$$

In which, D is an integral constant.

From (60), it is seen that $\dfrac{d\theta}{dT}$ must eventually vanish because of the presence of friction. It implies that in presence of friction, we can consider amplitude A as real one, because

$$A = Re^{i\theta} = R\cos\theta + iR\sin\theta.$$

When $\dot{\theta} \to 0$, $A \to R\cos\theta$ holds

V. CONCLUSIONS

We have absorbed the basic ideas of the high truncated spectral method used by Charney (1979) and amplitude evolution method set up by Hart (1979) and Pedlosky (1981), and combined Dodd's ideas with the problems of wave—flow interaction in the atmosphere. As a result, we set up A—B hybrid equation method. The method is a powerful tool for studying nonlinear stability of wave—flow interaction, especially in the baroclinic atmosphere.

This method has some rules to go by, and the transformation method is relatively fixed. It overcomes some difficulties existing in two methods mentioned above. Hence it is easy to treat with.

It is interesting to transfer the dynamic problems of atmosphere into complex Lorenz system, because complex Lorenz system has not been touched upon by meteorologists yet. Exploring complex Lorenz system, undoubtedly, will promote the researches of atmospheric dynamics.

The author would like to thank Prof. Zeng Qingcun for his kind encouragement and valuable suggestions.

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