A Simple Quasi–Geostrophic Coupled Ocean–Atmosphere Model①

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ABSTRACT

The quasi–geostrophic atmospheric and oceanic equations of momentum and thermodynamics with dissipation factors are used to create a simple coupled ocean–atmosphere model describing the large–scale shallow–water motion. We discuss the ocean–atmosphere coupling effect in mid–high and low latitudes separately and analyze characteristics of which the oscillatory periods of coupled low–frequency modes (ocean mode) vary with the coupling frequency and latitudinal number. This can interpret the correlation between low–frequency oscillation and ocean–atmosphere interaction. Then from the dispersion curves of atmosphere and ocean, we reveal effect of the coupling strength on the propagation of Rossby waves. The convection mechanism between the two modes is also discussed in view of the slowly varying wave train.

The results show that Newtonian cooling and Rayleigh friction play a stable rule in oceanic Rossby waves, the period of coupled low–frequency mode grows with the increment of the coupling frequency. The larger the latitudinal number is, the more rapidly it grows. When the coupling frequency tends to critical value, the oceanic Rossby waves become static. When the ocean–atmosphere coupling strength grows to some degree, the propagation of oceanic Rossby waves will become opposite to its original direction. One part of the oceanic Rossby waves is converted into atmospheric Rossby waves, the energy conversion coefficient is also solved out.

I. INTRODUCTION

Since Matsuno(1966) analyzed the atmospheric waves in low latitudes, great advances have been made in the research on low–latitude dynamics. Many researches (Holton and Lindzen, (1968), (1972); Lau, (1988) show that low–frequency oscillation is concerned with low–latitude waves. The main display is that there exists a positive feedback mechanism between atmosphere and ocean to extend the damping period, and the ocean–atmosphere coupling strength can also change the unstable damping time of oscillations. The wide researches on ENSO events (Halpern et al., 1983; Rasmusson and Wallace, (1983); Philander, 1983a, 1983b; Cane,1983) also show that ENSO events are concerned with the interaction between ocean and atmosphere. The main display of this action is that the wind stress force in the atmosphere can change the sea–surface temperature and the latent energy of ocean, specially high SST in ENSO events may influence the atmosphere enormously. The ocean–atmosphere coupling will create oceanic waves propagating westward or eastward separately, people are trying to use it to explain the dynamical mechanism in many ENSO events.

Lau (1981) used the shallow–water equations to create a coupled large–scale

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ocean–atmosphere model describing equatorial Kelvin waves. He obtained the modes of two kinds of Kelvin waves. The first mode is the fast moving Kelvin waves, hardly influenced by the ocean–atmosphere coupling. The second is the slow–moving Kelvin waves responding to ocean, this mode plays an important role in the teleccnnection and the ocean–atmosphere coupling may lead to its unstable. Philander et al., 1984 used similarly shallow–water equations to create a coupled model including steady atmosphere and unsteady ocean. He considered that because this ocean–atmosphere coupling can result in the instabilities of Kelvin and Rossby waves, he can use it to explain ENSO events and low–frequency oscillation.

To different oceanic thermodynamical parameters, Hirst (1985) linearized shallow–water equations at equatorial plane, and systematically solved four simple ocean–atmosphere coupling models. He found that the oceanic mode was affected violently by coupling. The coupling effect depends on the variation of all kinds of coefficients in model. Larger values of ocean thermal forcing and coupling coefficients and smaller values of damping coefficients and the atmospheric Rossby deformation radius are associated with increased growth rates for unstable modes. Most coupling affects long zonal wavelength and low–frequency modes. Coupling is ineffective when atmospheric heating is located within 5° of the equator. Schopf 1987 takes a 35–year integration on a coupled model to produce ENSO–like variability on time scale of 3–5 years. Provided that the heat fluxes between ocean and atmosphere are linear in ocean–atmosphere temperature differences in the calculation. Since there is no external time–dependent forcing, there are self–sustained intrinsic vacillations of the nonlinear system.

Lau (1988) expounded again the dynamical mechanism of producing intraseasonal oscillations and ENSO events from shallow–water equations which use potential temperature as an independent variable. The results show that the ocean SST advection and oceanic upwelling produce two Kelvin–like unstable modes being called the advection mode and the upwelling mode separately. They can be used to interpret the dynamical processes of ENSO events during their growth and mature phases. Lau studied the elemantary properties of tropical low–frequency phenomena collectively and underlined the importance of moist processes. The presence of moisture (condensation convergence feedback) in the tropical atmosphere causes atmospheric motions to slow down through the reduction of the moist stability of the lower troposphere. This influence becomes increasingly strong as the moist content of the lower troposphere increases due to increasing SST. Meanwhile, the evaporation–wind feedback can also strengthen the condensation convergence mechanism, Since the surface heat balance controlling the temperature of a tropical oceanic mixed layer implies very short damping time for SST anomalies, Lau considered that the calculated results from GCMS might show that tropical SST anomalies would decay in <100 days if no positive feedback processes excited. It is just this positive feedback mechanism that produces ENSO events.

In addition, Mccreary (1983) and Anderson (1984) considered a model with ocean dynamics and crude parameterization of surface wind response to SST changes. Ziviak (1984) and Cane (1985) developed an ocean–atmosphere coupling model in which the ocean also evolves as linear and geostrophic equation with a single layer and the atmosphere is that of Gill(1980), the relation between SST and heating of the atmosphere allows for a temperature dependence of the coupling coefficient, all dynamics are linearized. The coupling, however. is dependent on oceanic temperature predicted by a fully nonlinear thermodynamic equation. Ocean temperature is thus affected by both climatological currents and linearly predicted perturbations. Through the prescribed climatology, they include effects of the seasonal cycle.

Depending on the choice of parameters, their model produced a variety of low–frequency behaviors, including long–period ENSO–like vacillations.

The above results suggest that the air–sea interaction has great effects on oceanic and atmospheric waves. The change of some parameterizations, especially the coupling coefficient between ocean and atmosphere will make effect on the instability of modes, and extend the damping time of the disturbance and turn out low frequencies. But their models are too complex and most results come from numerical simulations, hence they can not make the coupling mechanism be understood well in analytic ways. A simple quasi–geostrophic model is created to describe the large–scale coupling between ocean and atmosphere in more detail in this paper, and the variation of Rossby waves led by coupling action is analyzed.

II. QUASI–GEOSTROPHIC COUPLED MODEL

Firstly, we use the equations of shallow–water model including the Rayleigh friction and Newtonian cooling.

For the atmosphere, these equations are

$$\frac{\partial u_1}{\partial t} - f v_1 = -\frac{\partial \varphi_1}{\partial x} - a_1 u_1 , \tag{2.1a}$$

$$\frac{\partial v_1}{\partial t} + f u_1 = -\frac{\partial \varphi_1}{\partial y} - a_1 v_1 , \tag{2.1b}$$

$$\frac{\partial \varphi_1}{\partial t} + c_1^2 \left(\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} \right) = -b_1 \varphi_1 - Q , \tag{2.1c}$$

where t is time, x and y are the coordinate axes, u_1 and v_1 are zonal and meridional wind velocity components respectively, φ is the perturbations of gravitational potential at free–surface height of atmosphere, f is Coriolis parameter, a_1 and b_1 are coefficients for Rayleigh friction and Newtonian cooling separately.

$$c_1 = \sqrt{g H_1} , \tag{2.2}$$

where g is the acceleration of gravity. The equivalent height of atmosphere is measured by H_1 for making the shallow–water model more close to practical situations.

In Eqs. (2.1), Q represents the heat forcing action of the ocean to the atmosphere. According to Philander's analysis (1984), it may be given by

$$Q = A \varphi_2 , \tag{2.3}$$

where φ_2 denotes the gravitational potential of the perturbations of ocean depth. The coefficient for ocean acting on atmosphere is A.

For the ocean, the shallow–water model is described by equations:

$$\frac{\partial u_2}{\partial t} - f v_2 = -\frac{\partial \varphi_2}{\partial x} - a_2 u_2 + \tau^x , \tag{2.4a}$$

$$\frac{\partial v_2}{\partial t} + f u_2 = -\frac{\partial \varphi_2}{\partial y} - a_2 v_2 + \tau^y , \tag{2.4b}$$

$$\frac{\partial \varphi_2}{\partial t} + c_2^2 \left(\frac{\partial u_2}{\partial x} + \frac{\partial v_2}{\partial y} \right) = -b_2 \varphi_2 , \tag{2.4c}$$

where u_2 and v_2 are zonal and meridional current velocity components separately, a_2 and b_2 are coefficients for Rayleigh friction and Newtonian cooling of ocean respectively.

$$c_2 = \sqrt{g H_2} , \tag{2.5}$$

where H_2 is the equivalent depth of ocean.

In Eqs. (2.4), (τ^x,τ^y) indicates the forces acted by the wind stress forcing in the region of atmosphere to ocean, it may be shown as

$$(\tau^x,\tau^y)=B(u_1,v_1) \ . \tag{2.6}$$

B is the coefficient of atmosphere acting on ocean based on the quasi–geostrophic conception, and for simplicity, we assume that

$$a_1=b_1 \ , \ a_2=b_2 \ . \tag{2.7}$$

Equations (2.1) and (2.4) then can be reduced separately to

$$\left(\frac{\partial}{\partial t}+a_1\right)\left(\nabla_h^2-\frac{f^2}{c_1^2}\right)\varphi_1+\beta_0\frac{\partial\varphi_1}{\partial x}=\frac{f^2}{c_1^2}A\varphi_2 \ , \tag{2.8a}$$

$$\left(\frac{\partial}{\partial t}+a_2\right)\left(\nabla_h^2-\frac{f^2}{c_2^2}\right)\varphi_2+\beta_0\frac{\partial\varphi_2}{\partial x}=B\nabla_h^2\varphi_1 \ , \tag{2.8b}$$

where

$$\nabla_h^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2} \tag{2.9}$$

and β is the Rossby parameter (a constant), according to the quasi–geostrophic conception,

$$f=f_0 \ , \quad \text{in mid}-\text{high latitudes} \ , \tag{2.10a}$$

$$f=\beta_0 y \ , \quad \text{in low}-\text{latitudes} \ . \tag{2.10b}$$

Eliminating φ_1 from (2.8a) and (2.8b), we get

$$L\varphi_2=0 \ , \tag{2.11}$$

where

$$L=\left[\left(\frac{\partial}{\partial t}+a_1\right)\left(\nabla_h^2-\frac{f^2}{c_1^2}\right)+\beta_0\frac{\partial}{\partial x}\right]\left[\left(\frac{\partial}{\partial t}+a_2\right)\left(\nabla_h^2-\frac{f^2}{c_2^2}\right)+\beta_0\frac{\partial}{\partial x}\right]-\frac{f^2}{c_1^2}AB\nabla_h^2 \tag{2.12}$$

Notice that Eq. (2.11) is an approximate one when $f=\beta_0 y$.

Eq. (2.11) is just the basic one to discuss coupled ocean and atmosphere in large–scale motion. Now, we illustrate the two cases in mid and low–latitudes respectively.

III. THE QUASI–GEOSTROPHIC COUPLING IN MID–HIGH LATITUDES

In mid–high latitudes, we take $f=f_0$ and set that

$$\lambda_1^2=\frac{f_0^2}{c_1^2} \ , \quad \lambda_2^2=\frac{f_0^2}{c_2^2} \tag{3.1}$$

λ_1^{-1} and λ_2^{-1} are Rossby deformation radii of atmosphere and ocean separately. So the differential operator (2.12) becomes

$$L=\left[\left(\frac{\partial}{\partial t}+a_1\right)\left(\nabla_h^2-\lambda_1^2\right)+\beta_0\frac{\partial}{\partial x}\right]\left[\left(\frac{\partial}{\partial t}+a_2\right)\left(\nabla_h^2-\lambda_2^2\right)+\beta_0\frac{\partial}{\partial x}\right]-\lambda_1^2AB\nabla_h^2 \ . \tag{3.2}$$

Applying the normal mode method, we assume the wavelike solution of Eq. (2.11) to be

$$\varphi_2=\Phi_2e^{i(kx+ly-\omega t)} \ , \tag{3.3}$$

where k and l are wave numbers in x and y directions respectively, ω is angular frequency.

Substituting (3.3) into (2.11), we have

$$[(i\omega-a_1)(K_h^2+\lambda_1^2)+\beta_0ki][(i\omega-a_2)(K_h^2+\lambda_2^2)+\beta_0ki]+\lambda_1^2ABK_h^2=0 \tag{3.4}$$

where

$$K_h^2 = k^2 + l^2 \ . \tag{3.5}$$

In the absence of the coupling between ocean and atmosphere, $A = 0$, $B = 0$, then from Eq. (3.4), we get

$$\omega_1 = -\frac{\beta_0 k}{K_h^2 + \lambda_1^2} - a_1 i \ , \tag{3.6a}$$

$$\omega_2 = -\frac{\beta_0 k}{K_h^2 + \lambda_2^2} - a_2 i \ . \tag{3.6b}$$

Obviously, ω_1 and ω_2 represent the angular frequencies of Rossby waves for the atmosphere and ocean with dissipations. And dissipation factors (including Rayleigh friction and Newtonian cooling) play stable action in Rossby waves. This can be easily understood on physics. Thus in the following we won't interpret the function of dissipations. Now (3.6) can be written as

$$\omega_1 = -\frac{\beta_0 k}{K_h^2 + \lambda_1^2} \ , \tag{3.7a}$$

$$\omega_2 = -\frac{\beta_0 k}{K_h^2 + \lambda_2^2} \ . \tag{3.7b}$$

When the coupling between ocean and atmosphere is considered, $A \neq 0$, $B \neq 0$, in the absence of the dissipation, Eq. (3.4) becomes

$$(\omega - \omega_1)(\omega - \omega_2) = \eta \ , \tag{3.8}$$

where

$$\eta = \frac{K_h^2 \lambda_1^2 \omega_c^2}{(K_h^2 + \lambda_1^2)(K_h^2 + \lambda_2^2)} \tag{3.9}$$

and

$$\omega_c = \sqrt{AB} \tag{3.10}$$

is known as the coupling frequency.

From the quaratic algebraic equation (3.8), we can get

$$\omega^{(1)} = \frac{1}{2}\left\{(\omega_1 + \omega_2) + \sqrt{(\omega_1 - \omega_2)^2 + 4\eta}\right\} \ , \tag{3.11a}$$

$$\omega^{(2)} = \frac{1}{2}\left\{(\omega_1 + \omega_2) - \sqrt{(\omega_1 - \omega_2)^2 + 4\eta}\right\} \ . \tag{3.11b}$$

Obviously, $\omega^{(1)}$ indicates coupled angular frequency of atmosphere Rossby waves, $\omega^{(2)}$ is the coupling angular frequency of ocean Rossby waves, and at same latitude

$$c_1^2 \gg c_2^2 \ , \quad \lambda_1^2 \ll c_2^2 \ . \tag{3.12}$$

Thus, when h has a small value

$$|\omega_2| < |\omega_1| \ , \quad |\omega^{(2)}| < |\omega^{(1)}| \ . \tag{3.13}$$

That means that $\omega^{(2)}$ further indicates the features of coupled low–frequency mode.

We will discuss the oscillatory period of coupled low–frequency mode in the following.

Following the analyses of Madden and Julian (1972) and McWilliams and Gent (1978), we take

$$c_1 = 15\text{ms}^{-1}, \quad c_2 = 2\text{ms}^{-1} \tag{3.14}$$

and let

$$L = \pi \times 10^6\,\text{m}, \quad k = \frac{\pi}{L}, \quad l = \frac{n\pi}{L} \tag{3.15}$$

then we have

$$\lambda_1^2 = 4.4 \times 10^{-11}\,\text{m}^{-2}, \quad \lambda_2^2 = 2.5 \times 10^{-9}\,\text{m}^{-2} \tag{3.16}$$

$$\eta = \frac{44(n^2+1)\omega_c^2}{(n^2+45)(n^2+2501)} \;. \tag{3.17}$$

From (3.17) and (3.11b) it is seen that $\omega^{(2)}$ tends to be the uncoupled frequency ω_2 gradually with the infinite increasement of latitudinal wave number 1 (or n), and we obtain the oscillatory period

$$T^{(2)} = \frac{2\pi}{\omega^{(2)}} \;. \tag{3.18}$$

For reflecting the practical case better, we add the basic flow $\bar{u}_1$ of atmosphere and the oceanic basic current $\bar{u}_2$ in mid–high latitudes. Now, the angular frequencies of atmosphere and ocean Rossby waves without coupling are

$$\omega_1 = k\bar{u}_1 - \frac{\beta_0 k}{K_h^2 + \lambda_1^2} \;, \tag{3.19a}$$

$$\omega_2 = k\bar{u}_2 - \frac{\beta_0 k}{K_h^2 + \lambda_2^2} \;. \tag{3.19b}$$

we take

$$\bar{u}_1 = 10\text{ms}^{-1}, \quad \bar{u}_2 = 0.5\text{ms}^{-1}, \quad \beta_0 = 2 \times 10^{-11}\,\text{m}^{-1}\text{s}^{-1} \;. \tag{3.20}$$

From Eqs. (3.16), (3.17), (3.19) and (3.18), we can draw the curves that the period $T^{(2)}$ of low–frequency ocean mode varies with the square value of coupling frequency ω_c, as shown in Fig.1, and the dotted lines whose corresponding values at the horizontal coordinate are critical ones ω_{cr}^2 and ω_c^2 when $\omega^{(2)} = 0$ are asymptotes of all corresponding curves. Let $\omega^{(2)} = 0$, we get

$$\omega_{cr}^2 = \frac{[k\bar{u}_1(K_h^2+\lambda_1^2) - \beta_0 k][k\bar{u}_2(K_h^2+\lambda_2^2) - \beta_0 k]}{K_h^2 \lambda_1^2} \tag{3.21}$$

From Fig.1, we can see that the periods of oceanic Rossby waves are about 5 months

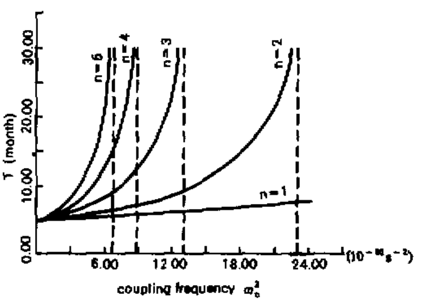

Fig.1. Curves in which $T^{(2)}$ varies with ω_c^2.

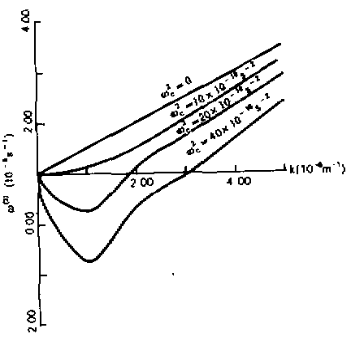

Fig.2. Curves in which ω_2 varies with k.

when the atmosphere–ocean coupling ($\omega_c = 0$) was not considered was when the square value of coupling frequency ω_c increases, the period $T^{(2)}$ of the coupled low–frequency mode increases correspondingly. Particularly, $T^{(2)}$ increases rapidly with the increment of ω_c^2 , when latitudinal number is larger. The curves in the figure are drawn only for $T^{(2)} = 30$ (months). This result provides the theoretical basis for the correlation between low–frequency oscillation and ocean–atmosphere coupling. When ω_t^2 tends to be the critical value ω_{cr}^2, $T^{(2)}$ increases rapidly and tends to infinity, now the oceanic Rossby waves become static.

Fig.2 depicts the curves in which the angular frequencies $\omega^{(2)}$ of oceanic Rossby waves vary with zonal wave number k for different square values of coupling frequency ω_c , and definite the latitudinal wave number $n = 3$, then

$$\omega_{cr}^2 = \omega_{cr}^2(k^2) \ . \tag{3.22}$$

After substitution, (3.11b) will become

$$\omega^{(2)} = \frac{1}{2}\left\{(\omega_1 + \omega_2) - \sqrt{(\omega_1 + \omega_2)^2 - 4\omega_1\omega_2(1 - \frac{\omega_c^2}{\omega_{cr}^2})}\right\} \ . \tag{3.23}$$

When there is no coupling, $\omega^{(2)} = \omega_2$ is always positive. When $\omega_c^2 < \omega_{cr}^2$, $\omega^{(2)} > 0$; and when $\omega_c^2 > \omega_{cr}^2$, $\omega^{(2)} < 0$. The results can also be confirmed from Fig.2. Because of the relation between ω_c^2 and k^2, it is easy to let $\omega_c^2 > \omega_{cr}^2$, $\omega^{(2)}$ shows negative values in the figure. It means that the oceanic Rossby waves with smaller zonal wave numbers will propagate eastward from westward, and when ω_c^2 increases further, the oceanic Rossby waves with larger wave numbers can also change their propagations. On the other hand, for a larger value of ω_c^2, the waves originally propagating westward will become propagating eastward with the increasing wave number. But, coupling frequency gives little affection to the angular frequency of atmospheric Rossby waves. $\omega^{(1)}$—k curves hardly vary with ω_c^2.

IV. THE QUASI–GEOSTROPHIC COUPLING IN LOW–LATITUDES

In low–latitudes, we let

$$f = \beta_0 y \ . \tag{4.1}$$

Now, when dissipations are disregarded ($a_1 = a_2 = 0$), the operator (2.12) becomes

$$L = \left[\frac{\partial}{\partial t}\left(\nabla_h^2 - \frac{\beta_0^2 y^2}{c_1^2}\right) + \beta_0 \frac{\partial}{\partial x}\right]\left[\frac{\partial}{\partial t}\left(\nabla_h^2 - \frac{\beta_0^2 y^2}{c_2^2}\right) + \beta_0 \frac{\partial}{\partial x}\right] - \frac{\beta_0^2 y^2}{c_1^2}\omega_c^2 \nabla_h^2 \ . \tag{4.2}$$

Using the normal mode method, we take the solution of Eq.(2.11) as

$$\varphi_2 = \Phi_2(y) e^{i(kx - \omega t)} \ . \tag{4.3}$$

Substituting it into Eq. (2.11), we obtain

$$\left[\frac{d^2}{dy^2} + \left(-\frac{\beta_0 k}{\omega} - k_2 - \frac{\beta_0^2 y^2}{c_1^2}\right)\right]\left[\frac{d^2}{dy^2} + \left(-\frac{\beta_0 k}{\omega} - k_2 - \frac{\beta_0^2 y^2}{c_2^2}\right)\right]\Phi_2 + \frac{\omega_c^2 \beta_0^2 y^2}{\omega^2 c_1^2}\left(\frac{d^2}{dy^2} - k^2\right)\Phi_2 = 0 \ . \tag{4.4}$$

In the absence of the coupling between ocean and atmosphere, $\omega_c = 0$, then Eq. (4.4) may be reduced to

$$\left[\frac{d^2}{dy^2} + \left(-\frac{\beta_0 k}{\omega} - k_2 - \frac{\beta_0^2 y^2}{c_1^2}\right)\right]\left[\frac{d^2}{dy^2} + \left(-\frac{\beta_0 k}{\omega} - k_2 - \frac{\beta_0^2 y^2}{c_2^2}\right)\right]\Phi_2 = 0 \ . \tag{4.5}$$

For

$$\left[\frac{d^2}{dy^2} + \left(-\frac{\beta_0 k}{\omega} - k_2 - \frac{\beta_0^2 y^2}{c_1^2}\right)\right]\Phi_2 = 0 \ , \tag{4.6}$$

We shall nondimensionalize Eq.(4.6) by setting

$$y = L_1 y_1 \ , \quad k = \frac{1}{L_1} k_1 \ , \quad \omega = \beta_0 L_1 \nu_1 \ , \quad \left(L_1 = \sqrt{\frac{c_1}{\beta_0}}\right) , \tag{4.7}$$

where y_1 , k_1 , ν_1 are the non–dimensional quantities, then Eq. (4.6) becomes

$$\frac{d^2 \Phi_2}{dy_1^2} + \left(-\frac{k_1}{\nu_1} - k_1^2 - y_1^2\right)\Phi_2 = 0 \ , \tag{4.8}$$

which is the Weber equation, when it satisfies

$$\Phi_2 |_{y_1 \to \pm\infty} < \infty \tag{4.9}$$

and its eigenvalues are

$$-\frac{k_1}{\nu_1 - k_1^2} = 2m + 1 \quad (m = 0,1,2,...) \ . \tag{4.10}$$

Then we can obtain

$$\nu_1 = -\frac{k_1}{k_1^2 + 2m + 1} \quad (m = 0,1,2,...) \tag{4.11}$$

to restore it to dimensional quantity we have

$$\omega_1 = -\frac{\beta_0 k}{k^2 + (2m+1)\dfrac{\beta_0}{c_1}} \quad (m = 0,1,2,...) \ , \tag{4.12}$$

which is obviously for the atmospheric Rossby waves in low–latitudes. Similarly, from

$$\left[\frac{d^2}{dy^2} + \left(-\frac{\beta_0 k}{\omega} - k_2 - \frac{\beta_0^2 y^2}{c_2^2}\right)\right]\Phi_2 = 0 \ , \tag{4.13}$$

the angular frequency of oceanic Rossby waves in low–latitudes is

$$\omega_2 = -\frac{\beta_0 k}{k^2 + (2m+1)\dfrac{\beta_0}{c_2}} \quad (m = 0,1,2,...) \tag{4.14}$$

Similarly, Eq. (4.4) indicates that the atmospheric and oceanic Rossby waves can be distinguished when there is interaction between the ocean and the atmosphere ($\omega_c \neq 0$). Considering that the front factors of Φ_2 on the left hand side of Eq. (4.4) are two operators of the Weber equation, it implies that $d^2/dy^2 - \beta_0^2 y^2/c_1^2$ and $d^2/dy^2 - \beta_0^2 y^2/c_2^2$ can be replaced by $-(2m+1)\beta_0/c_1$ and $-(2m+1)\beta_0/c_2$ separately. At the same time, the last term on the left hand side of Eq. (4.4), $-\beta_0^2 y^2/c_1^2$, is also replaced by $-(2m+1)\beta_0/c_1$, and removing the term $\partial^2/\partial y^2$, from Eq.(4.4) we obtain

$$(\omega - \omega_1)(\omega - \omega_2) = \eta \ , \tag{4.15}$$

where

$$\eta = \frac{\omega_1 \omega_2 \omega_c^2 (2m+1)\dfrac{\beta_0}{c_1}}{\beta_0^2} = \frac{k^2 (2m+1)\dfrac{\beta_0}{c_1}\omega_c^2}{\left[k^2 + (2m+1)\dfrac{\beta_0}{c_1}\right]\left[k^2 + (2m+1)\dfrac{\beta_0}{c_2}\right]} \ . \tag{4.16}$$

Eq.(4.5) has the same form as Eq.(3.8) completely, and when no coupling exists, it is degenerated as

$$(\omega - \omega_1)(\omega - \omega_2) = 0 \ , \tag{4.17}$$

this just leads to formulae (4.12) and (4.14).

Comparing (4.16) with (3.9), the form is similar, only the latitudinal wavenumber l is replaced by factors which include $2m+1$. So with the increment of m, $\omega^{(2)}$ tends to uncoupled frequency ω_2 gradually. The oscillatory period of the low–frequency mode is

$$T^{(2)} = \frac{2\pi}{\omega^{(2)}} \ . \tag{4.18}$$

To discuss the relation between $T^{(2)}$ and m is equivalent to discuss T varying with latitudinal wavenumber. In low–latitudes, we do not introduce the basic flow (oceanic current) of atmosphere and ocean, because there generally exist weak easterly zones in tropical atmosphere. From (3.11b), (4.16) and (4.18), we can draw the curves in which the period of oceanic Rossby waves varies with the square values of the coupling frequency ω_c for different values of latitudinal wave number n (or m). They are shown in Fig.3 in which the dotted lines are the asymptotes of all corresponding curves.

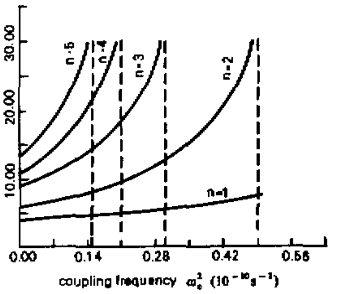

Fig.3. Curves in which $T^{(2)}$ varies with ω_2^c.

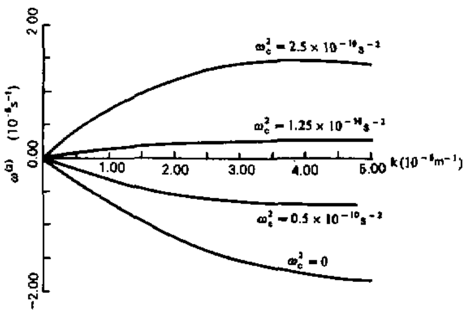

Fig.4. Curves which ω_2 varies with k.

Let $\omega^{(2)}=0$, we can get the critical value of ω_c^2,

$$\omega_{cr}^2=\frac{\beta_0 c_2}{2m+1} \quad . \tag{4.19}$$

The basic variation and tendency in Fig.3 is the same as Fig.1's. When there is no coupling, the main element affecting the period $T^{(2)}$ in mid–latitudes is the Rossby radius of deformation λ_1^{-1} , the latitudinal wave number m hardly plays any rule. But in low latitudes, different values of n make different non–coupling periods. Besides, we can also see that ω_c^2 corresponding the same period in Fig.3 has a lot of smaller values than those in Fig.1. It shows that the atmosphere–ocean coupling in low latitudes is easier and more effective than in mid–latitudes.

Corresponding to Fig.2, Fig.4 gives the curves in which $\omega^{(2)}$ varies with k for several definite ω_c^2 values. We can see from (4.19) that ω_{cr}^2 is not related to k. Let $m=1$, we can get the critical value $\omega_{cr}^2=2.0\times10^{-10}s^{-2}$ of ω_c^2 . Combinning Eq. (3.23) and Fig.4, we can see that $\omega^{(2)}<0$ when $\omega_c^2<\omega_{cr}^2$ and $\omega^{(2)}>0$ when $\omega_c^2>\omega_{cr}^2$. It means that the equatorial oceanic Rossby waves which originally propagate westward will change their propagations and become propagating eastward when the atmosphere–ocean coupling increases to some definite strength.

Fig.5 despicts the impact which the coupling frequency gives to $\omega^{(1)}$—k curves. We can see from the figure that $\omega^{(1)}$ increases continually with the increasing ω_c^2, but the equatorial atmospheric Rossby waves propagate eastward forever.

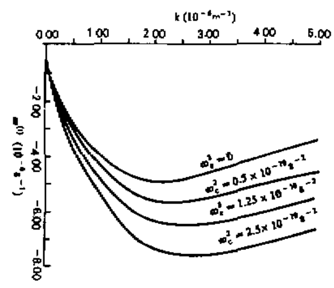

Fig.5. Curves in which ω_1 varies with k.

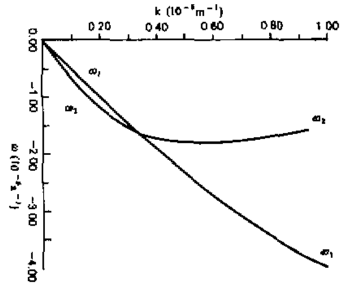

Fig.6. Curves in which ω_1, ω_2 vary with k.

V. THE CONVERSION BETWEEN EQUATORIAL MODES

Considering that the basic scale of atmosphere is a quantity larger than that of ocean, we draw the dispersion figure of ω_1 and ω^2 in low latitudes shown in Fig.6. Because of the different scales, the dispersional relations of atmospheric mode and oceanic mode are different obviously. However, under conditions of wave numbers and frequencies of the two modes being very close, they can "intersect" and get coupled and lose their original identity.

In view of slowly varying wave train, the angular frequencies ω_1 and ω_2 of Rossby waves for the atmosphere and ocean are slowly varying functions of time and space. For example, in the case of one dimension we have

$$\omega_1 = \omega_1(k,x) \ , \qquad \omega_2 = \omega_2(k,x) \ . \tag{5.1}$$

Because of the coupled action between ocean and atmosphere at some points of $x = x_0$ the two modes will be in resonance, now

$$\omega_1(k_0,x_0) = \omega_2(k_0,x_0) = \omega_0 \ . \tag{5.2}$$

x_0 is called mode–crossing point, where

$$k_0 = k(x_0) \ . \tag{5.3}$$

In the neighborhood of the crossing point, the two modes couple, and mode conversion and energy conversion take place. To analyze the property of mode conversion, we write

$$x = x_0 + \xi \ , \qquad k = k_0 + \delta \ , \tag{5.4}$$

where ξ and δ are small. In the neighborhood of the point (k_0, x_0) expanding ω_2 and ω_2 in Taylor series to(k_0, x_0), we have

$$\omega_1 = \omega_0 + \alpha_1\delta + \beta_1\xi \ , \qquad \omega_2 = \omega_0 + \alpha_2\delta + \beta_2\xi \ , \tag{5.5}$$

where

$$\alpha_1 = \left(\frac{\partial\omega_1}{\partial k}\right)_{k=k_0,x=x_0} \ , \qquad \alpha_2 = \left(\frac{\partial\omega_2}{\partial k}\right)_{k=k_0,x=x_0} \ , \tag{5.6}$$

$$\beta_1 = \left(\frac{\partial\omega_1}{\partial k}\right)_{k=k_0,x=x_0} \ , \qquad \beta_2 = \left(\frac{\partial\omega_2}{\partial k}\right)_{k=k_0,x=x_0} \ . \tag{5.7}$$

α_1, α_2 are the group velocities of Rossby waves for atmosphere and ocean separately.

By substituting (5.5) into (3.8) and using conditions (5.2) and (5.4), the formula listed be-low can be yielded

$$(\alpha_1 k - \alpha_1 k_0 + \beta_1 \xi)(\alpha_2 k - \alpha_2 k_0 + \beta_2 \xi) = \eta_0 \quad , \tag{5.8}$$

where η_0 is the value of η evaluated at (k_0, x_0). By setting

$$\lambda^2 = \frac{\eta_0}{\alpha_1 \alpha_2} \tag{5.9}$$

(5.8) then can be reduced as

$$\left(k - k_0 + \frac{\beta_1}{\alpha_2}\xi\right)\left(k - k_0 + \frac{\beta_2}{\alpha_1}\xi\right) = \lambda^2 \quad . \tag{5.10}$$

We know that the dispersion relation associates with certain partial differential equation, so that the local dispersion relation (5.10) may be associated with a differential equation for the disturbance amplitude. But following the analysis of Cairns et al. (1982, 1983), the second order differential equation obtained with k identified with the operator $id / d\xi$ will not satisfy energy conservation law. Since we are dealing with the coupled process between two modes, the two factors on the left hand side of (3.29) represent the different waves respectively. To satisfy (3.29), each of them should occupy one λ . Thus, assuming the amplitudes of the two waves to be A_1 and A_2, and using operator $id / d\xi$ to replace k, we can obtain the following coupled system of two first-order differential equations:

$$i\frac{dA_1}{d\xi} - \left(k_0 - \frac{\beta_1}{\alpha_1}\xi\right)A_1 = \lambda A_2 \quad , \tag{5.11a}$$

$$i\frac{dA_2}{d\xi} - \left(k_0 - \frac{\beta_2}{\alpha_1}\xi\right)A_1 = \lambda A_1 \quad . \tag{5.11b}$$

The system satisfies the energy conservation law. This is due to that the coupled system can be rewritten in the form

$$\frac{dA_1}{d\xi} - i\left(k_0 - \frac{\beta_1}{\alpha_1}\xi\right)A_1 = -i\lambda A_2 \quad , \tag{5.12a}$$

$$\frac{dA_2}{d\xi} - i\left(k_0 - \frac{\beta_2}{\alpha_1}\xi\right)A_1 = -i\lambda A_1 \quad . \tag{5.12b}$$

In the case of $\lambda^2 > 0$ (now λ is real, and it shows that group velocities α_1 and α_2 have the same sign), A_1^* *and* A_2^* which are conjugate complex numbers of A_1 and A_2 satisfy the following coupled system:

$$\frac{dA_1^*}{d\xi} - i\left(k_0 - \frac{\beta_1}{\alpha_1}\xi\right)A_1^* = -i\lambda A_2^* \quad , \tag{5.13a}$$

$$\frac{dA_2^*}{d\xi} - i\left(k_0 - \frac{\beta_2}{\alpha_1}\xi\right)A_1^* = -i\lambda A_1^* \quad . \tag{5.13b}$$

Hence from Eqs. (5.12) and (5.13), we obtain

$$\frac{d|A_1|^2}{d\xi} = i\lambda(A_1 A_2^* - A_1^* A_2) \quad , \tag{5.14a}$$

$$\frac{d|A_2|^2}{d\xi} = -i\lambda(A_1 A_2^* - A_1^* A_2) \quad , \tag{5.14b}$$

so that

$$\frac{d}{d\xi}(|A_1|^2 + |A_2|^2) = 0 \quad . \tag{5.15}$$

This is just the energy conservation law for the two coupled waves.

Eliminating A_1 from Eqs. (5.12a) and (5.12b), we obtain

$$\frac{d^2A_2}{d\xi^2}+i\left[2k_0-\left(\frac{\beta_1}{\alpha_1}+\frac{\beta_2}{\alpha_2}\right)\xi\right]\frac{dA_2}{d\xi}+\left[\lambda^2-\left(k_0-\frac{\beta_1}{\alpha_1}\xi\right)\left(k_0-\frac{\beta_2}{\alpha_2}\xi\right)-i\frac{\beta_2}{\alpha_2}\right]A_2=0. \quad (5.16)$$

To eliminate the first-order derivative term from Eq. (5.16), we make the transformation

$$A_2=\psi_2 e^{-ik_0\xi+\frac{i}{4}\left(\frac{\beta_1}{\alpha_1}+\frac{\beta_2}{\alpha_2}\right)\xi^2}\ , \quad (5.17)$$

so that Eq. (5.16) becomes

$$\frac{d^2\psi_2}{d\xi^2}+\left[\frac{\eta_0}{\alpha_1\alpha_2}-\frac{i}{2}\left(\frac{\beta_2}{\alpha_2}-\frac{\beta_1}{\alpha_1}\right)+\frac{1}{4}\left(\frac{\beta_2}{\alpha_2}-\frac{\beta_1}{\alpha_1}\right)^2\xi^2\right]\psi_2=0\ . \quad (5.18)$$

Again by setting

$$\zeta=\sqrt{\frac{\beta_2}{\alpha_2}-\frac{\beta_1}{\alpha_1}}\,\xi e^{i\frac{\pi}{2}}\ ,\quad \left(\frac{\beta_2}{\alpha_2}>\frac{\beta_1}{\alpha_1}\right) \quad (5.19)$$

then the equation becomes

$$\frac{d^2\psi_2}{d\zeta^2}+\left(\frac{-i\eta_0}{\alpha_1\beta_2-\alpha_2\beta_1}+\frac{1}{2}-\frac{1}{4}\zeta^2\right)\psi_2=0\ . \quad (5.20)$$

Eq. (5.20) is the Weber equation of ψ_2 in ζ. When it satisfies

$$\psi_2|_{\zeta\to\pm\infty}<\infty\ , \quad (5.21)$$

its eigenvalues are

$$-\frac{i\eta_0}{\alpha_1\beta_2-\alpha_2\beta_1}=n\quad (n=0,1,2,\ldots)\ . \quad (5.22)$$

the corresponding eigenfunction is the following Weber function $D_n(\xi)$

$$\psi_2\sim D_n(\zeta)=2^{-\frac{n}{2}}e^{-\frac{1}{4}\zeta^2}H_n\left(\frac{1}{\sqrt{2}}\zeta\right)\ , \quad (5.23)$$

where $H_n(x)$ is the Hermite polynomial of nth-order.

Substituting (5.23) into (5.17) we obtain

$$A_2\sim D_n\left(i\sqrt{\frac{\beta_2}{\alpha_2}-\frac{\beta_1}{\alpha_1}}\,\xi\right)e^{-ik_0\xi+\frac{i}{4}\left(\frac{\beta_2}{\alpha_2}+\frac{\beta_1}{\alpha_1}\right)\xi^2}\ . \quad (5.24)$$

Which is just the changes in wave amplitude of ocean Rossby waves at the crossing point of resonance regions due to the interaction between two modes.

For $\xi<0$, we have

$$\zeta=\sqrt{\frac{\beta_2}{\alpha_2}-\frac{\beta_1}{\alpha_1}}\,|\xi|e^{i\frac{\pi}{2}}e^{-i\pi}\ , \quad (5.25)$$

so that, $\arg\zeta=-\pi/2$, The asymptotic expansion of D_n in this region is

$$D_n(\zeta)\sim\zeta^n e^{-\frac{1}{4}\zeta^2}\ . \quad (5.26)$$

This is the oceanic mode on one side of the coupling region ($\xi<0$). On the other side of the coupling region $\xi>0$, we have

$$\zeta=\sqrt{\frac{\beta_2}{\alpha_2}-\frac{\beta_1}{\alpha_1}}\,\xi e^{i\frac{\pi}{2}} \quad (5.27)$$

and $\arg\zeta=\pi/2$, the asymptotic expansion in this case is

$$D_n(\zeta) \sim \zeta^n e^{-\frac{1}{4}\zeta^2} - \frac{(2\pi)^{\frac{1}{2}}}{(-n-1)!} e^{in\pi} \zeta^{(-n-1)} e^{\frac{1}{4}\zeta^2} \quad . \tag{5.28}$$

The first term can again be representing the oceanic mode, while the second term represents the atmospheric mode produced by the mode conversion. The ratio of the oceanic mode amplitude at $\xi = x$ to that at $\xi = -x$ is $|e^{in\pi}|$, the square of this quantity gives the energy transmission coefficient T

$$T = e^{i2n\pi} = e^{\frac{2\pi\eta_0}{\alpha_1\beta_2 - \alpha_2\beta_1}} \quad . \tag{5.29}$$

The quantity $1 - T$ is the conversion coefficient for the oceanic mode being converted to the atmospheric mode. The energy flux of the oceanic mode before coupling times this coefficient $1 - T$ is just the energy needed in the conversion from the oceanic mode to the atmospheric one owing to the coupling.

VI. CONCLUSION REMARKS

A simple shallow–water mode including momentum and thermodynamic equations in which the atmosphere and ocean are built in one layer is used in this paper. Using quasi–geostrophic filter and traditional coupling form, we analytically solved the coupling frequency of ocean and atmosphere and drew conclusions as follows:

1. Rossby waves in coupling are always stable, Newtonian cooling and Reyleigh friction play a stable rule to Rossby waves.

2. In mid–high latitudes, the period of coupled low–frequency mode grows corresponding to the increment of coupling frequency. The larger the latitudinal wave number is, the more rapidly the period grows with the increment of coupling frequency. No matter the value of n, does the period grow rapidly to be infinite when the coupling frequency tends to be the critical value; the oceanic Rossby waves become static. The coupling making the growth of the period shows the correlation between low–frequency oscillation and atmosphere–ocean interaction. In low latitudes, the latitudinal wave number 1 is replaced by the factor containing the eigenvalue $2m + 1$, the results are corresponding to those in mid–high latitudes the only difference is that the ocean–atmosphere coupling is more powerful and more effective than that in mid–high latitudes.

3. With the increment of ocean–atmosphere coupling strength even though and exceeding the critical value, the oceanic Rossby waves in mid–high or low latitudes appear that their propagations will become opposite to original directions, however, there does not exist this case for atmospheric Rossby waves. This can be used to interpret the eastward (westward) propagation of high SST anomalies in El Nino events.

4. The conversion mechanism between equatorial modes has been discussed in Section 5. Owing to the ocean–atmosphere interaction, at the crossing point in resonance regions, one part of the oceanic Rossby waves is converted into atmospheric waves by followed some energy conversion.

The model in this paper is rather simple, so the description of the coupling between the ocean and atmosphere is also quite rude. The analytical result and weather phenomena being not combined effectively is also the weakness of the paper. We expect that it can be useful for the future research of the coupling between ocean and atmosphere.

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