Planetary Stationary Waves in the Atmosphere
Part I: Orographic Stationary Waves

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ABSTRACT

It is proposed that the orographic stationary waves are required by long–term balance of momentum in the atmosphere with zonally asymmetric orographic forcing. This hypothesis may be confirmed successfully with the theoretical model of geostrophic waves. In the Part I, we will explain the observed phase distributions of orographic stationary waves at middle and high latitudes of the Northern Hemisphere, according to the long–term balance of zonal momentum over the stationary orographic forcing. It is revealed that the geographic distribution of stationary waves depends not only on local topgraphy but also on mean circulation fields and angular momentum flux in the atmosphere. So these waves cannot be simulated by the models in a restricted area.

I. INTRODUCTION

The maintenance of climatological stationary waves in the atmosphere requires the necessary *mechanisms of heat and momentum* balances over zonal asymmetries of external forcings in a long time period. So the study of stationary waves must be able to explain these balances physically and incorporate them in a mathematical model. However, the theoretical studies before did not approach toward this direction because of various difficulties. The most remarkable representative work on the study of orographic stationary waves was made by Charney and Eliassen (1949) in a limited β–plane channel. The phase of stationary waves was determined in their study by conservation of potential vorticity and intensity of Ekman pumping on vorticity. As the damping coefficient was taken as a constant, asymmetry of orographic torque made no influence on their wave solutions. Moreover, as momentum balance over the zonally asymmetric orographic forcing was not considered, it would be doubtful whether the obtained stationary waves may exist over a significant period in the atmosphere.

Since then, most studies of orographic stationary waves have been made by numerical experiments, which may simulate successfully in many aspects the observed planetary stationary waves (e. g., Manabe and Tenpstra, 1974; Blackmon and Lau, 1980; Tokioka and Noda, 1986 and Blackmon et al., 1987). The results have made it beyond doubt that these waves are induced mainly by orographic and thermal forcings near the lower boundary. However, the numerical experiments did not add more understanding substantially in the physical mechanism of stationary waves, except the discovery that orographically forced stationary waves, particularly at high levels, may be simulated well by barotropical models (Held, 1983). Thus, construction of a theory explaining the atmospheric asymmetries still remains a difficult challenge, even if some important feedback processes in the atmosphere are ignored and boundary conditions are simplified.

Until recent time, only few analytical studies on stationary waves can be found. Properties of the forced waves are used to be studied with variant travelling waves, resulting from specially simplified equations. The most impressive consequences of these studies are the vertical propagation of Rossby waves in a channeled β–plane (Charney and Drazin, 1961) and the meridional ray traces of two–dimensional Rossby waves at a sphere (Hoskins et al., 1977). Since the model atmospheres employed by these theories were greatly different, they could not draw a general picture of the circulations associated with stationary waves. We will find from this study that trap of vertical propagation and the meridional variation of wave phases can be interpreted in another way if using a different wave model.

We suppose, in the present investigation, that the stationary waves are produced by requirement of long–term heat and momentum balances over zonally asymmetric forcings. This mechanism will be incorporated in our mathematical model. According to the fundamental fact that large scale waves in the atmosphere are characterized by hydrostatic equilibrium and geostrophic balance, we will introduce the geostrophic perturbation equations using small–oscillation approximation to determine the main pattern of perturbations. The wave solutions are more geostrophic than Rossby waves, and so called particularly as the geostrophic waves. However, as the long–term balances of heat, momentum and mass are dominated by the primitive equations other than perturbation equations alone, these balances in the geostrophic waves will be investigated with the primitive equations.

In the atmosphere, thermal and mechanic forcings of topography are combined together and complementary to each other, so that contribution of topography cannot be considered simply as the sum of the effects caused by different forcings. But for convenience, the two forcings will be discussed separately. The momentum and heat balances in stationary geostrophic waves under these forcings will be considered using primitive equations in Part I and II, respectively. The results of Part I show that for maintenance of momentum balance in the atmosphere, zonally asymmetric variation of zonal momentum produced by zonal asymmetries of orographic forcing must be counteracted by the asymmetric generations of zonal acceleration in the atmosphere. The stationary waves in the scales comparable with the topography may produce the required asymmetric generation of zonal momentum. As the asymmetries of momentum generation depend on the phases of stationary waves, the phase distribution of orographic stationary waves may be determined by distribution of orographic forcing.

II. SMALL–OSCILLATION APPROXIMATION

One of the most essential properties of large–scale circulation at middle and high latitudes is geostrophic balance. In view of this, we may simplify the primitive equations for the study of geostrophic perturbations using small–oscillation approximation. This approximation will be proposed by examining firstly the geostrophic balance in classical Rossby waves.

Usually, the simplified momentum equations of Rossby waves in isobaric coordinates are written as:

$$\frac{\partial u'}{\partial t}+\bar{u}\frac{\partial u'}{\partial x}-fv'=-\frac{\partial \varphi'_1}{\partial x} \quad , \tag{1}$$

$$\frac{\partial v'}{\partial t}+\bar{u}\frac{\partial v'}{\partial x}+fu'=-\frac{\partial \varphi'_1}{\partial y} \quad . \tag{2}$$

With the usual assumption of no horizontal divergence, they are combined to be the conservation relationship of vertical vorticity.

$$\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)\nabla^2\psi+\beta\frac{\partial\psi}{\partial x}=0 .$$

It may be proved that the zonally averaged meridional fluxes of heat and zonal momentum depend on meridional wavenumber, so we use the two dimensional perturbation form of the streamfunction:

$$\psi = A\sin(kx+my-kct) , \tag{3}$$

where, c denotes zonal phase speed:

$$c=\bar{u}-\frac{\beta}{k^2+m^2} . \tag{4}$$

Consequently, we gain

$$u'=-\frac{\partial\psi}{\partial y}=-Am\cos(kx+my-kct) ,$$

$$v'=\frac{\partial\psi}{\partial x}=Ak\cos(kx+my-kct) .$$

Inserting them into (1) and (2) yields

$$\varphi'_1=A'\sin(kx+my-kct+\gamma) , \tag{5}$$

with

$$A'=A\frac{\sqrt{f^2(k^2+m^2)^2+\beta^2m^2}}{k^2+m^2} ,$$

$$\cos\gamma=\frac{f(k^2+m^2)}{\sqrt{f^2(k^2+m^2)^2+\beta^2m^2}} ,$$

$$\sin\gamma=\frac{\beta m}{\sqrt{f^2(k^2+m^2)^2+\beta^2m^2}} .$$

Comparing (3) with (5) finds that the geopotential perturbation is out of phase of the streamlines by γ. This phase departure is produced by meridional variation in Coriolis parameter f.

If we set $|m|=b|k|$ where b is a positive constant, the meridional variation in γ may be shown clearly by

$$\tan\gamma=\frac{bc\tan\varphi}{ak(1+b^2)} ,$$

in which, a measures the radius of the earth and φ signifies latitude. The distance of phase departure between geopotential field and streamfunction is then calculated from

$$D=a\gamma\cos\varphi / N ,$$

where $N=ak\cos\varphi$ indicates nondimensional zonal wavenumber. The phase departure is maximized when $b=1$ in the same waves. Fig.1 shows the dependence of phase departure on the magnitude of b. It may be proved that the phase departure exists also for the Rossby–Haurwitz waves, represented in the form of

$$\psi=A\cos my\sin(kx-kct) ,$$

but in the meridional direction.

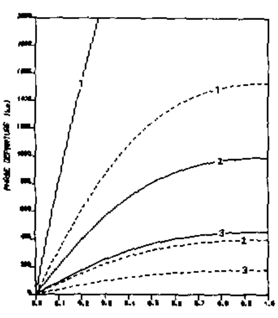

Fig.1. Phase departures between geopotential and streamfunction in discussed Rossby waves at 30° and 40°of latitude, represented respectively by solid and dashed lines. The numbers on lines indicate nondimensional zonal wavenumber.

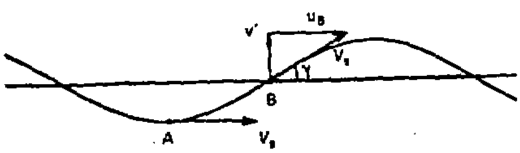

Fig.2. Small–oscillation approximation.

For the planetary Rossby waves on β–plane of which meridional wavelength is generally greater than zonal one, a good geostrophic balance requires that $b \ll 1$ or $m \ll k$. It implies that horizontal tilt of the geostrophic waves is small, which is consistent with the observations. So, the zonal perturbation velocity u' in large scale waves is much less than meridional perturbation velocity v'.

To support this argument, we consider a streamline depicted in Fig.2, of which, amplitude is relatively small compared with zonal wavelength. Provided that air speed V_s is almost the same along the streamline, we have

$$v'_{\max} = V_s \sin\gamma_{\max} \ .$$

While, the maximum deviation of u is

$$\Delta u_{\max} = V_s(1-\cos\gamma_{\max}) \ .$$

Supposing $u' = \Delta u_{\max} / 2$, namely,

$$u' = V_s \sin^2\frac{\gamma}{2} \ ,$$

we obtain

$$\frac{u'_{max}}{v'_{max}} = \frac{1}{2}\tan\frac{\gamma}{2} .$$

When $\gamma = 30°$, it gives $u'_{max} \approx 0.13 v'_{max}$. So u' is about one order less than v'. This result will be utilized as the small–oscillation approximation to produce the perturbation equations in next section. The waves described by these equations are therefore more geostrophic than Rossby waves.

III. GEOSTROPHIC PERTURBATION EQUATIONS

In the spherical coordinates with pressure instead of potential height as vertical coordinate, primitive equations for hydrostatic circulation in the atmosphere may be given by

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \omega\frac{\partial u}{\partial p} - \frac{uv}{a}\tan\varphi - fv = -\frac{\partial \varphi_1}{\partial x} + F_x , \tag{6}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \omega\frac{\partial v}{\partial p} + \frac{u^2}{a}\tan\varphi + fu = -\frac{\partial \varphi_1}{\partial y} + F_y , \tag{7}$$

$$\frac{\partial \varphi_1}{\partial p} = -\alpha , \tag{8}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} - \frac{v}{a}\tan\varphi = 0 , \tag{9}$$

$$\frac{\partial \alpha}{\partial t} + u\frac{\partial \alpha}{\partial x} + v\frac{\partial \alpha}{\partial y} - \sigma\omega = \frac{R}{C_p p}H , \tag{10}$$

$$P\alpha = RT. \tag{11}$$

Here

$$\sigma = -\frac{\alpha}{\theta}\frac{\partial \theta}{\partial p}, \qquad \theta = T\left(\frac{1000}{p}\right)^{R/C_p} ,$$

and H measures the rate of diabatic heating. The other symbols in these equations have their usual meanings.

Following the perturbation method, each of the field variables is divided into two portions, a mean state portion (which is independent of time and longitude) denoted by an overbar, and a perturbation portion (which is the local deviation from the mean) denoted by a prime, namely,

$$u = \bar{u} + u', \qquad v = \bar{v} + v', \qquad \omega = \bar{\omega} + \omega',$$
$$\alpha = \bar{\alpha} + \alpha', \qquad \theta = \bar{\theta} + \theta', \qquad T = \bar{T} + T'.$$

Usually, magnitudes of the mean states are assumed to be at least one order greater than those of perturbations. But at middle and high latitudes, mean meridional and vertical velocities are generally much less than perturbation components. For brevity, the mean vertical velocity will be ignored in this study.

It is assumed commonly that the mean portions satisfy the mean state equations:

$$f\bar{u} + \frac{\partial \bar{\varphi}_1}{\partial y} + \frac{\bar{u}^2}{a}\tan\varphi = 0 , \tag{12}$$

$$\frac{\partial \bar{\varphi}_1}{\partial p} = -\bar{\alpha}$$

and

$$p\bar{\alpha} = R\bar{T}.$$

(12) is the geostrophic relationship for mean zonal flow on a sphere. If we use the small–oscillation approximation described previously, the perturbation of zonal momentum may be represented by the geostrophic perturbation relationship

$$fv'_g = \frac{\partial \varphi'_1}{\partial x} \ . \tag{13}$$

While, the other perturbation equations read

$$\frac{\partial v'}{\partial t} + \bar{u}\frac{\partial v'}{\partial x} + fu' = -\frac{\partial \varphi'_1}{\partial y} \ , \tag{14}$$

$$\frac{\partial \varphi'_1}{\partial p} = -\alpha' \ , \tag{15}$$

$$\frac{\partial \alpha'}{\partial t} + \bar{u}\frac{\partial \alpha'}{\partial x} + \sigma_y v' - \sigma_z \omega' = 0 \ , \tag{16}$$

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial \omega'}{\partial p} - \frac{v'}{a}\tan\varphi = 0 \ , \tag{17}$$

$$p\alpha' = RT' \ , \tag{18}$$

where

$$\sigma_y = \frac{\partial \bar{\alpha}}{\partial y} \ , \qquad \sigma_z = -\frac{\bar{\alpha}}{\bar{\theta}}\frac{\partial \bar{\theta}}{\partial p} \ ,$$

and v' includes ageostrophic component, that is, $v' = v'_g + v'_a$. In deriving these equations, we have supposed that the terms of friction and diabatic heating are much smaller than the others in the same equations, so that they may be not involved to determine the main pattern of perturbations. But, these forcings have great effects on the long–term averaged balances of heat and momentum. So they must be considered when we discuss the stationary wave distributions in terms of the balances.

IV. GEOSTROPHIC WAVES

Without consideration of vertical variations in wave amplitudes, geopotential perturbation may be provided in the general form

$$\varphi'_1 = i\Phi(y)e^{i(\nu t - kx + my + lp)} \ , \tag{19}$$

Inserting it into (13) yields

$$v'_g = \frac{k}{f}\Phi e^{i(\nu t - kx + my + lp)} \ .$$

Moreover, the ageostrophic component is parameterized for simplicity by $v'_a = \delta v'_g$. Hence, (14)–(16) give

$$u' = \frac{m}{f}\Phi e^{i(\nu t - kx + my + lp)} \ ,$$

$$\alpha' = l\Phi e^{i(\nu t - kx + my + lp)} \ ,$$

$$\omega' = \frac{1}{\sigma_z}\left(\frac{1+\delta}{f}k\sigma_y - i\frac{f}{k}Ml\right)\Phi e^{i(\nu t - kx + my + lp)}. \quad \nu = \bar{u}k - \frac{f}{k\Phi}\frac{d\Phi}{dy} \ . \tag{20}$$

Subsequently, applying these solutions for (17) shows

$$l = -\frac{\delta\sigma_z}{(1+\delta)\sigma_y}m, \qquad M = \frac{\beta + \dfrac{f^2}{a^2\beta}}{f(k^2+k_T^2)}k^2 ,$$

inwhich

$$k_T^2 = \frac{\delta^2 f^2 \sigma_z m^2}{(1+\delta)^3 \sigma_y^2} \ , \qquad \beta = 2\frac{\Omega}{a}\cos\varphi \ .$$

While, the meridional variation in geopotential amplitude shows

$$\frac{d\Phi}{dy} = M\Phi \ .$$

It is caused by β–effect and earth's sphericity, which will be discussed further in another study.

Generally in planetary perturbations, there is $\delta \ll 1$. So, we have approximately

$$\omega' = \left(\frac{\sigma_y k}{f\sigma_z} + i\delta\frac{fm}{\sigma_y k}M\right)\Phi e^{i(\nu t - kx + my + lp)}. \quad l = -\delta\frac{\sigma_z}{\sigma_y}m, \quad k_T^2 = \frac{\delta^2 f^2 \sigma_z m^2}{\sigma_y^2} \ .$$

These results suggest that the ageostrophic component may be ignored except in the continuity equations. Because divergence of atmosphere depends essentially on ageostrophic motions. In general, the horizontal tilt of planetary waves is eastward and poleward, while the vertical phase slope is westward and upward. In these waves, there is $\delta > 0$.

The zonal phase speed of geostrophic waves may be obtained from (20), showing

$$c = \bar{u} - \frac{1}{k^2 + k_T^2}\left(\beta + \frac{f^2}{a^2\beta}\right) .$$

It is different from the phase speed of the classical Rossby waves given by (4), since it includes the effect of earth's sphericity. This effect is greater than β–effect at high latitudes, so cannot be ignored there. The phase speed of Rossby waves increases at higher latitude. This, however, is not consistent with observations. Thus, effect of earth's sphericity may be of significance for propagation of the large scale waves.

In a mean westerly flow, the zonal phase speed may be equal to or less than zero, if wavelength equals or exceeds the stationary wavelength

$$L_s = \frac{2\pi}{\sqrt{\dfrac{\beta}{\bar{u}}\left(1 + \dfrac{f^2}{a^2\beta^2}\right) - k_T^2}}$$

together with the conditions $\bar{u} > 0$ and

$$\sigma_y^2 \geqslant \delta^2 \frac{f^2 \sigma_z m^2 \bar{u}}{\beta + \dfrac{f^2}{a^2\beta}} \ .$$

When there are mean westerlies or baroclinity is sufficiently small in the upper atmosphere, stationary waves will not exist. The absence of stationary waves therein may be explained as well without using the concept of trap of vertical wave propagation (Charney and Drazin, 1961).

V. ZONAL MOMENTUM BALANCE IN STATIONARY GEOSTROPHIC WAVES

The small terms neglected in derivation of geostrophic perturbation equations have little contribution to the main wave patterns. However, they may be of great importance for heat and momentum balances over a long period. Thus, long-term zonal momentum balance in stationary geostrophic waves must be discussed with the primitive equation of zonal momentum.

The primitive equation (6) minus (13) produces the remainder equation of zonal momentum:

$$0 = -\frac{\partial u'}{\partial t} - u\frac{\partial u'}{\partial x} - v\frac{\partial u}{\partial y} - \omega'\frac{\partial u}{\partial p} + \frac{uv}{a}\tan\varphi + f(\bar{v} + v'_a) + F_x \ . \tag{21}$$

Unlike the study of Charney and Eliassen, the external forcing is incorporated here in the remainder equation. So, this forcing may be considered in its right order and will not be amplified.

The phase plane of stationary waves is, now, denoted by

$$\Psi_s = -kx + my + lp \ .$$

Substituting the stationary wave solutions into (21) yields

$$\overline{DU} + A_1\cos\Psi_s + A_2\sin\Psi_s + B\sin2\Psi_s + F_x = 0 \ , \tag{22}$$

in which,

$$\overline{DU} = \frac{\delta^2\sigma_z k m^3\Phi^2}{f\sigma_y^2(k^2+k_T^2)}\left(\beta + \frac{f^2}{a^2\beta}\right) + \left(f(1 + \frac{\bar{u}}{a^2\beta}) - \frac{\partial\bar{u}}{\partial y}\right)\bar{v}$$

represents zonal symmetric acceleration, while

$$A_1 = \left(\frac{\bar{u}}{a^2\beta} + \delta - \frac{\sigma_y}{f\sigma_z}\frac{\partial\bar{u}}{\partial p} - \frac{1}{f}\frac{\partial\bar{u}}{\partial y}\right)k\Phi \ ,$$

$$A_2 = \left(-\frac{\bar{u}}{f} + \frac{\delta}{\sigma_y(k^2+k_T^2)}(\beta + \frac{f^2}{a^2\beta})\frac{\partial\bar{u}}{\partial p}\right)km\Phi.$$

$$B = \delta^2\frac{km^2}{2f^2}\Phi^2 \ . \tag{23}$$

The small terms connected with mean meridional flow in the expressions of A_1 and A_2 have been neglected. Furthermore, with the relationship of thermal wind

$$\frac{\partial\bar{u}}{\partial p} = \frac{\sigma_y}{f} \ ,$$

they are replaced, respectively, by

$$A_1 = \left(\frac{\bar{u}}{a^2\beta} + \delta - \frac{\sigma_y^2}{f^2\sigma_z} - \frac{1}{f}\frac{\partial\bar{u}}{\partial y}\right)k\Phi \tag{24}$$

and

$$A_2 = \frac{1}{f}\left(-\bar{u} + \frac{\delta}{k^2 + k_T^2}(\beta + \frac{f^2}{a^2\beta})\right) km\Phi \ . \tag{25}$$

If we use the zonal average

$$\overline{(\ )} = \frac{1}{L}\int_x^{x+L} (\ )dx$$

where L denotes a zonal wavelength, (22) may be replaced by

$$\overline{DU} + \bar{F}_x + A_1\cos\Psi_s + A_2\sin\Psi_s + B\sin2\Psi_s + F'_x = 0 \ .$$

Here, F'_x indicates deviation from the zonal mean friction $\bar{F}_x$. Obviously, this relationship may be separated into

$$\overline{DU} + \bar{F}_x = 0 \tag{26}$$

and

$$A_1\cos\Psi_s + A_2\sin\Psi_s + B\sin2\Psi_s + F'_x = 0 \ . \tag{27}$$

Equation (26) is the balance relationship of zonally averaged zonal momentum, which gives the mean meridional circulation

$$\bar{v} = -\frac{\delta^2\sigma_z km^3(\beta + \frac{f^2}{a^2\beta})\Phi^2 - f\sigma_y^2(k^2 + k_T^2)\bar{F}_x}{f\sigma_y^2(k^2 + k_T^2)\left(f(1 + \frac{\bar{u}}{a^2\beta}) - \frac{\partial\bar{u}}{\partial y}\right)}$$

or, approximately,

$$\bar{v} = -\frac{\delta^2\sigma_z km^3(\beta + \frac{f^2}{a^2\beta})\Phi^2 - f\sigma_y^2(k^2 + k_T^2)\bar{F}_x}{f^2\sigma_y^2(k^2 + k_T^2)} \ .$$

It depends greatly on ageostrophic motions. Usually, zonal mean flux of absolute angle momentum is poleward at middle latitudes of the Northern Hemisphere (Newell et al., 1972), so the troughs of large scale waves tilt in the northeast–southwest direction (Starr, 1947) or $km > 0$. For these waves, the first term on the right hand side is negative. Additionally, the zonal mean friction is opposite to the direction of mean zonal flow. Thus, the two terms possess opposite signs in mean westerlies. If frictional force is sufficiently large, mean meridional flow will be along the direction of meridional pressure force. Whereas at high levels, it may be in reversed direction. Therefore, from the point of zonal momentum balance, mean meridional circulation at middle latitudes must be poleward at lower levels but equatorward at high levels to form the Ferrel cells.

Moreover, (27) shows that the stationary waves may produce zonal asymmetry of zonal acceleration. This asymmetric momentum generation must be counteracted by the opposite contribution resulting from zonally asymmetric orographic forcing, when momentum balance is established. As the asymmetric generation of zonal momentum has a specific phase relationship in stationary waves, phases of the stationary waves may be determined in terms of asymmetric topography. This will be discussed in later sections.

VI. LINEAR AND NONLINEAR RESPONSES TO OROGRAPHIC FORCING

Rearranging (27) gives

$$A\cos(\Psi_s - \gamma_1) + B\cos(2\Psi_s + \frac{1}{2}\pi) = -F'_x \ , \tag{28}$$

where

$$\mathrm{A} = \sqrt{\mathrm{A}_1^2 + \mathrm{A}_2^2} \ ,$$

and

$$\cos\gamma_1 = \frac{A_1}{A} \ , \qquad \sin\gamma_1 = \frac{A_2}{A} \ . \tag{29}$$

The first term on the left hand side of (28) comes from the linear terms in (21), so it will be referred to as the linear response. While, the other term represents nonlinear response, which results from the nonlinear terms.

Generally, for a continuous distribution of F'_x, more harmonic components of linear and nonlinear responses with different wave numbers are required, so that F'_x may be expanded by them. As we are concentrated on the principal mechanism of maintenance of stationary waves here, we will consider only the idealized F'_x represented approximately by a single harmonic function:

$$F'_x = G\cos(nx + \eta_f) \ , \qquad (n > 0)$$

where, η_f is a constant. For this idealized forcing, the linear and nonlinear responses may be discussed simply with (28).

To find the phase relationships of linear and nonlinear responses only, we may separate (28) into

$$\cos(\Psi_s - \gamma_1) = -\cos(nx + \eta_f)$$

and

$$\cos(2\Psi_s + \frac{1}{2}\pi) = -\cos(nx + \eta_f) \ .$$

Thus, the linear response is in the phase of

$$\Psi_s = \pm(nx - \eta_f) + \gamma_1 + \pi \tag{30}$$

and so has the same zonal wavelength as that of external forcing. While, the phase of nonlinear response shows

$$\Psi_s = \pm\frac{1}{2}(nx + \eta_f) + \frac{1}{4}\pi \ .$$

Its zonal wavelength is twice as long as that of the forcing.

VII. OROGRAPHIC STATIONARY WAVES

We find, after scale analyses, that the linear response in (28) is greater in order than the nonlinear response. If only the linear response is considered, (28) may be rewritten as:

$$A\cos(\Psi_s - \gamma_1) \approx -F'_x \ .$$

It tells that the amplitude of stationary geopotential perturbation associated with linear response is proportional to the intensity of orographic forcing, and the phase is expressed by

$$\varphi'_1 = \begin{cases} -\Phi\sin(nx + \eta_f - \gamma_1) & (k > 0) \\ \Phi\sin(nx + \eta_f + \gamma_1) & (k < 0) \end{cases}$$

derived from (30) and (19).

In other studies, distributions of stationary waves were used to be determined by choosing specific values for the included parameters. So the realistic picture depended crucially on the choice and could be distorted greatly if using different values. In the present study the phases are discussed in a general way, so that the differences will not exceed $\pi / 2$.

We will discuss, in the following, only the orographic stationary waves at middle and high latitudes of the Northern Hemisphere. The same physical mechanism and mathematical method may also be used for the study in the Southern Hemisphere. It has been noted that at middle latitudes where zonal mean flux of angular momentum is poleward, we have $km > 0$. In this study, we consider only the case of $k > 0$. It can be verified generally that the identical distributions may be obtained also by assuming $k < 0$.

From the relationships of linear response above, we see that at middle latitudes, especially on the polar side of westerly jet, there are $\cos\gamma_1 > 0$ and $\sin\gamma_1 < 0$, and so $3\pi / 2 < \gamma_1 < 2\pi$. Moreover, in mean westerlies, $F'_x < 0$ on the windward sides of mountains and $F'_x > 0$ on the lee sides. Thus in zonal momentum balance, the induced stationary geopotential perturbations associated with the linear response to the idealized orographic forcing at middle and high latitudes may be displayed respectively in Fig.3.

It is shown that troughs of orographic stationary waves are located to the east of mountain tops, while ridges to the west. This phase relationship may be proved by comparing with Fig.4, which gives the longitude–height cross section of stationary geopotential height, derived from 11 winters and 12 summers of NMC operational analyses. In the whole troposphere, two major troughs at middle latitude are located respectively over the east coast of North America and Asia, and two major ridges are off the west coasts of North America and North Africa. The ridge and trough corresponding to the Tibetan Plateau are more intense and intensive than the other major systems associated with Rocky Mountains.

In the lower troposphere, as orographic effect is enhanced, the high over North Africa is split into two centres corresponding to the two major highlands below, respectively. Thus, the stationary waves are identified with wave 3 as shown by the climatological mean 500 hPa height field in Fig.5. Since the surface between these two separated ridges is covered by land, the zonal contrast of F'_x is relatively weak, so that the trough over is shallow.

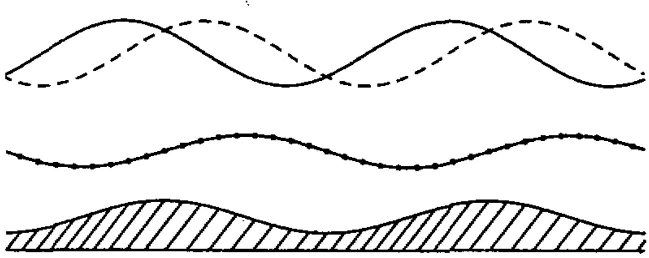

Fig.3. The stationary geopotential perturbations associated with linear response to the idealized zonal asymmetry of orographic forcing (dotted line) at middle latitudes ($\gamma_1 = 1.75\pi$, solid line) and high latitudes ($\gamma_1 = 0.25\pi$, dashed line), respectively. The simplified topography is illustrated on the bottom.

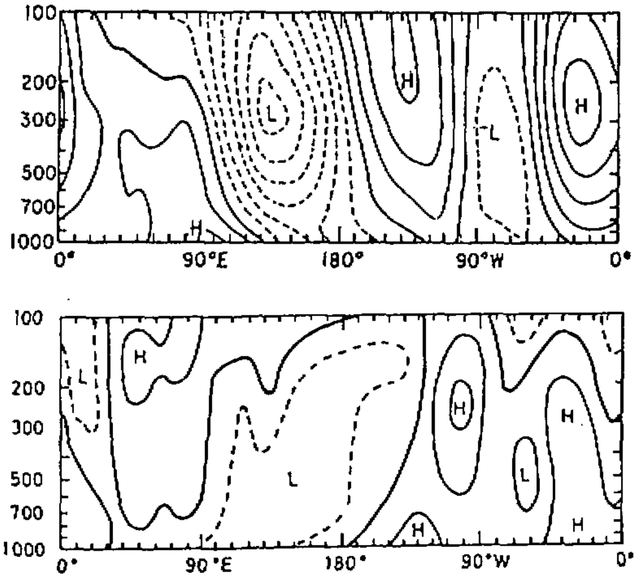

Fig.4. Longitude–height cross sections of stationary wave geopotential height at 45°N in (a) winter and (b) summer. The zero contour is thickened. (After Wallace, 1983).

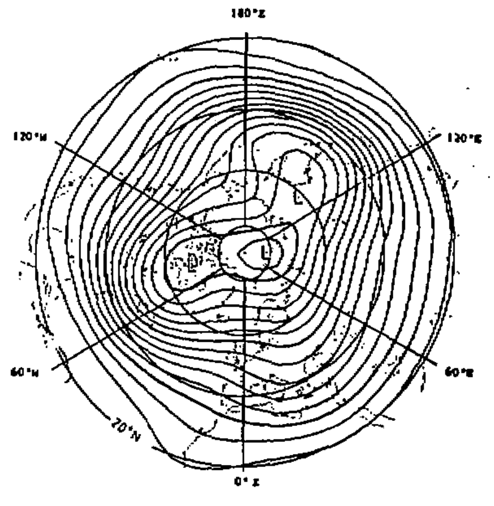

Fig.5. Climatological mean 500 hPa height field for January. Contour interval 60 m; the 5100, 5400 and 5700 m contours are thickened. The outer latitude circle is 20°N. (After Wallace, 1983).

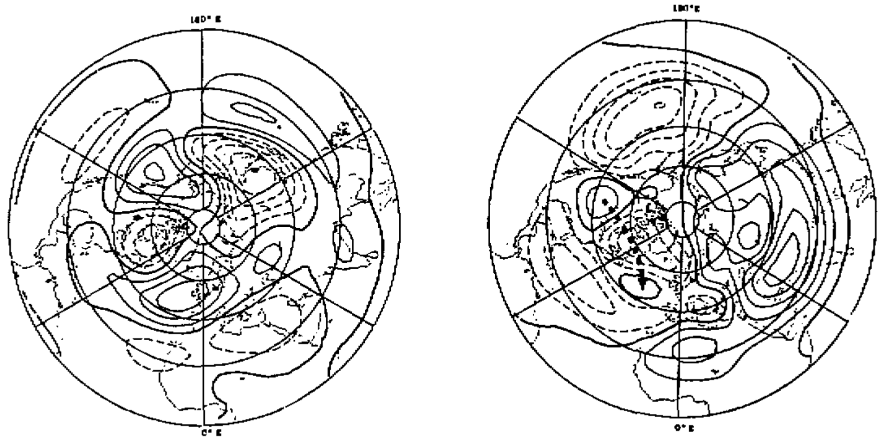

Fig.6. Northern Hemisphere climatological mean distributions of stationary wave geopotential height at the 200 hPa level. Contour interval 60m; the zero contour is thickened; positive contours are solid and negative ones are dashed. (a) January and (b) July (After Wallace, 1983).

At high latitudes of the Northern Hemisphere while, mean eddy flux of westerly momentum is generally equatorward from winter through summer (Newell et al., 1972) in the upper troposphere, so $km < 0$. The stationary geopotential perturbation at high latitudes may be represented by the dashed line in Fig.3, of which $\gamma_1 = 0.25\pi$. The troughs and ridges are centred respectively on the west and east sides of mountain tops. Referring to the climatological distributions of stationary geopotential height at 200 hPa level shown in Fig.6, we see apparently the effect of Greenland Plateau, particularly during winter when the equatorward momentum flux is strong. The low in the downstream of Rocky Mountains extends to the west side of Greenland, while the high in the upstream of the Tibetan Plateau stretches to the other side. The similar effect is also evident over the Cherskogo Mountains in eastern Siberia.

VIII. CONCLUSIVE REMARKS

The present investigation of orographic stationary waves is different from the study of Charney and Eliassen at least in four significant aspects. Firstly, the frictional force was included in their perturbation equations, so it was amplified artificially for obtained realistic result (Held, 1983). Secondly, distributions of stationary waves are discussed in our study according to the asymmetry of orographic forcing which, however, made no contribution to their stationary waves. Thirdly, we find that the orographic stationary waves are required by the long–term momentum balance manifested by primitive equations, which was not considered in their model. Finally, distributions of orographic waves in our study depend not only on orographic forcing, but also on mean circulations and angular momentum flux.

It is postulated that the orographically forced stationary waves may be well simulated by barotropic models (Held, 1983). We have found that the orographic stationary waves are related mostly to linear response, which is much larger than nonlinear response. Learned from (23)–(25) and (29), the zonal differences in phases of orographic stationary waves do not exceed $\pi / 2$ unless the baroclinity grows to a great extent. For mean zonal flow is strong and mean meridional temperature gradient is relatively small in the upper troposphere, we may expect that simulation with a barotropic model may gain better results for upper stationary waves.

If stationary waves in the atmosphere are forced mainly by the orographic forcing, their distributions will be independent of time. This is nearly true for the observed stationary waves in the upper troposphere of both hemispheres, especially at middle and high latitudes. Also, since the linear orographic effect is generally greater than the nonlinear one, stationary waves in these regions simulated by linear models may bear a good resemblance to the observations (e.g., Lin, 1982 and Jacqmin and Lindzen, 1985).

Finally, as stationary waves depend not only on topography but also on mean circulation and momentum transport, producing correct mean circulation fields and similar momentum fluxes in a model has the same importance as introducing realistic forcing fields for simulation of observed stationary waves. Thus, it is necessary to be prudent to draw any general conclusion with the results obtained from the experiments in a limited channel.

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