The Effects of Zonal Flow on Nonlinear Rossby Waves

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ABSTRACT

In this paper, we using phase plane method have derived the stability criteria of linear and nonlinear Rossby waves under the conditions of semi–geostrophic approximation and have gotten the solutions and geostrophic vorticity of corresponding solitary Rossby waves. It is pointed out that the wave stability is connected with the distribution of zonal flow and when the zonal flow is different the solitary wave trough or ridge is formed.

I. INTRODUCTION

Atmospheric long wave has been studied for quite a number of years. In 1939 Rossby first introduced the idea of Rossby wave and showed its dispersion relation. Afterwards the further studies about long waves have been done. Long (1964) discussed the effects of westerly flow shear on barotropic Rossby solitary wave under β–plane approximation. Redekopp (1977) and Redekopp and Weidman (1978) studied barotropic and baroclinic Rossby solitary waves using quasi–geostrophic model, and it was pointed out that the horizontal shear of zonal flow is the necessary condition of Rossby solitary existing in quasi–geostrophic model. Kuo (1949) using barotropic horizontal nondivergent model derived the stability criterion, and got the growth rate of unstable wave amplitude and the wave length of the most unstable waves. Liu (1987) applied the semi–geostrophic approximation and phase plane method to solve the barotropic and baroclinic atmospheric motions. It was pointed out that semi–geostrophic approximation can not only filter out the inertial gravity wave, but also find the nonlinear solution of Rossby wave. Lu (1987) also studied the stability criterion of linear and nonlinear Rossby wave in the barotropic nondivergent atmosphere, and wave growth rate was shown.

In this paper semi–geostrophic approximation and phase plane method were used to discuss the stability and solutions of linear and nonlinear long wave in shallow water wave equations which include the effect of zonal flow and its shear.

II. FUNDAMENTAL EQUATION

Without considering the orographic effects, nonlinear shallow water wave equations of describing atmospheric motions can be represented by

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=fv-\frac{\partial \varphi}{\partial x} \tag{2.1}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-fu-\frac{\partial \varphi}{\partial y} \tag{2.2}$$

$$\frac{\partial \varphi}{\partial t}+u\frac{\partial \varphi}{\partial x}+v\frac{\partial \varphi}{\partial y}+\Phi\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=0 \tag{2.3}$$

where u and v are wind velocities in x and y directions respectively: Φ is gravitational potential: $f = 2\Omega\sin\varphi$; Ω is the angular velocity of the earth rotation; φ is the earth latitude.

Seperating a meteorological element (A) into fundamental quantity $(\overline{A})$ and disturbed quantity (A'), that is

$$u(x,y,t) = \overline{u}(y) + u'(x,y,t), \quad v(x,y,t) = v'(x,y,t)$$
$$\varphi(x,y,t) = \overline{\varphi}(y) + \varphi'(x,y,t) \tag{2.4}$$

where $\overline{u} = -\dfrac{1}{f_0}\dfrac{\partial\overline{\varphi}}{\partial y}$; $f = 2\Omega\sin\varphi_0$; φ_0 is a reference latitude. If we only discuss northern hemisphere air, $f_0 > 0$.

Substituting (2.4) into (2.1)–(2.3) and neglecting the superscript " ' ", we have

$$\frac{\partial u}{\partial t} + \overline{u}\frac{\partial u}{\partial x} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + v\frac{\partial \overline{u}}{\partial y} = fv - \frac{\partial\varphi}{\partial x} \tag{2.5}$$

$$\frac{\partial v}{\partial t} + \overline{u}\frac{\partial v}{\partial x} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -fu - \frac{\partial\varphi}{\partial y} \tag{2.6}$$

$$\frac{\partial \varphi}{\partial t} + \overline{u}\frac{\partial \varphi}{\partial x} + u\frac{\partial \varphi}{\partial x} + v\frac{\partial \varphi}{\partial y} + v\frac{\partial \overline{\varphi}}{\partial y} + (\overline{\varphi} + \varphi)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0 \ . \tag{2.7}$$

Under β–plane approximation, from Eqs. (2.5) and (2.6) we may get the following vorticity equation

$$\left(\frac{\partial}{\partial t} + \overline{u}\frac{\partial}{\partial x} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right)\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) + \left[\left(f_0 - \frac{\partial\overline{u}}{\partial y}\right)\right.$$
$$\left. + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\right]\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \left(\beta - \frac{\partial^2\overline{u}}{\partial y^2}\right)v = 0 \ . \tag{2.8}$$

Under the semi–geostrophic approximation, if we suppose $\left(\beta - \dfrac{\partial^2\overline{u}}{\partial y^2}\right)v = \left(\beta - \dfrac{\partial^2\overline{u}}{\partial y^2}\right)v_g$, Eqs.(2.7) and (2.8) may be written as

$$\left(\frac{\partial}{\partial t} + \overline{u}\frac{\partial}{\partial x} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right)\zeta_g + \left[f_0 - \frac{\partial\overline{u}}{\partial y} + \zeta_g\right]$$
$$\times\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \left(\beta - \frac{\partial^2\overline{u}}{\partial y^2}\right)v_g = 0 \tag{2.9}$$

$$\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right)\varphi + (c_0^2 + \varphi)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0 \tag{2.10}$$

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \approx \zeta_g \tag{2.11}$$

where

$$\zeta_g = \frac{\partial v_g}{\partial x} - \frac{\partial u_g}{\partial y} \ , \quad u_g = -\frac{1}{f_0}\frac{\partial\varphi}{\partial y} \ , \quad v_g = \frac{1}{f_0}\frac{\partial\varphi}{\partial x} \ , \quad c_0^2 = \overline{\varphi} \ .$$

Now we solve (2.9)–(2.11) using the phase plane method.

We suppose the solutions of (2.9)–(2.11) are

$$\varphi(x,y,t) = \Phi(\theta), \quad u(x,y,t) = U(\theta), \quad v(x,y,t) = V(\theta) \tag{2.12}$$

where $\theta = kx + ly - \sigma t$; k and l are wave number in x and y directions respectively, and σ is frequency.

Substituting Eq.(2.12) into (2.9)–(2.11), we have

$$K_h^2(\sigma - k\bar{u})\Phi''' - K_h^2(kU + lV)\Phi''' - \left[f_0\left(f_0 - \frac{\partial \bar{u}}{\partial y}\right) + K_h^2\Phi''\right](kU + lV)' - k\left(\beta - \frac{\partial^2 \bar{u}}{\partial y^2}\right)\Phi' = 0 \tag{2.13}$$

$$\sigma\Phi' - (kU + lV)\Phi' - \varphi(kU + lV)' - C_0^2(kU + lV)' = 0 \tag{2.14}$$

$$(kV - lU)' = \frac{K_h^2}{f_0}\Phi'' \tag{2.15}$$

where the superscript " ' " denotes derivative to θ; and $K_h^2 = k^2 + l^2$.

In order to solve Eq. (2.13)–Eq. (2.15) conveniently, we supposed that if the variable interval of y is not too long and atmospheric fundamental quantities vary slowly in this interval, then these fundamental quantities are invariable in the interval. In this case, integrating (2.13)–(2.15) to θ and supposing integrated constants are zero, we obtain

$$kU + lV = \frac{K_h^2(\sigma - k\bar{u})\Phi'' - k\left(\beta - \frac{\partial^2 \bar{u}}{\partial y^2}\right)\Phi}{K_h^2\Phi'' + f_0\left(f_0 - \frac{\partial \bar{u}}{\partial y}\right)} \tag{2.16}$$

$$kU + lV = \frac{\sigma\Phi}{\Phi + C_0^2} \tag{2.17}$$

$$kV - lU = \frac{K_h^2}{f_0}\Phi' \tag{2.18}$$

From Eq.(2.16) and Eq.(2.17) in which U and V have been eliminated we have

$$\Phi'' = \frac{\left[C_x f_0\left(f_0 - \frac{\partial \bar{u}}{\partial y}\right) + C_0^2\left(\beta - \frac{\partial^2 \bar{u}}{\partial y^2}\right)\right]\Phi + \left(\beta - \frac{\partial^2 \bar{u}}{\partial y^2}\right)\Phi^2}{C_0^2 K_h^2(C_x - \bar{u}) - K_h^2\bar{u}\Phi} \tag{2.19}$$

where $C_x = \sigma / k$. It is easily seen that Eq.(2.19) is a nonlinear equation. We may solve Φ from the equation and then solve U and V from Eqs.(2.17) and (2.18).

When we do not consider the effect of the zonal flow, that means $\bar{U} \equiv 0$, Eq.(2.19) may be simplified as

$$\Phi'' = \frac{C_x f_0^2 + C_0^2\beta}{C_0^2 K_h^2 C_x}\Phi + \frac{\beta}{C_0^2 K_h^2 C_x}\Phi^2 \ . \tag{2.20}$$

If we differentiate Eq.(2.20) to θ, we have

$$\Phi''' - \frac{2\beta}{C_0^2 K_h^2 C_x}\Phi\Phi' - \frac{C_x f_0^2 + C_0^2\beta}{C_0^2 K_h^2 C_x}\Phi' = 0 \ .$$

Above–mentioned equation is a kind of KDV equation. Its solution may be written in elliptical cosine function. Thus it can be seen that under semi–geostrophic approximation we may get the completely analytical solution of shallow water wave equations of stationary basic flow using phase plane method. The difference from the result of Liu (1987) is that we consider the term of $\zeta_g\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$ in Eq.(2.9). But when we consider the effect of the zonal flow

Eq.(2.19) is complicated and is difficult to be solved. So in this paper only approximate solutions of Eq.(2.19) were discussed.

III. THE STABILITIES OF LINEAR AND NONLINEAR ROSSBY WAVES

Supposing $\Phi' = \Psi$. Eq.(2.19) may be shown as ordinary differential equation

$$\Phi' = \Psi$$
$$\Psi' = F(\Phi) \tag{3.1}$$

where

$$F(\Phi) = \frac{\left[C_x f_0\left(f_0 - \frac{\partial \bar{u}}{\partial y}\right) + C_0^2\left(\beta - \frac{\partial^2 \bar{u}}{\partial y^2}\right)\right]\Phi + \left(\beta - \frac{\partial^2 \bar{u}}{\partial y^2}\right)\Phi^2}{C_0^2 K_h^2 (C_x - \bar{u}) - K_h^2 \bar{u}\Phi}$$

Eq.(3.1) has two equilibrium points

Point A: $(\Phi, \Psi) = (0, 0)$

$$\text{Point } B: (\Phi, \Psi) = \left(-\frac{C_x f_0\left(f_0 - \frac{\partial \bar{u}}{\partial y}\right) + C_0^2\left(\beta - \frac{\partial^2 \bar{u}}{\partial y^2}\right)}{\beta - \frac{\partial^2 \bar{u}}{\partial y^2}},\ 0\right) \tag{3.2}$$

a. The stability near point A

We expand nonlinear function $F(\Phi)$ in Taylor series near point A

$$F(\Phi) = a\Phi + b\Phi^2 + \cdots \tag{3.3}$$

where

$$a = \frac{C_x f_0\left(f_0 - \frac{\partial \bar{u}}{\partial y}\right) + C_0^2\left(\beta - \frac{\partial^2 \bar{u}}{\partial y^2}\right)}{C_0^2 K_h^2 (C_x - \bar{u})}$$

$$b = \frac{\beta - \frac{\partial^2 \bar{u}}{\partial y^2}}{C_0^2 K_h^2 (C_x - \bar{u})} + \frac{\bar{u}\left[C_x f_0\left(f_0 - \frac{\partial \bar{u}}{\partial y}\right) + C_0^2\left(\beta - \frac{\partial^2 \bar{u}}{\partial y^2}\right)\right]}{C_0^4 K_h^2 (C_x - \bar{u})^2} .$$

We only retain the first–order term of Φ in Eq.(3.3), and Eq.(3.1) becomes

$$\Phi' = \Psi, \quad \Psi' = a\Phi . \tag{3.4}$$

The above–mentioned equations may be gotten by linearizing Eq.(2.5)–Eq.(2.7) in reality. If we also retain the second–order term of Φ in Eq.(3.3), Eq.(3.1) may become

$$\Phi' = \Psi, \quad \Psi' = a\Phi + b\Phi^2 . \tag{3.5}$$

This is a kind of nonlinear approximation of Rossby wave.

We first study the stability of Eq.(3.4). The characteristic equation of Eq.(3.4) is

$$\begin{vmatrix} -\lambda & 1 \\ a & -\lambda \end{vmatrix} = 0 .$$

Its roots are $\lambda_1 = \sqrt{a}$ and $\lambda_2 = -\sqrt{a}$. So we have

$$\text{Point } A \text{ is } \begin{cases} \text{saddle point when } a > 0 \\ \text{central point when } a < 0 \end{cases} \tag{3.6}$$

Because the phase velocity of Rossby wave is often satisfied with $C_x - \bar{u} < 0$, the stability condition of linear Rossby wave near point A becomes

$$\text{when} \quad C_x f_0\left(f_0 - \frac{\partial \bar{u}}{\partial y}\right) + C_0^2\left(\beta - \frac{\partial^2 \bar{u}}{\partial y^2}\right) < 0, \quad \text{Rossby wave is unstable}$$

$$\text{when} \quad C_x f_0\left(f_0 - \frac{\partial \bar{u}}{\partial y}\right) + C_0^2\left(\beta - \frac{\partial^2 \bar{u}}{\partial y^2}\right) > 0, \quad \text{Rossby wave is stable .} \tag{3.7}$$

If the zonal flow is not included in Eq.(2.5)–Eq.(2.7), condition (3.7) becomes

$$\text{when} \quad C_x f_0^2 + C_0^2 \beta < 0, \quad \text{Rossby wave is unstable}$$

$$\text{when} \quad C_x f_0^2 + C_0^2 \beta > 0, \quad \text{Rossby wave is stable .} \tag{3.8}$$

It may be proved that condition (3.8) is the stability criterion of Eq.(2.20). It is easily seen that β–effect makes Rossby wave stable, but Coriolis parameter (f_0) makes eastward–moving wave ($C_x > 0$) stable and makes westward–moving wave ($C_x < 0$) unstable.

If horizontal divergence is zero in Eq.(2.3), it may also be proved that condition (3.6) becomes

$$\text{when} \quad \frac{\beta - \dfrac{\partial^2 \bar{u}}{\partial y^2}}{C_x - \bar{u}} > 0, \quad \text{Rossby wave is unstable}$$

$$\text{when} \quad \frac{\beta - \dfrac{\partial^2 \bar{u}}{\partial y^2}}{C_x - \bar{u}} < 0, \quad \text{Rossby wave is stable .} \tag{3.9}$$

It is the same as that of Lu (1987). It is clearly shown that horizontal nondivergent approximation may make the stability criterion of Rossby wave be not associated with the one–order derivative of the zonal flow.

Now we further discuss the effect of different zonal flow patterns from condition (3.7). For the sake of convenience we order

$$M_1 = C_x f_0^2 + C_0^2 \beta \ , \ M_2 = -C_x f_0 \frac{\partial \bar{u}}{\partial y} \ , \ M_3 = -C_0^2 \frac{\partial^2 \bar{u}}{\partial y^2}$$

and then condition (3.7) may be rewritten as

$$\text{when} \quad M_1 + M_2 + M_3 < 0, \quad \text{Rossby wave is unstable}$$

$$\text{when} \quad M_1 + M_2 + M_3 > 0, \quad \text{Rossby wave is stable .} \tag{3.10}$$

The effects of M_2 are shown in Table 1. It can be seen that on the north side of a westerly jet axis M_2 makes eastward–moving wave stable, but westward–moving wave unstable: adversely, on the south side of the axis M_2 makes eastward–moving wave unstable, and westward–moving wave stable. The effects of M_3 are given in Table 2. It is shown that near a westerly jet axis M_3 makes wave stable, but between two westerly jet axes M_3 makes wave unstable.

Now we discuss the stability criterion of nonlinear equation (3.5). Because the nonlinear terms of Eq.(3.5) are

$$X(\Phi,\Psi) = 0, \quad Y(\Phi,\Psi) = b\Phi^2$$

and satisfy

$$X(0,0)=Y(0,0)=\frac{\partial X(0,0)}{\partial\Phi}=\frac{\partial Y(0,0)}{\partial\Phi}=\frac{\partial X(0,0)}{\partial\Psi}=\frac{\partial Y(0,0)}{\partial\Psi}=0\ .$$

The saddle point of Eq.(3.4) is also the saddle point of Eq.(3.5) according to Poincare–Bendxson theory. On the other hand we rewrite Eq.(3.5) as

$$\Phi'=\Psi=I(\Phi,\Psi),\quad \Psi'=a\Phi+b\Phi^2=J(\Phi,\Psi)\ .$$

Because $I(\Phi,\Psi)$ and $J(\Phi,\Psi)$ satisfy symmetry principle

$$J(\Phi,\Psi)=J(\Phi,-\Psi),\quad I(\Phi,\Psi)=-I(\Phi,-\Psi)$$

the central point of Eq.(3.4) is still the central point of Eq.(3.5). So, near the point A the stability condition of nonlinear Eq.(3.5) is the same as that of linear Eq.(3.4) so long as we replace the phase velocity of nonlinear wave into linear condition (3.10).

Table 1. Symbol of M_2

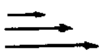
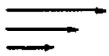
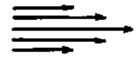
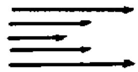
$\bar u$ \ M_2 \ C_x	<0	>0
	<0	>0
	>0	<0

Table 2. Symbol of M_3

$\bar u$	M_3
	>0
	<0

b. The stability near point B

Now we discuss the stability condition of Eq.(3.1) near point B. After adopting transformation

$$\Phi_*=\Phi+\Phi_0$$

we have

$$\Phi'_*=\Psi_*,\quad \Psi'_*=F_*(\Phi_*) \tag{3.11}$$

where

$$\Phi_0=\frac{C_xf_0\left(f_0-\frac{\partial\bar u}{\partial y}\right)+C_0^2\left(\beta-\frac{\partial^2\bar u}{\partial y^2}\right)}{\beta-\frac{\partial^2\bar u}{\partial y^2}} \tag{3.12}$$

$$F_*(\Phi_*)=\frac{\Phi_*(\Phi_*-\Phi_0)}{C_0^2K_h^2(C_x-\bar u)\left(\beta-\frac{\partial^2\bar u}{\partial y^2}\right)\left[1+\frac{\bar u}{C_0^2(C_x-\bar u)}\Phi_0-\frac{\bar u}{C_0^2(C_x-\bar u)}\Phi_*\right]}\ .$$

In this case point B becomes $B_*(\Phi_*,\Psi_*)=(0,0)$.

We expand $F_*(\Phi_*)$ near point B_* in Taylor series. If we retain first and second orders of Φ_* we may get nonlinear equation

$$\Phi'_*=\Psi_*,\quad \Psi'_*=a_*\Phi_*+b_*\Phi_*^2 \tag{3.13}$$

where

$$a_* = -\frac{\Phi_0}{C_0^2K_h^2(C_x-\bar{u})\left(\beta-\frac{\partial^2\bar{u}}{\partial y^2}\right)\left[1+\frac{\bar{u}}{C_0^2(C_x-\bar{u})}\Phi_0\right]}$$

$$b_* = \frac{1}{C_0^2K_h^2(C_x-\bar{u})\left(\beta-\frac{\partial^2\bar{u}}{\partial y^2}\right)\left[1+\frac{\bar{u}}{C_0^2(C_x-\bar{u})}\Phi_0\right]} - \frac{\bar{u}\Phi_0}{C_0^4K_h^2(C_x-\bar{u})^2\left(\beta-\frac{\partial^2\bar{u}}{\partial y^2}\right)\left[1+\frac{\bar{u}}{C_0^2(C_x-\bar{u})}\Phi_0\right]^2} \ .$$

Similarly it may be proved that

when $a_* > 0$, point B_* is saddle point

when $a_* < 0$, point B_* is central point (3.14)

So near point B we have

when $\dfrac{\Phi_0}{\left(\beta-\frac{\partial^2\bar{u}}{\partial y^2}\right)\left[C_0^2(C_x-\bar{u})+\bar{u}\Phi_0\right]} < 0$, Rossby wave is unstable

when $\dfrac{\Phi_0}{\left(\beta-\frac{\partial^2\bar{u}}{\partial y^2}\right)\left[C_0^2(C_x-\bar{u})+\bar{u}\Phi_0\right]} > 0$, Rossby wave is stable . (3.15)

If we do not consider the function of zonal flow, (3.15) becomes

when $C_xf_0^2 + C_0^2\beta > 0$, the wave is unstable

when $C_xf_0^2 + C_0^2\beta < 0$, the wave is stable . (3.16)

Obviously (3.16) is contrary to (3.9). It is shown that without considering the zonal flow, if wave is unstable near point A, the wave is stable near point B, and vice versa. But when the zonal flow exists this feature can not be gotten.

IV. SOLUTION OF NONLINEAR ROSSBY WAVE

We combine Eq.(3.5) and Eq.(3.13)

$$\tilde{\Phi}'' = \tilde{a}\tilde{\Phi} + \tilde{b}\tilde{\Phi}^2 \tag{4.1}$$

in which near point A $\tilde{\Phi}=\Phi$, $\tilde{a}=a$, $\tilde{b}=b$; and near point B $\tilde{\Phi}=\Phi_*$, $\tilde{a}=a_*$, $\tilde{b}=b_*$. Obviously Eq.(4.1) may be rewritten as KDV equation

$$\tilde{\Phi}''' - 2\tilde{b}\tilde{\Phi}\tilde{\Phi}' - \tilde{a}\tilde{\Phi}' = 0 \ . \tag{4.2}$$

When $\tilde{a} > 0$ (corresponding wave is unstable), we may get the following special solution in solitary wave form

$$\tilde{\Phi} = \tilde{A}\,\mathrm{sech}^2\left(\frac{\sqrt{\tilde{a}}}{2}\theta\right) \tag{4.3}$$

and when $\tilde{a} < 0$ (corresponding wave is stable), we may have

$$\tilde{\Phi} = -\tilde{A}\,\mathrm{sech}^2\left(\frac{\sqrt{-\tilde{a}}}{2}\theta\right) + \frac{2}{3}\tilde{A} \tag{4.4}$$

where $\tilde{A} = -3\tilde{a}/2\tilde{b}$. From Eq.(4.3) and Eq.(4.4) it may be seen that the amplitudes of nonlinear Rossby solitary wave are limited and connected with the zonal flow and wave features.

From Eq.(2.11) we may get the geostrophic vorticity of corresponding Eq.(4.3) and Eq.(4.4)

$$\zeta_g = \frac{3\tilde{a}^2}{4\tilde{b}} \operatorname{sech}^2\left(\frac{\sqrt{|\tilde{a}|}}{2}\theta\right)\left[3\operatorname{sech}^2\left(\frac{\sqrt{|\tilde{a}|}}{2}\theta\right)-2\right] . \tag{4.5}$$

For the sake of convenience we only discuss Eq.(4.5) near point B. In this case Eq.(4.5) becomes

$$\zeta_g = \frac{3}{4}\frac{C_0^2 K_h^2 a^2 (C_x - \bar{u})}{\left(\beta - \frac{\partial^2 \bar{u}}{\partial y^2}\right) + K_h^2 \bar{u} a} \operatorname{sech}^2\left(\frac{\sqrt{|a|}}{2}\theta\right)\left[3\operatorname{sech}^2\left(\frac{\sqrt{|a|}}{2}\theta\right)-2\right] . \tag{4.6}$$

Near the peak value of solitary waves we may have

$$3\operatorname{sech}^2\left(\frac{\sqrt{|a|}}{2}\theta\right) - 2 > 0 .$$

So we get

$$\zeta_g \propto -\frac{1}{\beta - \frac{\partial^2 \bar{u}}{\partial y^2} + K_h^2 \bar{u} a} .$$

If the zonal flow is not considered, $\zeta_g \propto -1/\beta$. It is shown that β effect only forms solitary wave ridge and the function of zonal flow may form solitary wave trough or solitary wave ridge.

V. CONCLUSIONS

(1) The stability criteria of linear and nonlinear Rossby waves are same in the formula, and are connected with the zonal flow and its one–order derivative and two–order derivative.

(2) β effect makes Rossby wave stable and forms solitary wave ridge; Coriolis parameter plays the role in Rossby wave through phase velocity.

(3) In different places of westerly jet, the functions of M_2 and M_3 are different. And the effects of different zonal flows may cause to form nonlinear Rossby solitary wave trough or ridge.

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